SMT-based Model Checking of Transition Systems

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Specifying Properties

SMT Solver Basics

Model Checking

Bounded Model Checking and k-induction

Model Checking Lustre Programs: Kind 2

Two types of properties

Safety property: "Something bad never happens"

I.e., a property is invariant and true in any accessible state. E.g.:

- "The variable *temp* is always less than 101."
- "The variable *temp* never increases by more than 5 in a single step."

Liveness property: "Something good eventually happens."

I.e., every execution will reach a state where the property holds.

- "If *heat* is on, *temp* eventually exceeds 10."

Remark:

"If *heat* is on, *temp* exceeds 10 within 5 minutes." is a safety property.

And remember that liveness properties are likely to be the least important part of your specification. You will probably not lose much if you simply omit them.

Lamport (2002): Specifying Systems: The TLA+ Language and Tools for Hardware and Software Engineers

Model Checking: Temporal Logics

Typical formulation of model checking problem: $M \models \phi$
where M is a transition system and ϕ is a formula in temporal logic.

Temporal Logic

- Characterize either
  - » sets of traces; LTL = Linear-Time Logic
  - » sets of trees; CTL = Computation-Tree Logic
- The idea is *not* to write complicated specifications in temporal logic.
  (too hard to write and understand—use synchronous observers.)
- *Nor* is it to study the properties of such logics.
  (we want to write and verify programs.)
- Rather use temporal logic to precisely formulate verification problems and the corresponding algorithms and proof patterns.
- (Also to reason algebraically about liveness [Lamport (2002): Specifying Systems: The TLA+ Language and Tools for Hardware and Software Engineers].)

LTL Basics (from [Biere, Cimatti, E. Clarke, and Zhu (1999): Symbolic Model Checking without BDDs])

Usually presented in terms of (finite) Kripke structures: (S, I, T, l)

- S is a finite set of states
- $I \subseteq S$ are the initial states
- $T \subseteq S \times S$ is the transition relation
- $l : S \to \mathcal{P}(\mathcal{A})$ labels each state with atomic propositions from a set $\mathcal{A}$.
- Require that every state has a successor, i.e., that T is total.
- Abstraction of transition system over states mapping variables to values.

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Usually presented in terms of (finite) Kripke structures: (S, I, T, l)

where $I \subseteq S$ and $T \subseteq S \times S$, $l : S \to \mathcal{P}(\mathcal{A})$.

$$
\begin{array}{llll}
\pi \models p & \textit{iff} & p \in \ell(\pi(0)) & \pi \models \neg p \quad \textit{iff} \quad p \notin \ell(\pi(0)) \\
\pi \models f \wedge g & \textit{iff} & \pi \models f \textit{ and } \pi \models g & \pi \models f \vee g \quad \textit{iff} \quad \pi \models f \textit{ or } \pi \models g \\
\pi \models \mathbf{G} f & \textit{iff} & \forall i.\, \pi^i \models f & \pi \models \mathbf{F} f \quad \textit{iff} \quad \exists i.\, \pi^i \models f \\
\pi \models \mathbf{X} f & \textit{iff} & \pi^1 \models f & \\
\pi \models f \,\mathbf{U}\, g & \textit{iff} & \exists i\, [\, \pi^i \models g \;\; \textit{and} \;\; \forall j,\, j < i.\, \pi^j \models f \,] & \\
\pi \models f \,\mathbf{R}\, g & \textit{iff} & \forall i\, [\, \pi^i \models g \;\; \textit{or} \;\; \exists j,\, j < i.\, \pi^j \models f \,] &
\end{array}
$$

- $\pi = s_0, s_1, \ldots$ is an infinite sequence of states called a *path* if $(s_i, s_{i+1}) \in T$ for all i.
- Basic safety: G f
- Basic liveness: F f

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\pi \models f \wedge g & \textit{iff} & \pi \models f \textit{ and } \pi \models g & \pi \models f \vee g \quad \textit{iff} \quad \pi \models f \textit{ or } \pi \models g \\
\pi \models \mathbf{G} f & \textit{iff} & \forall i.\, \pi^i \models f & \pi \models \mathbf{F} f \quad \textit{iff} \quad \exists i.\, \pi^i \models f \\
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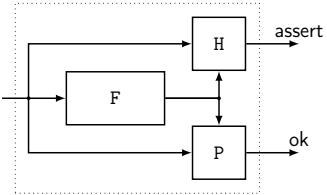
\pi \models f \,\mathbf{R}\, g & \textit{iff} & \forall i\, [\, \pi^i \models g \;\; \textit{or} \;\; \exists j,\, j < i.\, \pi^j \models f \,] &
\end{array}$$

- Write $M \models \mathrm{A} f$ iff for all paths π of M where $\pi(0) \in I$, $\pi \models f$. ('universal model checking problem')
- Write $M \models \mathrm{E} f$ iff there exists a path π of M with $\pi(0) \in I$ and $\pi \models f$. ('existential model checking problem')

Synchronous Observers

- if $y = F(x)$, we write $ok = P(x, y)$ for the property relating x and y
- and `assert`$(H(x, y))$ to states an hypothesis on the environment.

```
node check(x:t) returns (ok:bool);
let
  assert H(x,y);
  y = F(x);
  ok = P(x,y);
tel;
```



If *assert* remains indefinitely true then *ok* remains indefinitely true
`always`$(assert) \Rightarrow$ `always`(ok).

Any safety property can be expressed as a Lustre program. No need to introduce a temporal logic in the language

[Halbwachs, Lagnier, and Raymond (1993): Synchronous observers and the verification of reactive systems] [Halbwachs, Lagnier, and Ratel (1992): Programming and verifying real-time systems by means of the synchronous data-flow language LUSTRE];

Temporal properties are regular Lustre programs

Example of Temporal Properties

- "A is never true twice in a row": `never_twice(A)` where:

```
node never_twice(A : bool) returns (OK : bool);
let
  OK = true -> not(A and pre A);
tel;
```

- "Any event A is followed by an event B before C happens":
  followed_by(A, B) and followed_by(B, C)
  where:

```
node implies(A, B : bool) returns (OK : bool);
let
  OK = not(A) or B;
tel;

node once(A : bool) returns (OK : bool);
let
  OK = A -> A or pre OK;
tel;
```

```
node followed_by(A, B : bool)
returns (OK : bool);
let
  OK = implies(B, once(A));
tel;
```

Example of Temporal Properties (cont.)

Note: Several properties have a sequential nature, e.g., "The temperature should increase for at most 1 min or until the event stop occurs then it must decrease for 2 min".

They can be expressed as regular expressions and then translated into Lustre [Raymond (1996): Recognizing regular expressions by means of dataflow networks]

This is the basis of the language Lutin [Raymond, Roux, and Jahier (2008): Lutin: A Language for Specifying and Executing Reactive Scenarios]

For an encoding of past-time Linear Temporal Logic (LTL) see:
[Halbwachs, Fernandez, and Bouajjani (1993): An executable temporal logic to express safety properties and its connection with the language Lustre]

Exercise: implementing temporal properties

1. Returns false until A occurs, then returns true from the subsequent instant onward

   ```
   node after(a : bool) returns (o : bool);
   -- make clean; make MAIN=after TRACE=trace1.txt
   ```

2. Returns true if and only if its first input has been continuously true since the last time its second input was true

   ```
   node always_since(b, a : bool) returns (o : bool);
   -- make clean; make MAIN=always_since TRACE=trace2.txt
   ```

3. Returns true if and only if its first input has been true at least once since the last time its second input was true.

   ```
   node once_since(c, a : bool) returns (o : bool);
   -- make clean; make MAIN=once_since TRACE=trace3.txt
   ```

4. Any time A has occurred in the past, either B has been continuously true, or C has occurred at least once, since the last occurrence of A

   ```
   node always_from_to(b, a, c : bool) returns (x : bool);
   -- make clean; make MAIN=always_from_to TRACE=trace4.txt
   ```

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SAT solvers

Given a boolean formula b with free variables $x_1, ..., x_n$ from propositional logic, find a valuation $V : \{x_1, ..., x_n\} \to \{0, 1\}$ such that $V(b) = 1$.

- initial algorithm by Davis-Putnam-Logemann-Loveland (DPLL); various heuristics. Generalization of SAT to QBF (Quantified Boolean Formula)
- a very active/competitive research/industrial topic (see http://www.satlive.org/)
- Now, more interest for SMT (Satisfiability Modulo Theory) for first-order logic (quantified formula + interpreted/non-interpreted functions)
- Close interaction between a SAT solver and ad-hoc solvers (e.g., simplex. method for linear arithmetic constraints)

SMT: Satisfiability Modulo Theories

- SAT = Satisfiability (of Boolean formulas)
- SMT = SAT Modulo Theories

- Input: set of constraints (interpreted in a theory)
- Output: are the constraints satisfiable?
  - » `sat` and a *model* (an assignment to free variables that satisfies the constraints)
  - » `unsat`: no model exists
  - » `unknown`: could not determine due to resource limits, incompleteness, etcetera.

- Different solvers:
  - » z3 (see also: docs and version in browser)
  - » Alt-Ergo
  - » CVC5
  - » Yices
- Today we will use Z3 and SMT-LIB.

SMT-LIB 2.6

- SMT-LIB defines a common language for interfacing with SMT solvers

  [Barrett, Fontaine, and Tinelli (2021): The SMT-LIB Standard: Version 2.6] https://smtlib.cs.uiowa.edu/

- Developed to facilitate research and development in SMT (in particular, by providing an extensive benchmarking library)

- Lisp-like syntax for
  - » a many-sorted first-order logic with equality
  - » solver commands
  - » declaring theory interfaces
- Solvers like Z3 also provide programmatic interfaces (e.g., Python, OCaml)

Satisfiability

A `.smt2` file is a sequence of commands. (Fig. 3.6, p. 45 [Barrett, Fontaine, and Tinelli (2021): The SMT-LIB Standard: Version 2.6])

```
(declare-fun a () Bool)    ; uninterpreted function with zero arguments
(declare-const b Bool)     ; similar effect, easier to read

(assert (or a b))
(assert (= a false))

(echo "Is (a or b) and (a = false) satisfiable?")
(check-sat)
(get-model)
```

Copy this text into `test.smt2` and try `z3 test.smt2`...

z3 looks for a model (an interpretation of the functions) that satisfies all the constraints.

Validity

What about proving one of De Morgan's laws? $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

```
(declare-const P Bool)
(declare-const Q Bool)

(assert (= (not (or P Q)) (and (not P) (not Q))))
(check-sat)
```

z3 says `sat`. Have we proved the law?

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Now z3 says `unsat`. Have we proved the law?

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Yes. There are no values for P and Q such that the law is not true.

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```

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Yes. There are no values for P and Q such that the law is not true.

valid(P) $\Leftrightarrow$ not (satisfiable (not P))
satisfiable(P) $\Leftrightarrow$ not (valid (not P))

To determine valid($P \wedge Q \Rightarrow R$), we ask satisfiable($P \wedge Q \wedge \neg R$).

Interacting with the solver

- Typical to run several (check-sat) commands in series.
- Use (push) and (pop) to manage the environment of functions and assertions.

```
(declare-const P Bool)
(declare-const Q Bool)

(push)
(assert (not (= (not (or P Q)) (and (not P) (not Q)))))
(echo "Checking: !(P or Q) <=> !P and !Q (unsat = valid)")
(check-sat)
(pop)

(push)
(assert (not (= (not (and P Q)) (or (not P) (not Q)))))
(echo "Checking: !(P and Q) <=> !P or !Q (unsat = valid)")
(check-sat)
(pop)
```

- Usually interact with the solver using a programmatic interface. Query results determine future queries.
- Solvers are designed to work incrementally.

Functions

- Functions declared with `declare-fun` are uninterpreted.
- Functions from theories, like `xor`, are interpreted.

  see https://smtlib.cs.uiowa.edu/theories-Core.shtml

```
(declare-fun f (Bool Bool) Bool)
(assert (and (= (f false false) false)
             (= (f false true)  true)
             (= (f true  false) true)
             (= (f true  true)  false)))

(declare-const a Bool)
(declare-const b Bool)
(assert (not (= (f a b) (xor a b))))
(check-sat)
```

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```
(declare-fun f (Bool Bool) Bool)
(assert (and (= (f false false) false)
             (= (f false true)  true)
             (= (f true  false) true)
             (= (f true  true)  false)))

(declare-const a Bool)
(declare-const b Bool)
(assert (not (= (f a b) (xor a b))))
(check-sat)
```

- Can also define functions:

```
(define-fun f ((x Bool) (y Bool)) Bool (xor x y))
```

Terms and Formulas

⟨*qual_identifier*⟩ ::= ⟨*identifier*⟩ | (as ⟨*identifier*⟩ ⟨*sort*⟩)

⟨*var_binding*⟩ ::= (⟨*symbol*⟩ ⟨*term*⟩)

⟨*sorted_var*⟩ ::= (⟨*symbol*⟩ ⟨*sort*⟩)

⟨*pattern*⟩ ::= ⟨*symbol*⟩ | (⟨*symbol*⟩ ⟨*symbol*⟩$^+$)

⟨*match_case*⟩ ::= (⟨*pattern*⟩ ⟨*term*⟩)

⟨*term*⟩ ::= ⟨*spec_constant*⟩
| ⟨*qual_identifier*⟩
| (⟨*qual_identifier*⟩ ⟨*term*⟩$^+$)
| (let (⟨*var_binding*⟩$^+$) ⟨*term*⟩)
| (forall (⟨*sorted_var*⟩$^+$) ⟨*term*⟩)
| (exists (⟨*sorted_var*⟩$^+$) ⟨*term*⟩)
| (match ⟨*term*⟩ (⟨*match_case*⟩$^+$))
| (! ⟨*term*⟩ ⟨*attribute*⟩$^+$)

(p. 27, [Barrett, Fontaine, and Tinelli (2021): The SMT-LIB Standard: Version 2.6])

- Satisfiability without quantifiers is NP-Complete
- With quantifiers it is undecidable.
- The effectiveness of *quantifier elimination* depends on the shape of formulas.
- Take care with your encodings!

Exercise: model checking 1-bit adders

How to be sure that `full_add` and `full_add_h` are equivalent?

$$\forall a, b, c : \texttt{bool}.\ \texttt{full\_add}(a, b, c) = \texttt{full\_add\_h}(a, b, c)$$

Implement the following interface so that it returns `true` exactly when two full adder implementations return the same value for the same inputs.

```
-- file fulladder.lus
node equivalence(a,b,c:bool) returns (ok:bool);
  var o1, c1, o2, c2: bool;
  let
     (o1, c1) = full_add(a,b,c);
     (o2, c2) = full_add_h(a,b,c);
     ok = (o1 = o2) and (c1 = c2);
  tel;
```

Check equivalence with z3 and SMT-LIB!

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Model Checking Lustre Programs: Kind 2

Model Checking: (extremely) partial overview

1981 Explicit state enumeration

[E. M. Clarke and Emerson (1981): Design and Synthesis of Synchronization Skeletons using Branching Time Temporal Logic]

[Queille and Sifakis (1982): Specification and Verification of Concurrent Systems in CESAR]

1992 BDD-based algorithms

[Burch, E. Clarke, McMillan, Dill, and Hwang (1992): Symbolic Model Checking: 10^{20} States and Beyond]

1999 Bounded Model Checking

[Biere, Cimatti, E. Clarke, and Zhu (1999): Symbolic Model Checking without BDDs]

2000 K-induction

[Sheeran, Singh, and Stålmarck (2000): Checking Safety Properties Using Induction and a SAT-Solver]

2003 Interpolation-based

[McMillan (2003): Interpolation and SAT-based model checking]

2011 IC3 Algorithm

[Bradley (2011): SAT-Based Model Checking without Unrolling]

Model checking of Lustre

- Lesar: based on BDDs

  [Halbwachs, Lagnier, and Ratel (1992): Programming and verifying real-time systems by means of the synchronous data-flow language LUSTRE]

- Kind 2: based on SMT/k-induction/IC3

  [Champion, Mebsout, Sticksel, and Tinelli (2016): The Kind 2 Model Checker]

- DV of (Ansys) Scade based on Prover SAT/k-induction

Model checking: forward method

The set of reachable states never intersects the set of error states

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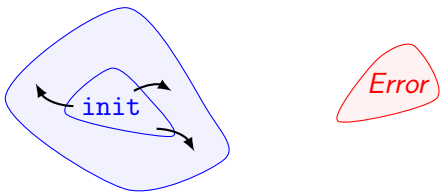

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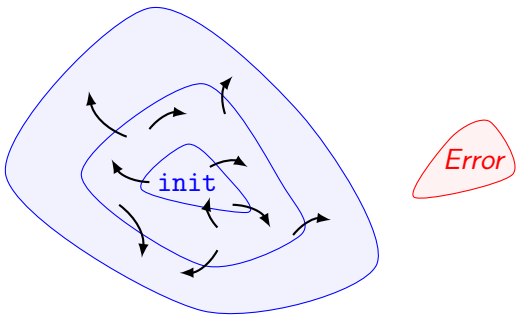

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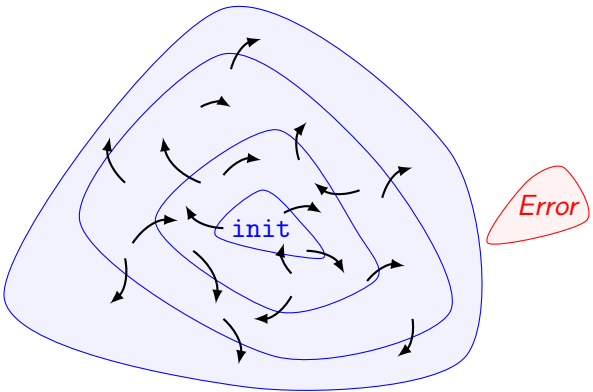

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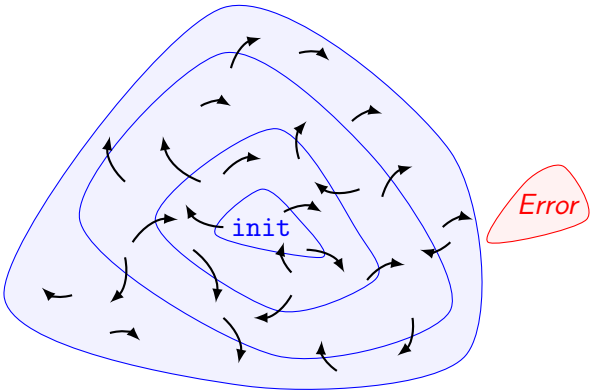

Model checking: backward method

The states that can reach an error state do not include the initial states

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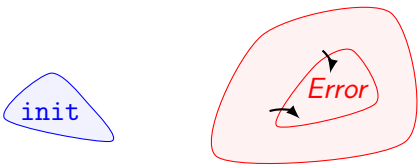

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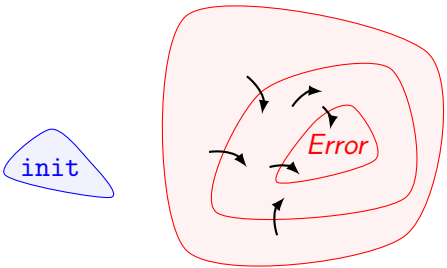

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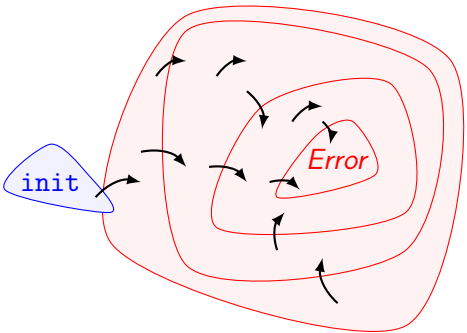
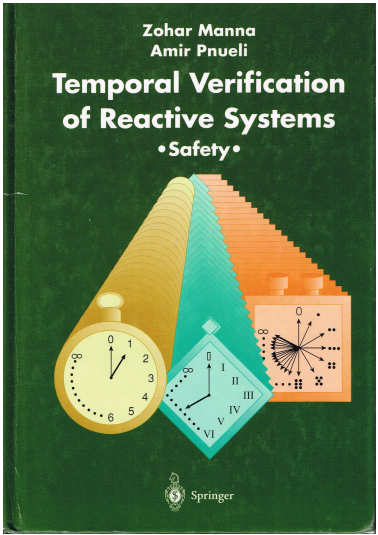


Verifying safety properties of reactive systems



- Published in 1995
  [Manna and Pnueli (1995): Temporal Verification of Reactive Systems: Safety]
- Companion to
  [Manna and Pnueli (1992): The Temporal Logic of Reactive and Concurrent Systems]
- Builds on Floyd's inductive invariants
- Temporal logic formulas as 'proof patterns'

The basic 'pattern' for showing invariance

For an assertion φ,

$$\frac{\begin{array}{ll}\text{B1.} & \Theta \rightarrow \varphi \\ \text{B2.} & \{\varphi\}\ \mathcal{T}\ \{\varphi\}\end{array}}{\Box\varphi}$$

Fig. 1.1. Rule INV-B (basic invariance).

The *verification condition* (or *proof obligation*) of φ and ψ, relative to transition τ, is given by the state formula

$$\rho_\tau \ \wedge\ \varphi \ \rightarrow\ \psi'.$$

We adopt the notation

$$\{\varphi\}\ \tau\ \{\psi\}$$

as an abbreviation for this verification condition.

The basic 'pattern' for showing invariance

For an assertion φ,

B1. $\Theta \rightarrow \varphi$

B2. $\{\varphi\}\ \mathcal{T}\ \{\varphi\}$

$\square\varphi$

Fig. 1.1. Rule INV-B (basic invariance).

show property of initial states

then for every transition:

- assume the property of the pre state (φ)
- show the property of the post state (φ')

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$$\rho_\tau \wedge \varphi \rightarrow \psi'.$$

We adopt the notation

$$\{\varphi\}\ \tau\ \{\psi\}$$

as an abbreviation for this verification condition.

Exercise: proving invariance of a simple transition system

- Consider a simple transition system with two integer state variables x and y:
  $\mathit{init}(x, y) := (x = 1) \wedge (y = 1)$
  $\mathit{trans}(x, y, x', y') := (x' = x + 1) \wedge (y' = y + x)$
- And the safety property $\mathit{prop}(x, y) = y \geq 1$.
- Encode this system and use Z3 to prove that the property is invariant.

General rule for showing invariance

For assertions φ, p,

$$\frac{\begin{array}{ll} \text{I1.} & \varphi \rightarrow p \\ \text{I2.} & \Theta \rightarrow \varphi \\ \text{I3.} & \{\varphi\}\ \mathcal{T}\ \{\varphi\} \end{array}}{\Box p}$$

Fig. 1.5. Rule INV (general invariance).

Not all *invariants* are *inductive invariants*.

Inductive invariants and model checking

- This idea works for manual/interactive proof.
- What about automatic proof (model checking)?
- (BTW, note that SMT solvers do not themselves do induction.)

- k-induction: strengthen P with information from last k steps.
  [Sheeran, Singh, and Stålmarck (2000): Checking Safety Properties Using Induction and a SAT-Solver]
- IC3: automate 'discovery' of strengthenings
  [Bradley (2011): SAT-Based Model Checking without Unrolling]

- Generic algorithms
  - » work with SAT solvers on boolean transition systems, or
  - » with SMT solvers on richer transition systems.
  - » avoid or minimize quantifiers, look for efficient encodings

Specifying Properties

SMT Solver Basics

Model Checking

Bounded Model Checking and k-induction

Model Checking Lustre Programs: Kind 2

k-induction

- Iterate BMC. Explained as a succession of algorithms.
  [Sheeran, Singh, and Stålmarck (2000): Checking Safety Properties Using Induction and a SAT-Solver]
- Focus completely on invariant properties (AG f)

k-induction

Algorithm 1 First algorithm to check if system P-safe

i=0
while True **do**
  if not Sat$(I(s_0) \wedge \mathit{loopFree}(s_{[0..i]}))$ or not Sat$((\mathit{loopFree}(s_{[0..i]}) \wedge \neg P(s_i))$ **then**
    return True
  end if
  if Sat$(I(s_0) \wedge \mathit{path}(s_{[0..i]}) \wedge \neg P(s_i))$ **then**
    return Trace $c_{[0..i]}$
  end if
  $i = i + 1$
end while

$$\mathit{path}(s_{[0..n]}) \;\hat{=}\; \bigwedge_{0 \le i < n} T(s_i, s_{i+1})$$

$$\mathit{loopFree}(s_{[0..n]}) \;\hat{=}\; \mathit{path}(s_{[0..n]}) \wedge \bigwedge_{0 \le i < j \le n} s_i \neq s_j$$

The restriction to loop-free paths is necessary for completeness.

k-induction

Algorithm 2 An improved algorithm to check if system P-safe

i=0
while True **do**
  if not Sat$(I(s_0) \wedge all.\neg I(s_{[1..i]}) \wedge loopFree(s_{[0..i]}))$
  or not Sat$((loopFree(s_{[0..i]}) \wedge all.P(s_{[0..(i-1)]}) \wedge \neg P(s_i))$ **then**
    return True
  end if
  if Sat$(I(s_0) \wedge path(s_{[0..i]}) \wedge \neg P(s_i))$ **then**
    return Trace $c_{[0..i]}$
  end if
  $i = i + 1$
end while

$$path(s_{[0..n]}) \;\hat{=}\; \bigwedge_{0 \le i < n} T(s_i, s_{i+1})$$

$$loopFree(s_{[0..n]}) \;\hat{=}\; path(s_{[0..n]}) \;\wedge \bigwedge_{0 \le i < j \le n} s_i \neq s_j$$

Tighter termination conditions.

k-induction

Algorithm 3 An algorithm that need not iterate from 0

i= some constant which can be greater than zero
while True **do**
  if Sat$(I(s_0) \wedge path(s_{[0..i]}) \wedge \neg all.P(s_{[0..i]}))$ **then**
    return Trace $c_{[0..i]}$
  end if
  if not Sat$(I(s_0) \wedge all.\neg I(s_{[1..(i+1)]}) \wedge loopFree(s_{[0..(i+1)]}))$
  or not Sat$((loopFree(s_{[0..(i+1)]}) \wedge all.P(s_{[0..i]}) \wedge \neg P(s_{i+1}))$ **then**
    return True
  end if
  $i = i + 1$
end while

$$path(s_{[0..n]}) \;\hat{=}\; \bigwedge_{0 \le i < n} T(s_i, s_{i+1})$$

$$loopFree(s_{[0..n]}) \;\hat{=}\; path(s_{[0..n]}) \wedge \bigwedge_{0 \le i < j \le n} s_i \neq s_j$$

Swap ordering of conditions.

k-induction

Algorithm 4 A forwards version of the algorithm

i= some constant which can be greater than zero
while True **do**
  if Sat$(\neg(I(s_0) \wedge path(s_{[0..i]}) \rightarrow all.P(s_{[0..i]})))$ **then**
    return Trace $c_{[0..i]}$
  end if
  if Taut$(\neg I(s_0) \leftarrow all.\neg I(s_{[1..(i+1)]}) \wedge loopFree(s_{[0..(i+1)]}))$
  or Taut$((loopFree(s_{[0..(i+1)]}) \wedge all.P(s_{[0..i]}) \rightarrow P(s_{i+1}))$ **then**
    return True
  end if
  $i = i + 1$
end while

$$path(s_{[0..n]}) \;\hat{=}\; \bigwedge_{0 \leq i < n} T(s_i, s_{i+1})$$

$$loopFree(s_{[0..n]}) \;\hat{=}\; path(s_{[0..n]}) \;\wedge \bigwedge_{0 \leq i < j \leq n} s_i \neq s_j$$

The base and transition cases of the induction become evident.

k-induction and completeness

- The algorithm is complete for finite transition systems.
- Diameter = length of the longest shortest path in transition system.

$$shortest(s_{[0..n]}) \;\hat{=}\; path(s_{[0..n]}) \;\wedge\; \neg(\bigvee_{0 \leq i < n} path_i(s_0, s_n))$$

Specifying Properties

SMT Solver Basics

Model Checking

Bounded Model Checking and k-induction

Model Checking Lustre Programs: Kind 2

Model checking Lustre programs: Kind 2

- http://kind2-mc.github.io/kind2/ (or use web interface: http://kind.cs.uiowa.edu:8080/app/)
- SMT-based Model Checker for Lustre: BMC, k-induction, IC3, ...
- Specify properties to check as comments:

  ––%PROPERTY ok;

```
> kind2 toggles.lus
kind2 v1.1.0-214-g00b3d21d

================================================================
Analyzing compare
  with First top: "compare"
             subsystems
               | concrete: toggle2, toggle1

<Success> Property ok is valid by inductive step after 0.164s.

----------------------------------------------------------------
Summary of properties:
----------------------------------------------------------------
ok: valid (at 1)
================================================================
> kind2 --enable BMC --enable IND --lus_main compare toggles.lus
```

Kind 2

- Consider integers (not machine words)
- and infinite-precision rationals (not floating-point)

- Optimize existing techniques for Lustre programs and features of modern SMT solvers.

Encoding Lustre in SMT

[Hagen and Tinelli (2008): Scaling Up the Formal Verification of Lustre Programs with SMT-based Techniques]

- Represent streams as uninterpreted functions $\mathbb{N} \to \tau$
- Examples:

x = y + z	$\forall n : \mathbb{N},\ x(n) = y(n) + z(n)$
x = y -> y + pre z	$\forall n : \mathbb{N},\ x(n) = ite(n = 0, y(0), y(n) + z(n-1))$

Encoding Lustre in SMT

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- Examples:

  x = y + z $\quad \forall n : \mathbb{N},\ x(n) = y(n) + z(n)$

  x = y -> y + pre z $\quad \forall n : \mathbb{N},\ x(n) = ite(n = 0, y(0), y(n) + z(n-1))$

- Let N be a node with stream variables $\mathrm{x} = \langle x_1, \ldots, x_p, y_1, \ldots, y_q \rangle$ ($x_1, \ldots, x_p$ are inputs, and $y_1, \ldots, y_q$ are outputs)
- $$\Delta(n) = \begin{cases} y_1(n) = t_1[\mathrm{x}(n), \mathrm{x}(n-1), \ldots, \mathrm{x}(n-d)] \\ \quad\quad\vdots \\ y_q(n) = t_q[\mathrm{x}(n), \mathrm{x}(n-1), \ldots, \mathrm{x}(n-d)] \end{cases}$$

```
node thermostat (actual_temp, target_temp, margin: real)
returns (cool, heat: bool);
let
  cool = (actual_temp − target_temp) > margin;
  heat = (actual_temp − target_temp) < −margin;
tel

node therm_control (actual: real; up, down: bool) returns (heat, cool: bool);
var target, margin: real;
let
  margin = 1.5;
  target = 70.0 −> if down then (pre target) − 1.0
                  else if up then (pre target) + 1.0
                  else pre target;
  (cool, heat) = thermostat (actual, target, margin);
tel
```

$$\Delta(n) = \begin{cases} m(n) = 1.5 \\ t(n) = ite(n = 0, 70.0, ite(d(n), t(n-1) - 1.0, \ldots)) \\ c(n) = (a(n) - t(n)) > m(n) \\ h(n) = ((a(n) - t(n)) < -m(n) \end{cases}$$

SMT-based k-induction

$$\Delta_0 \wedge \Delta_1 \wedge \cdots \wedge \Delta_k \models_{\mathcal{IL}} P_0 \wedge P_1 \wedge \cdots \wedge P_k \quad (1)$$

$$\Delta_n \wedge \Delta_{n+1} \wedge \cdots \wedge \Delta_{n+(k+1)} \wedge P_n \wedge P_{n+1} \wedge \cdots \wedge P_{n+k} \models_{\mathcal{IL}} P_{n+(k+1)} \quad (2)$$

where $k \geq 0$ and n is an uninterpreted integer constant.

Kind 2 optimizations: path compression

$C_{n,k}$ is a predicate over state variables that is satisfied iff no two configurations in a path have the same state and none of them, except possibly the first is the initial state.

$$\begin{array}{l} \Delta_{\mathsf{n}} \wedge \Delta_{\mathsf{n}+1} \wedge \cdots \wedge \Delta_{\mathsf{n}+(k+1)} \wedge \\ P_{\mathsf{n}} \wedge P_{\mathsf{n}+1} \wedge \cdots \wedge P_{\mathsf{n}+k} \wedge C_{\mathsf{n},k} \end{array} \quad \models_{\mathcal{IL}} \quad P_{\mathsf{n}+(k+1)} \qquad (2')$$

Allows the addition of a termination condition.

$$\Delta_0 \wedge \cdots \wedge \Delta_k \quad \models_{\mathcal{IL}} \quad \neg C_{0,k+1}$$

Kind 2 optimizations: abstraction

- Drop equations defining variables that are not mentioned in the property P.
  Sound: those variables are unconstrained (like inputs).
- Add them back one-by-one if checking fails.
  Take one (removed) variable appearing in counter-example and recursively add removed variables from its defining expression (work towards input variables).

Summary

- Express programs, (safety) properties, and assumptions on the environment in a single language.
- Model-checking ideal:
  - » 'push-button' verification gives ok or counter-example;
  - » no need to understand why (i.e., write invariants).
- SAT-based techniques for BMC, complete with k-induction.
- Extend SAT to SMT to handle integers and directly encode Lustre programs.
- Lots of tools for automating induction and interfacing with SMT solvers
  - » Mikino tutorial [Champion, Oliveira, and Didier (2022): Mikino: Induction for Dummies]
  - » F* [Swamy et al. (2016): Dependent Types and Multi-monadic Effects in F*], Why3 [Bobot, Filliâtre, Marché, and Paskevich (2011): Why3: Sheperd your herd of provers], Boogie [Barnett, Chang, DeLine, Jacobs, and Leino (2005): Boogie: A Modular Reusable Verifier for Object-Oriented Programs], ...
- Just the tip of the iceberg (IC3/PDR, interactive theorem provers, ...)

References I

- Barnett, M., B.-Y. E. Chang, R. DeLine, B. Jacobs, and K. R. M. Leino (Nov. 2005). "Boogie: A Modular Reusable Verifier for Object-Oriented Programs". In: vol. 4111. LNCS. Amsterdam, The Netherlands: Springer, pp. 364–387.
- Barrett, C., P. Fontaine, and C. Tinelli (2021). *The SMT-LIB Standard: Version 2.6*.
- Biere, A., A. Cimatti, E. Clarke, and Y. Zhu (Mar. 1999). "Symbolic Model Checking without BDDs". In: *5th Int. Conf. on Tools and Algorithms for the Construction and Analysis of Systems (TACAS 1999)*. Ed. by W. R. Cleaveland. Vol. 1579. LNCS. Amsterdam, The Netherlands: Springer, pp. 193–207.
- Bobot, F., J.-C. Filliâtre, C. Marché, and A. Paskevich (Aug. 2011). "Why3: Sheperd your herd of provers". In: *Boogie 2011: First Int. Workshop on Intermediate Verification Languages*. Wrocław, Poland, pp. 53–64.
- Bradley, A. R. (Jan. 2011). "SAT-Based Model Checking without Unrolling". In: *Proc. 12th Int. Conf. on on Verification, Model Checking, and Abstract Interpretation (VMCAI 2011)*. Ed. by R. Jhala and D. Schmidt. Vol. 6538. LNCS. Austin, TX, USA: Springer, pp. 70–87.

References II

- Burch, J., E. Clarke, K. McMillan, D. Dill, and J. Hwang (June 1992). "Symbolic Model Checking: 10^{20} States and Beyond". In: *Information and Computation* 98.2, pp. 142–170.
- Champion, A., A. Mebsout, C. Sticksel, and C. Tinelli (July 2016). "The Kind 2 Model Checker". In: *Proc. 28th Int. Conf. on Computer Aided Verification (CAV 2016), Part II*. Ed. by S. Chaudhuri and A. Farzan. Vol. 9780. LNCS. Toronto, Canada: Springer, pp. 510–517.
- Champion, A., S. de Oliveira, and K. Didier (June 2022). "Mikino: Induction for Dummies". In: ed. by C. Keller and T. Bourke. Saint-Médard-d'Excideuil, France, pp. 254–260.
- Clarke, E. M. and E. A. Emerson (May 1981). "Design and Synthesis of Synchronization Skeletons using Branching Time Temporal Logic". In: *Workshop on Logics of Programs*. Ed. by D. Kozen. Vol. 131. LNCS. Yorktown Heights, NY, USA: Springer, pp. 52–71.
- Hagen, G. and C. Tinelli (Nov. 2008). "Scaling Up the Formal Verification of Lustre Programs with SMT-based Techniques". In: *Proc. 8th Int. Conf. on Formal Methods in Computer-Aided Design (FMCAD 2008)*. Ed. by A. Cimatti and R. B. Jones. IEEE. Portland, OR, USA, Article 15.

References III

- Halbwachs, N., F. Lagnier, and P. Raymond (June 1993). "Synchronous observers and the verification of reactive systems". In: *Proc. 3rd Int. Conf. on Algebraic Methodology and Software Technology (AMAST'93)*. Ed. by M. Nivat, C. Rattray, T. Rus, and G. Scollo. Twente: Workshops in Computing, Springer Verlag.
- Halbwachs, N., J.-C. Fernandez, and A. Bouajjani (Apr. 1993). "An executable temporal logic to express safety properties and its connection with the language Lustre". In: *Proc. 6th Int. Symp. Lucid and Intensional Programming (ISLIP'93)*. Quebec, Canada.
- Halbwachs, N., F. Lagnier, and C. Ratel (Sept. 1992). "Programming and verifying real-time systems by means of the synchronous data-flow language LUSTRE". In: *IEEE Trans. Software Engineering* 18.9, pp. 785–793.
- Lamport, L. (2002). *Specifying Systems: The TLA+ Language and Tools for Hardware and Software Engineers*. Addison Wesley.
- Manna, Z. and A. Pnueli (1992). *The Temporal Logic of Reactive and Concurrent Systems*. Springer.
- — (1995). *Temporal Verification of Reactive Systems: Safety*. Springer.

References IV

- McMillan, K. (July 2003). "Interpolation and SAT-based model checking". In: *Proc. 15th Int. Conf. on Computer Aided Verification (CAV 2003)*. Ed. by W. A. Hunt Jr. and F. Somenzi. Vol. 2725. LNCS. Boulder, CO, USA: Springer, pp. 1–13.
- Queille, J.-P. and J. Sifakis (Apr. 1982). "Specification and Verification of Concurrent Systems in CESAR". In: *Proc. 5th Int. Symp. Programming*. Ed. by M. Dezani-Ciancaglini and U. Montanari. Vol. 137. LNCS. Turin, Italy: Springer, pp. 337–351.
- Raymond, P. (July 1996). "Recognizing regular expressions by means of dataflow networks". In: *Proc. 23rd Int. Colloq. on Automata, Languages and Programming*. Ed. by F. Meyer auf der Heide and B. Monien. LNCS 1099. Paderborn, Germany: Springer, pp. 336–347.
- Raymond, P., Y. Roux, and E. Jahier (2008). "Lutin: A Language for Specifying and Executing Reactive Scenarios". In: *EURASIP Journal of Embedded Systems*.

References V

- Sheeran, M., S. Singh, and G. Stålmarck (Nov. 2000). "Checking Safety Properties Using Induction and a SAT-Solver". In: *Proc. 3rd Int. Conf. on Formal Methods in Computer-Aided Design (FMCAD 2000)*. Ed. by W. A. Hunt Jr. and S. D. Johnson. IEEE. Austin, TX, USA, pp. 127–144.
- Swamy, N., C. Hriţcu, C. Keller, A. Rastogi, A. Delignat-Lavaud, S. Forest, K. Bhargavan, C. Fournet, P.-Y. Strub, M. Kohlweiss, J. K. Zinzindohoue, and S. Zanella Béguelin (Jan. 2016). "Dependent Types and Multi-monadic Effects in F*". In: *Proc. 43rd ACM SIGPLAN-SIGACT Symp. Principles of Programming Languages (POPL 2016)*. St. Petersburg, FL, USA: ACM Press, pp. 256–270.