

Programming Embedded Systems with Lustre

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Introduction

Lustre: Combinatorial Programs

Sequential operators (adding state)

Sampling operators (conditional activation)

“Lustre-like” languages

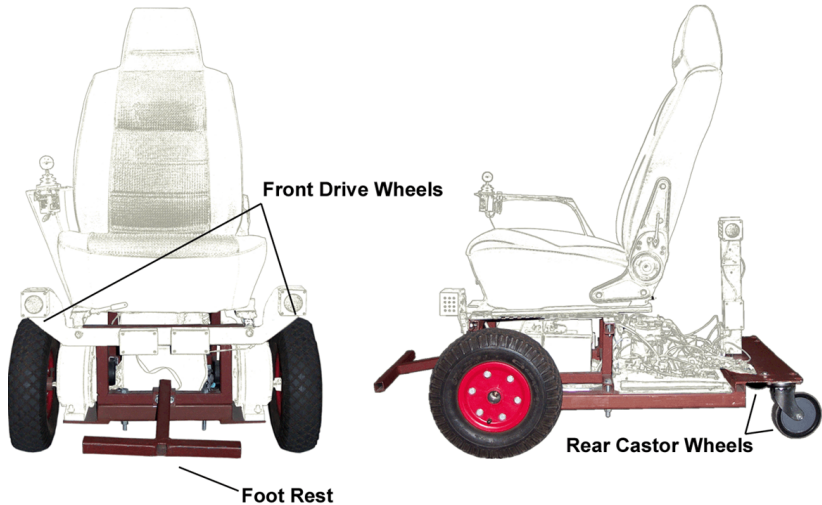
Conclusion

Wheelchair: An old, simple, but concrete example

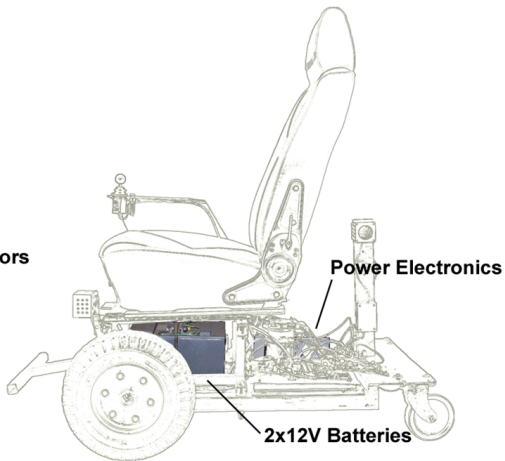
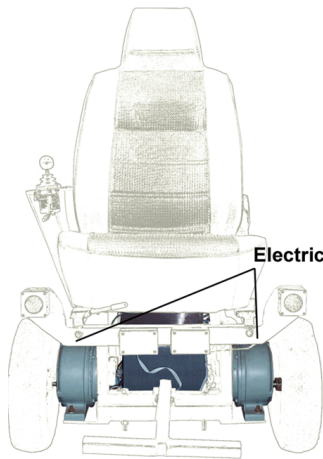
- The UOW 'robotic' wheelchair
- Goal: low-cost mobility assistance
- Target of engineering student projects



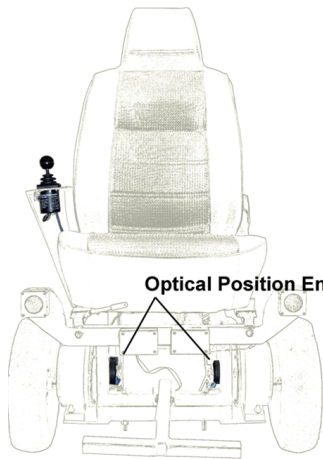
Wheelchair: mechanical structure



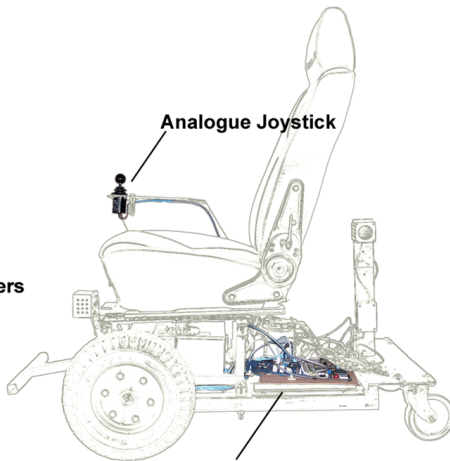
Wheelchair: power electronics



Wheelchair: drive controller



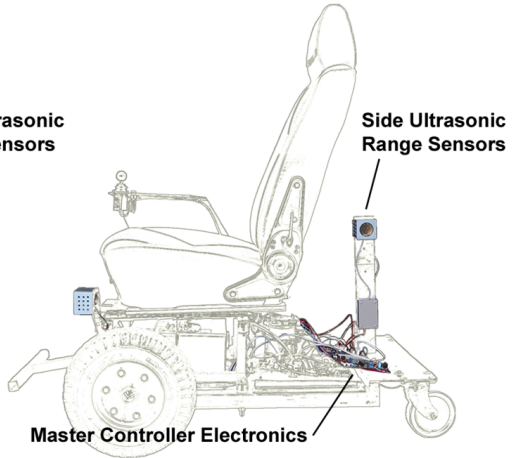
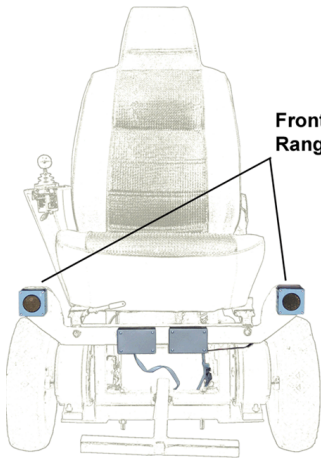
Optical Position Encoders



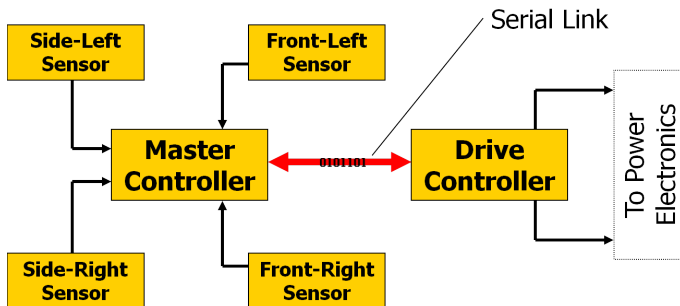
Analogue Joystick

Drive Controller Electronics

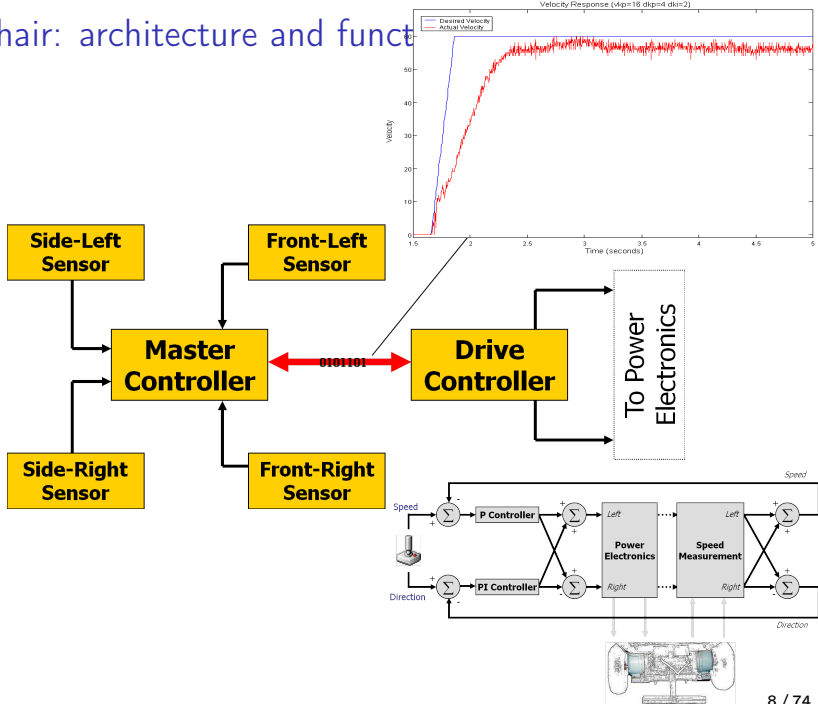
Wheelchair: master controller



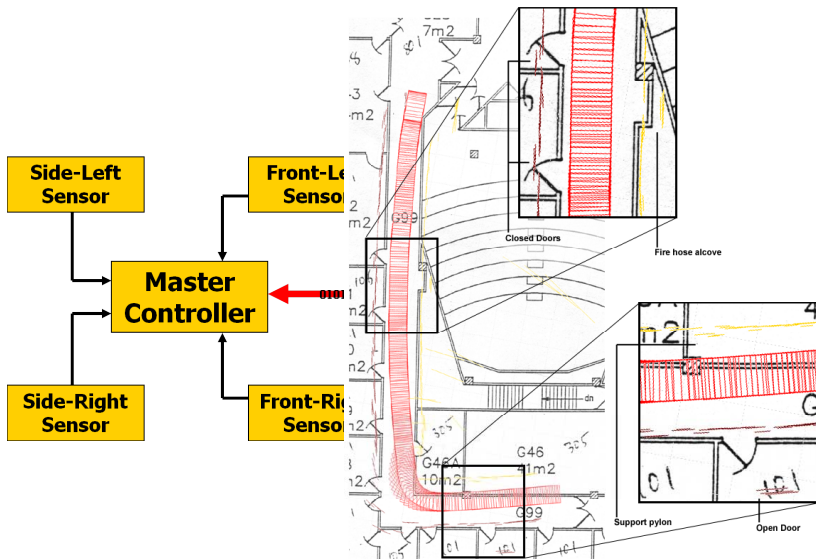
Wheelchair: architecture and functions



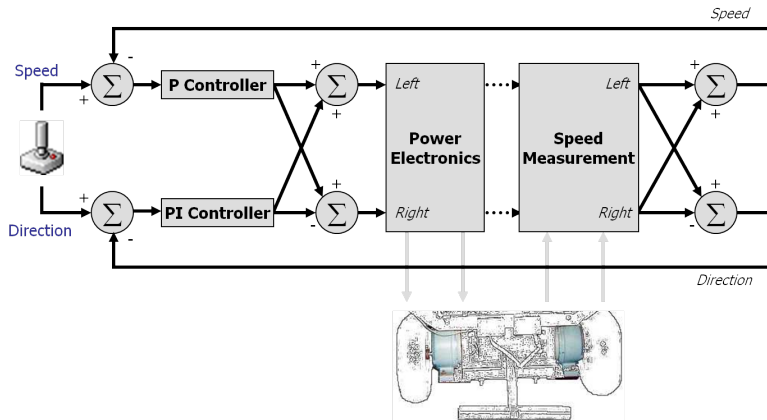
Wheelchair: architecture and function



Wheelchair: architecture and functions



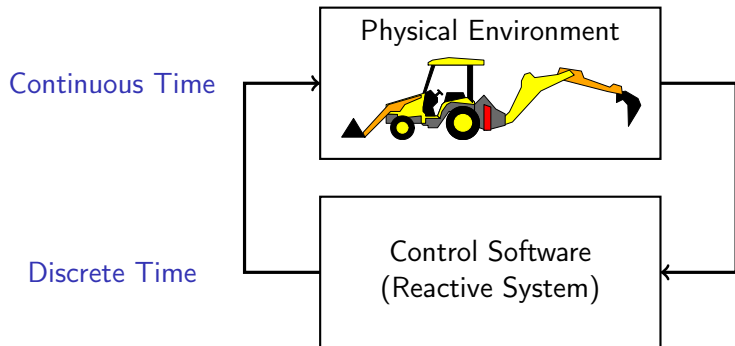
Wheelchair: drive control feedback loop



- fly-by-wire: $\approx 1,5\text{MLOC}$
- critical software
- “dynamic” = “too late”
- cyclic execution



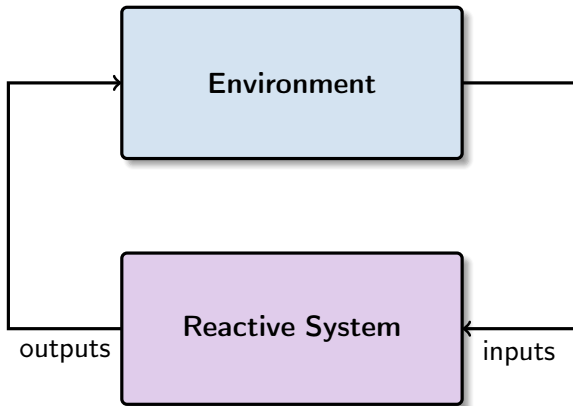
Embedded Real-Time Software



- 2005 Mars Reconnaissance Orbiter: 545 000 LoC
- 2010 Lockheed Martin F-22 Raptor: 2 500 000 LoC
- 2012 Drug infusion pump 170 000 LoC
- 2017 Primary Flight Controller A350-1000: 1 500 000 LoC generated

[Dvorak (ed.) (2009): NASA Study on Flight Software Complexity] [The Economist (2012): Open-source medical devices: When code can kill or cure]

Program and control a reactive system?



Write Assembly / C / C++ / JavaScript / Python /
OCaml / Coq code by hand?

Write Assembly / C / C++ / JavaScript / Python / OCaml / Coq code by hand?

- Concurrent Designs

Multiple OS tasks communicating via shared memory with locking to ensure atomicity?

- » Complicated implementations for complicated applications?
- » Effect of non-determinism (interleaving/preemption) on reasoning/testing?
- » What about subtle effects like deadlocks and priority inversion?

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- What about lighter-weight compositions?

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- What about lighter-weight compositions?

- What is the programmer's model of time?
- How are timing details expressed, implemented, validated?

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- Concurrent Designs

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- What about lighter-weight compositions?

- What is the programmer's model of time?

- How are timing details expressed, implemented, validated?

- What about writing reusable libraries?

- What about targeting different platforms?

Arduino demo

1. Blinking an LED
2. Blinking two LEDs

In any case, what should the implementation be compared to?

What is the specification?

Introduction

Lustre: Combinatorial Programs

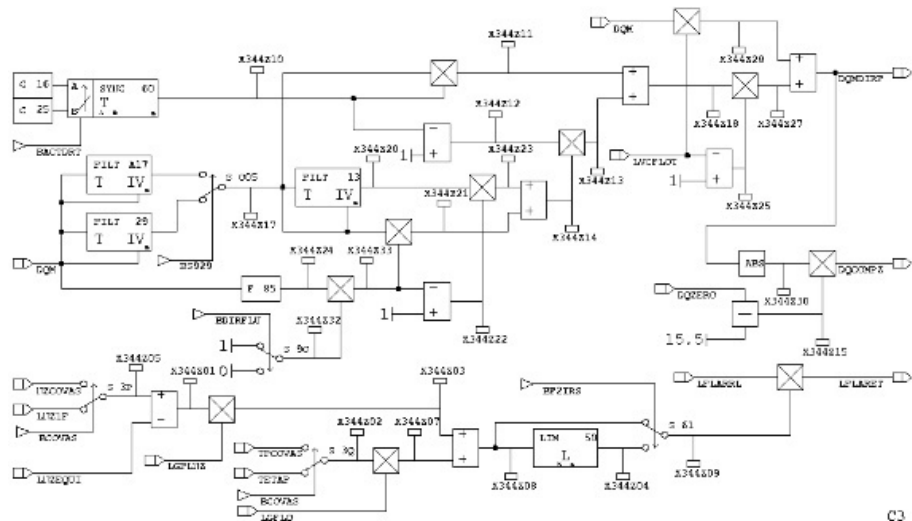
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Conclusion

SAO (Spécification Assistée par Ordinateur) — Airbus 80's



C3

They were very precise drawings...

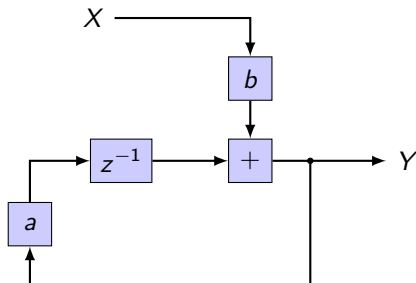
Control and signal engineers described control/command systems with very precise mathematics before computers were used.

Stream equations, z-transforms, (discrete) difference equations

Example: a linear filter

$$Y_0 = b \cdot X_0$$

$$\forall n, Y_{n+1} = a \cdot Y_n + b \cdot X_{n+1}$$



- Important idea: block diagrams can be given a precise meaning in terms of equations on stream variables.
- ...but they're not executable.
- Should we just write code and convince ourselves that it is correct?

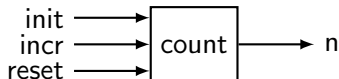
How to make those mathematics executable?

Somewhere in Grenoble. . . the language Lustre (1984)

[Caspi, Pilaud, Halbwachs, and Plaice (1987):
LUSTRE: A declarative language for program-
ming synchronous systems]

[Halbwachs, Caspi, Raymond, and Pilaud (1991):
The synchronous dataflow programming lan-
guage LUSTRE]

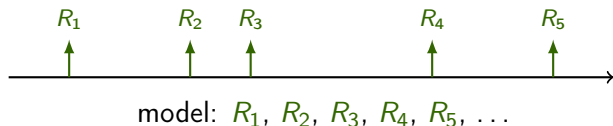
[Jahier, Raymond, and Halbwachs (2019): The Lustre V6 Reference Manual]



```
node COUNT (init, incr: int; reset: bool)
  returns (n: int);
let
  n = init ->
    if reset then init else pre(n) + incr;
tel;
```

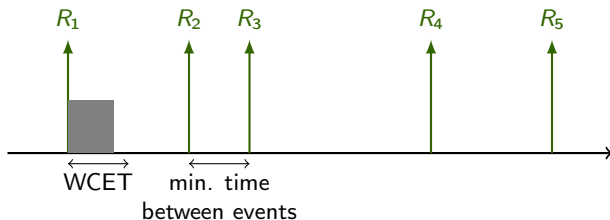
Basic model: cyclic execution

every trigger:
 read inputs;
 compute;
 write outputs



- Often periodic with sampling, but not always.
- Reactions are **atomic**: buffer new inputs during compute.
- Assume that reactions occur at least as frequently as inputs.
- Reason in **logical** time rather than **physical** time.
 - » The execution model (semantics) of a program is a sequence of reactions.
 - » It abstracts from the time between reactions
—even if physical time still matters to programmers.
- This is a significant simplification; it divides the problem in two:
 1. Specify and reason about logical behaviour;
 2. Separately validate assumptions on real-time behaviour.

Validating Timing Assumptions



- Calculate an upper-bound on execution time:
Worst Case Execution Time (WCET)
- Check that $WCET < \text{minimum time between any two events}$
- Typical case: $WCET < \text{sample period}$
- The WCET is calculated on the generated assembly code for a given processor configuration (pipeline depth, cache sizes).
- Overapproximation at function calls and branching instructions.

Program by writing stream equations

A discrete-time system: a **stream function**; streams are **synchronous**.

X	1	2	1	4	5	6	...
Y	2	4	2	1	1	2	...
1	1	1	1	1	1	1	...
$X + Y$	3	6	3	5	6	8	...
$X + 1$	2	3	2	5	6	7	...

The equation $Z = X + Y$ means $\forall n, Z_n = X_n + Y_n$.

Time is **logical**: the inputs X and Y arrive “**at the same time**”; the output Z is produced “**at the same time**”.

For simple expressions, we effectively just added a loop around a sequence of assignment statements.

It gets more interesting when we add **abstraction** and **state**.

The idea will be to **compose** stream functions to build more and more sophisticated systems.

Syntax of Lustre nodes

1. Declare a block (a “node”) by giving its **interface**:
name, list of input variables, list of output variables

```
node full_add(a : bool; b : bool; c : bool) returns (s : bool; co : bool);
```

or

```
node full_add(a, b, c : bool) returns (s, co : bool);
```

There are three basic types: **bool**, **int**, **real/float**.

2. The interface is optionally followed by a list of **local variables**.

```
var t1, t2 : bool; t3 : int;
```

3. The relation between inputs and outputs is defined by a list of equations. Each output and local variable must be defined exactly once.

```
let
```

```
  s = e1;
```

```
  co = e2;
```

```
  (t1, t2, t3) = e3;
```

```
tel;
```

The order of the equations is not significant.

Combinatorial expressions

- Boolean operators

not, and, or, xor

- Comparisons

<, <=, =, <>, >=, >

- Arithmetic

+ - * / div mod

+. -. *. /.

- Conversions

int, float

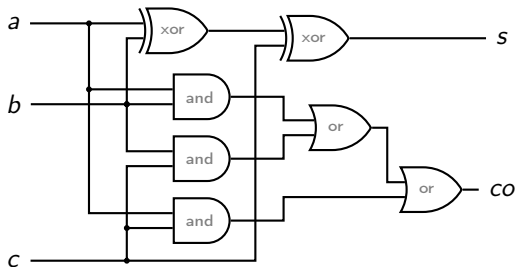
- Multiplexing

if e1 then e2 else e3

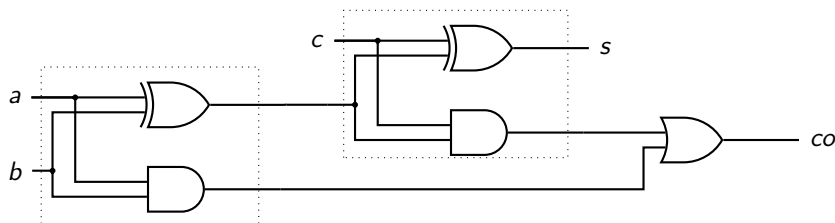
Exercise: programming 1-bit adders

Full adder (full_add)

<i>a</i>	<i>b</i>	<i>c</i>	<i>s</i>	<i>co</i>
0	0	0	0	0
0	1	0	1	0
1	0	0	1	0
1	1	0	0	1
0	0	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1



Composition of two half adders (full_add_h)



Simulate with `make runtrace_full_add` using the provided Makefile

Exercise: model checking 1-bit adders

How to be sure that `full_add` and `full_add_h` are equivalent?

$$\forall a, b, c : \text{bool}. \text{full_add}(a, b, c) = \text{full_add_h}(a, b, c)$$

Implement the following interface so that it returns `true` exactly when two full adder implementations return the same value for the same inputs.

— file `fulladder.lus`

`node` `equivalence(a, b, c : bool)` `returns` (`ok : bool`);

Use the model-checking tool `lesar` to check for all possible inputs:

```
% lesar fulladder.lus equivalence -diag  
--Pollux Version 2.2
```

TRUE PROPERTY

Type `make check` to run `lesar`. Try introducing a bug and rechecking.

Exercise: more combinatorial nodes

Return the maximum at each instant

```
node max(i1, i2 : int) returns (o : int);
```

Return the absolute value at each instant

```
node abs(i : int) returns (o : int);
```

Saturate a signal when it exceeds bounds

```
node saturate(lb, i, ub : int) returns (o : int);
```

```
let
```

```
  -- assert (lb <= ub);
```

```
  ...
```

```
tel;
```

- Build up a library of stream functions to write more abstract programs.
- Ideal: **specify** a system in Lustre, rely on the compiler to produce efficient low-level code.

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The Unit Delay (pre) and Initialization Operator (->)

An operator for referring to the “previous” value of a stream.

(A **register** in a digital circuit; the z^{-1} operator in digital signal processing.)

X	1	2	1	4	5	6	...
$\text{pre } X$	<i>nil</i>	1	2	1	4	5	...
Y	2	4	2	1	1	2	...
$Y \rightarrow \text{pre } X$	2	1	2	1	4	5	...
S	1	3	4	8	13	19	...

nil represents an arbitrary value. It is *not* represented explicitly at runtime. The actual value is whatever happens to be in memory.

Calculate the running sum of values on X ,
i.e., the stream $(S_n)_{n \in \mathbb{N}}$ with $S_0 = X_0$ and for $n > 0$, $S_n = S_{n-1} + X_n$.

$$S = X \rightarrow (\text{pre } S) + X$$

Introducing intermediate equations does not change the meaning:

$$S = X \rightarrow I; I = \text{pre } S + X$$

Programming with $\rightarrow$ and pre

Rising edge detection

```
node redge (b : bool) returns (edge : bool);  
let  
  edge = false  $\rightarrow$  (b and not (pre b));  
tel
```

Modulo counter

```
node mod_count (m : int) returns (out : int);  
let  
  out = (0  $\rightarrow$  (pre out + 1)) mod m;  
tel
```

Count of trues and falses?

```
node tf_count (in : bool) returns (out : int);  
let  
  out = if in then mod_count(512) else mod_count(512);  
tel;
```

What is wrong with this program?

Exercise: programming with `->` and `pre`

Return true if an input has always been true

```
node always(i : bool) returns (o : bool);
```

Count true values

```
node count_true(i : bool) returns (o : int);
```

Count successive true values (i.e., false = restart from zero)

```
node count_succ_true(i : bool) returns (o : int);
```

Tracking minimum and maximum bounds of a stream

```
node bounds(i : int) returns (min, max : int);
```

The Initialized Unit Delay (fby)

- Separating delay (pre) and initialization (->) can be useful.
- But, an extra analysis is necessary to check that pre is used correctly, i.e., that outputs never depend on an undefined (nil) value.

[Colaço and Pouzet (2004): Type-based initialization
analysis of a synchronous dataflow language]

- The fby (*followed by*) operator combines delay and initialization.
$$X \text{ fby } Y \quad \text{defines the same stream as} \quad X \rightarrow (\text{pre } Y)$$
- A fby with a constant at left is easy to compile into efficient code since the generated state variable can be initialized directly.
- $e_0 \rightarrow e_1$ and $\text{if (true fby false) then } e_0 \text{ else } e_1$ define the same stream.
- NB: Lustre v4 does not have fby.

Exercise: simple filtering with fbys in Heptagon

- Consider the input signal:

$$x(n) = \begin{cases} (1.02)^n + 0.5 \cos(2\pi n/8 + \pi/4) & \text{if } 0 \leq n \leq 40 \\ 0 & \text{otherwise} \end{cases}$$

- We want to recover the exponential component by eliminating the sinusoidal component (the noise).
- Filter with a 3-point running average:

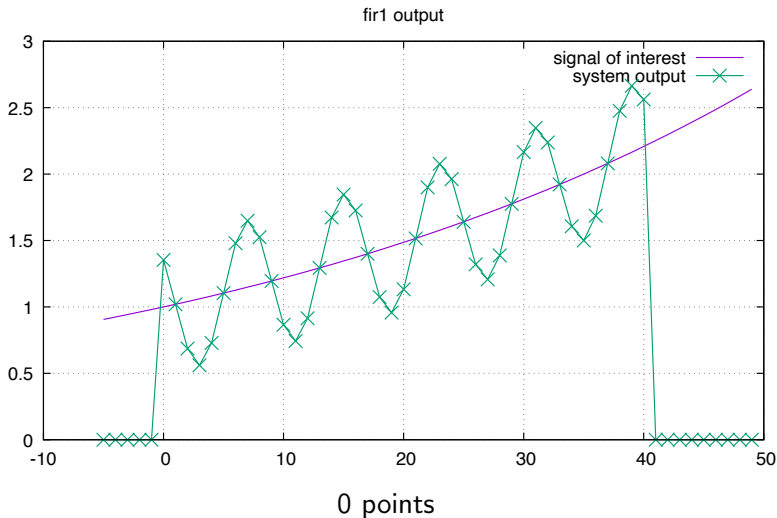
$$y(n) = \frac{1}{3} \left(\sum_{k=0}^2 x(n-k) \right)$$

- ... and a 7-point running average:

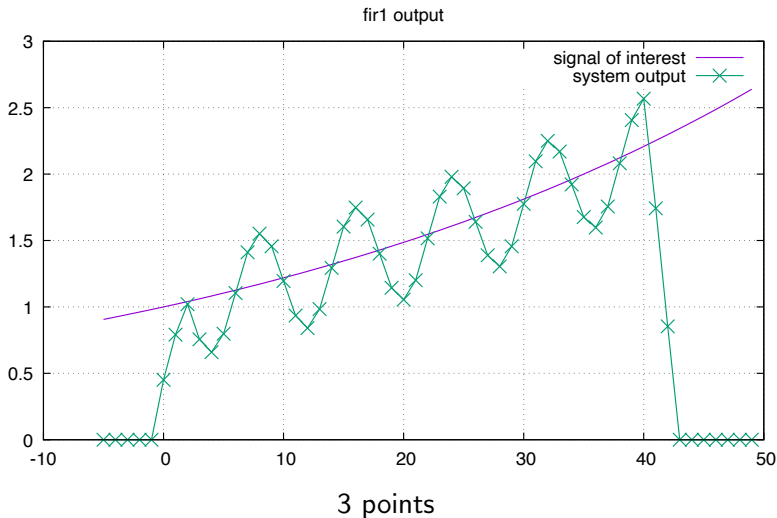
$$y(n) = \frac{1}{7} \left(\sum_{k=0}^6 x(n-k) \right)$$

- Both are basic FIR (Finite Impulse Response) filters.
- NB: use `+`, `.` and `/` on values of type `float`.

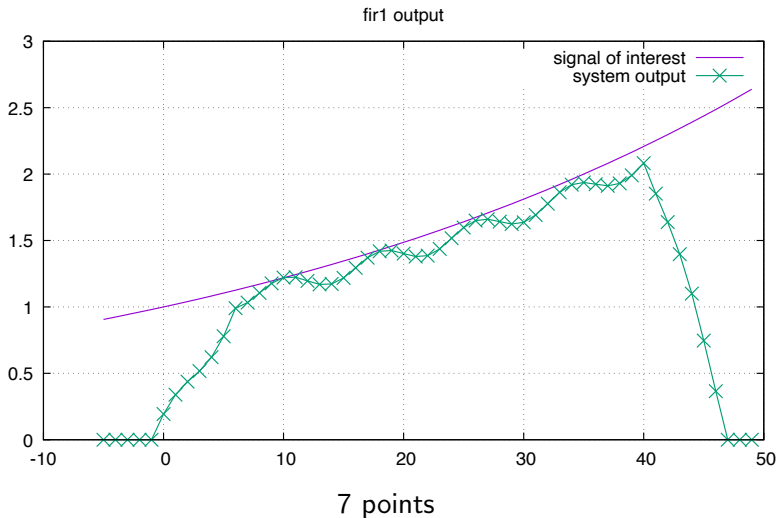
Exercise: simple filtering with `fbys` in Heptagon



Exercise: simple filtering with `fir1` in Heptagon



Exercise: simple filtering with `fbys` in Heptagon



Cyclic Definitions

A program may have

- **no solutions:** $x = x + 1$ or $x = y + 1$ and $y = x + 2$
- **many solutions:** $x = x$ or $y = x$ and $x = y$
(nondeterminism)

Nondeterminism may be useful for abstract specifications and it induces a rich theory, i.e., different notions of equivalence in process algebra.

But, for critical embedded systems, we must guarantee at compile time that a program has exactly one solution, i.e., trace equivalence suffices.

The programmer chooses what happens, not a compiler or scheduler.

Trace equivalence simplifies testing, reasoning, and reproducibility.

Causality

The term **causality** refers to the relations between “events” in a program.

It's a rich subject, but for basic Lustre programs it suffices to

- Generate a dependency graph and
- Check, by topological sort, that it contains no cycles.

No dependencies are recorded for `pre` or the right-hand-side of a `fb`.

In Lustre, every dependency cycle must be broken by one or more delays.

Dependency analysis

```
node regulator (cruise_speed, speed : float; rst : bool)
returns (throttle : float);
var delta, aux : float;
let
  delta = cruise_speed - speed;
  throttle = delta * 10.0 + aux * 0.2;
  aux = if rst then delta else delta + aux;
tel
```

Dependency analysis

```
node regulator (cruise_speed, speed : float; rst : bool) ⇐  
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var delta, aux : float;  
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  delta = cruise_speed - speed;  
  throttle = delta * 10.0 + aux * 0.2;  
  aux = if rst then delta else delta + aux;  
tel  
  
cruise_speed      speed      rst
```

Dependency analysis

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tel
```

cruise_speed

speed

rst

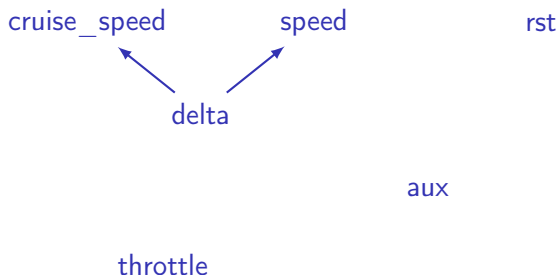
delta

aux

throttle

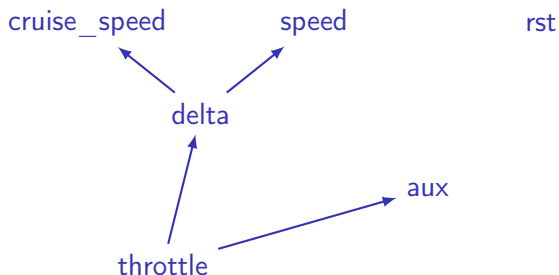
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tel
```



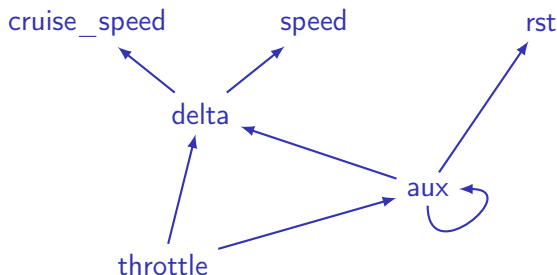
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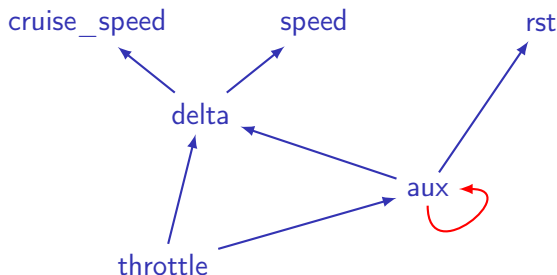
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tel
```



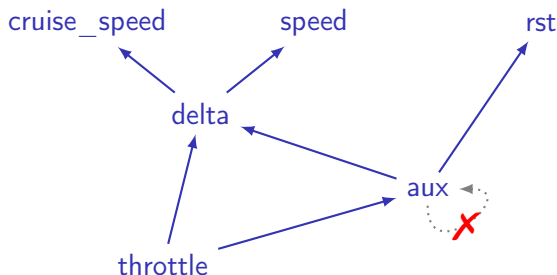
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tel
```



Dependency analysis

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node regulator (cruise_speed, speed : float; rst : bool)
returns (throttle : float);
var delta, aux : float;
let
  delta = cruise_speed - speed;
  throttle = delta * 10.0 + aux * 0.2;
  aux = if rst then delta else delta + (0.0 -> pre aux);
tel
```



Modular causality analysis

Accept this program?

```
node direct(x : int)
returns (y : int);
let
  y = x;
tel
```

```
node main1(w : int)
returns (z : int);
let
  z = direct(z);
tel
```

Modular causality analysis

Accept this program?

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node direct(x : int)
returns (y : int);
let
  y = x;
tel
```

```
node main1(w : int)
returns (z : int);
let
  z = direct(z);
tel
```

What about this one?

```
node delayed(x : int)
returns (y : int);
let
  y = 0 fby x;
tel
```

```
node main2(w : int)
returns (z : int);
let
  z = delayed(z);
tel
```

Modular causality analysis

Accept this program?

```
node direct(x : int)
returns (y : int);
let
  y = x;
tel
```

```
node main1(w : int)
returns (z : int);
let
  z = direct(z);
tel
```

What about this one?

```
node delayed(x : int)
returns (y : int);
let
  y = 0 fby x;
tel
```

```
node main2(w : int)
returns (z : int);
let
  z = delayed(z);
tel
```

The causality of a program can be analysed statically and modularly using a dedicated **type system**.

Modular causality analysis and compilation

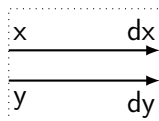
Should this program be rejected?

```
node plumbing(x, y : int)  node main3(w : int)
returns (dx, dy : int);    returns (z : int);
let                        var v : int;
  dx = x;                  let
  dy = y;                  z, v = plumbing(v, w);
tel                        tel
```

Modular causality analysis and compilation

Should this program be rejected?

```
node plumbing(x, y : int)  node main3(w : int)
returns (dx, dy : int);   returns (z : int);
let                       var v : int;
  dx = x;                 let
  dy = y;                 z, v = plumbing(v, w);
tel                       tel
```

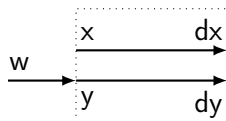


Modular causality analysis and compilation

Should this program be rejected?

```
node plumbing(x, y : int)
returns (dx, dy : int);
let
  dx = x;
  dy = y;
tel
```

```
node main3(w : int)
returns (z : int);
var v : int;
let
  z, v = plumbing(v, w);
tel
```



Modular causality analysis and compilation

Should this program be rejected?

```
node plumbing(x, y : int)
```

```
returns (dx, dy : int);
```

```
let
```

```
  dx = x;
```

```
  dy = y;
```

```
tel
```

```
node main3(w : int)
```

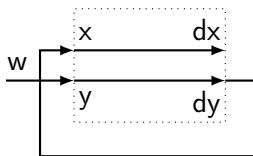
```
returns (z : int);
```

```
var v : int;
```

```
let
```

```
  z, v = plumbing(v, w);
```

```
tel
```



Modular causality analysis and compilation

Should this program be rejected?

```
node plumbing(x, y : int)
```

```
returns (dx, dy : int);
```

```
let
```

```
  dx = x;
```

```
  dy = y;
```

```
tel
```

```
node main3(w : int)
```

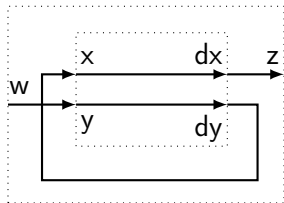
```
returns (z : int);
```

```
var v : int;
```

```
let
```

```
  z, v = plumbing(v, w);
```

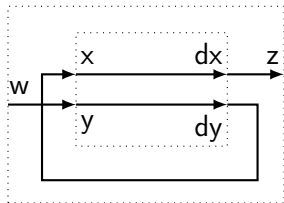
```
tel
```



Modular causality analysis and compilation

Should this program be rejected?

```
node plumbing(x, y : int)  node main3(w : int)
returns (dx, dy : int);   returns (z : int);
let                        var v : int;
  dx = x;                  let
  dy = y;                  z, v = plumbing(v, w);
tel                        tel
```



It depends on the compilation schema.

- Inlining and (minimize) automaton generation (Lustre v4)

[Plaice (1988): Sémantique et compilation de
LUSTRE, un langage déclaratif synchrone]

[Raymond (1991): Compilation efficace d'un lan-
gage déclaratif synchrone: le générateur de code
Lustre-V3]

- Modular compilation with a single step function (Scade 6/Heptagon)

[Biernacki, Colaço, Hamon, and Pouzet (2008): Clock-directed modular code
generation for synchronous data-flow languages]

- Modular compilation with multiple step functions

[Pouzet and Raymond (2009): Modular Static
Scheduling of Synchronous Data-flow Networks:
An efficient symbolic representation]

[Lublinerman, Szegedy, and Tripakis (2009):
Modular Code Generation from Synchronous
Block Diagrams: Modularity vs. Code Size]

Transition systems and formal specification

Transition systems are the basic formal model for reactive systems:
 $(S, \rightarrow)$, where $\rightarrow \subseteq S \times S$.

Mealy machines: distinguish inputs and (instantaneous) outputs:
 (S, S_0, I, O, T, G) , where $T : S \times I \rightarrow S$ and $G : S \times I \rightarrow O$.

Many specification formalisms (e.g., I/O Automata and TLA+) essentially provide expressive languages for writing Mealy Machines.

Lustre can be viewed in a similar way...

```
automaton IncDec()  
signature  
    external increment,  
        decrement
```

```
states  
    x : Integer := 5
```

```
transitions  
    external increment  
    pre  
        x < 10  
    eff  
        x := x + 1
```

```
    external decrement  
    pre  
        x > 0  
    eff  
        x := x - 1
```

Sometimes also write

```
x' = x + 1  
next x = x + 1
```

```

automaton IncDec()
signature
    external increment,
        decrement

```

```

states
    x : Integer := 5

```

```

transitions
    external increment
    pre
        x < 10
    eff
        x := x + 1

    external decrement
    pre
        x > 0
    eff
        x := x - 1

```

Sometimes also write

```

x' = x + 1
next x = x + 1

```

```

node IncDec(increment, decrement : bool)
returns (nx : int);
let
    x = 5 -> pre nx;
    nx = if increment and x < 10 then x + 1
        else if decrement and x > 0 then x - 1
        else x;
tel;

```

Assuming that `increment` and `decrement` are never true at the same time.

```

node f(i0, ..., in) returns (o0, ..., om);
var s0, ..., sk, n0, ..., nk;
let
    s0, ..., sk = f_init(i0, ..., in) -> pre (n0, ..., nk);
    n0, ..., nk = f_trans(s0, ..., sk, i0, ..., in);
    o0, ..., om = f_out(s0, ..., sk, i0, ..., in);
tel

```

```
automaton IncDec()  
signature  
  external increment,  
    decrement
```

```
states  
  x : Integer := 5
```

```
transitions  
  external increment  
  pre  
    x < 10  
  eff  
    x := x + 1
```

```
  external decrement  
  pre  
    x > 0  
  eff  
    x := x - 1
```

Sometimes also write

```
x' = x + 1  
next x = x + 1
```

```
node IncDec(increment, decrement : bool)  
returns (nx : int);  
let  
  x = 5 -> pre nx;  
  nx = if increment and x < 10 then x + 1  
       else if decrement and x > 0 then x - 1  
       else x;  
tel;
```

Assuming that `increment` and `decrement` are never true at the same time.

Lustre and other synchronous languages

- Deterministic
- Synchronous (versus asynchronous interleaving)
- Programming Languages (compile and execute)
- Composition: functions on streams
- Alternative view: iterated transition functions

Composing stream functions

Basic Lustre: a very simple language.

The idea is to build up hierarchies of functions, combining them to specify increasingly sophisticated behaviours.

With `pre/fby`, we can specify functions whose output is determined solely by the “history” of inputs received.

These **subsystems** (“blocks”) can be connected together to communicate over time. A **signal** between two subsystems is represented as a (infinite) sequence, a “stream”, of values.

Think λ -calculus but for specifying cyclic behaviours in finite memory.

Exercise: LED blinking in Lustre

Reimplement the Arduino example in Lustre.

Blink `led1` every six cycles, and `led2` every 3 cycles.

```
node main() returns (led1, led2 : bool);
```

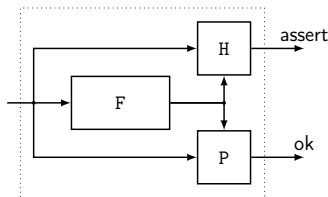
(Suggestion: break the problem down into at least three distinct nodes.)

Synchronous Observers

The comparison of programs is a particular case of a synchronous observer.

- if $y = F(x)$, we write $ok = P(x, y)$ for the property relating x and y
- and $\text{assert}(H(x, y))$ to states an hypothesis on the environment.

```
node check(x:t) returns (ok:bool);  
let  
  assert H(x,y);  
  y = F(x);  
  ok = P(x,y);  
tel;
```



If *assert* remains indefinitely true then *ok* remains indefinitely true
 $\text{always}(\text{assert}) \Rightarrow \text{always}(ok)$.

Any temporal safety property can be expressed as a Lustre program. No need to introduce a temporal logic in the language

[Halbwachs, Lagnier, and Raymond (1993):
Synchronous observers and the verification of
reactive systems]

[Halbwachs, Lagnier, and Ratel (1992): Programming
and verifying real-time systems by means of the syn-
chronous data-flow language LUSTRE];

Temporal properties are regular Lustre programs

Example of Temporal Properties

- “A is never true twice in a row”: `never_twice(A)` where:

```
node never_twice(A : bool) returns (OK : bool);
```

```
let
```

```
    OK = true  $\rightarrow$  not(A and pre A);
```

```
tel;
```

- “Any event A is followed by an event B before C happens”:

```
followed_by(A, B) and followed_by(B, C)
```

```
where:
```

```
node implies(A, B : bool) returns (OK : bool);
```

```
let
```

```
    OK = not(A) or B;
```

```
tel;
```

```
node once(A : bool) returns (OK : bool);
```

```
let
```

```
    OK = A  $\rightarrow$  A or pre OK;
```

```
tel;
```

```
node followed_by(A, B : bool)
```

```
returns (OK : bool);
```

```
let
```

```
    OK = implies(B, once(A));
```

```
tel;
```

Example of Temporal Properties (cont.)

Note: Several properties have a sequential nature, e.g., “The temperature should increase for at most 1 min or until the event stop occurs then it must decrease for 2 min”.

They can be expressed as **regular expressions** and then translated into

Lustre [Raymond (1996): Recognizing regular expressions by
means of dataflow networks]

This is the basis of the language Lutin

[Raymond, Roux, and Jahier (2008): Lutin: A Language
for Specifying and Executing Reactive Scenarios]

For an encoding of past-time Linear Temporal Logic (LTL) see:

[Halbwachs, Fernandez, and Bouajjani (1993): An executable temporal logic to express safety properties and its connection with the language Lustre]

Exercise: implementing temporal properties

1. Returns false until A occurs, then returns true from the subsequent instant onward

```
node after(a : bool) returns (o : bool);  
-- make clean; make MAIN=after TRACE=trace1.txt
```

2. Returns true if and only if its first input has been continuously true since the last time its second input was true

```
node always_since(b, a : bool) returns (o : bool);  
-- make clean; make MAIN=always_since TRACE=trace2.txt
```

3. Returns true if and only if its first input has been true at least once since the last time its second input was true.

```
node once_since(c, a : bool) returns (o : bool);  
-- make clean; make MAIN=once_since TRACE=trace3.txt
```

4. Any time A has occurred in the past, either B has been continuously true, or C has occurred at least once, since the last occurrence of A

```
node always_from_to(b, a, c : bool) returns (x : bool);  
-- make clean; make MAIN=always_from_to TRACE=trace4.txt
```

Introduction

Lustre: Combinatorial Programs

Sequential operators (adding state)

Sampling operators (conditional activation)

“Lustre-like” languages

Conclusion

Mixing slow and fast processes

A slow process is made by undersampling with **when**;

A fast process is made by oversampling with **current** or **merge**.

	b	<i>false</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
x		x ₀	x ₁	x ₂	x ₃	x ₄	x ₅
y		y ₀	y ₁	y ₂	y ₃	y ₄	y ₅
z = x when b			x ₁		x ₃		
k = y when not b		y ₀		y ₂		y ₄	y ₅
t = current z		<i>nil</i>	x ₁	x ₁	x ₃	x ₃	x ₃
o = merge b z k		y ₀	x ₁	y ₂	x ₃	y ₄	y ₅

Note that we leave “holes” in the execution trace: streams remain synchronized even when sampled.

The value of a stream is said to be **present** whenever it has a value at an instant and **absent** otherwise.

Absence is *not* represented explicitly at runtime. Instead, a special type system guarantees that an absent value is never read. A program cannot react to the absence of a value alone.

Clocks: simple example

```
node rtotal(x : int) returns (y : int);
```

```
let
```

```
  y = (0 -> pre y) + x;
```

```
tel;
```

```
node count_true(x : bool) returns (n : int);
```

```
var count : int when x;
```

```
// <- need to specify variable "clock"
```

```
let
```

```
  count = rtotal(1 when x);
```

```
  n = current count;
```

```
tel;
```

Clocks: simple example

```
node rtotal(x : int) returns (y : int);
```

```
let
```

```
  y = (0 -> pre y) + x;
```

```
tel;
```

```
node count_true(x : bool) returns (n : int);
```

```
var count : int when x; // ← need to specify variable "clock"
```

```
let
```

```
  count = rtotal(1 when x);
```

```
  n = current count;
```

```
tel;
```

Sampling inputs is not the same as sampling outputs:

```
node bad_count_true(x : bool) returns (n : int);
```

```
var count : int when x;
```

```
let
```

```
  count = rtotal(1) when x;
```

```
  n = current count;
```

```
tel;
```

Sampling inputs versus sampling outputs

x	x ₀	x ₁	x ₂	x ₃	x ₄	x ₅	...
x when c		x ₁		x ₃		x ₅	...
pre x	<i>nil</i>	x ₀	x ₁	x ₂	x ₃	x ₄	...
pre (x when c)		<i>nil</i>		x ₁		x ₃	...
(pre x) when c		x ₀		x ₂		x ₄	...

The problem with current

The current operator is convenient. In implementation terms, it simply represents a buffer that is written less frequently than it is read.

But, its value may not be determined for several cycles, if ever.

	b	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
x		x_0	x_1	x_2	x_3	x_4	x_5
y		y_0	y_1	y_2	y_3	y_4	y_5
z = x when b					x_3		
t = current z		<i>nil</i>	<i>nil</i>	<i>nil</i>	x_3	x_3	x_3

Unlike for $\rightarrow$, the initialization instant may depend on inputs or arbitrary expressions. A useful static analysis, e.g., type system, is impossible.

The problem with current

The `current` operator is convenient. In implementation terms, it simply represents a buffer that is written less frequently than it is read.

But, its value may not be determined for several cycles, if ever.

	b	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
x		x ₀	x ₁	x ₂	x ₃	x ₄	x ₅
y		y ₀	y ₁	y ₂	y ₃	y ₄	y ₅
z = x when b					x ₃		
t = current z		<i>nil</i>	<i>nil</i>	<i>nil</i>	x ₃	x ₃	x ₃

Unlike for $\rightarrow$, the initialization instant may depend on inputs or arbitrary expressions. A useful static analysis, e.g., type system, is impossible.

What about a runtime error?

- Less efficient: need state to track whether initialization has occurred and extra branching to test the (transitive) use of `current` values.
- Not ideal for embedded systems; better to detect as many errors as possible before a program goes into production.

Safer oversampling with `merge`

The `merge` operator requires the specification of value(s) for the complementary sampling condition.

Syntax in Heptagon: `merge c (true  $\rightarrow$  et) (false  $\rightarrow$  ef)`

Alternative: `merge c et ef` (Syntax in Scade: `merge(c; et; ef)`)

1. Use a default (often constant) value

```
merge x (rtotal(1 when x)) 0
```

2. Program a well-initialized current

```
node current(vi : int; ck : bool; v : int :: ck) returns (b : int);  
let  
  b = merge ck v ((vi fby b) when not ck);  
tel
```

(Sadly, not polymorphic.)

Clocks: simple example (again)

```
node current(vi : int; ck : bool; v : int :: ck) returns (b : int);
```

```
let  
  b = merge ck v ((vi fby b) when not ck);  
tel
```

```
node rtotal(x : int) returns (y : int);
```

```
let  
  y = (0 -> pre y) + x;  
tel;
```

```
node count_true(x : bool) returns (n : int);
```

```
var count : int when x;  
let  
  count = rtotal(1 when x);  
  n = current(0, x, count);  
tel;
```

- No risk of using an arbitrary value.
- Less convenient for the programmer: explicit initial value and clock.
- Initial conditions (and state variables) are often duplicated.

Alternatives to when and merge

Modern languages, like SCADE 6, provide polymorphic constructs that hide clocks, sampling, and buffering from the programmer.

1. Use a default value

$o = (\text{activate } f \text{ every } ec \text{ default } ed)(e1, \dots, en)$

means

$h = ec;$

$o = \text{merge } h \ (f((e1, \dots, en) \text{ when } h)) \ (ed \text{ when not } h)$

2. Use an initial value or the previous value

$o = (\text{activate } f \text{ every } ec \text{ initial default } ed)(e1, \dots, en)$

means

$h = ec;$

$o = \text{merge } h \ (f((e1, \dots, en) \text{ when } h)) \ ((ed \rightarrow \text{pre } o) \text{ when not } h)$

Exercise: Correct count of trues and falses

```
node tf_count (in : bool) returns (out : int);  
let  
  out = if in then mod_count(512) else mod_count(512); (* wrong *)  
tel;
```

The Gilbreath trick

The Gilbreath shuffle (from Wikipedia):

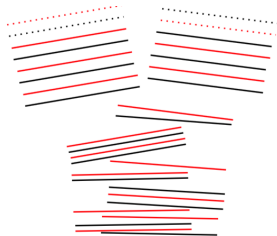
1. Deal off any number of the cards from the top of a deck onto a new pile.
2. Riffle the new pile with the remainder of the deck.

A trick based on the resulting Gilbreath permutations was formalized and verified in Coq by G. Huet. [Huet (1991): The Gilbreath trick: a case study in axiomatisation and proof development in the Coq proof assistant]

Presentation of the magic trick in G.Huet paper:

Why is this a card trick? Our boolean words are card decks, with true for red and false for black. Take an even deck x , arranged alternatively red, black, red, black, etc. Ask a spectator to cut the deck, into sub-decks u and v . Now shuffle u and v into a new deck w . When shuffling, note carefully whether u and v start with opposite colors or not. If they do, the resulting deck is composed of pairs red-black or black-red; otherwise, you get the property by first rotating the deck by one card. The trick is usually played by putting the deck behind your back after the shuffle, to perform “magic”. The magic is either rotating or doing nothing. When showing the pairing property, say loudly “red black red black...” in order to confuse in the spectator’s mind the weak *paired* property with the strong *alternate* one.

There is a variant. If the cut is favorable, that is if u and v are opposite, just go ahead showing the pairing, without the “magic part.” If the spectator says that he understands the trick, show him the counter-example in the non-favorable case. Of course now you have to leave him puzzled, and refuse to redo the trick.



Input: two decks of alternating colours (red, black, red, black, ...) whose bottom cards have different colours.

Output: one deck of alternating red/black pairs.

The Gilbreath trick in Scade/Lustre (thanks to J.-L. Colaço)

The property is implied by the following one on Boolean streams:

*if s_1 and s_2 be two alternating streams starting with different values;
let o be a stream built by “riffle shuffling” s_1 and s_2 , then o is such
that it is the succession of pairs of different values.*

The Gilbreath trick in Scade 6

```
node Gilbreath_stream (clock c : bool) returns (prop: bool; o:bool);
var
  s1 : bool when c;
  s2 : bool when not c;
  half : bool;
let
  s1 = (false when c) -> not (pre s1);
  s2 = (true when not c) -> not (pre s2);
  o = merge (c; s1; s2);
  half = false -> (not pre half);

  prop = true -> not (half and (o = pre o));
tel;
```

The Gilbreath trick in Lustre

```
node Gilbreath_stream (c:bool) returns (OK: bool; o:bool);
var ps1, s1 : bool;
    ps2, s2 : bool;
    half : bool;
let
  s1 = if c then not ps1 else ps1;
  ps1 = false -> pre s1;
  s2 = if not c then not ps2 else ps2;
  ps2 = true -> pre s2;

  o = if c then s1 else s2;

  half = false -> not (pre half);

  OK = true -> not (half and (o = pre o));
tel;
```

Proved automatically using Lesar or Kind 2.

Different time scales

Synchronise slow and fast processes?

X	1	2	3	4	5	...
$half$	T	F	T	F	T	...
$X \text{ when } half$	1		3		5	...
$X + (X \text{ when } half)$	2		5		8	...

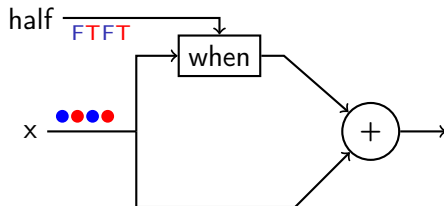
let

half = true $\rightarrow$ not (pre half);

o = x + (x when half);

tel

Defines the stream $\forall n \in \mathbb{N}, o_n = x_n + x_{2n}$



Different time scales

Synchronise slow and fast processes?

X	1	2	3	4	5	...
$half$	T	F	T	F	T	...
$X \text{ when } half$	1		3		5	...
$X + (X \text{ when } half)$	2		5		8	...

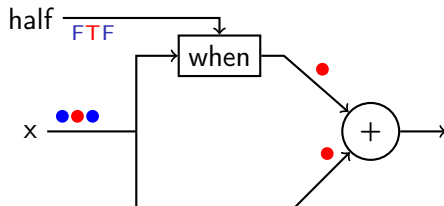
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Different time scales

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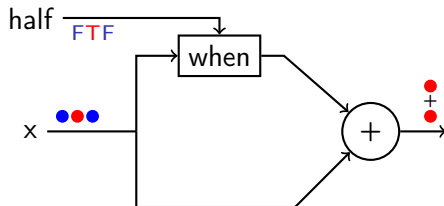
let

half = true $\rightarrow$ not (pre half);

o = x + (x when half);

tel

Defines the stream $\forall n \in \mathbb{N}, o_n = x_n + x_{2n}$



Different time scales

Synchronise slow and fast processes?

X	1	2	3	4	5	...
$half$	T	F	T	F	T	...
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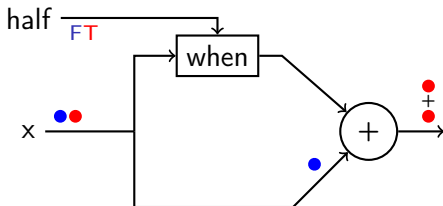
let

half = true $\rightarrow$ not (pre half);

o = x + (x when half);

tel

Defines the stream $\forall n \in \mathbb{N}, o_n = x_n + x_{2n}$



Different time scales

Synchronise slow and fast processes?

X	1	2	3	4	5	...
$half$	T	F	T	F	T	...
$X \text{ when } half$	1		3		5	...
$X + (X \text{ when } half)$	2		5		8	...

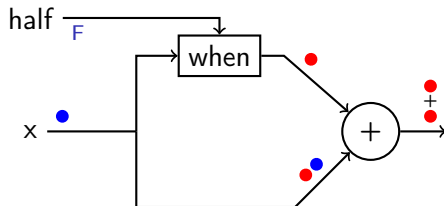
let

half = true $\rightarrow$ not (pre half);

o = x + (x when half);

tel

Defines the stream $\forall n \in \mathbb{N}, o_n = x_n + x_{2n}$



Different time scales

Synchronise slow and fast processes?

X	1	2	3	4	5	...
<i>half</i>	T	F	T	F	T	...
$X \text{ when } half$	1		3		5	...
$X + (X \text{ when } half)$	2		5		8	...

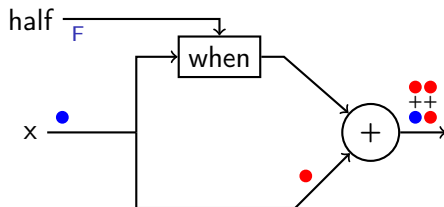
let

half = true $\rightarrow$ not (pre half);

o = x + (x when half);

tel

Defines the stream $\forall n \in \mathbb{N}, o_n = x_n + x_{2n}$



Different time scales

Synchronise slow and fast processes?

X	1	2	3	4	5	...
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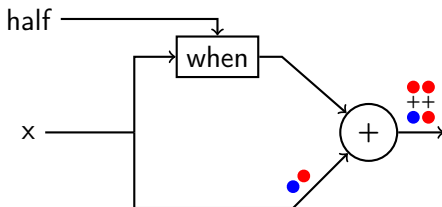
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let

half = true $\rightarrow$ not (pre half);

o = x + (x when half);

tel

Defines the stream $\forall n \in \mathbb{N}, o_n = x_n + x_{2n}$

- It cannot be implemented with bounded buffers;
(The corresponding Kahn network requires an unbounded buffer.)
- Reject such programs statically using a dedicated **type system** based on **clock annotations**. [Colaço and Pouzet (2003): Clocks as First Class Abstract Types]
- Why not just just sample? I.e., $\forall n \in \mathbb{N}, o_n = x_n + x_{\lfloor n/2 \rfloor}$

Clock calculus

The static analysis that rejects programs that cannot be executed in finite memory is called a **clock calculus**. It can be expressed and implemented as a type system [Colaço and Pouzet (2003): Clocks as First Class Abstract Types].

Stream clocks s

base	base rate relative to the current node
α	abstracted base rate / undetermined clock
s on c	subsampling of s when $c = \text{true}$
s on not c	subsampling of s when $c = \text{false}$

Clock schemes σ

Abstraction for operators and nodes

$\forall \alpha, \forall X_1, \dots, X_k, cl \times \dots \times cl \rightarrow cl \times \dots \times cl$

where $cl ::= s \mid (c : s)$

The type system produces judgements of the form $H \vdash e : cl$, meaning “the expression e has clock cl in the environment H ”.

Clock calculus (cont.)

$+, *, \dots : \forall \alpha, \alpha \times \alpha \rightarrow \alpha$

pre : $\forall \alpha, \alpha \rightarrow \alpha$

$\rightarrow$: $\forall \alpha, \alpha \rightarrow \alpha$

fby : $\forall \alpha, \alpha \times \alpha \rightarrow \alpha$

when : $\forall \alpha, \forall X, \alpha \times (X : \alpha) \rightarrow \alpha \text{ on } X$

when not : $\forall \alpha, \forall X, \alpha \times (X : \alpha) \rightarrow \alpha \text{ on not } X$

merge : $\forall \alpha, \forall X, (X : \alpha) \times \alpha \text{ on } X \times \alpha \text{ on not } X \rightarrow \alpha$

Expressions are analyzed as in any type system.

Abstraction is applied to node declarations to bind free variables.

Clock schemes are instantiated when operators or nodes are applied.

Clocks schemes in Heptagon/SCADE 6 have a single base rate (α).

Lucid synchronic [Pouzet (2006): Lucid Synchrone,] allows multiple rates and higher-order functions. E.g., plumbing : $\forall \alpha \beta, \alpha \rightarrow \beta \rightarrow (\alpha \times \beta)$.

Clocks in Heptagon

The Heptagon compiler can infer clocks using type variables and unification as for standard Hindley-Milner inference (the core of OCaml).

-- test.ept

```
node sometimes_add(c : bool; x : int) returns (sum : int);
```

```
let
```

```
  sum = (0 fby sum) + (merge c x 0);
```

```
tel
```

heptc -i test.ept gives

```
val sometimes_add(c : bool :: .; x : int :: . on true(c))
  returns (sum : int :: .)
```

Clocks are generalized: e.g., *base on not c* is written `. on false(c)`.

Compare to (obligatory) clock declarations in Lustre v4: `x int when c`.

Generalization to other enumerated types

- The booleans are just an enumerated type.
- The `when` and `merge` operators, and associated clock annotations generalize naturally.

```
type mode = Rising | Falling | Stable
```

```
node counter(v : mode) returns (y : int);  
var py : int;  
let  
  y = merge v (Rising -> (py when Rising(v)) + 1)  
            (Falling -> (py when Falling(v)) - 1)  
            (Stable -> py when Stable(v));  
  py = 0 fby y;  
tel
```

Sampling operators

- Provide a means of **conditional activation**.
- Programming directly with them can be tricky.
- Serve as a target for more complicated structures.

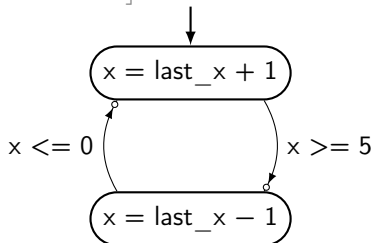
[Colaço, Pagano, and Pouzet (2005): A Conservative Extension of Synchronous Data-flow with State Machines]

```
node main (go : bool) returns (x : int)
  var last_x : int;
  let
    last_x = 0 fby x;

  automaton
    state Up
      do x = last_x + 1
      until x >= 5 then Down

    state Down
      do x = last_x - 1
      until x <= 0 then Up

  end;
tel
```



Sampling operators

- Provide a means of **conditional activation**.
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Synchronous Data-flow with State Machines]

```
node main (go : bool) returns (x : int)
```

```
  var last_x : int;
```

```
  type st = St_Up | St_Down
```

```
let
```

```
  last_x = 0 fby x;
```

```
  (* ... *)
```

```
  automaton
```

```
    last_x = 0 fby x
```

```
    state Up
```

```
      do x = last_x + 1
```

```
      until x >= 5 then Down
```

```
    x_St_Down = (last_x when St_Down(ck)) - 1
```

```
    x_St_Up = (last_x when St_Up(ck)) + 1
```

```
    x = merge ck (St_Down: x_St_Down)
```

```
              (St_Up: x_St_Up);
```

```
    state Down
```

```
      do x = last_x - 1
```

```
      until x <= 0 then Up
```

```
    ck = St_Up fby ns
```

```
    ns = ...
```

```
  end;
```

```
tel
```

Programming with sampling [Pouzet (2006): Lucid Synchrone, v. 3.]

[Tutorial and reference manual]

Rising-edge Retrigger: “Every rising edge on input i ,
hold the output o true for n cycles.”

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Rising-edge Retrigger: “Every rising edge on input i ,
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```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
let
  ...
tel
```

i	F	T	T	F	F	F	F	T	F	...
n	3	3	3	3	3	3	3	3	3	...

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Rising-edge Retrigger: “Every rising edge on input i ,
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```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
var edge : bool;
let
  edge = i and (false fby (not i));
...
tel
```

i	F	T	T	F	F	F	F	T	F	...
n	3	3	3	3	3	3	3	3	3	...
edge	F	T	F	F	F	F	F	T	F	...

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Rising-edge Retrigger: “Every rising edge on input i ,
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```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
var edge : bool; v : int;
let
  edge = i and (false fby (not i));
  v = /* count down from n whenever edge is true */
  o = v > 0;
tel
```

i	F	T	T	F	F	F	F	T	F	...
n	3	3	3	3	3	3	3	3	3	...
edge	F	T	F	F	F	F	F	T	F	...
v	0	3	2	1	0	0	0	3	2	...
o	F	T	T	T	F	F	F	T	T	...

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returns (o : bool)
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```
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```
let
```

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  edge = i and (false fby (not i));
```

```
  v = /* count down from n whenever edge is true */
```

```
  o = v > 0;
```

```
tel
```

```
node dcount(res : bool; n : int)
```

```
returns (cpt : int)
```

```
let
```

```
  cpt = if res then n
```

```
        else (n fby (cpt - 1));
```

```
tel
```

i	F	T	T	F	F	F	F	T	F	...
n	3	3	3	3	3	3	3	3	3	...
edge	F	T	F	F	F	F	F	T	F	...
v	0	3	2	1	0	0	0	3	2	...
o	F	T	T	T	F	F	F	T	T	...

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Rising-edge Retrigger: “Every rising edge on input i ,
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```
node rising_edge_retrigger(i : bool; n : int)
returns (o : bool)
var edge, ck : bool; v : int;
let
  edge = i and (false fby (not i));
  ck = edge or (false fby o);
  v = dcount((false, n) when ck);
  o = v > 0;
tel
```

```
node dcount(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n
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```

i	F	T	T	F	F	F	F	T	F	...
n	3	3	3	3	3	3	3	3	3	...
edge	F	T	F	F	F	F	F	T	F	...
v	0	3	2	1	0	0	0	3	2	...
ck	F	T	T	T	T	F	F	T	T	...
n when ck		3	3	3	3			3	3	...
dcount(n when ck)		3	2	1	0			-1	-2	...
o	F	T	T	T	F	F	F	T	T	...

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var edge, ck : bool; v : int;
let
  edge = i and (false fby (not i));
  ck = edge or (false fby o);
  v = merge ck
      (true -> dcount((false, n) when ck))
      (false -> 0);
  o = v > 0;
tel
```

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node dcount(res : bool; n : int)
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edge	F	T	F	F	F	F	F	T	F	...
v	0	3	2	1	0	0	0	3	2	...
ck	F	T	T	T	T	F	F	T	T	...
n when ck		3	3	3	3			3	3	...
dcount(n when ck)		3	2	1	0			-1	-2	...
o	F	T	T	T	F	F	F	T	T	...

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    (false -> 0);
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```

i	F	T	T	F	F	F	F	T	F	...
n	3	3	3	3	3	3	3	3	3	...
edge	F	T	F	F	F	F	F	T	F	...
v	0	3	2	1	0	0	0	3	2	...
ck	F	T	T	T	T	F	F	T	T	...
n when ck		3	3	3	3			3	3	...
dcount(n when ck)		3	2	1	0			3	2	...
o	F	T	T	T	F	F	F	T	T	...

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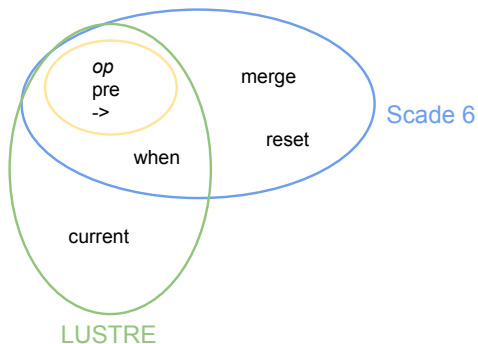
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- **Ptolemy II** [(2014): System Design, Modeling, and Simulation using Ptolemy II]
- **MathWorks Simulink (and Stateflow)** <https://www.mathworks.com/products/simulink/>
- **National Instruments LabView** <https://www.ni.com/en-us/shop/labview.html>



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Further reading

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