An Introduction to Lustre

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The language Lustre

- Invented by Paul Caspi and Nicolas Halbwachs around 1984, in Grenoble (France).
- A program is a set of equations. An equation defines an infinite sequence of values.
- Boolean operators applied point-wise, a unit-delay, and sampling operators.
- Equivalent graphical representation by block diagrams.
- Feedback loops must cross a unit delay.
- Time is synchronous: at every tick of a global clock, every operation does a step.
- Code generation to sequential code and formal verification techniques.
- An industrial success: SCADE (Esterel-Technologies company) is used for programming critical control software (e.g., planes, nuclear plants).
Lustre

Program by writing stream equations.

\[
\begin{array}{ccccccc}
X & 1 & 2 & 1 & 4 & 5 & 6 & \ldots \\
Y & 2 & 4 & 2 & 1 & 1 & 2 & \ldots \\
\  & 1 & 1 & 1 & 1 & 1 & 1 & \ldots \\
X + Y & 3 & 6 & 3 & 5 & 6 & 8 & \ldots \\
X + 1 & 2 & 3 & 2 & 5 & 6 & 7 & \ldots \\
\end{array}
\]

Equation \( Z = X + Y \) means that at any instant \( n \in \mathbb{N} \), \( Z_n = X_n + Y_n \).

Time is logical: inputs \( X \) and \( Y \) arrive at the same time; the output \( Z \) is produced at the same time.

Synchrony means that at instant \( n \), all streams take their \( n \)-th value.

In practice, check that the current output is produced before the next input arrives.
Example: 1-bit adder

```plaintext
node full_add(a, b, c:bool) returns (s, co:bool);
    let
        s = (a xor b) xor c;
        co = (a and b) or (b and c) or (a and c);
    tel;

or:

node full_add(a, b, c:bool) returns (s, co:bool);
    let
        co = if a then b or c else b and c;
        s = (a xor b) xor c;
    tel;
```
Full Adder

Compose two “half adder”

```plaintext
define node half_add(a, b: bool)
    returns (s, co: bool);
    let s = a xor b;
    co = a and b;
end
```

Instanciate it twice:

```plaintext
define node full_add_h(a, b, c: bool)
    returns (s, co: bool);
    var s1, c1, c2: bool;
    let
    (s1, c1) = half_add(a, b);
    (s, c2) = half_add(c, s1);
    co = c1 or c2;
end
```
Verify properties

How to be sure that `full_add` and `full_add_h` are equivalent?

\[ \forall a, b, c : \text{bool}. \; \text{full}_\text{add}(a, b, c) = \text{full}_\text{add}_\text{h}(a, b, c) \]

Write the following program and prove that it returns `true` at every instant!

```lucinda
-- file prog.lus
node equivalence(a,b,c:bool) returns (ok:bool);
    var o1, c1, o2, c2: bool;
    let
        (o1, c1) = full_add(a,b,c);
        (o2, c2) = full_add_h(a,b,c);
        ok = (o1 = o2) and (c1 = c2);
    tel;
```

Then, use the model-checking tool `lesar`:

```
% lesar prog.lus equivalence
--Pollux Version 2.2
TRUE PROPERTY
```

TRUE PROPERTY
The Unit Delay

One can refer to the value of a input at the “previous” step.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
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<td></td>
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<td></td>
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<tr>
<td>$pre \ X$</td>
<td>nil</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>...</td>
</tr>
<tr>
<td>$Y$</td>
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<td>4</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>...</td>
</tr>
<tr>
<td>$Y \rightarrow pre \ X$</td>
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<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>...</td>
</tr>
<tr>
<td>$S$</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>13</td>
<td>19</td>
<td>...</td>
</tr>
</tbody>
</table>

The stream $(S_n)_{n \in \mathbb{N}}$ with $S_0 = X_0$ and $S_n = S_{n-1} + X_n$, for $n > 0$ is written:

$$S = X \rightarrow pre \ S + X$$

Introducing intermediate equations does not change the meaning of programs:

$$S = X \rightarrow I; \ I = pre \ S + X$$
Example: convolution

Define the sequence:

\[ Y_0 = \frac{X_0}{2} \quad \land \quad \forall n > 0. Y_n = \frac{(X_n + X_{n-1})}{2} \]

\[
\begin{array}{c}
X \\
\downarrow \\
\text{+} \\
\text{2} \\
\downarrow \\
0 \\
\downarrow \\
/ \\
\downarrow \\
Y
\end{array}
\]

node convolution(X:real) returns (Y:real);
let Y = (X + (0 -> pre X)) / 2.0;
tel;

or:

node convolution(X:real) returns (Y:real);
var pY:int;
let Y = (X + pY) / 2;
pY = 0 -> pre X;
tel;
Linear filters

FIR (Finite Impulse Response)

\[
y(n) = \sum_{m=0}^{L-1} x(n - m)b(m)
\]

IIR (Infinite Impulse Response) or recursive filter

\[
y(n) = \sum_{m=0}^{L-1} x(n - m)b(m) + \sum_{m=1}^{M-1} y(n - m)a(m)
\]
FIR

Build a block-diagram with three operators: a gain (multiplication by a constant), a sum and a unit delay (register).

\[
\forall n \geq 0. y(n) = 1/2 (x(n) + x(n - 1))
\]
Example: follow $x$ with a 20% gain.

$$\forall n \geq 0. y(n) = 0.2(x(n) - y(n-1)) + y(n-1)$$

node filter(x: real) returns (y:real);
  let y = 0.0 -> 0.2 * (x - pre y) + pre y; tel;

Retiming:

Optimise by moving unit delays around combinatorial operators.

**DEMO:** type luciole filter.lus filter
Counting events

Count the number of instants where the input signal tick is true between two top.

node counter(tick, top:bool) returns (cpt:int);
let
    cpt = if top then 0
        else if tick then (0 -> pre cpt) + 1 else pre cpt;
    tel;

Is this program well defined? Is-it deterministic? No: initialization issue.

<table>
<thead>
<tr>
<th>tick</th>
<th>f</th>
<th>f</th>
<th>t</th>
<th>t</th>
<th>t</th>
<th>t</th>
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<tbody>
<tr>
<td>top</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>t</td>
<td>t</td>
<td>t</td>
<td>f</td>
<td>...</td>
</tr>
<tr>
<td>cpt</td>
<td>nil</td>
<td>nil</td>
<td>nil</td>
<td>nil</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Write instead:

cpt = if top then 0
    else if tick then (0 -> pre cpt) + 1 else 0 -> pre cpt;
An explicit Euler integrator

node integrator(const step: real; x0, x’:real) returns (x:real);
let
  x = x0 -> pre(x) + pre(x’) * step;
tel;

step is a constant stream computed at compile-time.

Sinus/cosine functions

node sinus_cosinus(theta:real) returns (sin,cos:real);
let sin = theta * integrator(0.01, 0.0, cos);
  cos = theta * integrator(0.01, 1.0, 0.0 -> pre sin);
tel;
Initial Value Problem (IVP)

$f$ is a combinatorial function with $y$ of type $ty$. $t$ is the current time. $x(t)$ be defined by the IVP:

$$\dot{x} = f(y, t, x) \quad \text{with} \quad x(0) = x_0$$

node ivp(const step: real; y: ty; read) returns (x: real)

var t: real;
let
  x = integr(step, x0, f(y, t, x));
  t = 0.0 -> pre t + step;
tel;

Exercice

- Program a classical explicit Runge Kutta method (e.g., order 4).
- More difficult: program a variable step Runge Kutta method (RK45). Hint: use a control bit `error_too_large` to shrink the step dynamically.
Counting Beacons

Counting becons and seconds to decide whether a train is on time.

Use an **hysteresis** with a low and high threshold to reduce oscillations.

node beacon(sec, bea: bool) returns (ontime, late, early: bool);
var diff, pdiff: int; pontime: bool;
let
  pdiff = 0 -> pre diff;
  diff = pdiff + (if bea then 1 else 0) +
    (if sec then -1 else 0);
  early = pontime and (diff > 3) or
    (false -> pre early) and (diff > 1);
  late = pontime and (diff < -3) or
    (false -> pre late) and (diff < -1);
  ontime = not (early or late);
  pontime = true -> pre ontime;
tel;

---

1This example is due to Pascal Raymond
Two types of properties

Safety property
“Something wrong never happen”, i.e., a property is invariant and true in any accessible state. E.g.:
- “The train is never both early and late”, it is either on time, late or early;
- “The train never passes immediately from late to early”; “It is impossible to stay late only a single instant”.

Liveness property
“Something good with eventually happen.”, i.e., any execution will reach a state verifying the property. E.g., “If the trains stop, it will eventually be late.”

Remark:
“If the train is on time and stops for ten seconds, it will be eventually late” is a safety property.
Safety properties are critical ones in practice.
Formal verification and modeling of systems

A safety property ("something bad will never happen") is a boolean proved to be true at every instant.

Example: the alternating bit protocol
A transmitter $A$; a receiver $B$. Two unreliable lines $A2B$ and $B2A$ that may lose messages.

- $A$ asks for one input. It re-emits the data with $bit = true$ until it receives $ack = true$.
- It asks for another input and emits the data with $bit = false$ until it receives $ack = false$.
- $B$ sends $ack = false$ until it receives $bit = true$; it sends $ack = true$ until it receives $bit = false$;
- initialization: send anything with $bit = true$. The first message arriving with $bit = false$ is valid.
Objective:

Model and prove the protocol is correct, i.e., the network is the identity function (input sequence = output sequence) with two unreliable lines.

Model the asynchronous communication by adding a “presence” bit to every data: a pair \((data, enable)\) is meaningful when \(enable = true\).
The Sender

- A asks for one input. It re-emits the data with bit = true until it receives ack = true.
- It asks for an other input and emits the data with bit = false until it receives ack = false.

```plaintext
node A(dataIn: int; recB: bool; ack: bool)
returns (reqData: bool; send: bool; data: int; bit: bool);

var
  buff: int; chstate : bool;

let
  buff = if reqData then dataIn else (0 -> pre buff);
  chstate = recB and (bit = ack);
  reqData, send, bit =
    (false, true, true) ->
    pre (if chstate then (true, true, not bit)
      else (false, send, bit));
  data = buff;
tel
```
The Receiver

- *B* sends $ack = false$ until it receives $bit = true$; it sends $ack = true$ until it receives $bit = false$;

node B(recA : bool; data: int; bit: bool;) returns (sendOut: bool; dataOut: int; send2A: bool; ack: bool);

var chstate : bool;

let

  chstate = recA and (ack xor bit);

  sendOut, send2A, ack =  
  (false, true, true) -> 
    pre (if chstate then (true, true, not ack) 
        else (false, true, ack));

  dataOut = data;

tel
Modeling the channel and the main property

node unreliable(loose: bool; presIn: bool) returns (presOut: bool);
let
    presOut = presIn and not loose;
tel

-- The property that two signals [r] and [s] alternate.
node altern(r,s: bool) returns (ok: bool);
var
    s0, s1 : bool;
    ps0, ps1 : bool;
let
    ps0 = true -> pre s0;
    ps1 = false -> pre s1;
    s0 = ps0 and (r = s) or ps1 and s and not r;
    s1 = ps0 and r and not s or ps1 and not r and not s;
    ok = (true -> pre ok) and (s0 or s1);
tel
The main system

node obs(dataIn: int; looseA2B, looseB2A : bool;) returns (ok : bool; reqData: bool; sendOut: bool);
var
dataOut: int;
sendA2B: bool; data: int; bit: bool;
recA2B, recB2A : bool;
sendB2A: bool; ack: bool;

let
ok = altern(reqData, sendOut);
recA2B = unreliable(looseA2B, sendA2B);
recB2A = unreliable(looseB2A, sendB2A);

reqData, sendA2B, data, bit = A(dataIn, recB2A, ack);
sendOut, dataOut, sendB2A, ack = B(recA2B, data, bit);
tel

%aneto.local: lesar ba.lus obs
TRUE PROPERTY
Clocks: mixing slow and fast processes

A slow process is made by sampling its inputs; a fast one by oversampling its inputs.

The operators when, current and merge

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>false</th>
<th>true</th>
<th>false</th>
<th>true</th>
<th>false</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>x_0</td>
<td>x_1</td>
<td>x_2</td>
<td>x_3</td>
<td>x_4</td>
<td>x_5</td>
<td></td>
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<tr>
<td>Y</td>
<td>y_0</td>
<td>y_1</td>
<td>y_2</td>
<td>y_3</td>
<td>y_4</td>
<td>y_5</td>
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<tr>
<td>Z = X when B</td>
<td></td>
<td></td>
<td>x_1</td>
<td></td>
<td>x_3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K = Y when not B</td>
<td></td>
<td>y_0</td>
<td></td>
<td>y_2</td>
<td></td>
<td>y_4</td>
<td>y_5</td>
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<tr>
<td>T = current Z</td>
<td>nil</td>
<td>x_1</td>
<td>x_1</td>
<td>x_3</td>
<td>x_3</td>
<td>x_3</td>
<td></td>
</tr>
<tr>
<td>O = merge B T K</td>
<td>y_0</td>
<td>x_1</td>
<td>y_2</td>
<td>x_3</td>
<td>y_4</td>
<td>y_5</td>
<td></td>
</tr>
</tbody>
</table>

The operator merge is not part of Lustre. It was introduced later in Lucid Synchrone and SCADE 6.
The Gilbreath trick in SCADE 6

Take a card deck where card alternate; split it in two; shuffle them arbitrarily. Then, if you take two successives cards, their colors are different (provided bottom cards have diff. colors).

node Gilbreath_stream (clock c:bool) returns (prop: bool; o:bool);
var
  s1 : bool when c;
  s2 : bool when not c;
  half : bool;
let
  s1 = (false when c) -> not (pre s1);
  s2 = (true when not c) -> not (pre s2);
  o = merge (c; s1; s2);
  half = false -> (not pre half);
prop = true -> not (half and (o = pre o));
tel;

The Gilbreath trick in Lustre

node Gilbreath_stream (c:bool) returns (OK: bool; o:bool);
var
    ps1, s1 : bool; ps2, s2 : bool; half : bool;
let

    s1 = if c then not ps1 else ps1;
    ps1 = false -> pre s1;
    s2 = if not c then not ps2 else ps2;
    ps2 = true  -> pre s2;

    o = if c then s1 else s2;

    half = false -> (not pre half);

    OK = true  -> not (half and (o = pre o));
tel;

Proved automatically using Lesar (Pascal Raymond) and KIND2 (Cesare Tinelli).
A classical use of clock: the activation condition

Run a process on a slower by sub-sampling its inputs; hold outputs.

node sum(i:int) returns (s:int);
let
    s = i -> pre s + i;
tel;

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>cond</td>
<td>true</td>
<td>false</td>
<td>true</td>
<td>true</td>
<td>false</td>
<td>true</td>
</tr>
<tr>
<td>sum(1)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>sum(1 when cond)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(sum 1) when cond</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td></td>
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</tr>
</tbody>
</table>

Sampling inputs vs sampling outputs

• current \((f(x \text{ when } c))\) is called an “activation condition”
• \(f(x \text{ when } c) \neq (f(x) \text{ when } c)\)
• current\((x \text{ when } c) \neq x\)
Why synchrony?

```
let half = true -> not (pre half);
o = x & (x when half);
tel
```

It defines the sequence: $\forall n \in \mathbb{N}. o_n = x_n & x_{2n}$

- It cannot be computed in bounded memory.
- Its corresponding Kahn networks has unbounded buffering.
- This is forbidden, a dedicated analysis for that: clock calculus
(Intuitive) Clocking rules in Lustre

Clocks must be declared and visible from the interface of a node.

node stables(i:int) ← base clock (true)
returns (s:int; ncond:bool;
    (ns:int) when ncond); ← clock declaration

var cond:bool;
(l:int) when cond; ← clock declaration

let
    cond = true -> i <> pre i;
    ncond = not cond;
    l = somme(i when cond);
    s = current(l);
    ns = somme(i when ncond); 

tel;
Constraints

Rules

- Constants are on the base clock of the node.
- By default, variables are on the base clock of the node.
- Unless a clock is associated to the variable definition.
- \( \text{clock}(e_1 \text{ op } e_2) = \text{clock}(e_1) = \text{clock}(e_2) \)
- \( \text{clock}(e \text{ when } c) = c \)
- \( \text{clock(current}; e) = \text{clock(clock}(e)) \)

Implementation choices

- Clocks are declared and verified. No automatic inference.
- Two clocks are equal if expressions that define them are syntactically equal.
One hot coding of Mealy machines

Represent a state by a Boolean variable.

Think in term of an invariant: what is the expression defining the current value of `on` at every instant?
Verification with assertions

Consider a second version.

```nim
node switch2(set, reset:bool) returns (ok:bool);
    var s1, s2: bool;
let
    s1 = true -> if reset and pre s2 then true
        else if pre s1 and set then false else pre s1;
    s2 = false -> if set and pre s1 then true
        else if pre s2 and reset then false else pre s2;
    ok = s2;
 tel;

node compare(set, reset: bool) returns (ok: bool);
    let ok = switch(set, reset) = switch2(set, reset); tel;
```

We get:

```bash
% lesar prog.lus compare
--Pollux Version 2.2
TRUE PROPERTY
```
Synchronous observers

Comparison is a particular case of a synchronous observer.

- Let $y = F(x)$, and $ok = P(x, y)$ for the property relating $x$ and $y$
- $assert(H(x, y))$ is an hypothesis on the environment.

```
node check(x:t) returns (ok:bool);
  let
    assert H(x,y);
    y = F(x);
    ok = P(x,y);
  tel;
```

If $assert$ is (infinitely) true, then $ok$ stay infinitely true
$(always(assert)) \Rightarrow (always(ok))$.

Any safety temporal property can be expressed as a Lustre program [6, 5]. No temporal logic/language is necessary.

**Safety temporal properties are regular Lustre programs**
Array and slices

Array are manipulated by slices with implicit point-wise extension of operations. \( t[0..N] \) defines a slice of \( t \) from index 0 to \( N \).

\[
\text{const } N = 10;
\]

\[
\text{node plus}(\text{const } N: \text{ int}; a1, a2: \text{ int}^N) \text{ returns } (o: \text{ int}^N);
\]

\[
\begin{align*}
\text{let} \\
& o[1..N] = a1[1..N] + a2[1..N]; \\
\text{tel};
\end{align*}
\]
Arrays

-- serial adder

define node add(a1: bool^N; a2: bool^N; carry: bool) returns (a: bool^N; new_carry: bool)
{
    var c: bool^N;
    let
        (a[0..N-1], c[0..N-1]) =
        bit_add(a1[0..N-1], a2[0..N-1], ([carry] | c[0..N-2]));
        new_carry = c[N-1];
    tel;
}

define node add_short(a1: bool^N; a2: bool^N; carry: bool) returns (a: bool^N; new_carry: bool)
{
    var c: bool^N;
    let
        (a, c) = bit_add(a1, a2, ([carry] | c[0..N-2]));
        new_carry = c[N-1];
    tel;
}
Conclusion

Compilation

- Static, compile-time checking to ensure the absence of deadlock, that the code behave deterministically.
- Execution in bounded memory and time.
- Code generation into sequential “single loop” code. More advanced methods into automata and/or modular.

Verification by Model-checking

- Synchronous observer: a safety property is a Lustre program
- Avoid to introduce an ad-hoc temporal logic.
- Tool Lesar (BDD technique) by Pascal Raymond (VERIMAG, France).
- KIND and KIND2 (k-induction, PDR based on SMT techniques) by Cesare Tinelli (Iowa State Univ., USA).
- Plug-in (k-induction based on SAT techniques) by Prover-Technologies (associated to SCADE 6).
Related languages and verification tools

Various teams have done their own variant of Lustre.

Language embedding in Haskell

- Copilot (Nasa, USA), an Embedding of Lustre.
- FRAN (images, animation), Functional Reactive Programming (FRP), Hawk (architecture), Lava (synchronous circuits).
- Based on a compilation-by-evaluation technique.

Language extensions, formal verification

- Heptagon: Extended Lustre (automata, arrays) with controller synthesis (Gwenael Delaval, Univ. Grenoble)
- Prelude: Lustre with periodic clocks and a compiler that generates tasks for a real-time OS (Julien Forget, Univ. Lille).
- Lustre compiler at Onera for verification purposes (Pierre-Loic Garoche)
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