Kurosawa-Desmedt Meets Tight Security

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Scenario

All illustrations by John Tenniel, Alice's Adventures in Wonderland, 1865.

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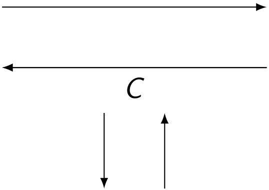

$(sk, pk) \leftarrow \mathbf{KeyGen}(1^\lambda)$

$M = \mathbf{Dec}_{sk}(C)$

$C \leftarrow \mathbf{Enc}_{pk}(M)$

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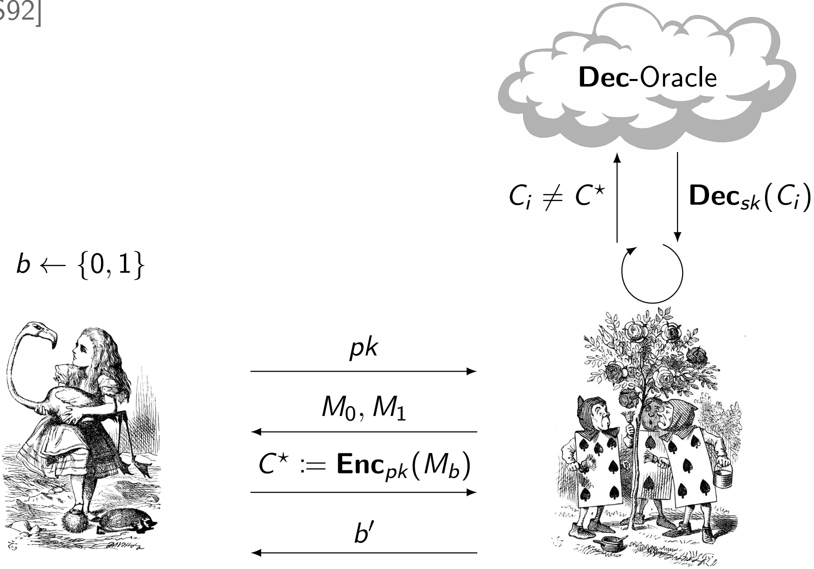
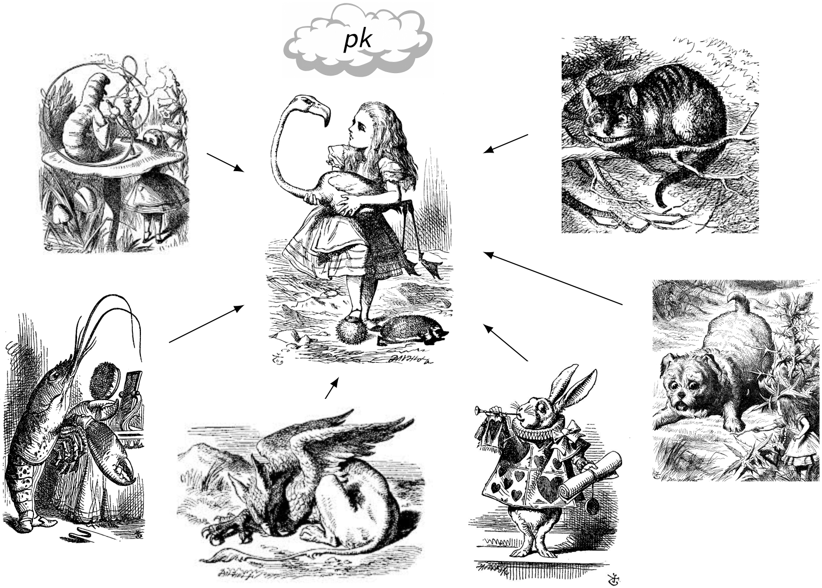
Security model (IND-CCA)

[RS92]



Multi-ciphertext scenario



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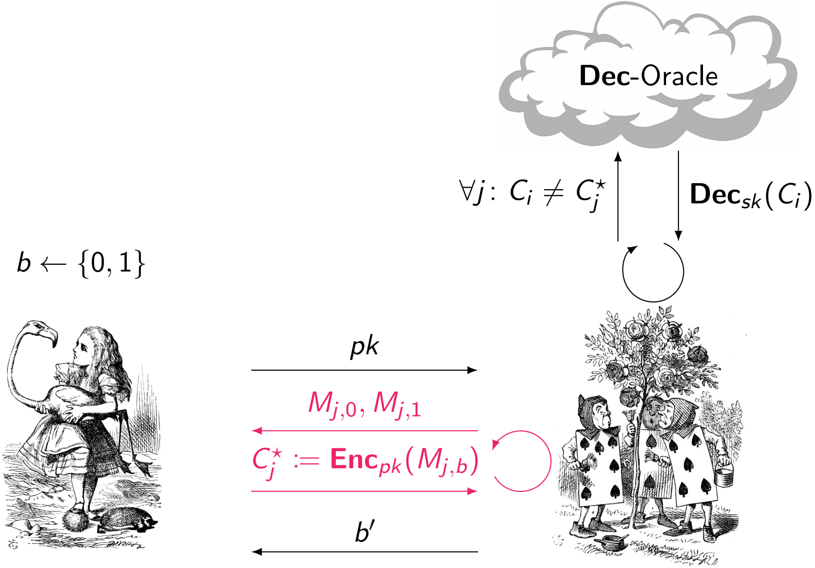

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[BBM00],[HJ12]

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Multi-ciphertext sec. via hybrid argument: $L \in \Omega(Q_{\mathbf{enc}})$

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Multi-ciphertext sec. via hybrid argument: $L \in \Omega(Q_{\mathbf{enc}})$

(Almost) tight reduction: $L = c_{\text{small}} \cdot \lambda$

$\Rightarrow$ **shorter concrete parameters**

CCA-secure encryption schemes

	$\lvert C \rvert - \lvert M \rvert$	$\lvert pk \rvert$	assumption	w/o pairing	security loss
[CS98]	3	3	DDH	✓	$\Omega(Q)$
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Kurosawa-Desmedt + OR-proof π (new!)

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Questions:

1. What is π good for?
2. How does π look like?

Recap: Decisional Diffie-Hellman assumption

$\mathbb{G}$ group, $\mathbf{a} \in \mathbb{G}^2$

Diffie-Hellman language:

$$\mathcal{L}_{\mathbf{a}}^{\text{lin}} = \left\{ \mathbf{x} \in \mathbb{G}^2 \mid \exists w \in \mathbb{Z}_{|\mathbb{G}|} : \mathbf{x} = \mathbf{a} \cdot w \right\}$$

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Useful for tightness: $\Downarrow$ Re-Randomizability $\Downarrow$

$$\left(\mathbf{a}, \mathbf{a} \cdot w_1, \mathbf{a} \cdot w_2, \cdots, \mathbf{a} \cdot w_n \right) \approx_c \left(\mathbf{a}, \mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_n \right)$$

Recap: Hash proof system
[CS98]

Purpose: prove $\mathbf{x} \in \mathcal{L}_{\mathbf{a}}^{\mathrm{lin}}$ to a designated verifier

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Completeness: $\forall \mathbf{x} \in \mathcal{L}_{\mathbf{a}}^{\mathrm{lin}}$ with witness w:

$$\mathbf{PubH}(pk, \mathbf{x}, w) = \mathbf{PrivH}(sk, \mathbf{x})$$

1- **Universality:** $\forall \mathbf{x} \notin \mathcal{L}_{\mathbf{a}}^{\mathrm{lin}}$

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$$\mathbf{PubEval}(pk, \mathbf{x}, w) = \mathbf{PubH}(pk_1, \mathbf{x}, w) + \mathsf{H}(\mathbf{x}) \cdot \mathbf{PubH}(pk_2, \mathbf{x}, w),$$

H collision resistant hash function

Recap: Kurosawa-Desmedt and its proof
[KD04]

$\mathbf{Enc}_{pk}(M)$:

- choose $\mathbf{x} \leftarrow_R \mathcal{L}^{\mathrm{lin}}_{\mathbf{a}}$ with witness w
- compute $k = \mathbf{PubEval}(pk, \mathbf{x}, w)$
- publish $C = (\mathbf{x}, \underbrace{\mathbf{E}_k(M)}_{=C_{sym}})$

$\mathbf{Dec}_{sk}(C = (\mathbf{x}, C_{sym}))$:

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Problem: entropy in *sk* limited $\Rightarrow$ reduction **non-tight**

Reminder: Our scheme

Kurosawa-Desmedt + OR-proof π (new!)

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Idea: use freshly randomized *sk* for each ciphertext

$\xRightarrow[\text{of DDH}]{\text{re-randomizability}}$ randomize all ciphertexts at once [GHKW16]

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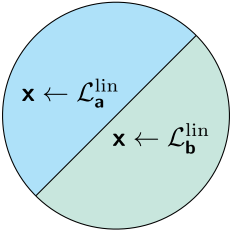

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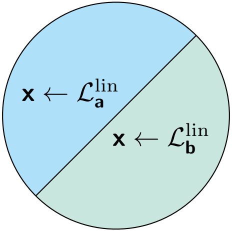

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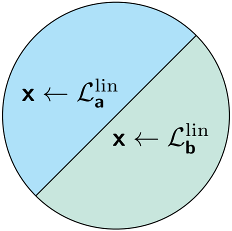

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Idea: use explicit proof π [Hof17], **but** w/o pairings

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Main challenge: construct **pairing-free** non-interactive OR-proof

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$\Rightarrow$ employ **hash proof system**

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But: during randomization of *sk*

- sometimes choose $\mathbf{x} \leftarrow_R \mathcal{L}_{\mathbf{b}}^{\mathrm{lin}}$, prove $\mathbf{x} \in \mathcal{L}_{\mathbf{a}}^{\mathrm{lin}} \cup \mathcal{L}_{\mathbf{b}}^{\mathrm{lin}}$

Difficulty: forging a proof for $\mathbf{x} \notin \mathcal{L}_{\mathbf{a}}^{\mathrm{lin}} \cup \mathcal{L}_{\mathbf{b}}^{\mathrm{lin}}$ must remain hard

OR-proof

Solution: protect hash proof system by encrypting its evaluation

- $\mathsf{enc}_{\mathbf{x}}$ encryption scheme depending on $\mathbf{x}$
- $\mathsf{enc}_{\mathbf{x}}$ will be **lossy** for $\mathbf{x} \in \mathcal{L}_{\mathbf{b}}^{\mathrm{lin}}$:

$$\forall \mathbf{x} \in \mathcal{L}_{\mathbf{b}}^{\mathrm{lin}}, \forall k \in \mathbb{Z}_{|\mathbb{G}|} : \mathsf{enc}_{\mathbf{x}}(k) \approx_s \mathsf{enc}_{\mathbf{x}}(\mathrm{rand})$$

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Our OR-proof:

- $\mathbf{x} = \mathbf{a} \cdot w \in \mathcal{L}_{\mathbf{a}}^{\mathrm{lin}}$:

$$\pi = \mathbf{enc_x}(\mathbf{PubH}(pk, \mathbf{x}, w))$$

Conclusion

	$\lvert C\rvert - \lvert M\rvert$	$\lvert pk\rvert$	assumption	w/o pairing	security loss
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Key building block: new efficient pairing-free NIDV OR-proof

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