Caulk: Lookup Arguments in Sublinear Time

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Ethereum Foundation

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Protocol Labs

MOTIVATION

MEMBERSHIP PROOFS

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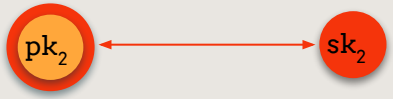

RANGE PROOFS

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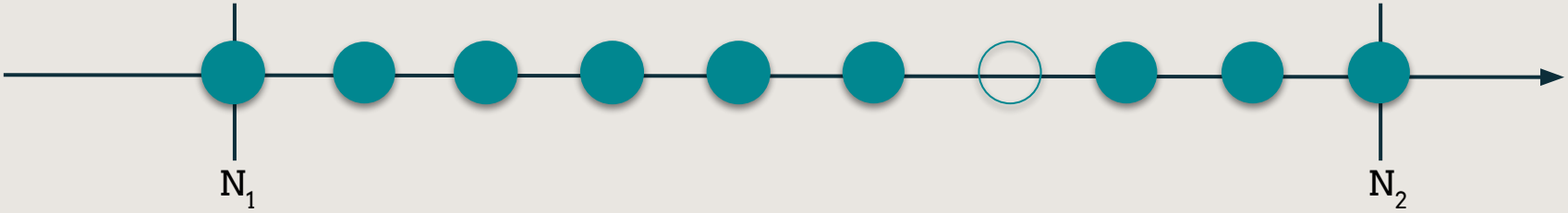
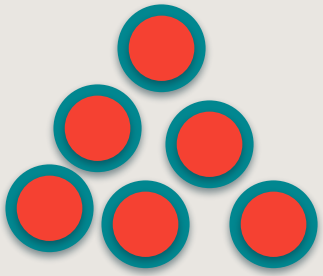


LOOKUP TABLES



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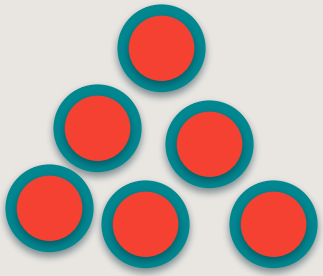

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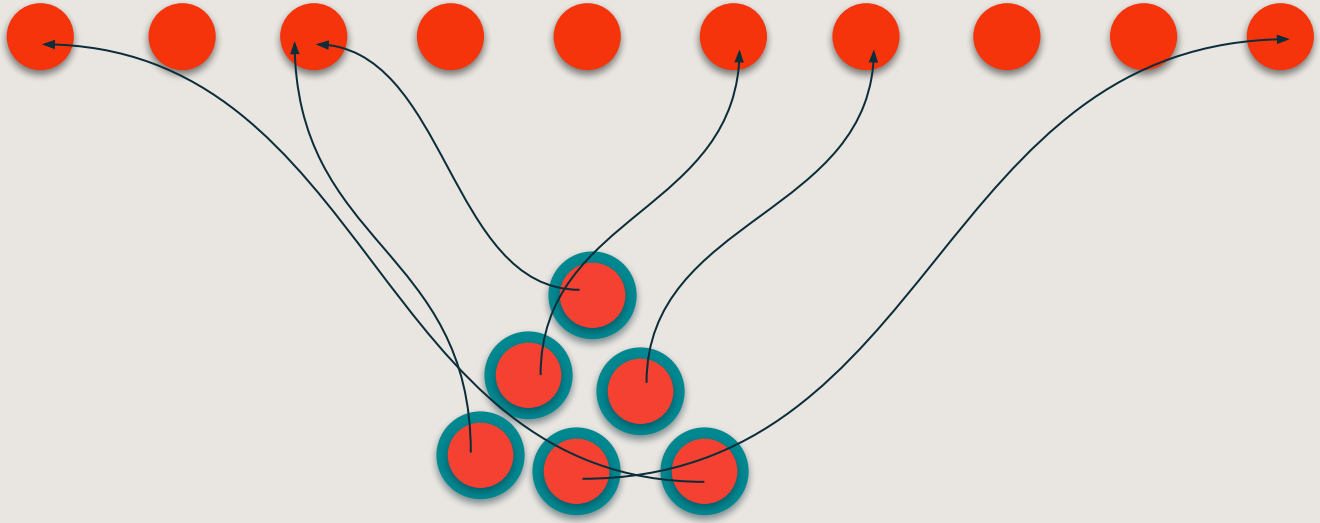

STATE OF THE ART

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Discrete-log

STATE OF THE ART

Discrete-log

- Transparent setup
- Linear prover and verifier

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RSA Accumulators

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Pairing-based

- Constant proof + verifier
- Linear prover

CAULK

Pairing-based

CAULK

Pairing-based

Logaritmic prover + constant proof

log(log) verifier

DEFINITION

DEFINITION

DEFINITION

Position-hiding linkability for two VC schemes

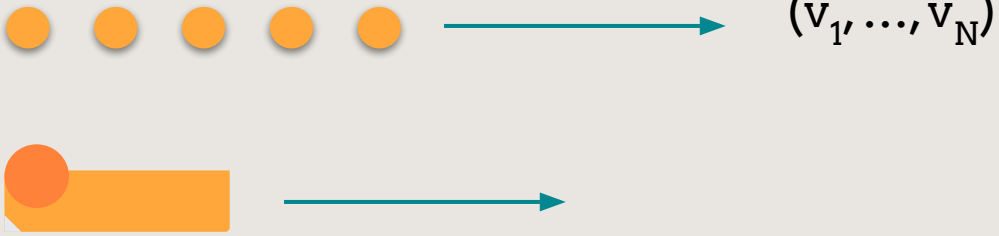
Everything in 

is also in 

KZG

KZG

KZG



KZG

$(v_1, \ldots, v_N)$

$H=\{1, \omega, \omega^2, \ldots, \omega^{N-1}\}, \quad \omega^N=1$

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$C(X)=\Sigma_i\, v_i\, \lambda_i(X)$

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$C(\omega^{i-1})=v_i$

ROOTS OF UNITY

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$z_H(X)=X^N-1 \qquad \lambda_i(X)=(\omega^{i-1}(X^N-1))((X-\omega^{i-1})N)^{-1}$

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If $u^N=1 \longrightarrow u=\omega^{something}$

KZG + TABDFK

$$C(X)=\Sigma_i\, v_i\, \lambda_i(X)$$

KZG + TABDFK

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$$C(X)=\Sigma_i\, v_i\, \lambda_i(X)$$

- Prover sends $[v_i]$, $[(x-\omega^{i-1})]$, $[Q_i(x)]$ s.t:

KZG + TABDFK

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Verifier checks

$$e([C(x)],[v_i])=e([(x-\omega^{i-1})],[Q_i(x)])$$

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Verifier checks

$$e([C(x)],\,[C_I(x)])=e([\Pi_{i\varepsilon I}(x-\omega^{i-1})],[Q_I(x)])$$

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$$C(X)-C_I(X)=\Pi_{i\varepsilon I}\,(X-\omega^{i-1})\,Q_I(X)$$

Verifier checks

$$e([C(x)],\ [C_I(x)])=e([\Pi_{i\varepsilon I}(x-\omega^{i-1})], [Q_I(x)])$$

$Q_i(X)$ and $Q_I(X)$ are linear in N!!!

KZG + TABDFK

Precompute $([Q_1(x)],\ldots,[Q_N(x)])$

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Precompute $([Q_1(x)],\ldots,[Q_N(x)])$

$$C(X)-v_i=(X-\omega^{i-1})Q_i(X)$$

$$\{Q_i(X)\}_{i\varepsilon I} \qquad Q_I(X)=\Sigma_{i\varepsilon I}\, k_i Q_i(X)$$

$$C(X)-C_I(X)=\Pi_{i\varepsilon I}\,(X-\omega^{i-1})\,Q_I(X)$$

KZG + TABDFK

Precompute $([Q_1(x)],\ldots,[Q_N(x)])$

$$C(X)-v_i=(X-\omega^{i-1})Q_i(X)$$

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$$Q_I(X)=\Sigma_{i\varepsilon I}\, k_i Q_i(X)$$

$$C(X)-C_I(X)=\Pi_{i\varepsilon I}\,(X-\omega^{i-1})\,Q_I(X)$$

Prover depends on $|I|=m$

KZG + TABDFK

$C(X)=\Sigma_i\ v_i\ \lambda_i(X)$ is public

$proof_i=([v_i], [(x-\omega^{i-1})], [Q_i(x)])$

$proof_I=([C_I(x)], [\Pi_{i\varepsilon\ I}\ (x-\omega^{i-1})], [Q_I(x)])$

KZG + TABDFK

$C(X)=\Sigma_i v_i \lambda_i(X)$ is public

Blind v_i, $C_I(X)$

$proof_i=([v_i], [(x-\omega^{i-1})], [Q_i(x)])$

$proof_I=([C_I(x)], [\Pi_{i\varepsilon I} (x-\omega^{i-1})], [Q_I(x)])$

CHALLENGES

$C(X)=\Sigma_i v_i \lambda_i(X)$ is public

Blind v_i, $C_I(X)$

$proof_i=([v_i+hr], [(x-\omega^{i-1})], [Q_i(x)])$

$proof_I=([C_I(x)+z_I(x)s(x)], [\Pi_{i\varepsilon I} (x-\omega^{i-1})], [Q_I(x)])$

CHALLENGES

$C(X)=\Sigma_i v_i \lambda_i(X)$ is public

Blind v_i, $C_I(X)$

$proof_i=([a], [(x-\omega^{i-1})], [Q_i(x)])$

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KZG + TABDFK

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Blind $Q_i(X), Q_I(X)$

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Blind $Q_i(X), Q_I(X)$

$proof_i=([a], [(x-\omega^{i-1})], [Q_i(x)+sh)])$

$proof_I=([a], [\Pi_{i\varepsilon I} (x-\omega^{i-1})], [r_1^{-1}Q_I(x)+r(x)])$

CHALLENGES

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Blind $Q_i(X), Q_I(X)$

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$C(X)=\Sigma_i v_i \lambda_i(X)$ is public

$proof_i=([v_i], [(x-\omega^{i-1})], [Q_i])$

Blind , $(X-\omega^{i-1})$, $\Pi_{i\varepsilon I}(X-\omega^{i-1})$

$proof_I=([C_I(x)], [\Pi_{i\varepsilon I}(x-\omega^{i-1})], [Q_I])$

CHALLENGES

$C(X)=\Sigma_i v_i \lambda_i(X)$ is public

$proof_i=([a], [a(x-\omega^{i-1})], [Q_i])$

Blind , $(X-\omega^{i-1}), \Pi_{i\varepsilon I} (X-\omega^{i-1})$

$proof_I=([a], [r_1\Pi_{i\varepsilon I} (x-\omega^{i-1})], [Q_I])$

CHALLENGES

$C(X)=\Sigma_i\ v_i\ \lambda_i(X)$ is public

Blind $v_i, C_I(X)$

Blind $Q_i(X), Q_I(X)$

$proof_i=([a], [z], [Q_i])$

Blind , $(X-\omega^{i-1}), \Pi_{i\varepsilon\ I}\ (X-\omega^{i-1})$

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$proof_I=([a], [z_I], [Q_I])$

- Blind v_i, $C_I(X)$
- Blind $Q_i(X)$, $Q_I(X)$
- Blind , $(X-\omega^{i-1})$, $\Pi_{i\varepsilon I}(X-\omega^{i-1})$
- Well formation

CHALLENGES

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$proof_i=([a], [z], [Q_i])$

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- Blind $v_i, C_I(X)$
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$$[z]=[a(x-\omega^{i-1})]$$

m=1

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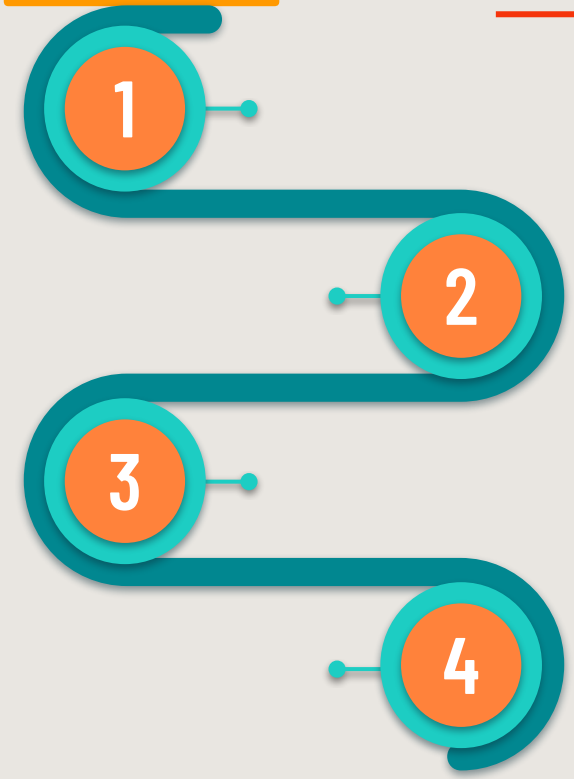

Well formation

$[z]=[a(x-\omega^{i-1})]$

m=1

1. Prove $[z]=[ax+b]$ $(b=a\omega^{i-1})$
2.
3.
4.

Well formation

$[z]=[a(x-\omega^{i-1})]$

m=1

1. Prove $[z]=[ax+b]$ $(b=a\omega^{i-1})$

2. Construct f(X) of degree log(N)+6,
   $f(X)=\Sigma_j f_j \mu_j(X)$

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1. Prove $[z]=[ax+b]$ $(b=a\omega^{i-1})$
2. Construct f(X) of degree log(N)+6, $f(X)=\Sigma_j f_j \mu_j(X)$ → New set of roots of unity! V Lagrange polynomials $\{\mu_j(X)\}$
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3. Prove $f_5=b/a$, and for j = 6,...,log(N)+5 $f_j=f_{j-1} f_{j-1}$

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   Should be ω^{i-1}

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   Should be ω^{i-1}

   f_{5+j} is the 2^j th power of ω^{i-1}

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3. Prove $f_5=b/a$, and for j = 6,...,log(N)+5 $f_j=f_{j-1} f_{j-1}$

4. Prove $f_{log(N)+5}=1$

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3. Prove $f_5=b/a$, and for j = 6,...,log(N)+5 $f_j=f_{j-1} f_{j-1}$
4. Prove $f_{\log(N)+5}=1$

$(b/a)^N=1$!!!

m>1

$$proof_I=([C_I(x)],[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})],[Q_I])$$

m>1

$$proof_I=([\Pi_{i\varepsilon I} v_i \tau_i(x)], [r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})], [Q_I])$$

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$$H_I=\{\omega^{i-1}\}_{i\varepsilon I}$$

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$$H_I=\{\omega^{i-1}\}_{i\varepsilon I}$$

"I" unknown to the verifier!

$[C(x)=\Sigma_i v_i \lambda_i(x)\,],\ [\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([\Pi_{i\varepsilon I} v_i \tau_i(x)], [r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})], [Q_I])$

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$a_j=v_i$ for some i, for all j

$[C(x)=\Sigma_i v_i \lambda_i(x)\,], [\phi(x)=\Sigma_j a_j \mu_j(x)]$

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$proof_I=([C_I], [z_I], [Q_I])$

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m>1

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01

02

03

04

$[C(x)=\Sigma_i v_i \lambda_i(x)\ ],\ [\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

02

03

04

$[C(x)=\Sigma_i v_i \lambda_i(x)\ ], [\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

02

03

04

$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

1. Set $\mathbf{u}=(\omega^{i-1})_{i\varepsilon I}$ but with repetitions

$[C(x)=\Sigma_i v_i \lambda_i(x)\,], [\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

02

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04

$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

1. Set $u(X)=\Sigma \omega^{i-1} \mu_{j,i}(x)=\Sigma u_j \mu_j(x)$

$[C(x)=\Sigma_i v_i \lambda_i(x)\ ], [\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

02

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04

$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

The roots used in z_I with repetitions

1. Set $u(X)=\Sigma \omega^{i-1} \mu_{j,i}(x)=\Sigma u_j \mu_j(x)$

$[C(x)=\Sigma_i v_i \lambda_i(x)\,], [\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

$[z_I]=[r_1 \Pi_{i\varepsilon I}(x-\omega^{i-1})]$

02

1. Set $u(X)=\Sigma \omega^{i-1} \mu_{j,i}(x)=\Sigma u_j \mu_j(x)$
2. Compute $\mathbf{u_s=u_{s-1}u_{s-1}}$

03

04

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

02

1. Set $u(X)=\Sigma \omega^{i-1} \mu_{j,i}(x)=\Sigma u_j \mu_j(x)$
2. Compute $u_s(X)=u_{s-1}(X)u_{s-1}(X) \bmod z_V(X)$

03

04

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

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02

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$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

1. Set $u(X)=\Sigma \omega^{i-1} \mu_{j,i}(x)=\Sigma u_j \mu_j(x)$
2. Compute $u_s(X)=u_{s-1}(X)u_{s-1}(X) \bmod z_V(X)$

The coeficients in $u_s(X)$ are the 2^s power of the u_js

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

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$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

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The coeficients in $u_s(X)$ are the 2^s power of the u_js

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1. Set $u(X)=\Sigma \omega^{i-1} \mu_{j,i}(x)=\Sigma u_j \mu_j(x)$
2. Compute $u_s(X)=u_{s-1}(X)u_{s-1}(X) \bmod z_V(X)$
3. Prove $\mathbf{u_{log(N)}}=(1,1,\ldots,1)$

03

04

$[C(x)=\Sigma_i v_i \lambda_i(x)\ ], [\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

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04

$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

1. Set $u(X)=\Sigma \omega^{i-1} \mu_{j,i}(x)=\Sigma u_j \mu_j(x)$
2. Compute $u_s(X)=u_{s-1}(X)u_{s-1}(X) \bmod z_V(X)$
3. Prove $u_{log(N)}(X)=\Sigma 1\mu_j(x)$

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

02

03

04

$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

1. Set $u(X)=\Sigma \omega^{i-1} \mu_{j,i}(x)=\Sigma u_j \mu_j(x)$
2. Compute $u_s(X)=u_{s-1}(X)u_{s-1}(X) \bmod z_V(X)$
3. Prove $u_{log(N)}(X)=\Sigma 1\mu_j(x)$

All the $2^{log(N)}$ th powers of the u_js are 1!

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

02

03

04

$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$

1. Set $u(X)=\Sigma \omega^{i-1} \mu_{j,i}(x)=\Sigma u_j \mu_j(x)$
2. Compute $u_s(X)=u_{s-1}(X)u_{s-1}(X) \bmod z_V(X)$
3. Prove $u_{log(N)}(X)=\Sigma 1\mu_j(x)$

All the Nth powers of the u_js are 1!

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

$[z_I]=[r_1 \Pi_{i\varepsilon I} (x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02

03

04

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02 $e([z_I(u(x))], [1])= e([z_V], [Q_2])$

03

04

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02 $e([z_I(\omega^{something})], [1])= e([0], [Q_2])$

03

04

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01

$[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02

$e([z_I(\omega^{something})], [1])= e([0], [Q_2])$

03

$[z_I]$ is well formed

04

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02 $e([z_I(u(x))], [1])= e([z_V], [Q_2])$

03 $e([C(x)]-[C_I])= e([Q_I], [z_I])$

04

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02 $e([z_I(u(x))], [1])= e([z_V], [Q_2])$

03 $e([C(\omega^{something})]-[C_{Isomething}])= e([Q_I], [0])$

04

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02 $e([z_I(u(x))], [1])= e([z_V], [Q_2])$

03 $e([v_{something}]-[C_{Isomething}])= e([Q_I], [0])$

04

$[C_I]$ contains a *subvector* of **v**

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02 $e([z_I(u(x))], [1])= e([z_V], [Q_2])$

03 $e([C(x)]-[C_I])= e([Q_I], [z_I])$

04 $e([C_I(u(x))]- [\phi(x)], [1])= e([z_V], [Q_3])$

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ $[u(x)]$ with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02 $e([z_I(u(x))], [1])= e([z_V], [Q_2])$

03 $e([C(x)]-[C_I])= e([Q_I], [z_I])$

04 $e([C_I(\omega^{something})]- [a_j], [1])= e([0], [Q_3])$

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02 $e([z_I(u(x))], [1])= e([z_V], [Q_2])$

03 $e([C(x)]-[C_I])= e([Q_I], [z_I])$

04 $e([v_{something})]- [a_j], [1])= [0]$

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

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$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

02 $e([z_I(u(x))], [1])= e([z_V], [Q_2])$

03 $e([C(x)]-[C_I])= e([Q_I], [z_I])$

04 $e([v_{something})]- [a_j], [1])= [0]$

Everyting in **a** is also in **v**

$[C(x)=\Sigma_i v_i \lambda_i(x)]$, $[\phi(x)=\Sigma_j a_j \mu_j(x)]$

m>1

$proof_I=([C_I], [z_I], [Q_I])$

01 $[z_I]=[r_1\Pi_{i\varepsilon I}(x-\omega^{i-1})]$ [u(x)] with Nth roots of unity as coefficients in $\{\mu_j(X)\}$

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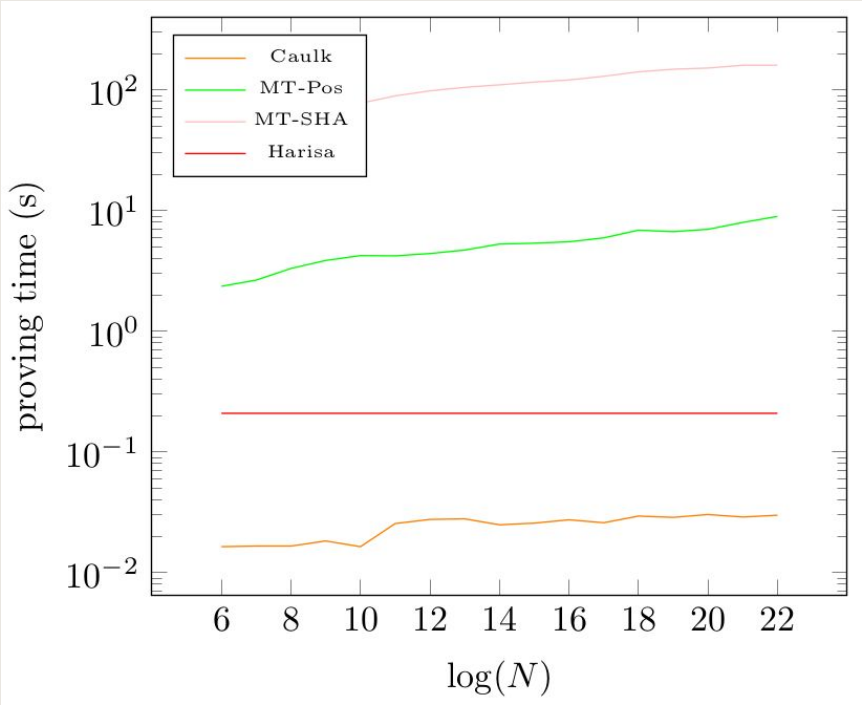
03 $e([C(x)]-[C_I])= e([Q_I], [z_I])$

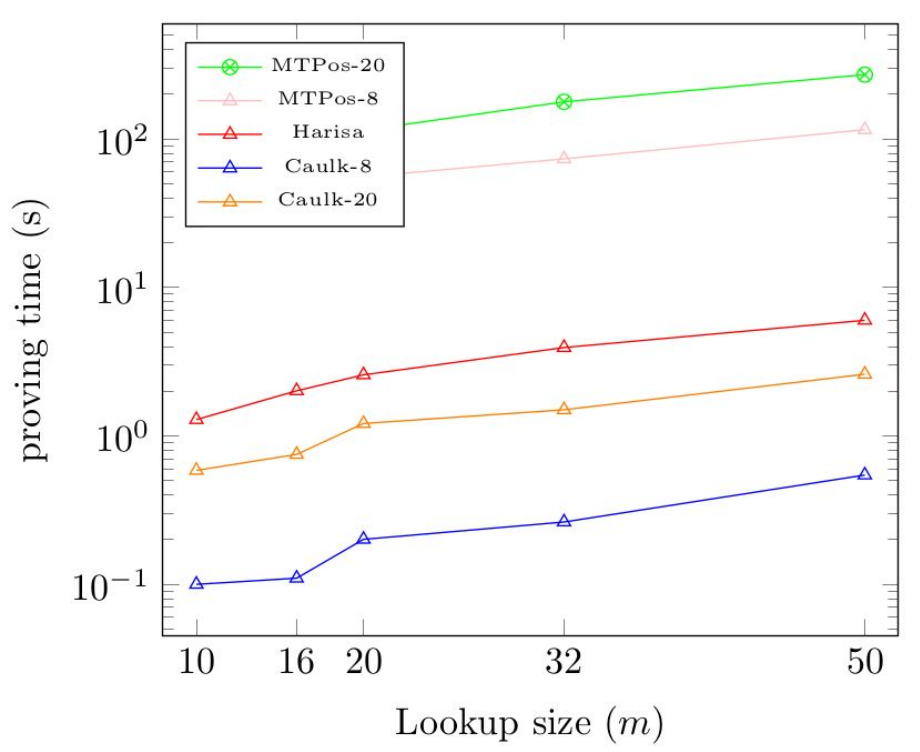
04 $e([v_{something})]- [a_j], [1])= [0]$

Position-hiding linkability!!!

IMPLEMENTATION



IMPLEMENTATION



THANKS!

https://eprint.iacr.org/2022/621