Minimum number of edges in k-critical digraphs on n vertices

Key words : Extremal graph theory, directed graphs, chromatic number
Lab Computer science departement of ENS Paris.
Team : TALGO
Supervisor : Pierre Aboulker - https://www.di.ens.fr/ paboulker/ - pierreaboulker@gmail.com

General context : The *chromatic number* $\chi(G)$ of a graph G is the minimum number of colors needed to color the vertices of G in such a way that two adjacent vertices receive distinct colors. It is certainly the most studied parameter in graph theory and many deep results and theories have been developped around this concept.

It is not an easy task to extend the notion of chromatic number to directed graphs in a meaningfull way. Anyway, since a few years, more and more results are showing that the (now) so-called notion of *dichromatic number* is the right concept to generalise chromatic number to directed graphs, and more and more efforts are made to extend coloring results from undirected graphs to directed graphs through this notion. The goal of this intership is to participate to this effort.

Definition : The *dichromatic number* $\vec{\chi}(D)$ of a digraph D is the minimum number of colors needed to color the vertices of D in such a way that no directed cycle is monochromatic [1]. A directed graph is *k-critical* if it has dichromatic number k and all its proper subgraphs have dichromatic number at most $k-1$.

Objective : Recently, Kostochka and Yancey [1] solved a long standing conjecture of Ore, giving a tight bound on the minimal number of edges of a k-critical (undirected) graphs on n vertices. In order to do so, they developped a new and very powerfull method, called the *potential method*. The goal of the internship will be to generalise their result to digraphs and dichromatic number, using the potential method.

The student should have a particular test for graph theory and combinatorics, and a strong desire to do a PhD.

Références

[1] A. Kostochka and M. Yancey. Ore's conjecture on color-critical graphs is almost true. *Journal of Combinatorial Theory, Series B*, Volume 109, p : 73-101 (2014).

1. In other words, it is the minimum number of acyclic subgraph needed to partition $V(D)$