Improved Dual System ABE in Prime-Order Groups via Predicate Encodings

Jie Chen – East China Normal University, Shanghai

Romain Gay – ENS, Paris

Hoeteck Wee – ENS, Paris

Attribute-Based Encryption

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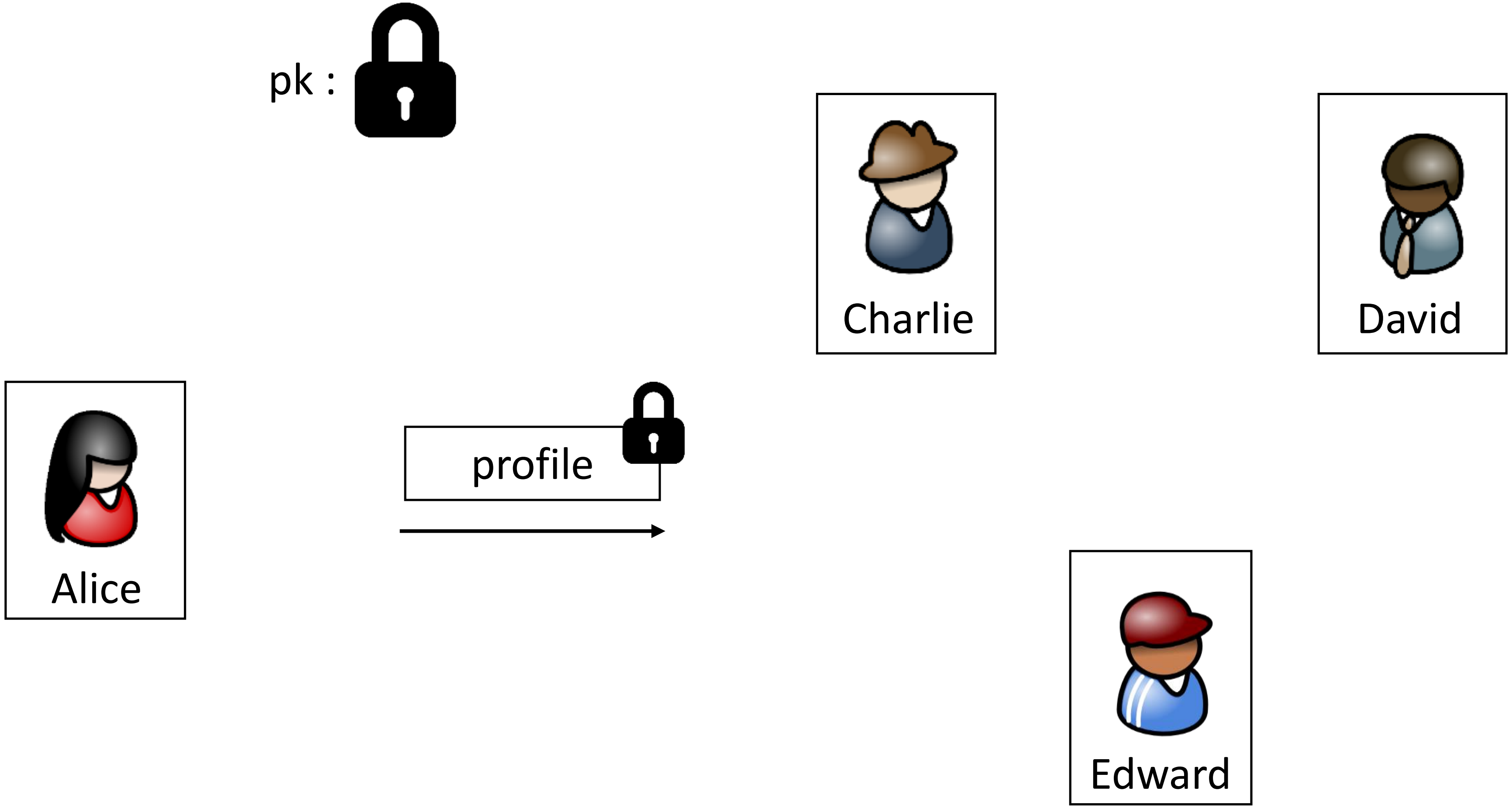
ABE: online dating

[Sahai,Waters'05; Goyal,Pandey,Sahai,Waters'06]



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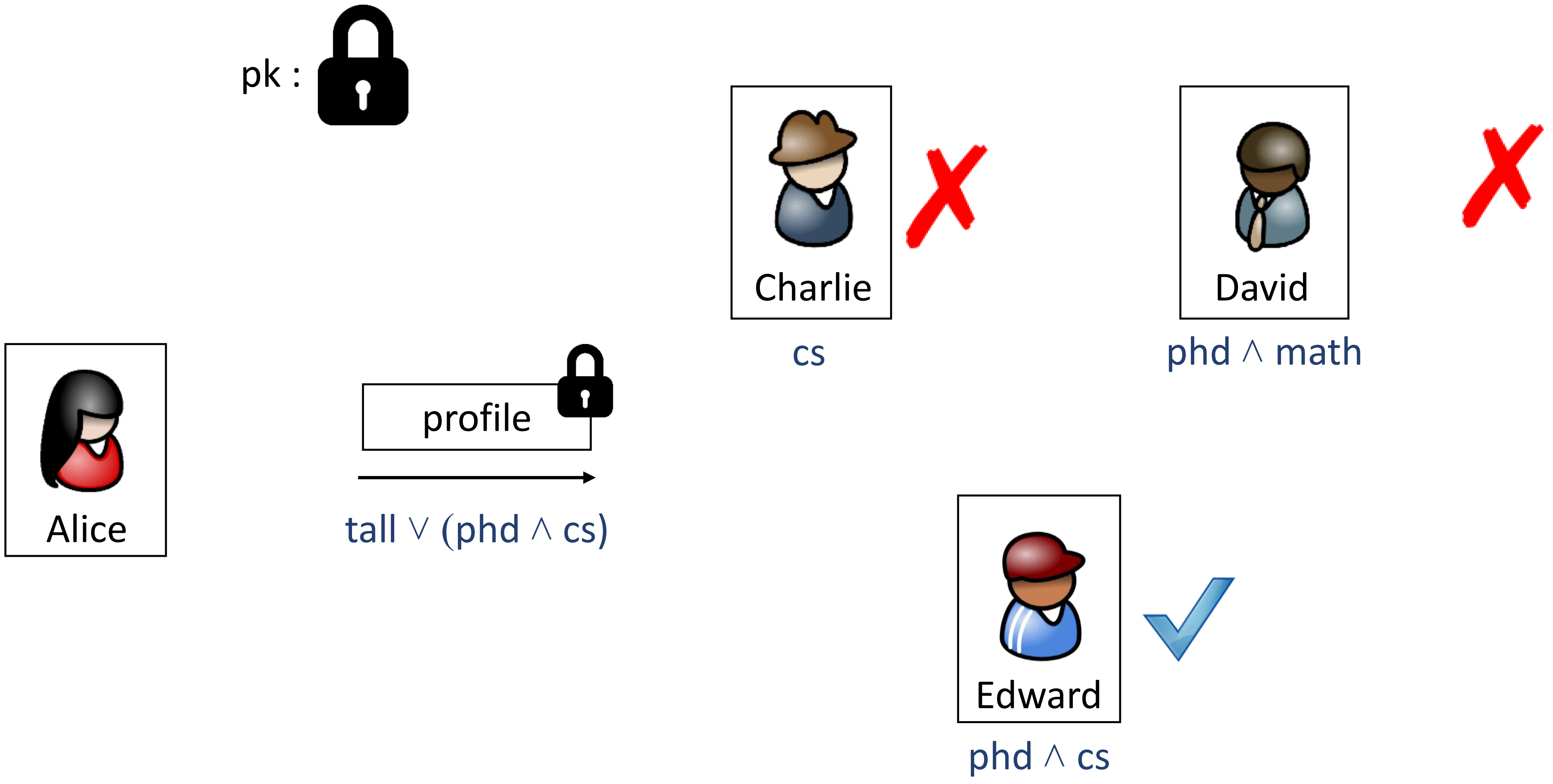

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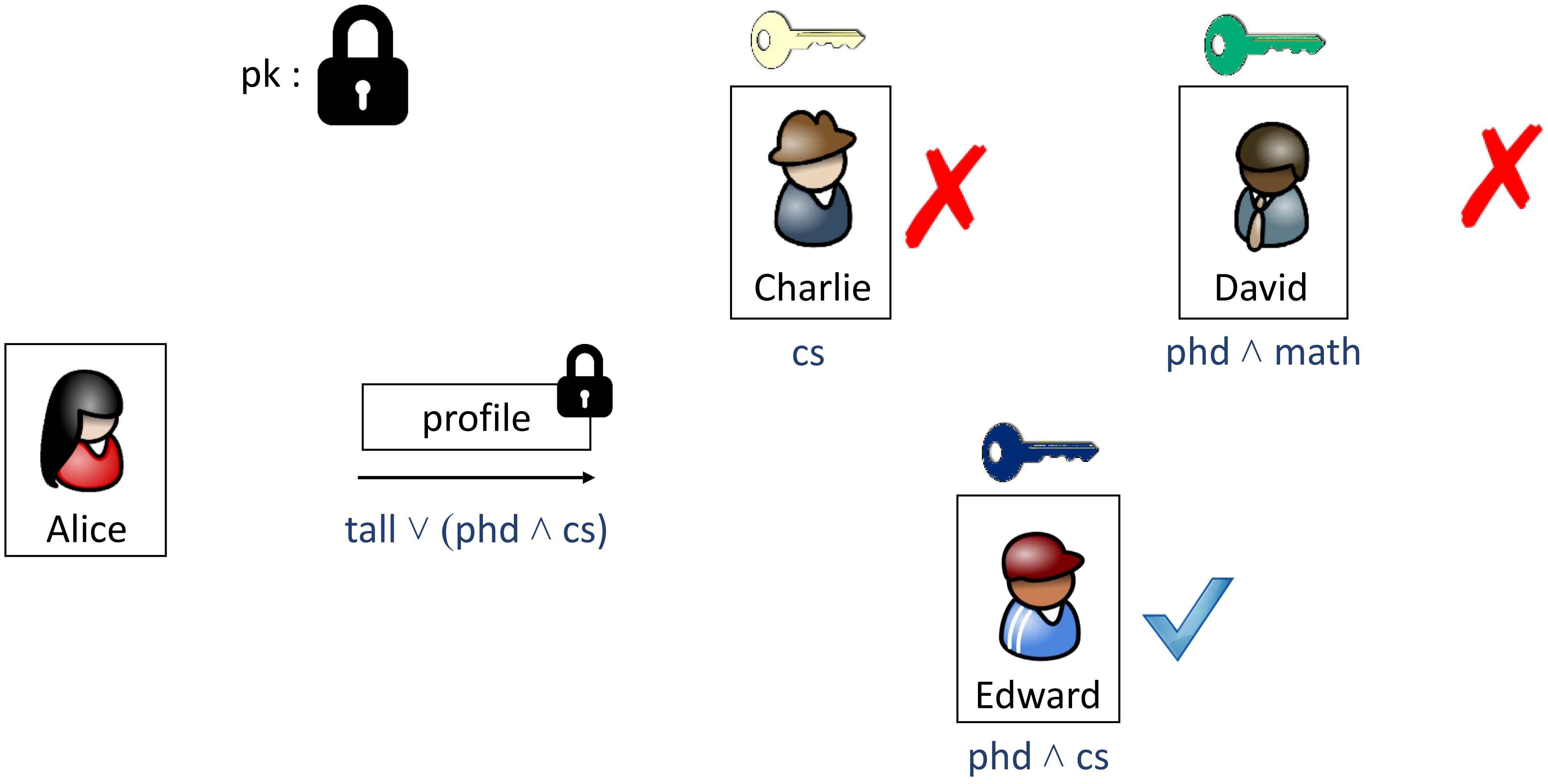

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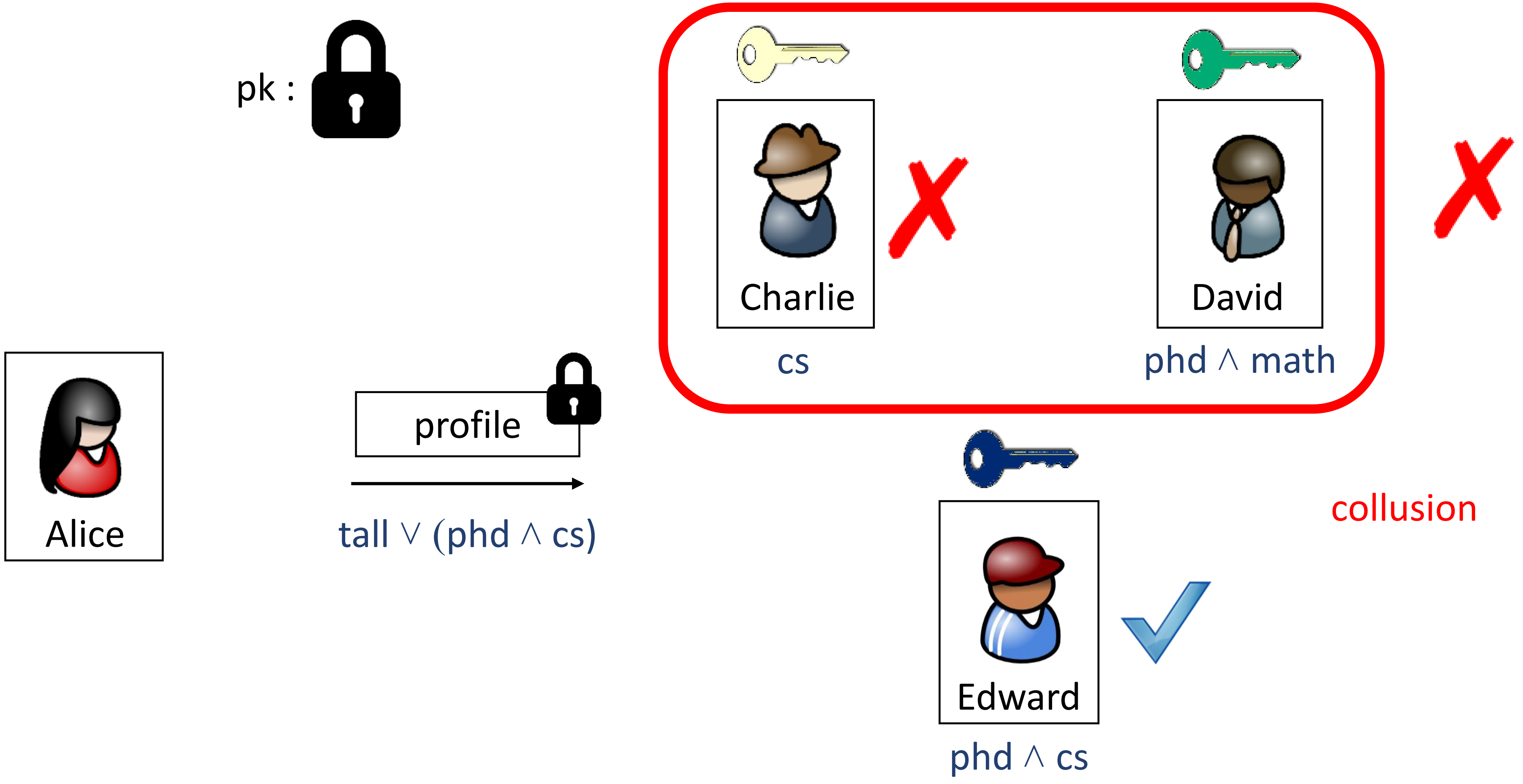

Modular framework for ABE

[Attrapadung 14, Wee 14]

Compiler

+

encoding ← P

=

Adaptively secure ABE for P

Modular framework for ABE

[Attrapadung 14, Wee 14]

Composite-order groups

+

encoding ← P

=

Adaptively secure ABE for P

Modular framework for ABE

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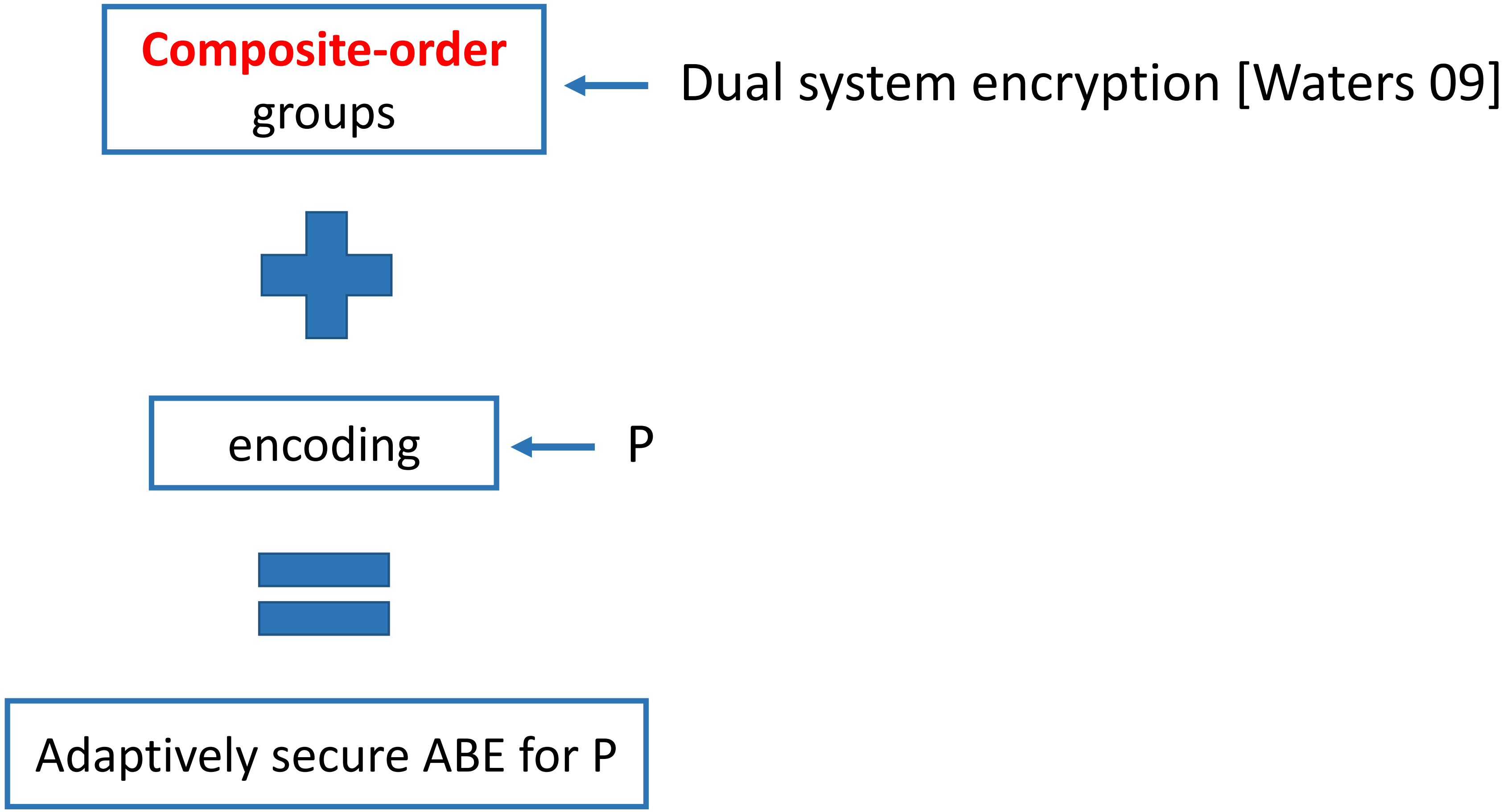
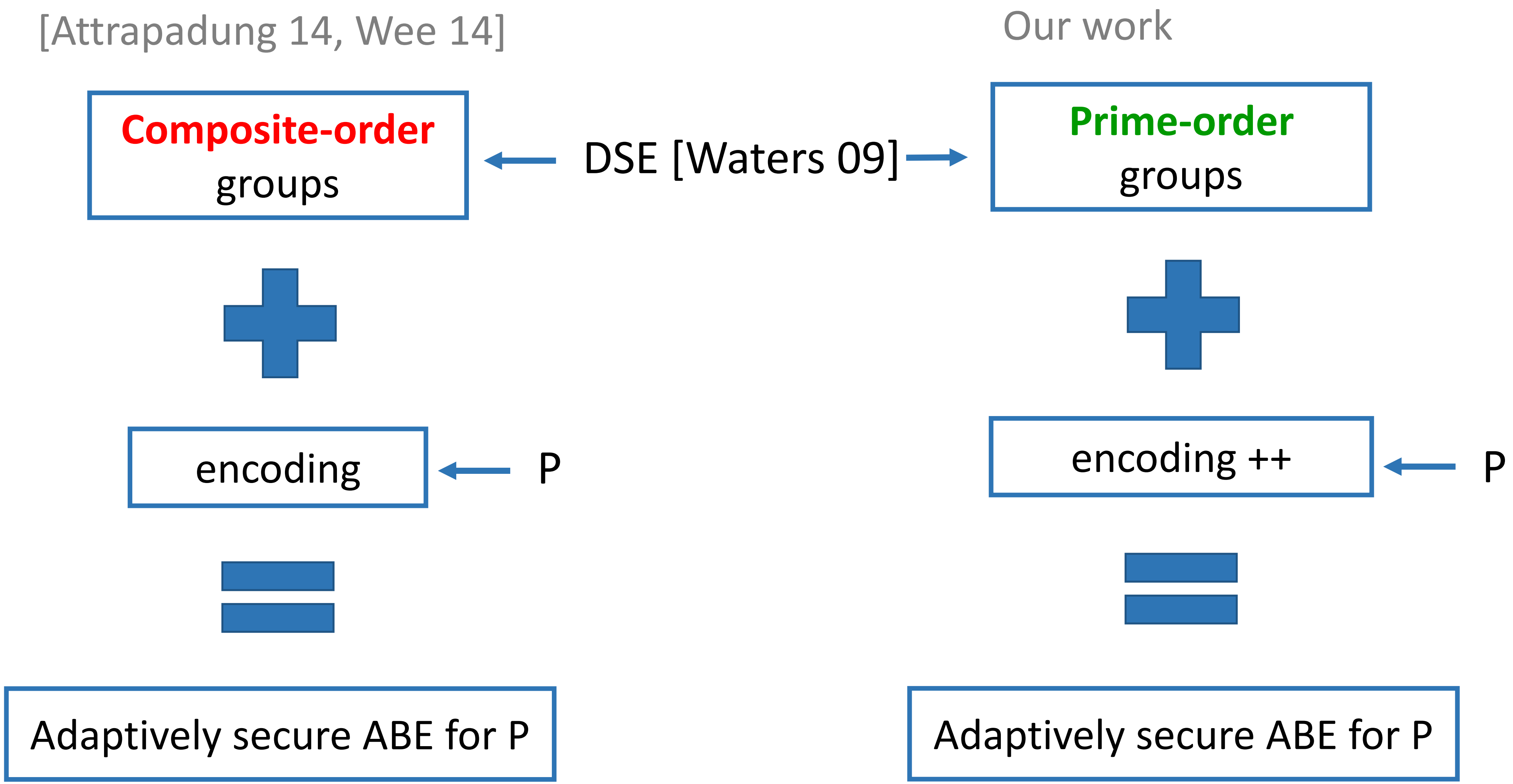


Modular framework for ABE



Our contributions

1. New techniques for simulating composite-order groups

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2. New efficient ABEs

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functionality	improvements
ABE for boolean formula	sk, ct 50% shorter

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1. New techniques for simulating composite-order groups

2. New efficient ABEs

functionality	improvements
ABE for boolean formula	sk, ct 50% shorter
ABE for arithmetic formula	First adaptively secure scheme

Composite-order groups

[Boneh, Goh, Nissim'05; Lewko, Waters'10]

p,q primes

$$e : (G_p \times G_q) \times (G_p \times G_q) \rightarrow G_T$$

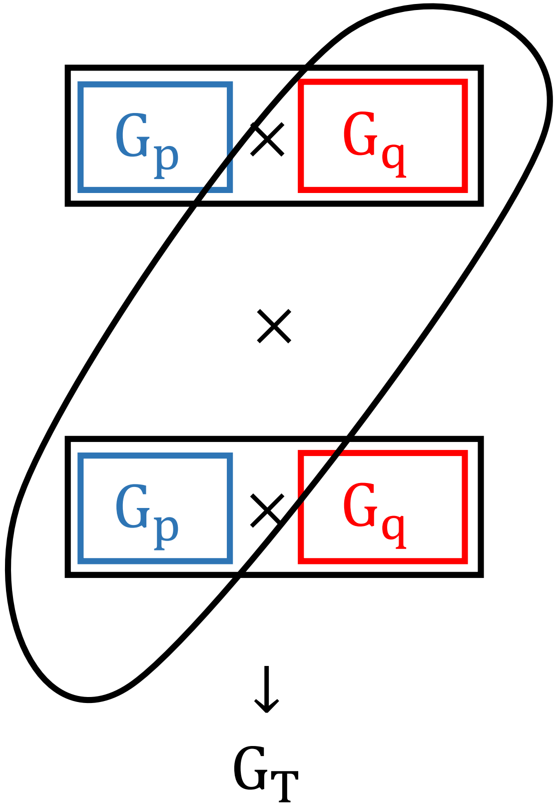
Composite-order groups

[Boneh, Goh, Nissim'05; Lewko, Waters'10]

p,q primes

e :

$e(G_q, G_p)=1$



Composite-order groups

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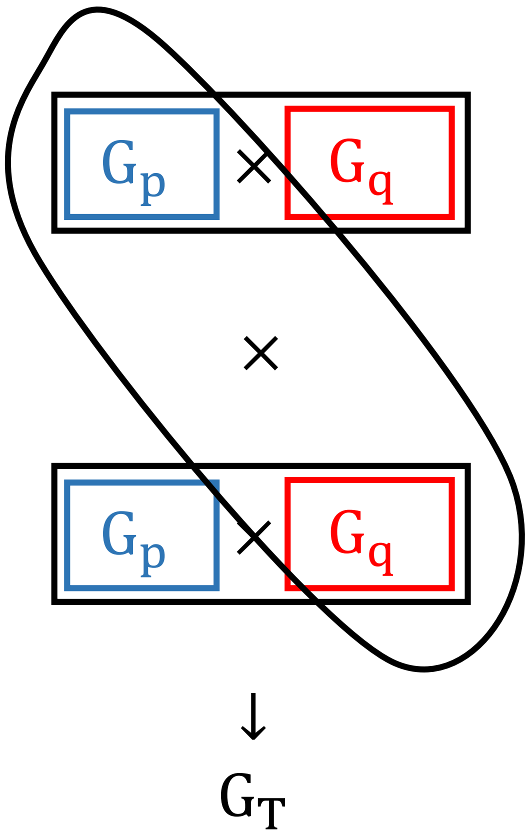

Composite-order groups

[Boneh, Goh, Nissim'05; Lewko, Waters'10]

p,q primes

$$e : \boxed{G_p \times G_q} \times \boxed{G_p \times G_q} \rightarrow G_T$$

Subgroup membership:

$$\underset{\in G_p}{\text{random}} \approx_c \underset{\in G_p}{\text{random}} \cdot \underset{\in G_q}{\text{random}}$$

Composite-order groups

[Boneh, Goh, Nissim'05; Lewko, Waters'10]

p,q primes

$$e : (G_p \times G_q) \times (G_p \times G_q) \downarrow G_T$$

Parameter hiding:

$G_p = < g_1 >$, $G_q = < g_2 >$

For all $w \in \mathbb{Z}_{pq}$

given $g_1^{\,w}$, $g_2^{\,w}$ is hidden

Composite-order groups

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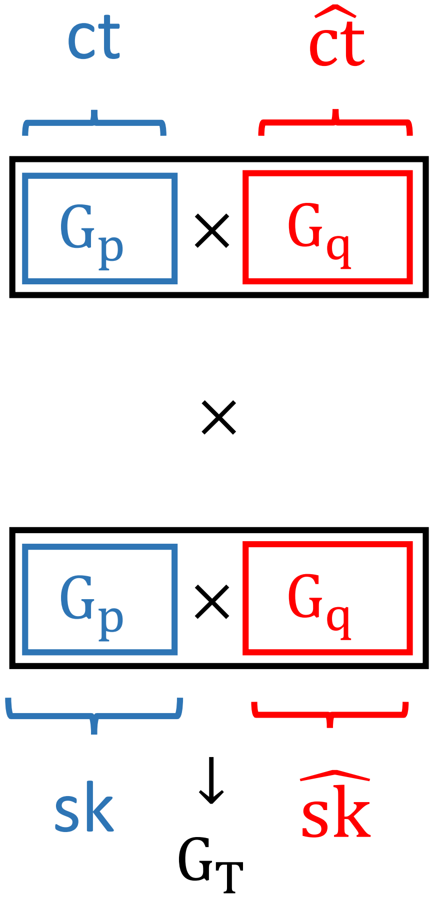


Parameter hiding:

$G_p = \langle g_1 \rangle$, $G_q = \langle g_2 \rangle$

For all $w \in \mathbb{Z}_{pq}$

given $g_1{}^w$, $g_2{}^w$ is hidden

DSE [Waters 09]

Simulating composite-order groups

- [Freeman 10, MSF 10, Seo 12, HHHRR14] -> parameter hiding?
- DPVS: [OT 08, OT 09, Lewko 12, CLLWW 12] -> not compact
- [CW 13, BKP 14] -> not all predicate

Simulating composite-order groups

G_1 = < g_1 > , G_2 = < g_2 > , G_T of order p ,

$$e: G_1 \times G_2 \to G_T$$

$$e\left(g_1^x, g_2^y\right) = e(g_1, g_2)^{xy}$$

Simulating composite-order groups

$G_1 = < g_1 >$, $G_2 = < g_2 >$, G_T of order p ,

$$e: G_1 \times G_2 \rightarrow G_T$$

$$e([x]_1, [y]_2) = [xy]_T$$

Simulating composite-order groups

$G_1 = < g_1 >$, $G_2 = < g_2 >$, G_T of order p ,

$$e: G_1 \times G_2 \rightarrow G_T$$

$$e([x]_1, [y]_2) = [xy]_T$$

Matrix assumptions [EHKRV 13, MRV15]:

$$[A\vec{r}]_1 \approx_c [\vec{u}]_1$$

$A \in \mathbb{Z}_p^{(k+1)\times k}$, $\vec{r} \leftarrow^R \mathbb{Z}_p^{\ k}$ $\qquad$ $\vec{u} \leftarrow^R \mathbb{Z}_p^{(k+1)}$

Simulating composite-order groups

$G_1 = < g_1 >, G_2 = < g_2 >, G_T$ of order p ,

$$e: G_1 \times G_2 \rightarrow G_T$$

$$e([x]_1, [y]_2) = [xy]_T$$

Matrix assumptions [EHKRV 13, MRV15]:

$$[A\vec{r}]_1 \approx_c [\vec{u}]_1$$

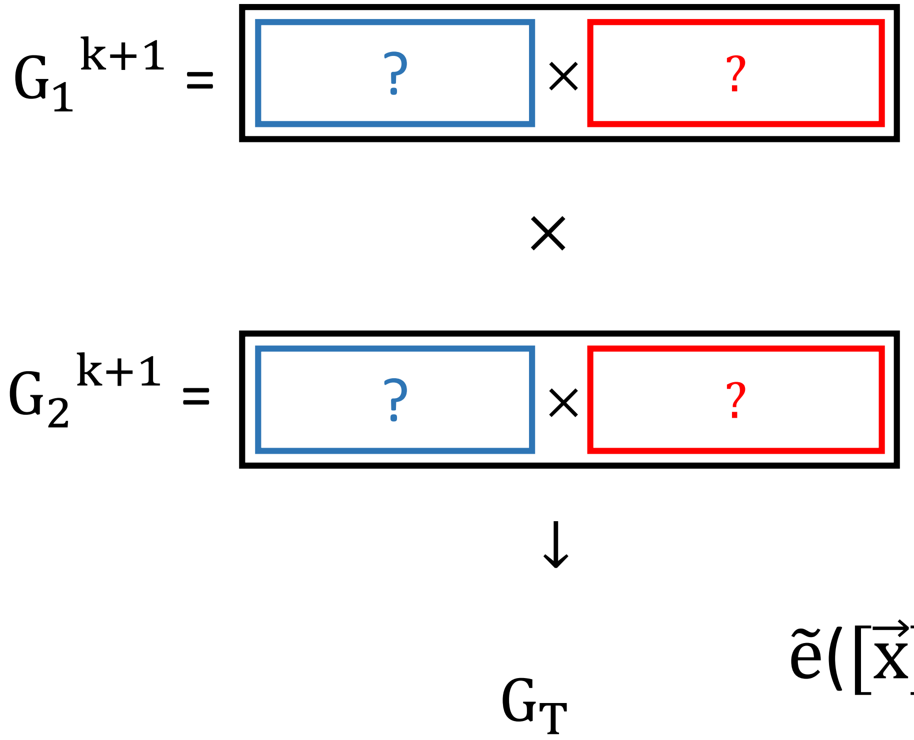
DDH: $A = \binom{1}{a}, a \leftarrow^R \mathbb{Z}_p$

k-Lin: $A = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \\ a_1 & \cdots & a_k \end{pmatrix}, a_1, \dots, a_k \leftarrow^R \mathbb{Z}_p$

Simulating composite-order groups

$e: G_1 \times G_2 \rightarrow G_T$ G_1, G_2 of order p

$\tilde{e}:$

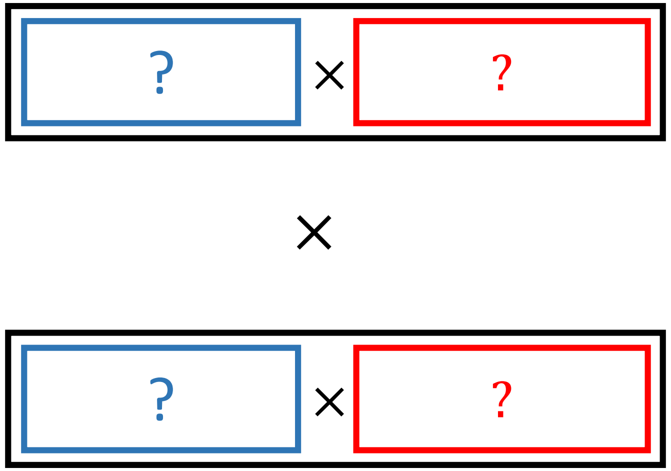


$$\tilde{e}([\vec{x}]_1, [\vec{y}]_2) = [\vec{x}^T \vec{y}]_T$$

Simulating composite-order groups

$e: G_1 \times G_2 \rightarrow G_T$ G_1, G_2 of order p

$\tilde{e}:$ $G_1^{k+1} =$



$G_2^{k+1} =$

$\downarrow$

G_T

$$\tilde{e}([X]_1, [Y]_2) = [X^T Y]_T$$

Simulating composite-order groups

$e: G_1 \times G_2 \to G_T$ $\quad G_1, G_2$ of order p

- $[A]_1, [B]_2 \leftarrow^R$ k-Lin

$\tilde{e}:$

$$G_1^{k+1} = \boxed{< [A]_1 >} \times \boxed{?}$$

$$\times$$

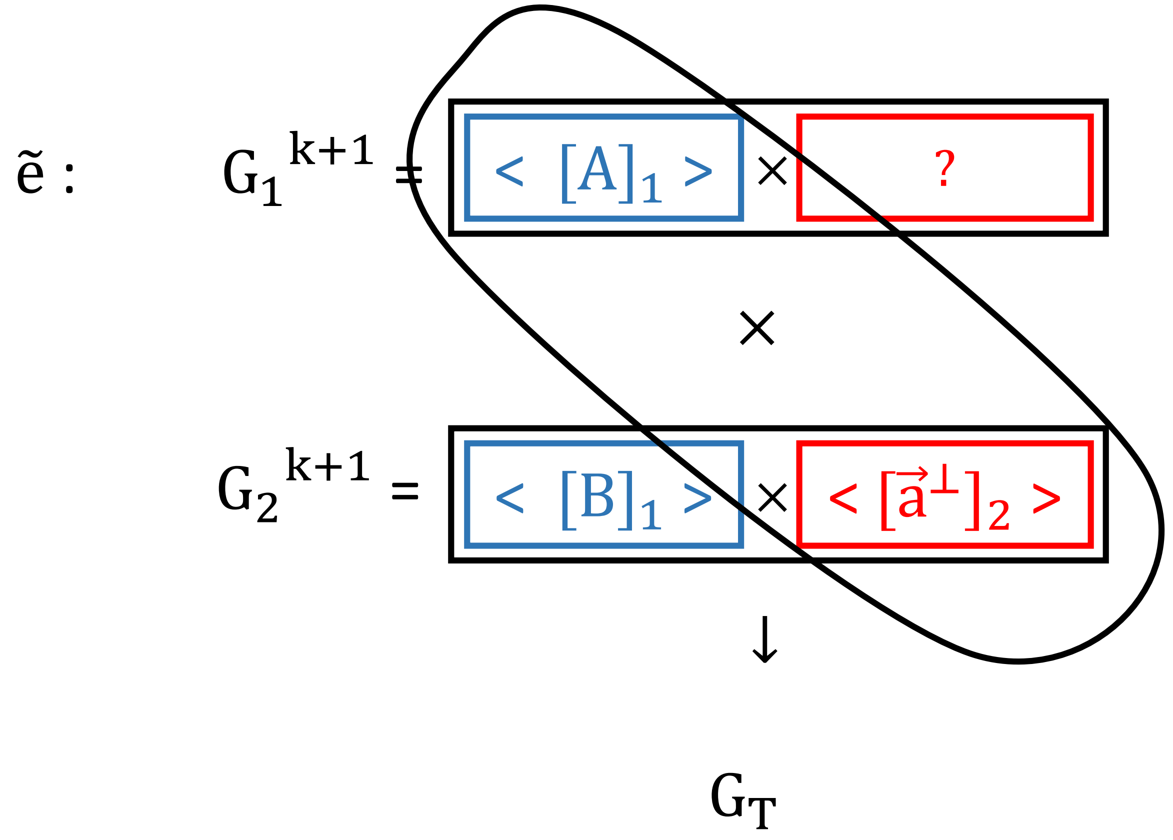
$$G_2^{k+1} = \boxed{< [B]_1 >} \times \boxed{?}$$

$$\downarrow$$

$$G_T$$

Simulating composite-order groups

$e: G_1 \times G_2 \rightarrow G_T$ G_1, G_2 of order p



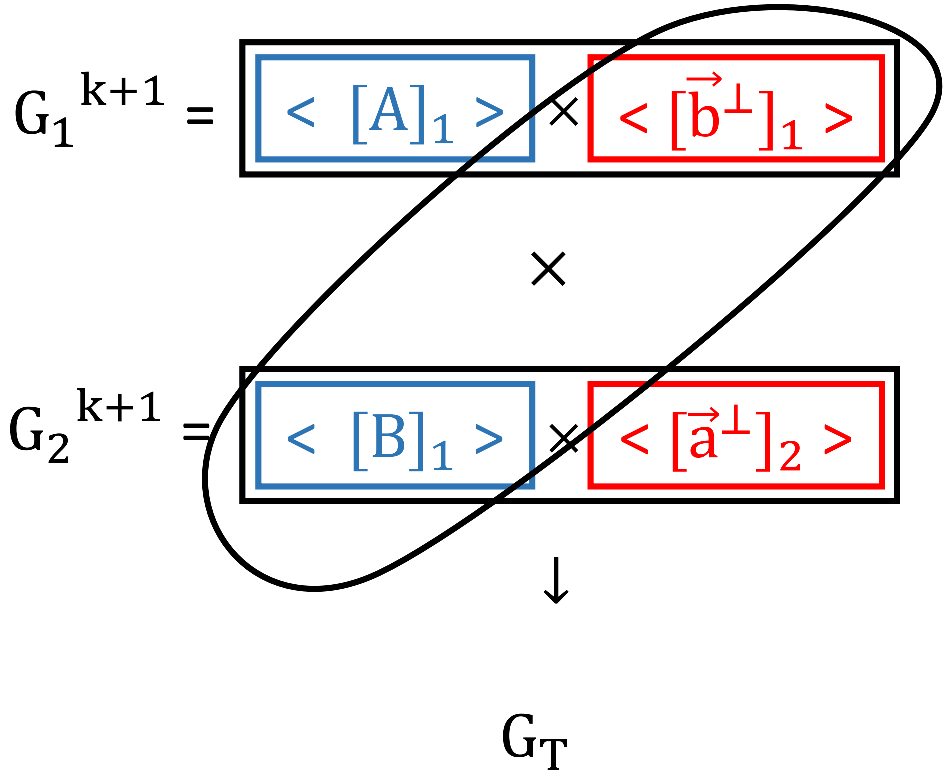
- $[A]_1, [B]_2 \leftarrow^R$ k-Lin
- $\vec{a}^{\perp} \leftarrow^R A^{\perp}$

$e([A]_1,[\vec{a}^{\perp}]_2)=1$

Simulating composite-order groups

$e: G_1 \times G_2 \rightarrow G_T$ G_1, G_2 of order p

$\tilde{e}$:



- $[A]_1, [B]_2 \leftarrow^R$ k-Lin
- $\vec{a}^\perp \leftarrow^R A^\perp$
- $\vec{b}^\perp \leftarrow^R B^\perp$

$e([\vec{b}^\perp]_1, [B]_2)=1$

Simulating composite-order groups

$e: G_1 \times G_2 \rightarrow G_T$ $\quad G_1, G_2$ of order p

$\tilde{e}:$

$$G_1^{k+1} = \langle [A]_1 \rangle \times \langle [\vec{b}^{\perp}]_1 \rangle$$

$$\times$$

$$G_2^{k+1} = \langle [B]_1 \rangle \times \langle [\vec{a}^{\perp}]_2 \rangle$$

$$\downarrow$$

$$G_T$$

- $[A]_1, [B]_2 \leftarrow^{R}$ k-Lin
- $\vec{a}^{\perp} \leftarrow^{R} A^{\perp}$
- $\vec{b}^{\perp} \leftarrow^{R} B^{\perp}$

Simulating composite-order groups

$e: G_1 \times G_2 \rightarrow G_T$ $\qquad$ G_1, G_2 of order p

$\tilde{e}:$ $\qquad G_1{}^{k+1} = \langle\ [A]_1\ \rangle \times \langle\ [\vec{b}^{\perp}]_1\ \rangle$

$\times$

$G_2{}^{k+1} = \langle\ [B]_1\ \rangle \times \langle\ [\vec{a}^{\perp}]_2\ \rangle$

$\downarrow$

G_T

$[A]_1, [\vec{b}^{\perp}]_1$: basis of G_1^{k+1}

$[B]_2, [\vec{a}^{\perp}]_2$: basis of G_2^{k+1}

Simulating composite-order groups

$e: G_1 \times G_2 \to G_T$ $\quad$ G_1, G_2 of order p

$\tilde{e}:$ $\quad G_1^{k+1} = \langle [A]_1 \rangle \times \langle [\vec{b}^{\perp}]_1 \rangle$

$\times$

$G_2^{k+1} = \langle [B]_1 \rangle \times \langle [\vec{a}^{\perp}]_2 \rangle$

$\downarrow$

G_T

Subgroup membership:

$$[A\vec{r}]_1 \approx_c [A\vec{r}]_1 \cdot [r'\vec{b}^{\perp}]_1 = [\vec{u}]_1$$

k-Lin in G_1

Simulating composite-order groups

$e: G_1 \times G_2 \to G_T$ $\quad$ G_1, G_2 of order p

$\tilde{e}:$ $\quad G_1^{k+1} = \langle [A]_1 \rangle \times \langle [\vec{b}^{\perp}]_1 \rangle$

$\times$

$G_2^{k+1} = \langle [B]_1 \rangle \times \langle [\vec{a}^{\perp}]_2 \rangle$

$\downarrow$

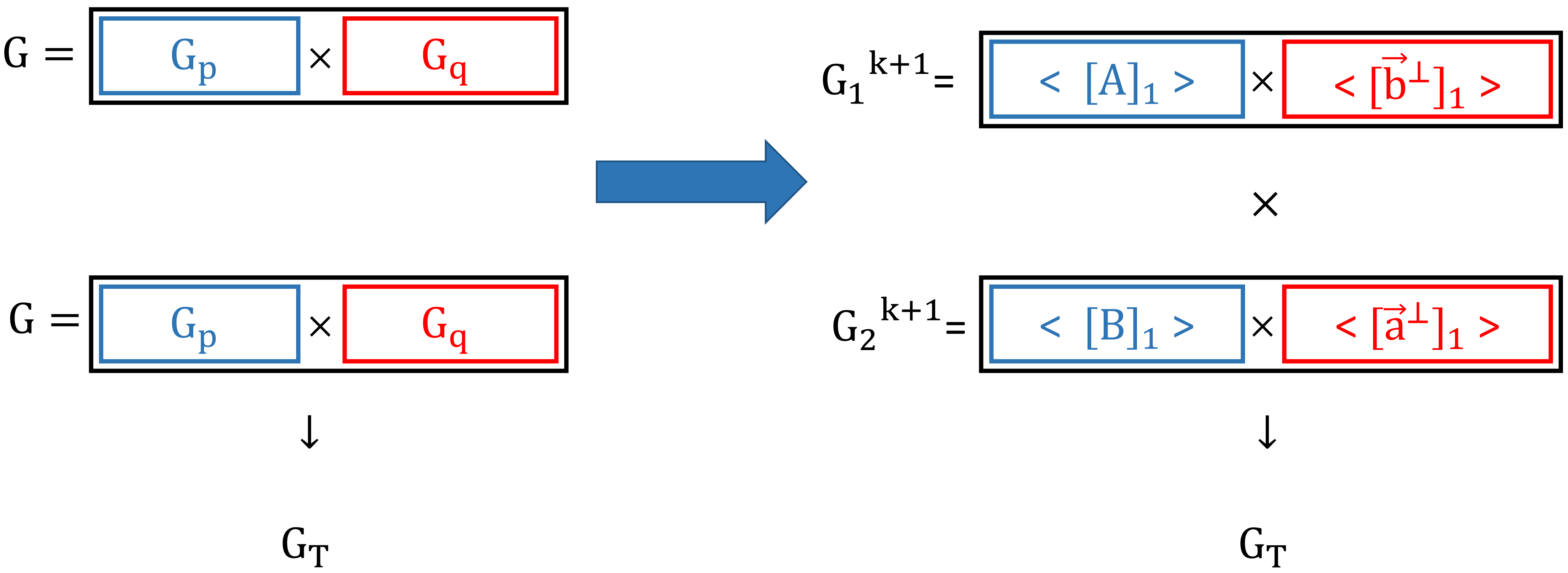
G_T

Subgroup membership:

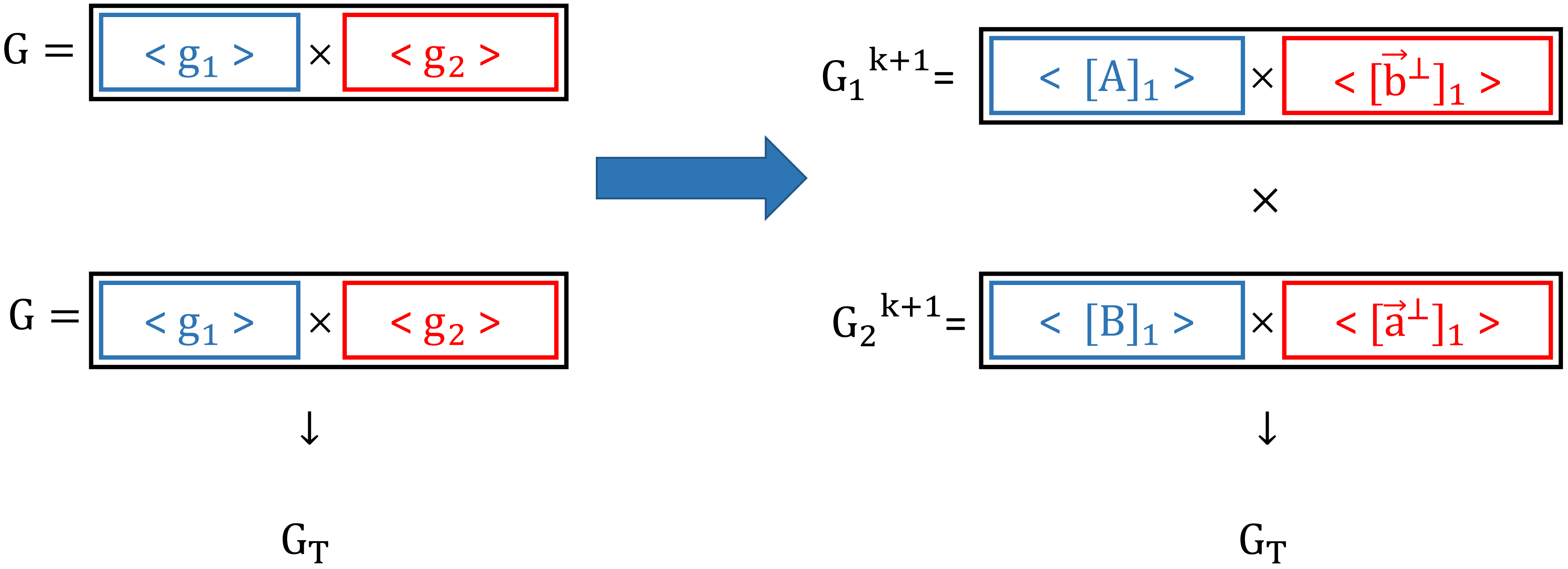
$[B\vec{s}]_1 \approx_c [B\vec{s}]_1 \cdot [s'\vec{a}^{\perp}]_1 = [\vec{v}]_1$

k-Lin in G_2

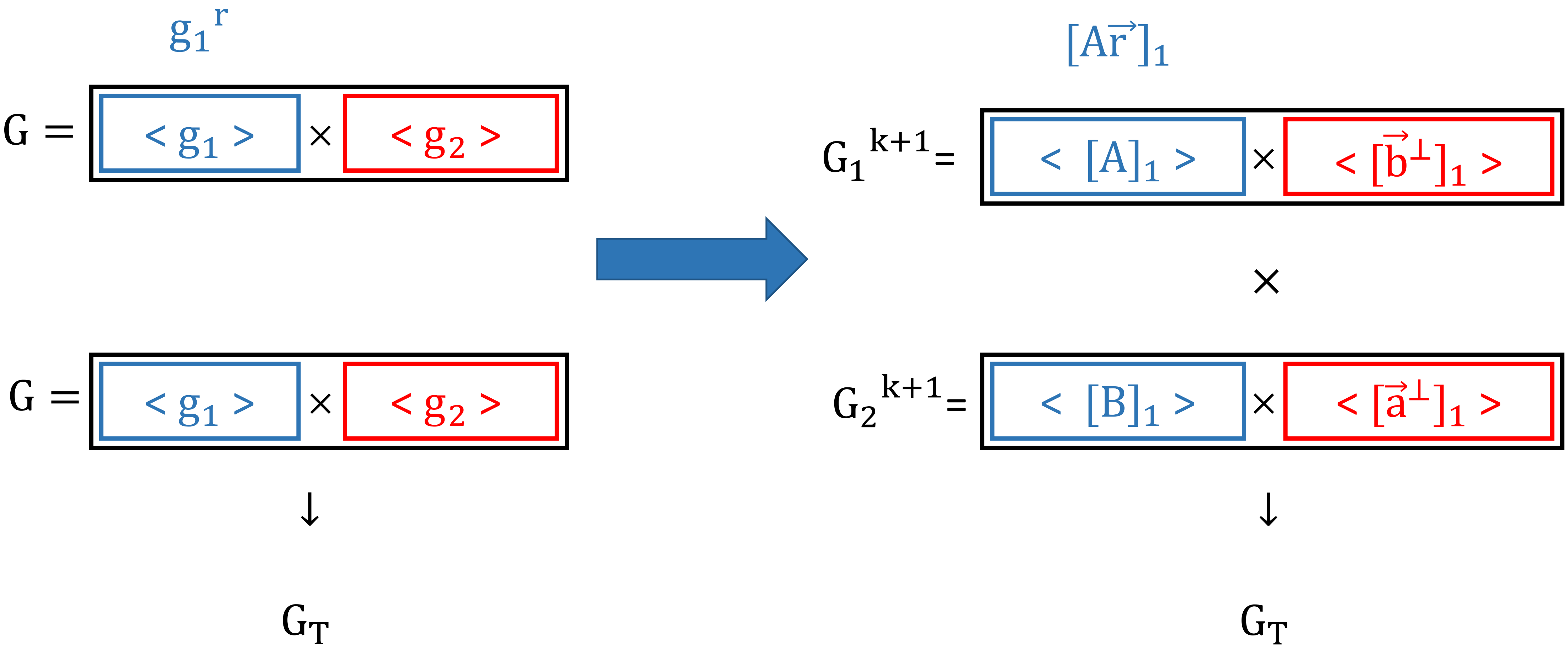
Simulating composite-order groups



Simulating composite-order groups



Simulating composite-order groups



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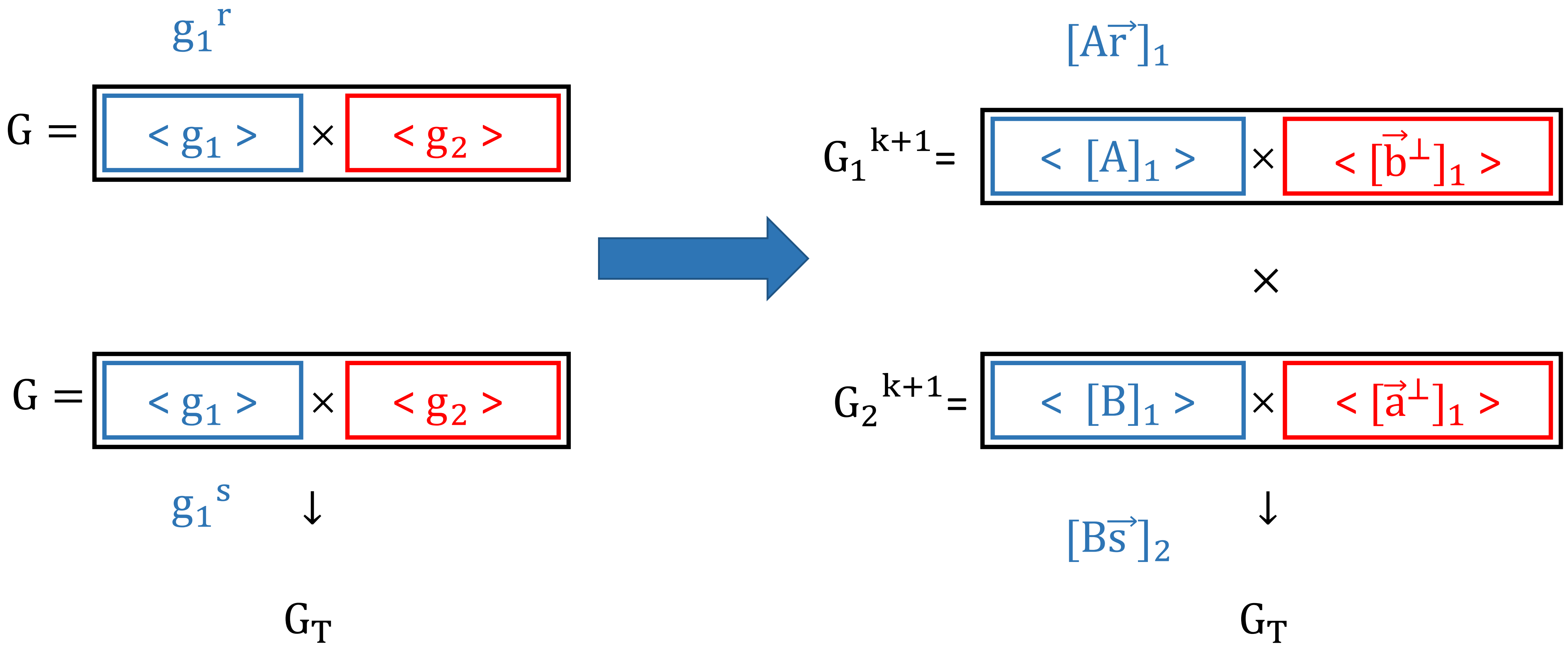

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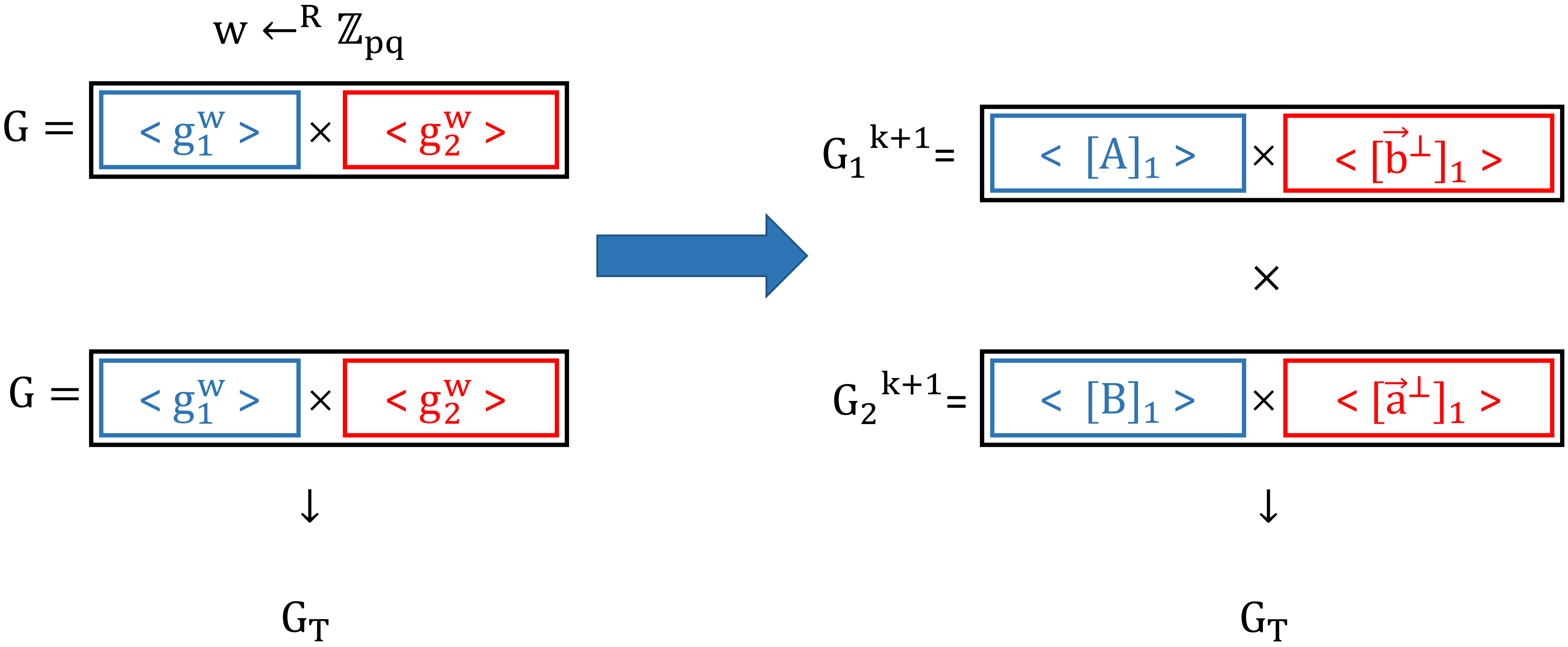


Simulating composite-order groups

$w \leftarrow^{R} \mathbb{Z}_{pq}$

$G = < g_1^w > \times < g_2^w >$

$G = < g_1^w > \times < g_2^w >$

$\downarrow$

G_T

$W \leftarrow^{R} \mathbb{Z}_p^{(k+1)\times(k+1)}$

$G_1{}^{k+1} = < [W^T A]_1 > \times < [W^T \vec{b}^{\perp}]_1 >$

$\times$

$G_2{}^{k+1} = < [WB]_1 > \times < [W\vec{a}^{\perp}]_1 >$

$\downarrow$

G_T

Simulating composite-order groups

Parameter hiding:

$w \leftarrow^{R} \mathbb{Z}_{pq}$ → $W \leftarrow^{R} \mathbb{Z}_{p}^{(k+1)\times(k+1)}$

Given g_1^{w} , g_2^{w} is hidden

Given $[A^{T}W]_1$ and $[WB]_2$ $(\vec{a}^{\perp})^{T}W\vec{b}^{\perp}$ is hidden

Simulating composite-order groups

$$w \rightarrow W \in \mathbb{Z}_p^{(k+1)\times(k+1)}$$

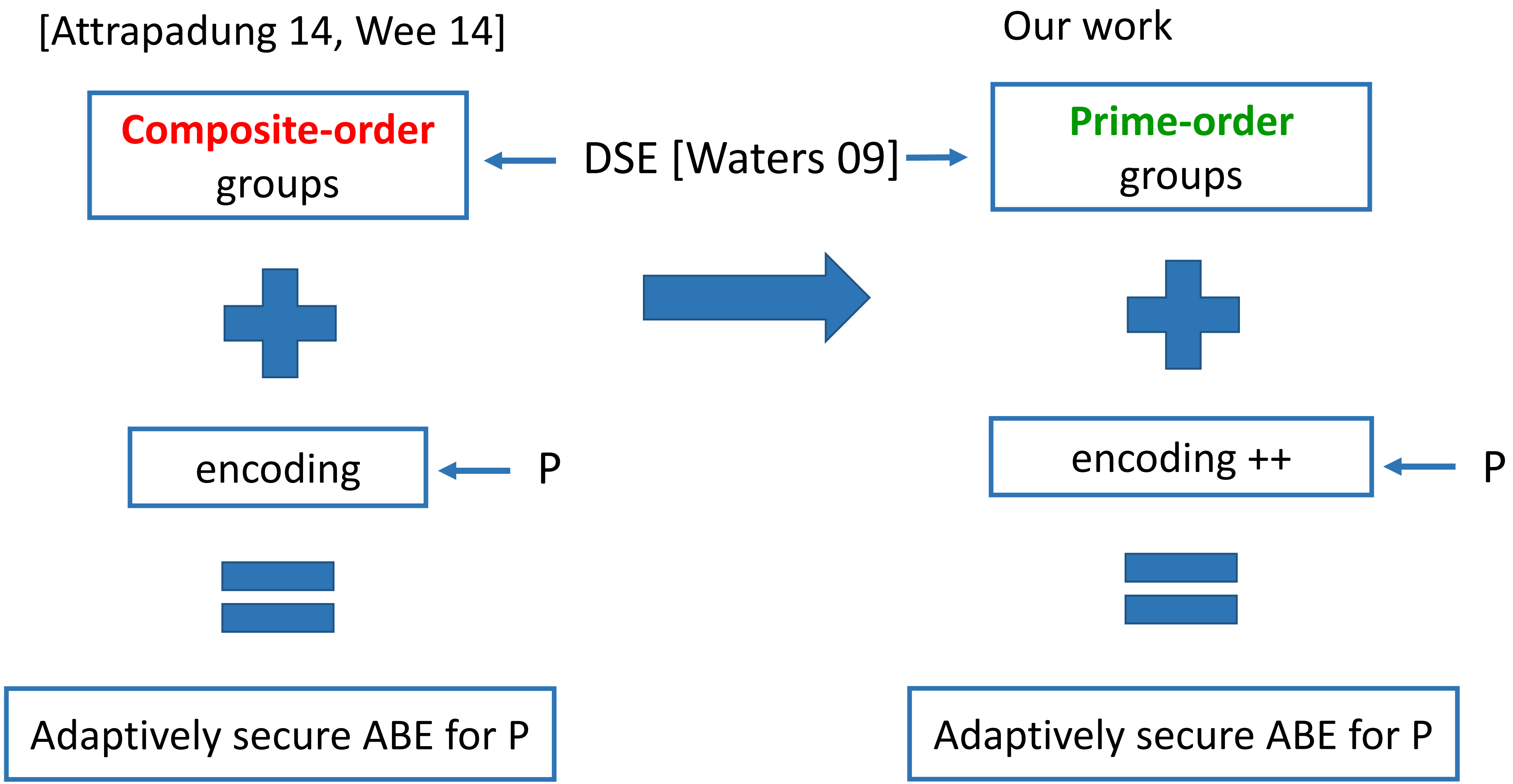
$$s \rightarrow \vec{s} \in \mathbb{Z}_p^k$$
$$g_1{}^s \rightarrow [A\vec{s}]_1$$
$$g_1{}^{ws} \rightarrow [W^T A\vec{s}]_1$$

ct

$$r \rightarrow \vec{r} \in \mathbb{Z}_p^k$$
$$g_1{}^r \rightarrow [B\vec{r}]_2$$
$$g_1{}^{wr} \rightarrow [WB\vec{r}]_2$$

sk

Modular framework for ABE



Conclusion

New efficient ABEs for boolean formula of size n:

reference	(static) assumption	|sk| , |ct|
[A14, W14]	Composite-order	|sk| , |ct| = n + O(1) g.e.

Conclusion

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[Lewko 12, CLL+ 12]	k-Lin	$\|sk\|$, $\|ct\|$ = O((k+1)(n + O(1))) g.e.

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[our work]	k-Lin	|sk| , |ct| = (k+1)(n + O(1)) g.e.

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reference	(static) assumption	$|sk|$, $|ct|$
[A14, W14]	Composite-order	$|sk|$, $|ct|$ = n + O(1) g.e.
[Lewko 12, CLL+ 12]	k-Lin	$|sk|$, $|ct|$ = O((k+1)(n + O(1))) g.e.
[our work]	k-Lin	$|sk|$, $|ct|$ = (k+1)(n + O(1)) g.e.
Open problem	k-Lin	$|sk|$, $|ct|$ = n + k + O(1) ? g.e.

Thank you!

Questions?