Robust Password-Protected Secret Sharing

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ENS

cnrs dépasser les frontières

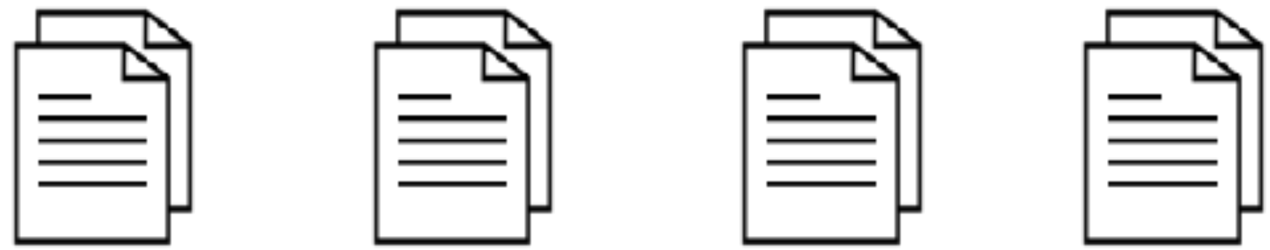
Inria INVENTORS FOR THE DIGITAL WORLD

PSL RESEARCH UNIVERSITY

PPSS: Motivation

PPSS: Motivation

Cloud provider



taxes

medical records

paychecks

top secret documents

PPSS: Motivation

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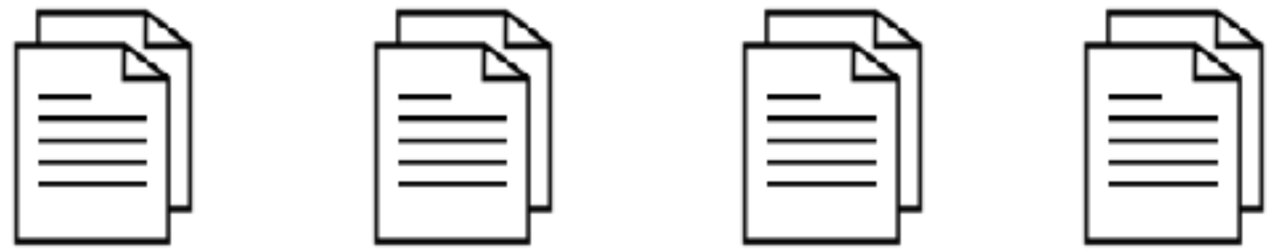

taxes

medical records

paychecks

top secret documents

Everyone might have access to the data

PPSS: Motivation

Cloud provider

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Provider *still* has access to the data

PPSS: Motivation

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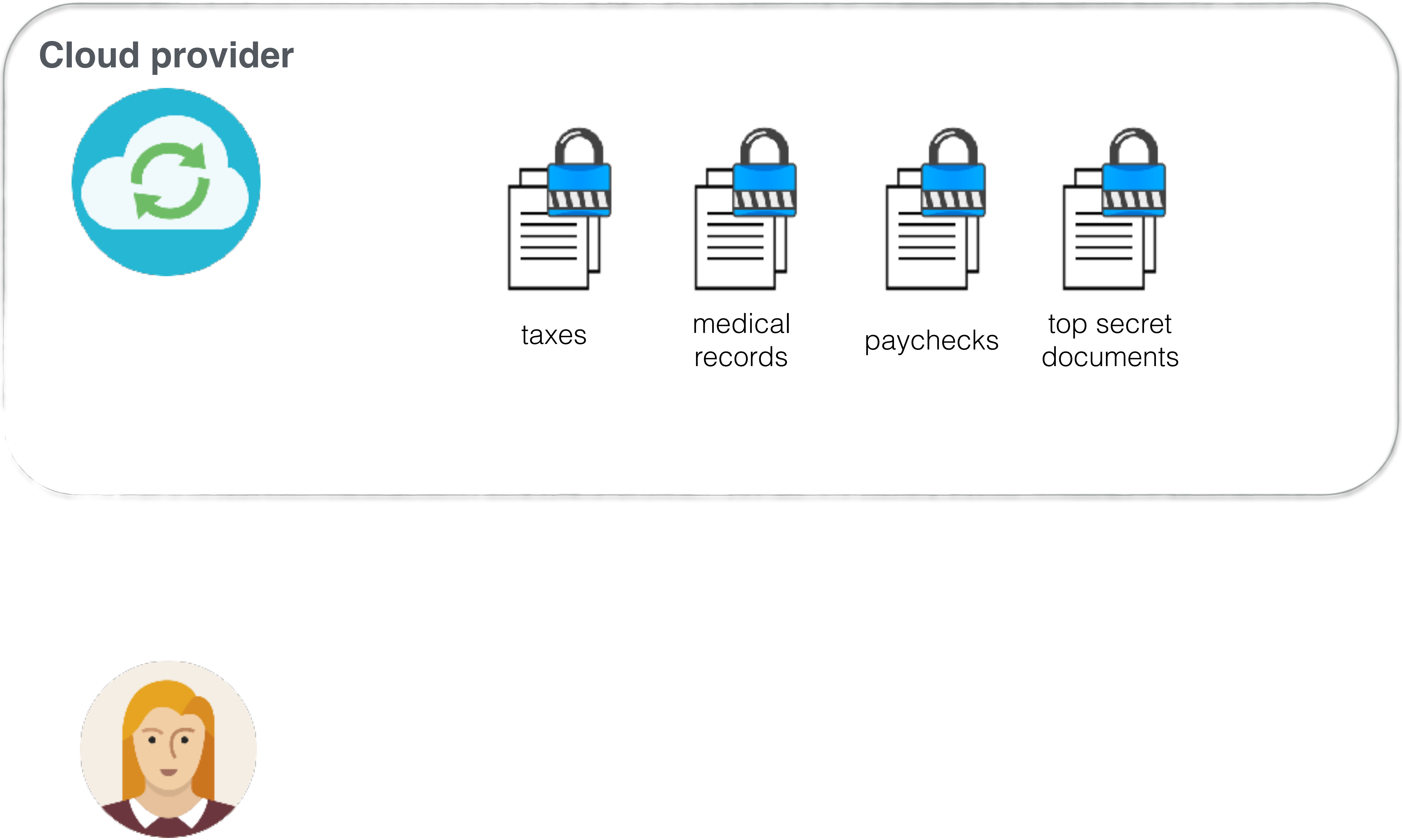

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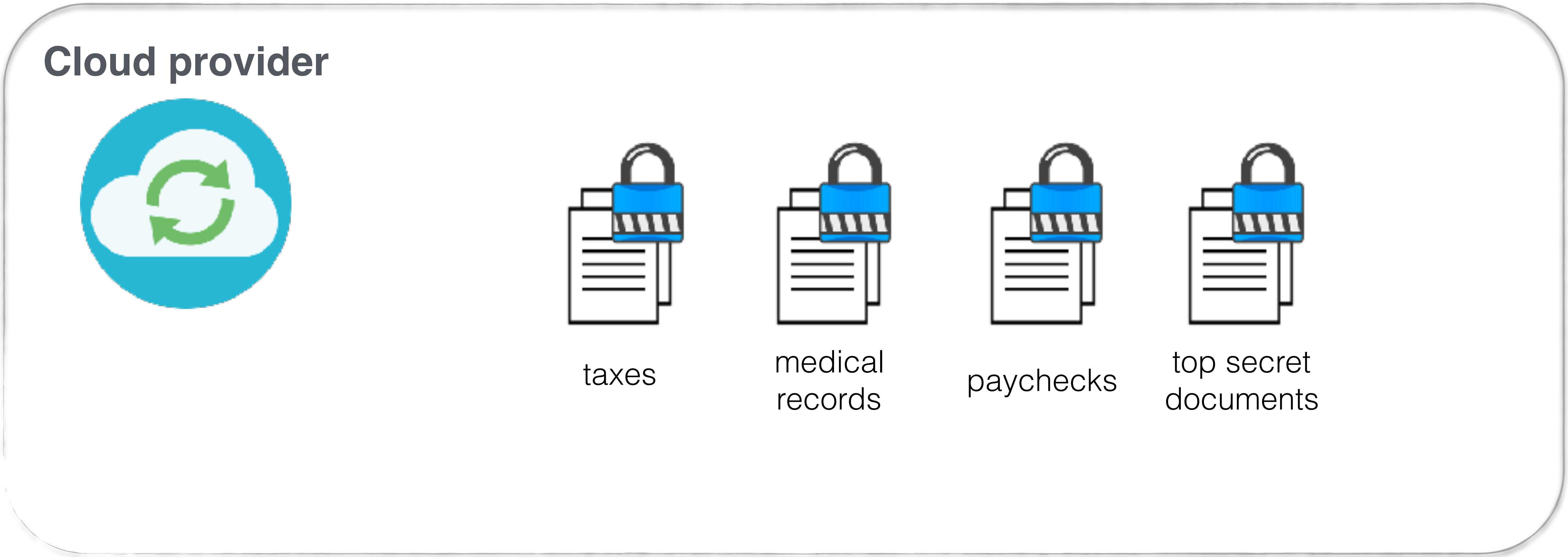
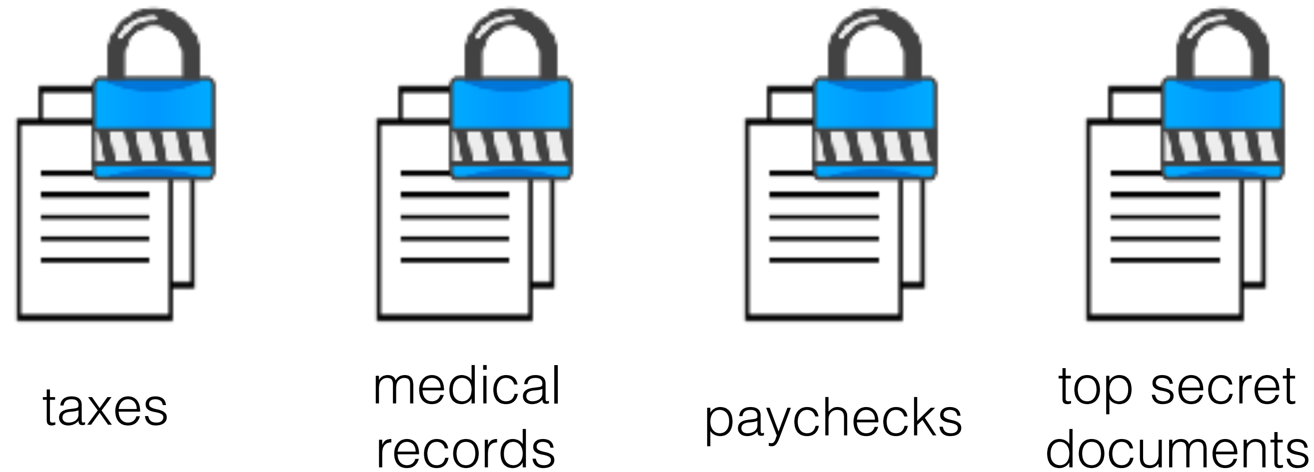


- We can remember just low-entropy passwords (and not too many).
- Humans cannot remember large secret keys.
- **Provider/authorities might perform an offline dictionary attack.**

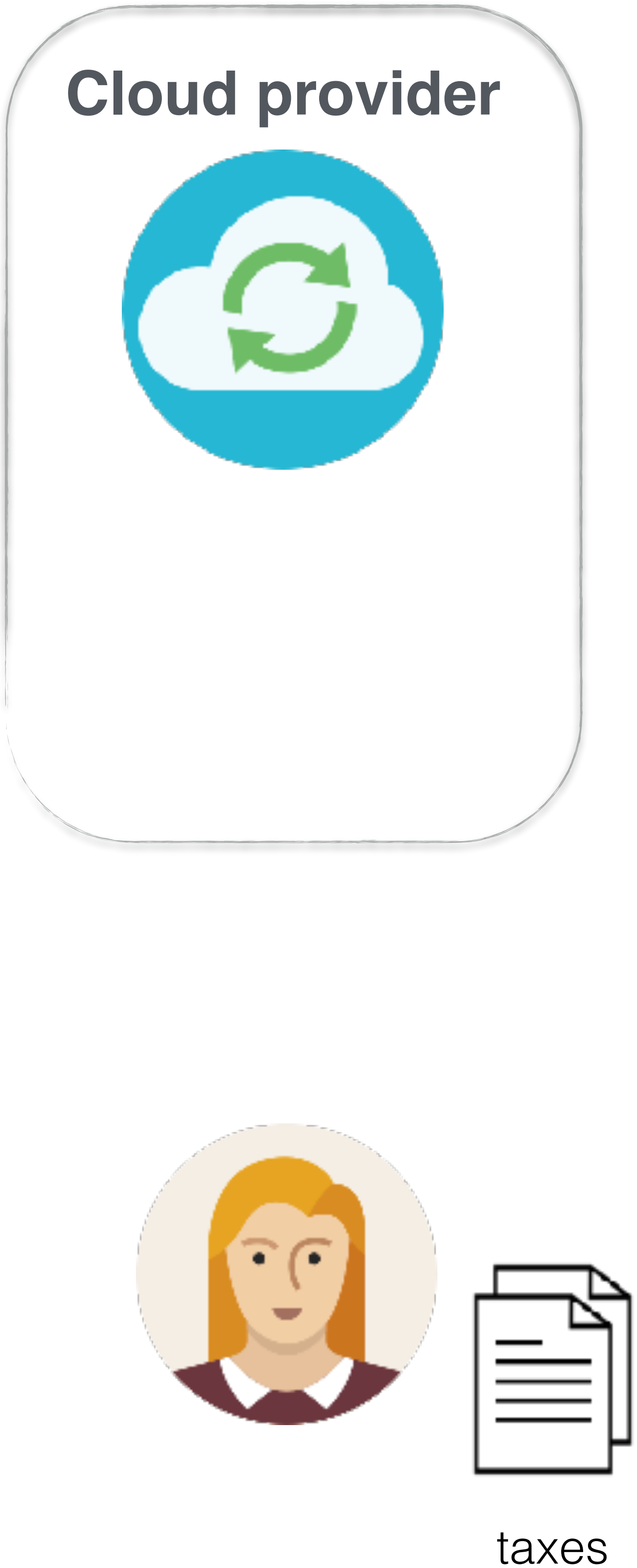
PPSS: Motivation

Cloud provider



- USB Tokens might not be always available.
- Tokens might fall into the wrong hands.
- **Large keys give better security.**

PPSS: Password-Protected Secret Sharing



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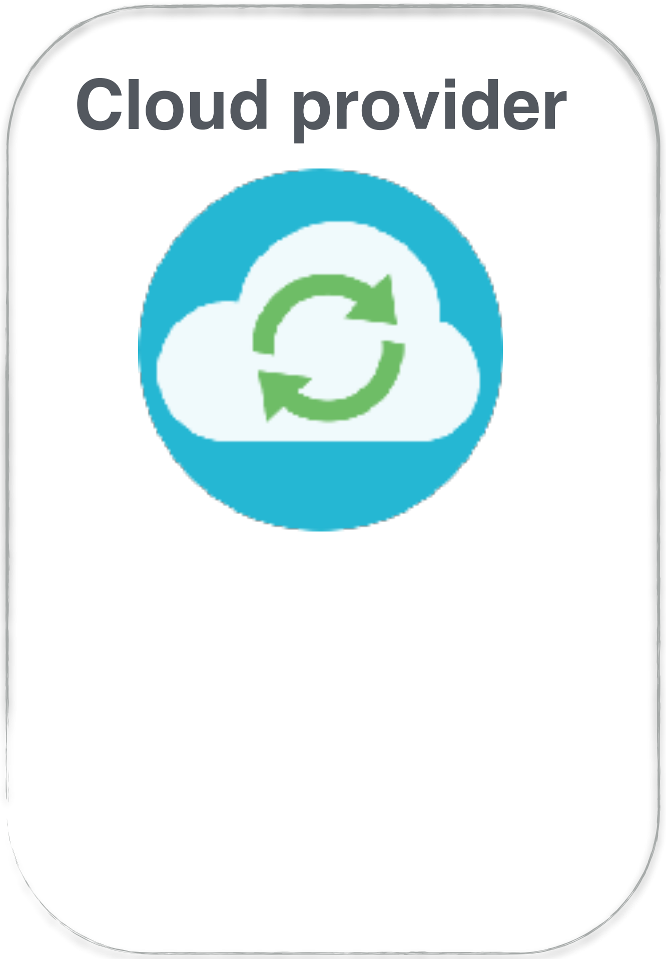
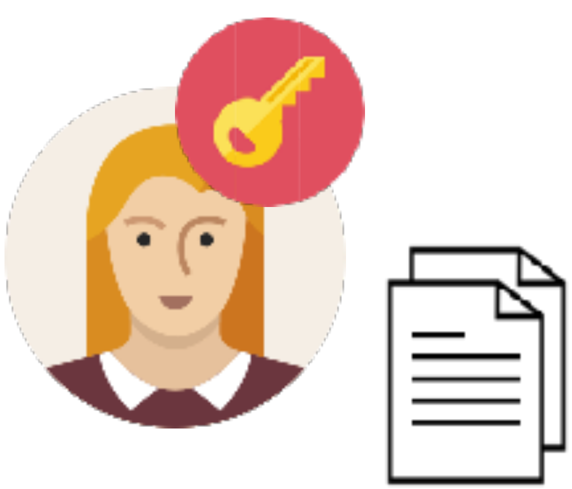


- User creates a cryptographic key.



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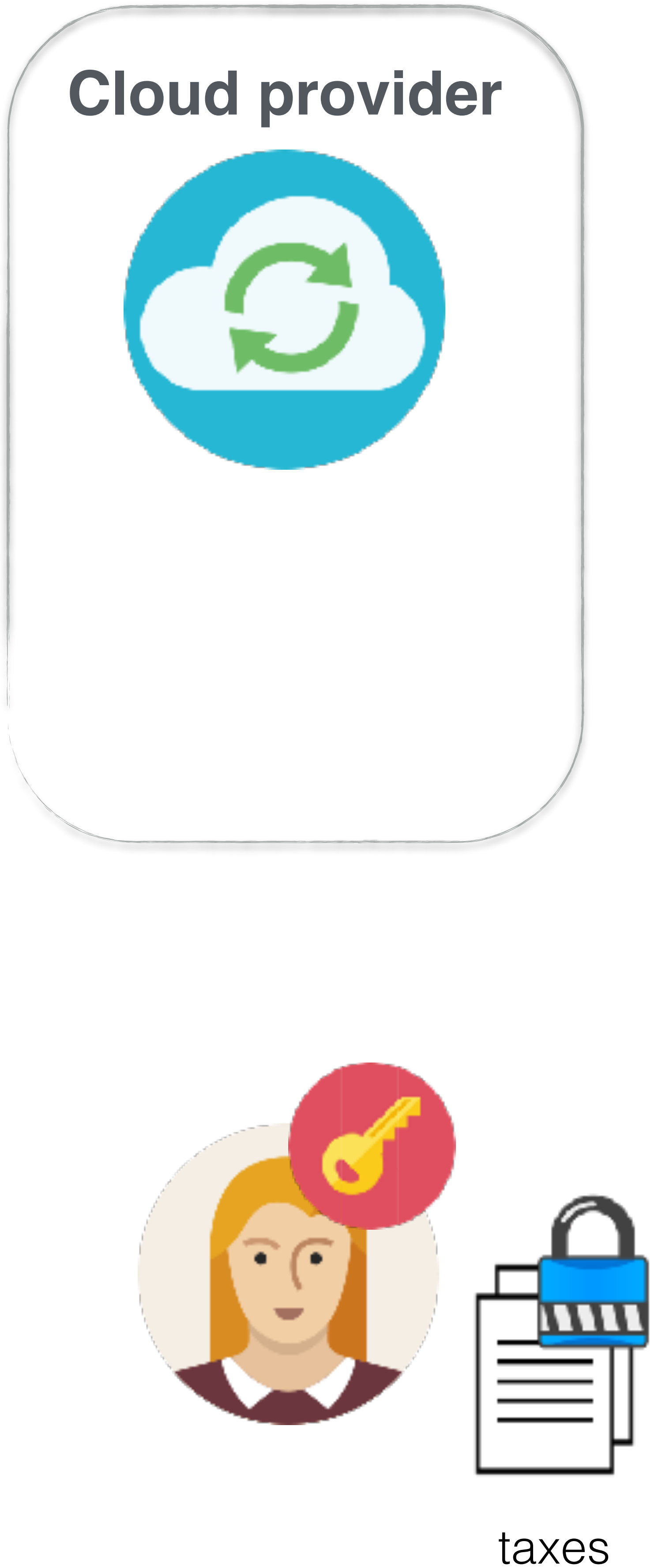
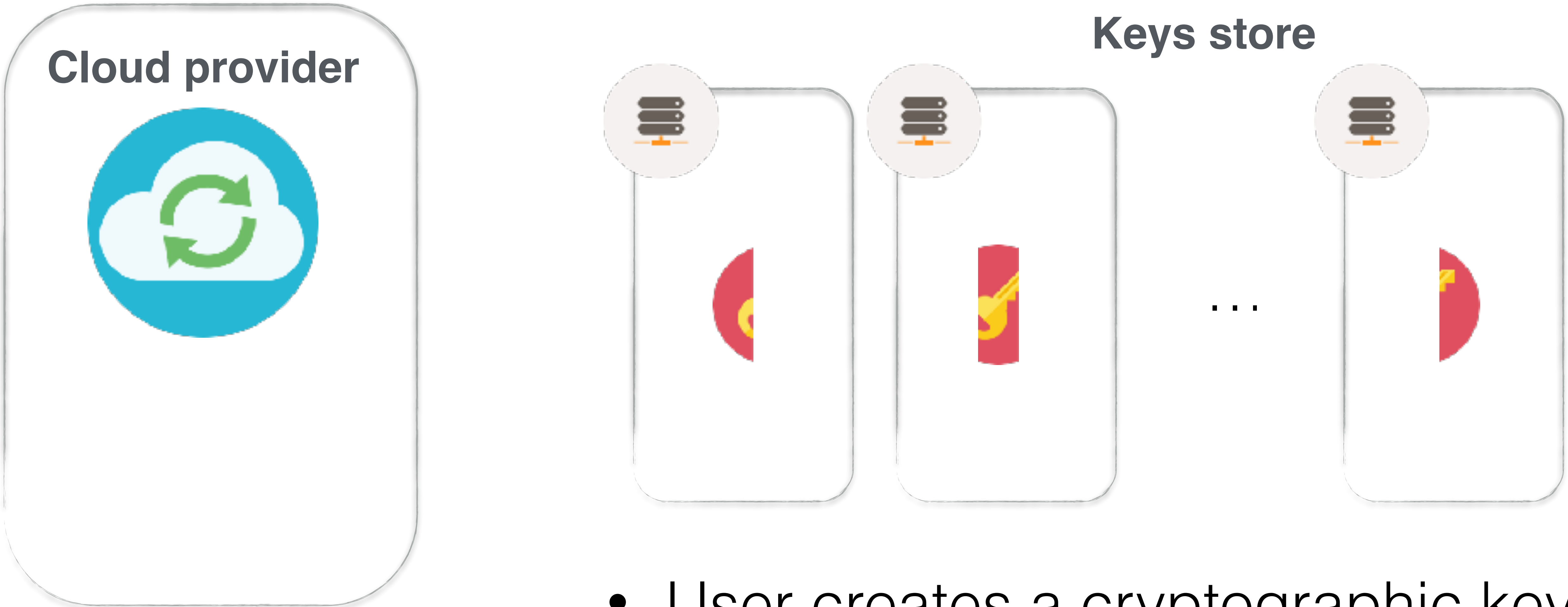
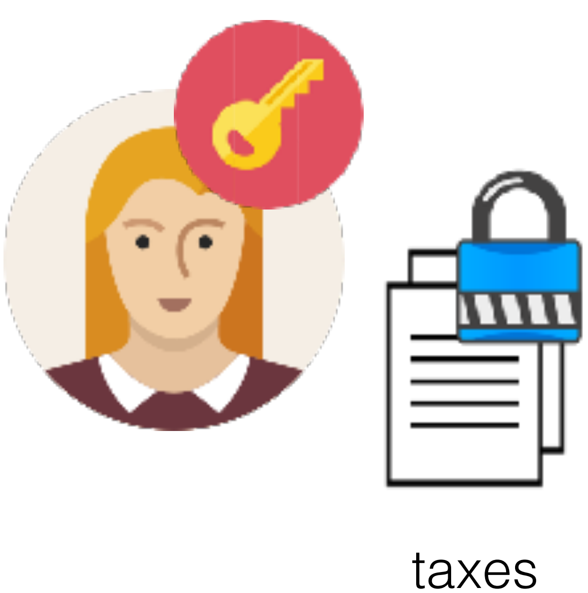


- User creates a cryptographic key.
- Encrypts her data using this key.

PPSS: Password-Protected Secret Sharing



- User creates a cryptographic key.
- Encrypts her data using this key.
- Stores her secret key into n servers by using her password and some public information.



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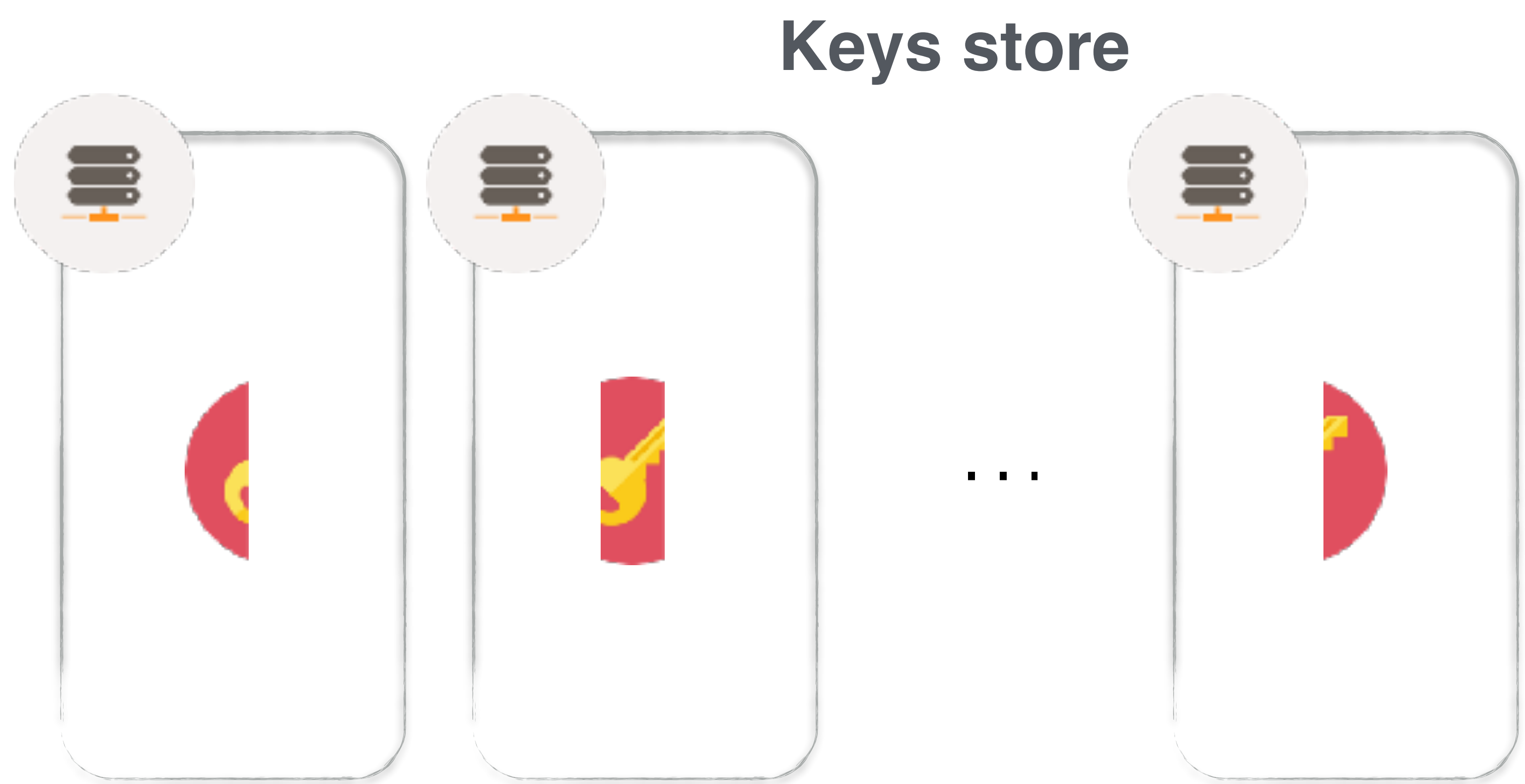
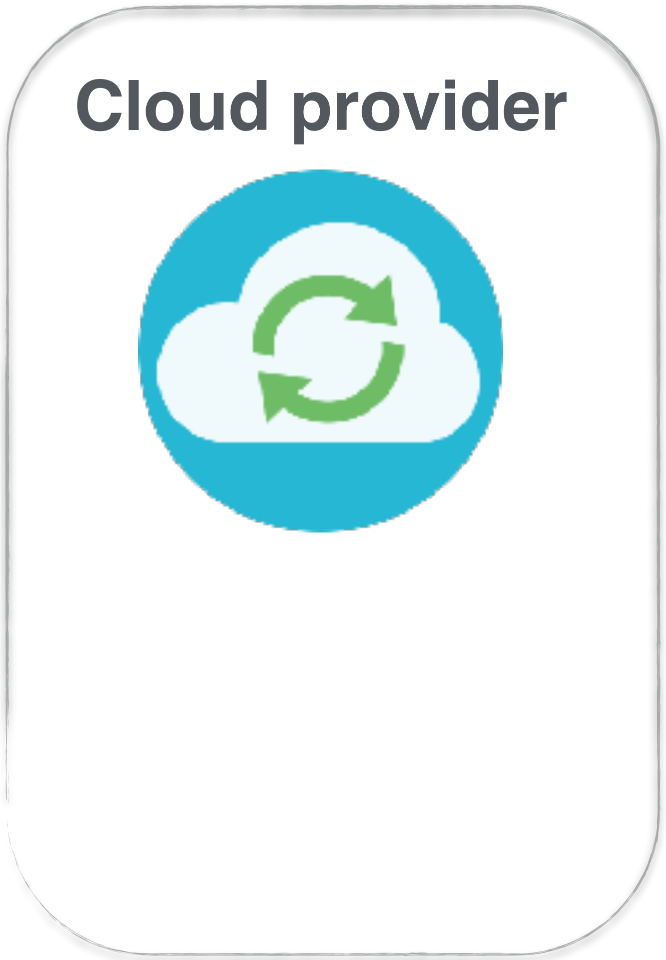
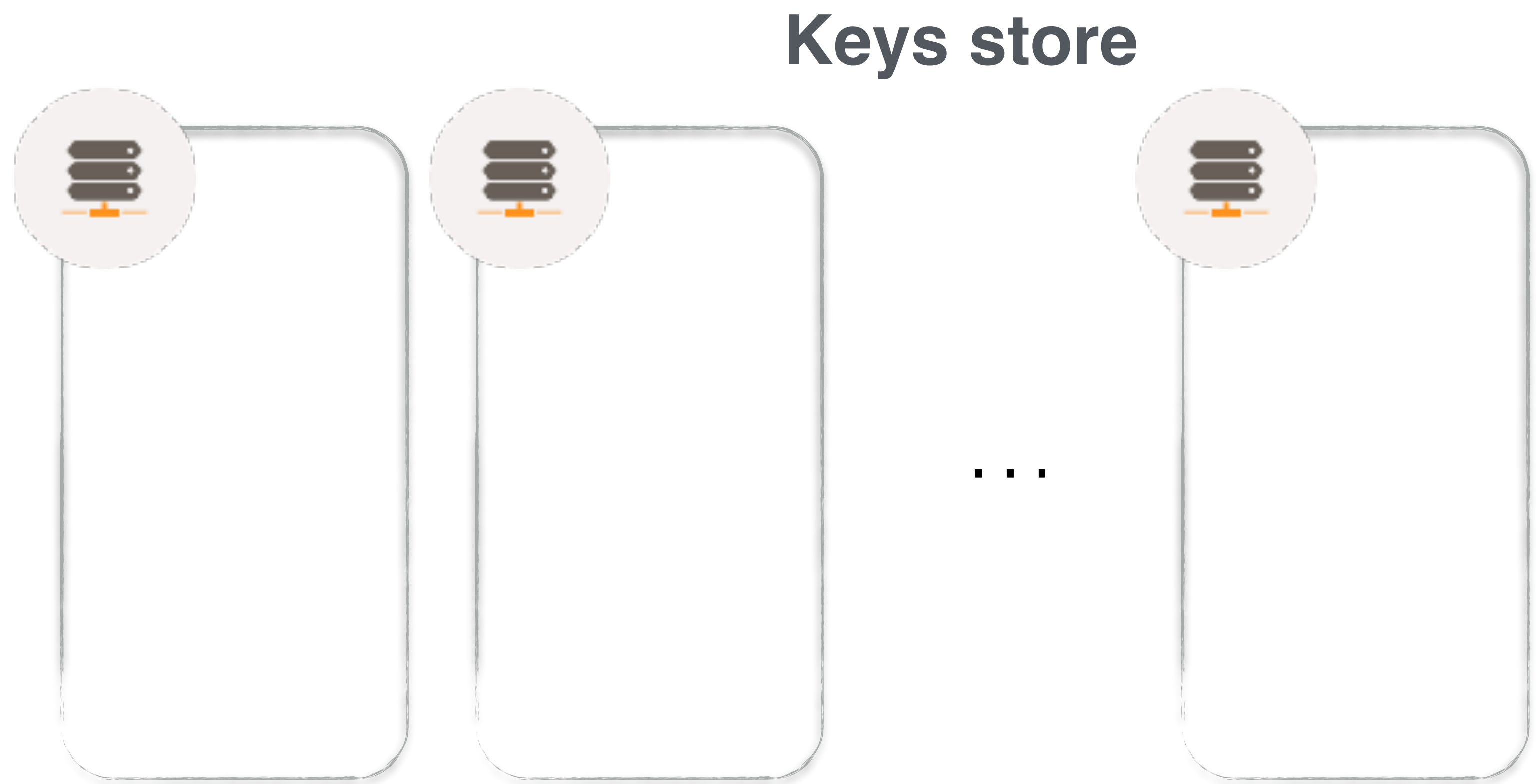


- User creates a cryptographic key.
- Encrypts her data using this key.
- Stores her secret key into n servers by using her password and some public information.
- Stores the data into the provider.

PPSS: Password-Protected Secret Sharing





- After $t+1$ interactions using her password, the user can recover her secret key

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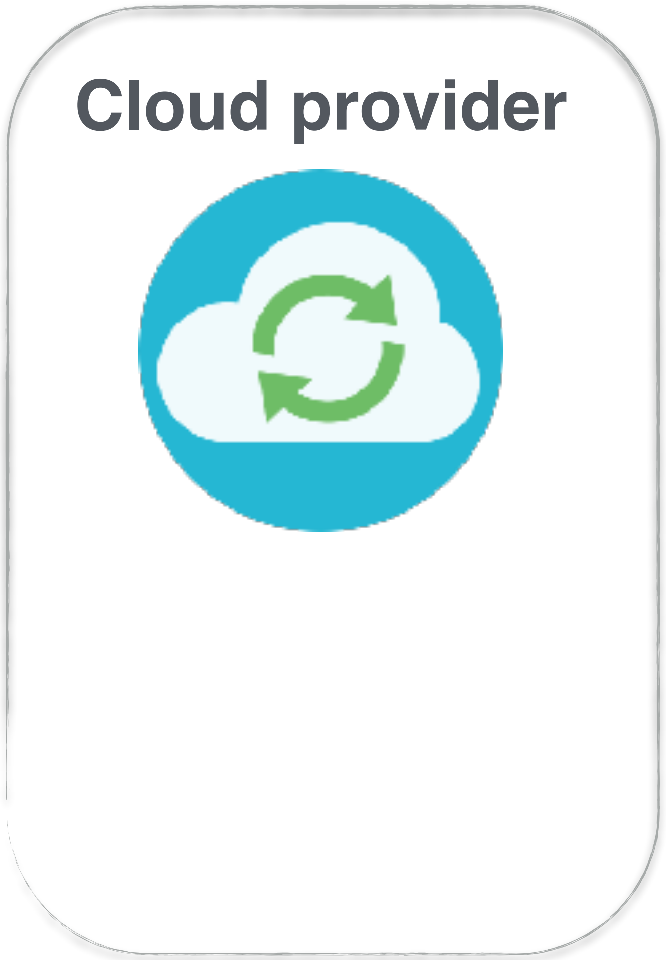

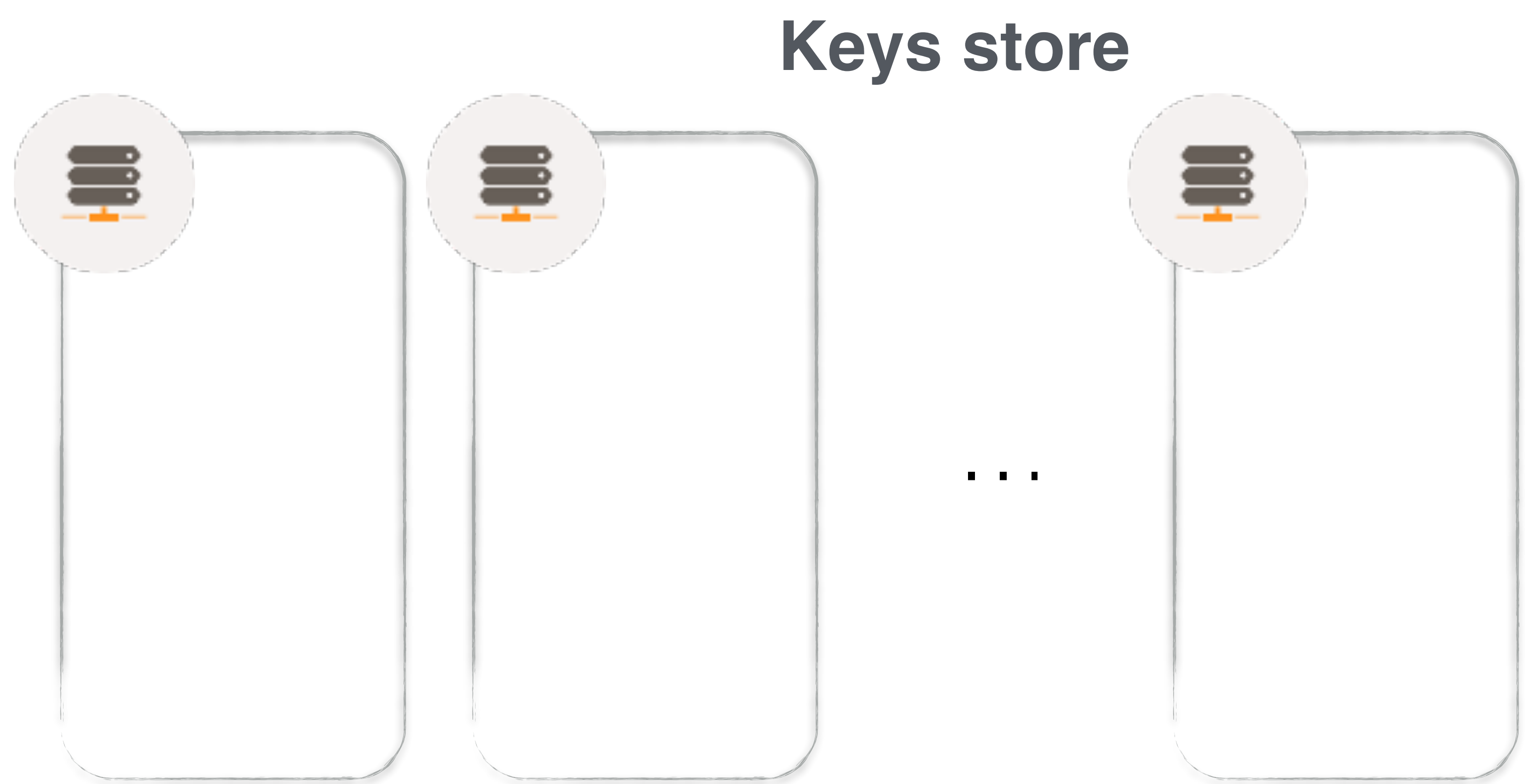

- After $t + 1$ interactions using her password, the user can recover her secret key

PPSS: Properties

- A PPSS scheme defines two steps:

Initialization: Secret & password are processed

Reconstruction: The user can recover the secret by interacting with a subset of $t+1$ servers.

- Additional properties:

Soundness: Even if the adversary cannot make the user recover a different secret.

Robustness: The recovery is guaranteed if there are $t+1$ non-corrupt servers.

PPSS: Instantiations of PPSS

Scheme	Messages	Client	inter-server	Robust	ZKP
BJSL11	4	PKI	PKI	No	Costly
CLLN14	10	Std	PKI	No	Costly
JKK14	2	CRS	None	Yes	Costly
JKKX16	2	CRS	None	No	No

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This work	2	CRS	None	Yes	No

Robust Password-Protected Secret Sharing

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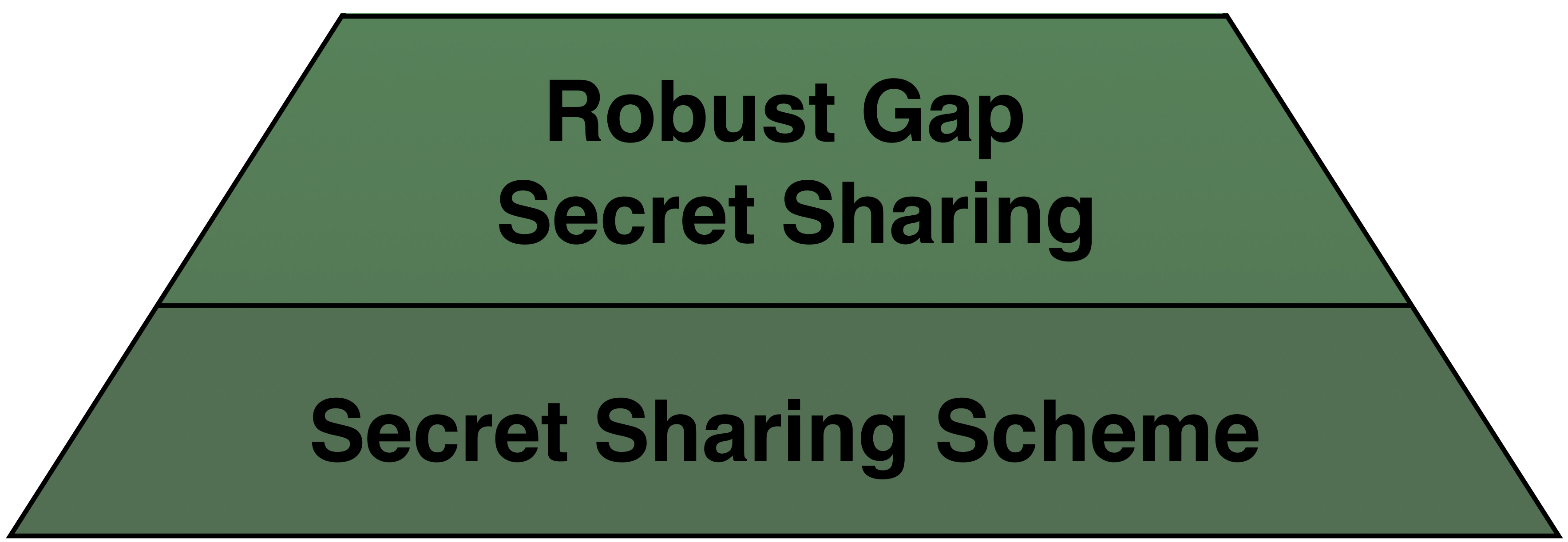

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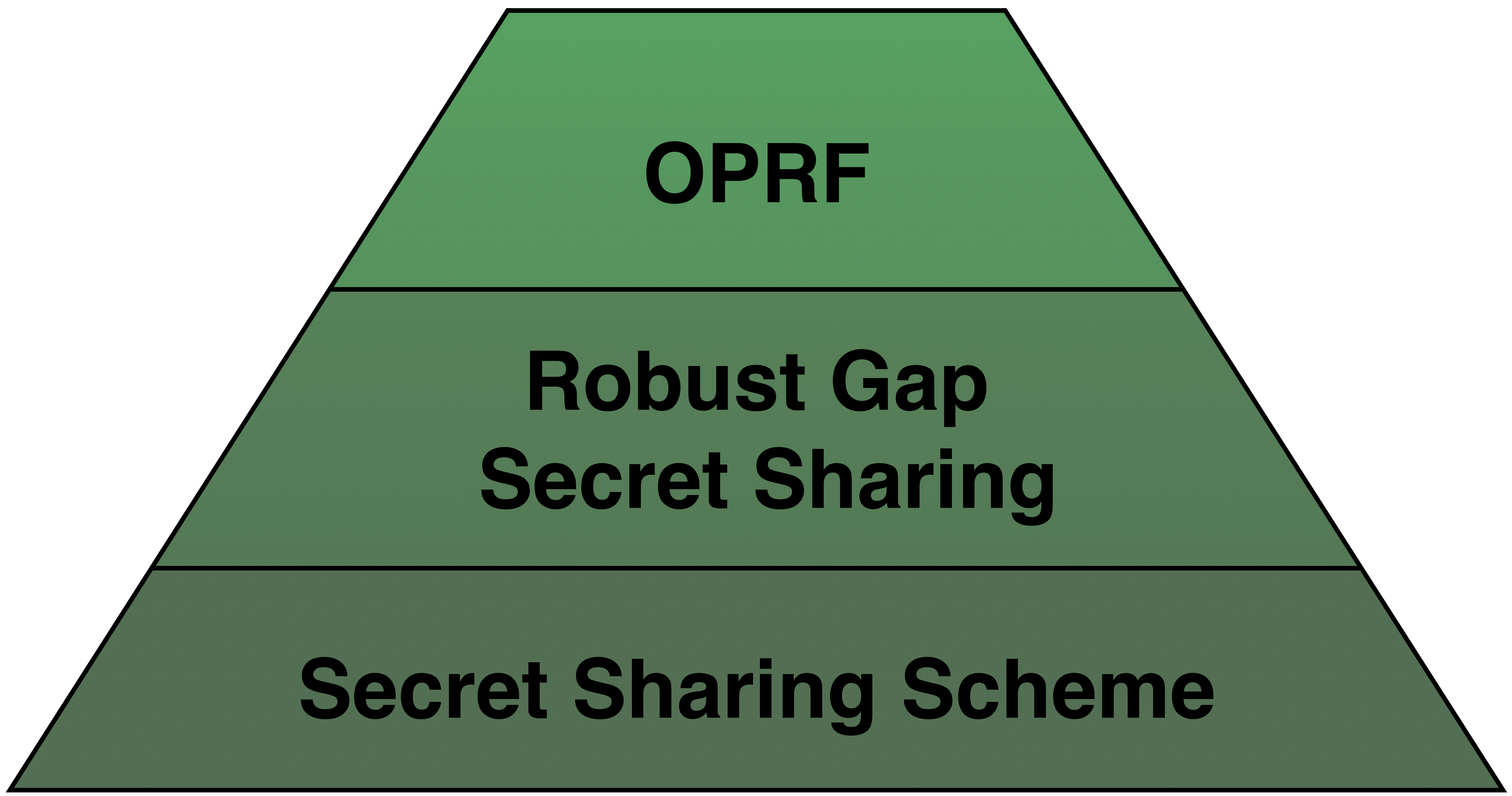

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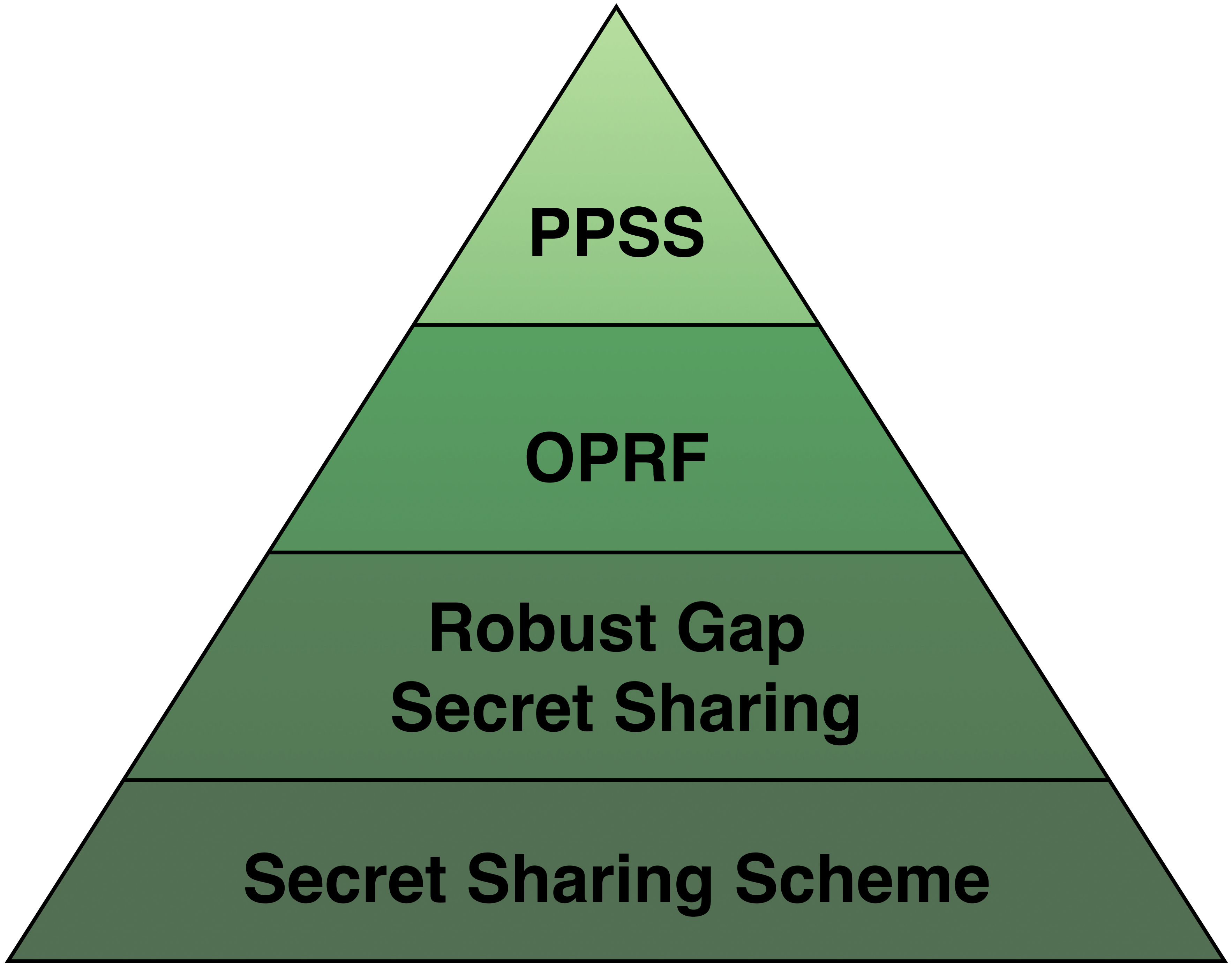
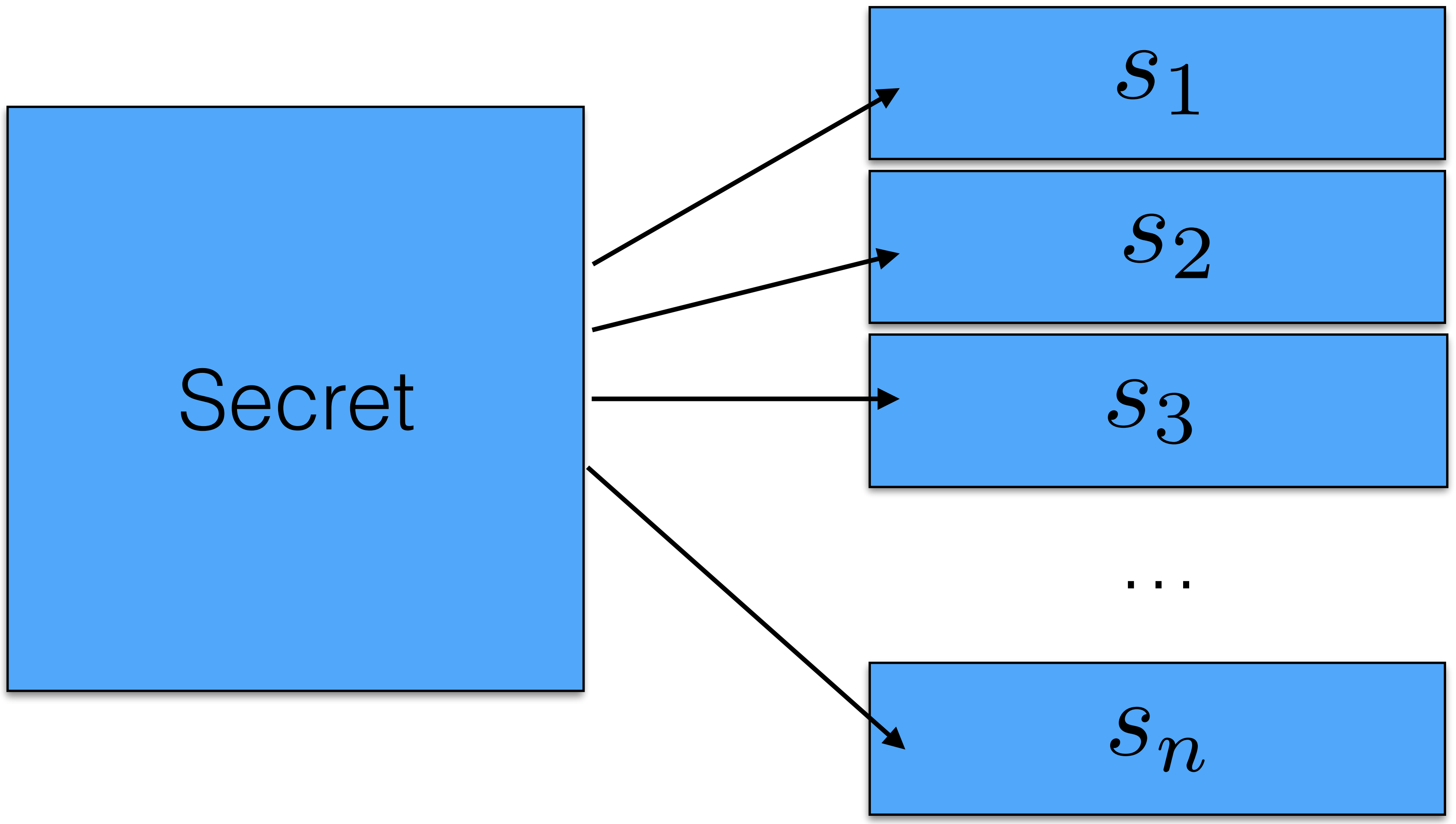


PPSS: Secret Sharing Scheme



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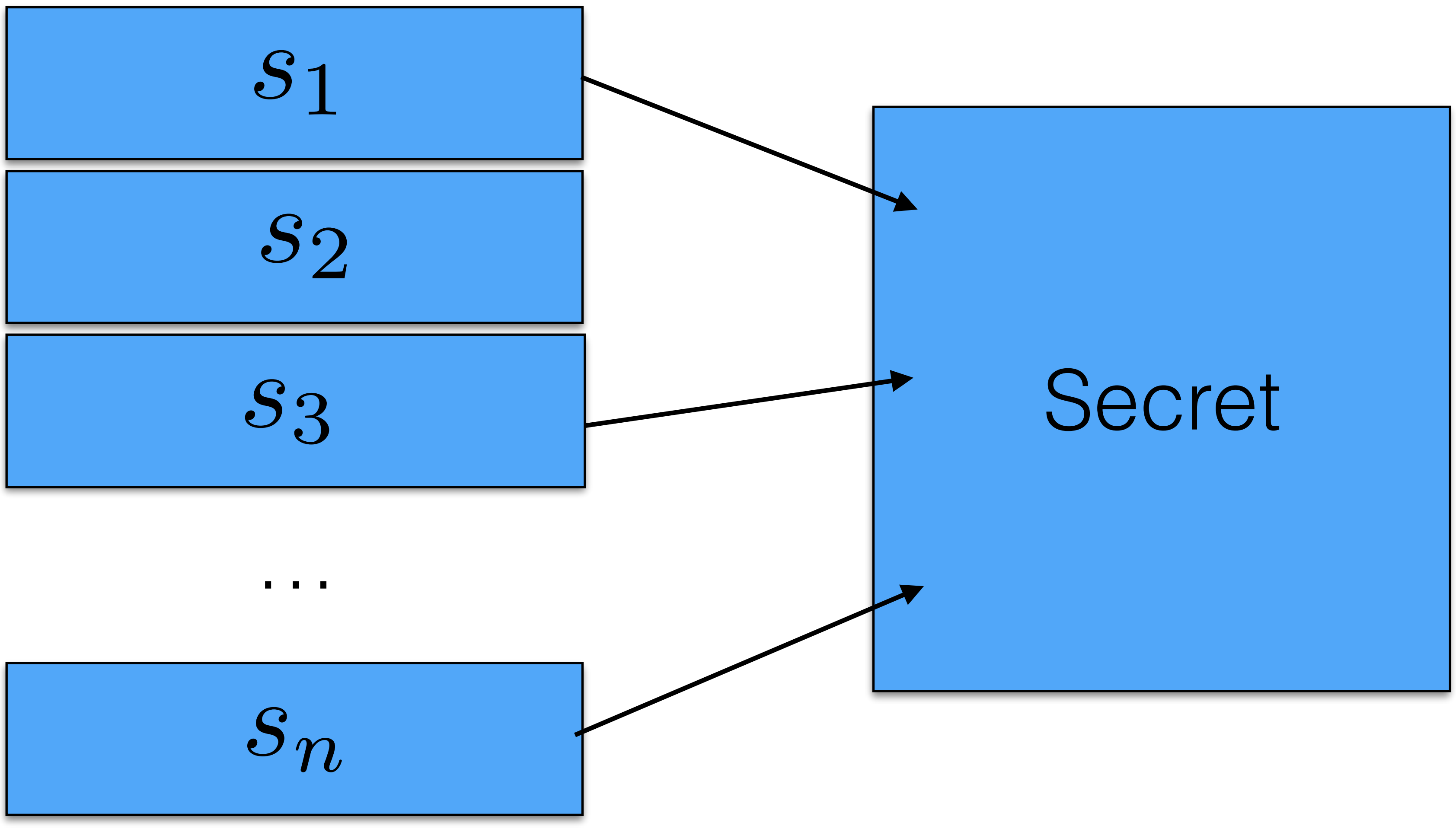


PPSS: Robust Gap Secret Sharing Scheme

How do we implement robustness?

PPSS: Robust Gap Secret Sharing Scheme

Assume a set of valid shares from a Threshold SSS

s_1

s_2

s_3

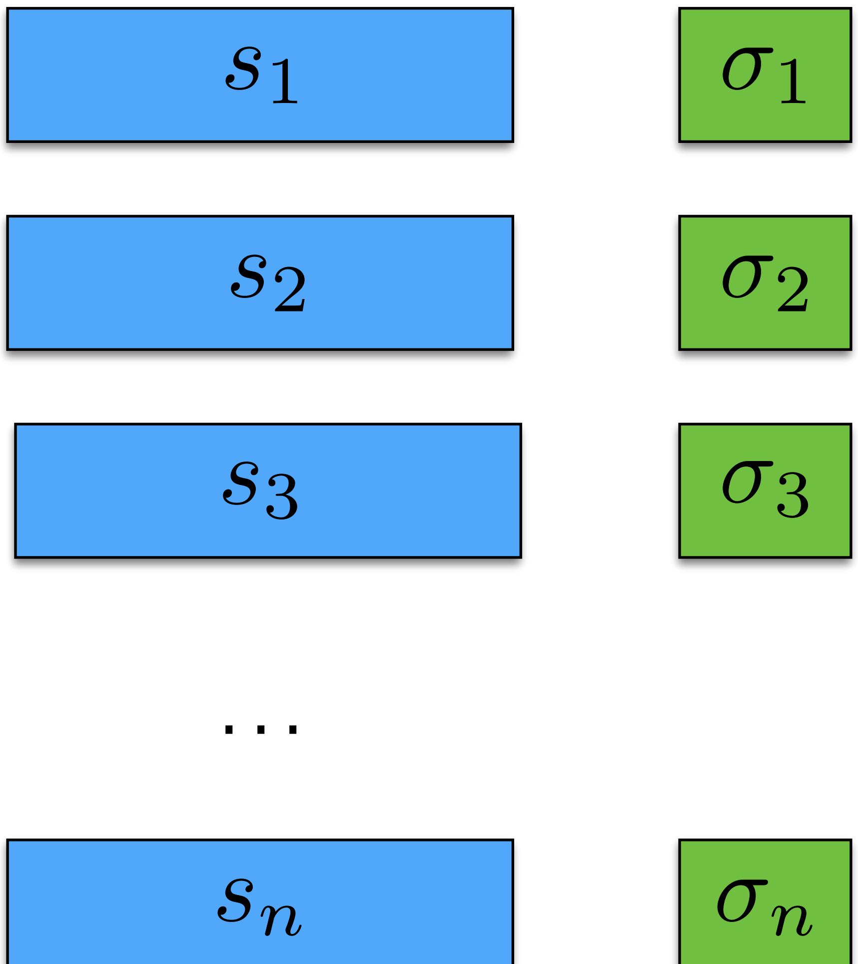
$\ldots$

s_n

$$(s_1, \ldots, s_n)$$

PPSS: Robust Gap Secret Sharing Scheme

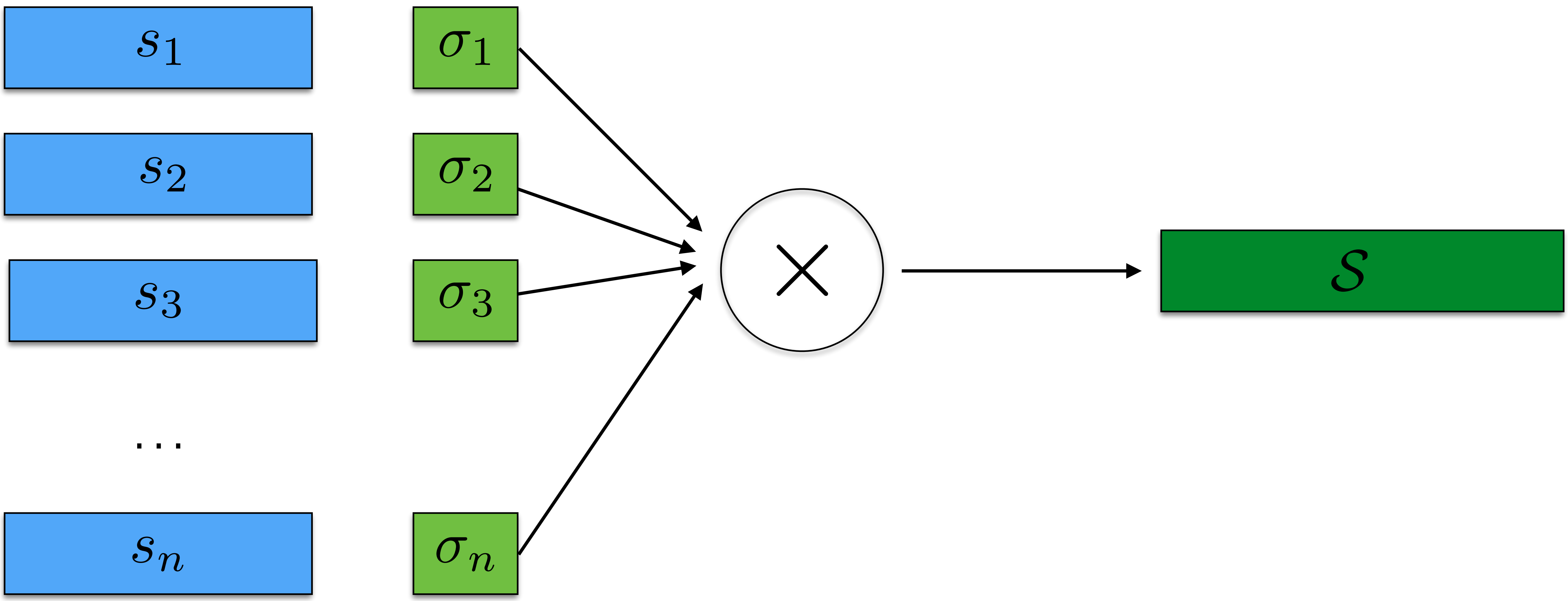
Fingerprint function: Hash function



$$(s_1, \ldots, s_n) \quad (\sigma_1, \ldots, \sigma_n)$$

PPSS: Robust Gap Secret Sharing Scheme

Generate a prime number N $\quad 2^{2k(n-t_r)+1} < N \leq 2^{2k(n-t_r)+2}$



$(s_1, \dots, s_n) \quad (\sigma_1, \dots, \sigma_n)$

$\mathcal{S} = \prod_{i=1}^{n} \sigma_i \mod N$

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Generate a prime number N $\quad 2^{2k(n-t_r)+1} < N \leq 2^{2k(n-t_r)+2}$

s_1

s_2

s_3

$\dots$

s_n

Output:

$\{s_k\}_n = \boxed{s_1}\ \boxed{s_2}\ \boxed{s_3} \dots \boxed{s_n}$

$\mathsf{SSInfo} = \{ \boxed{\mathcal{S}}, N \}$

$\mathcal{S}$

$(s_1, \dots, s_n) \quad (\sigma_1, \dots, \sigma_n)$

$\mathcal{S} = \prod_{i=1}^{n} \sigma_i \mod N$

PPSS: Robust Gap Secret Sharing Scheme

How can we decide which are the valid sets of shares to reconstruct?

PPSS: Robust Gap Secret Sharing Scheme

Given $\mathsf{SSInfo} = \{\ \mathcal{S}\ , N\ \}$

s_1

s_2

s_3

$\ldots$

s_n

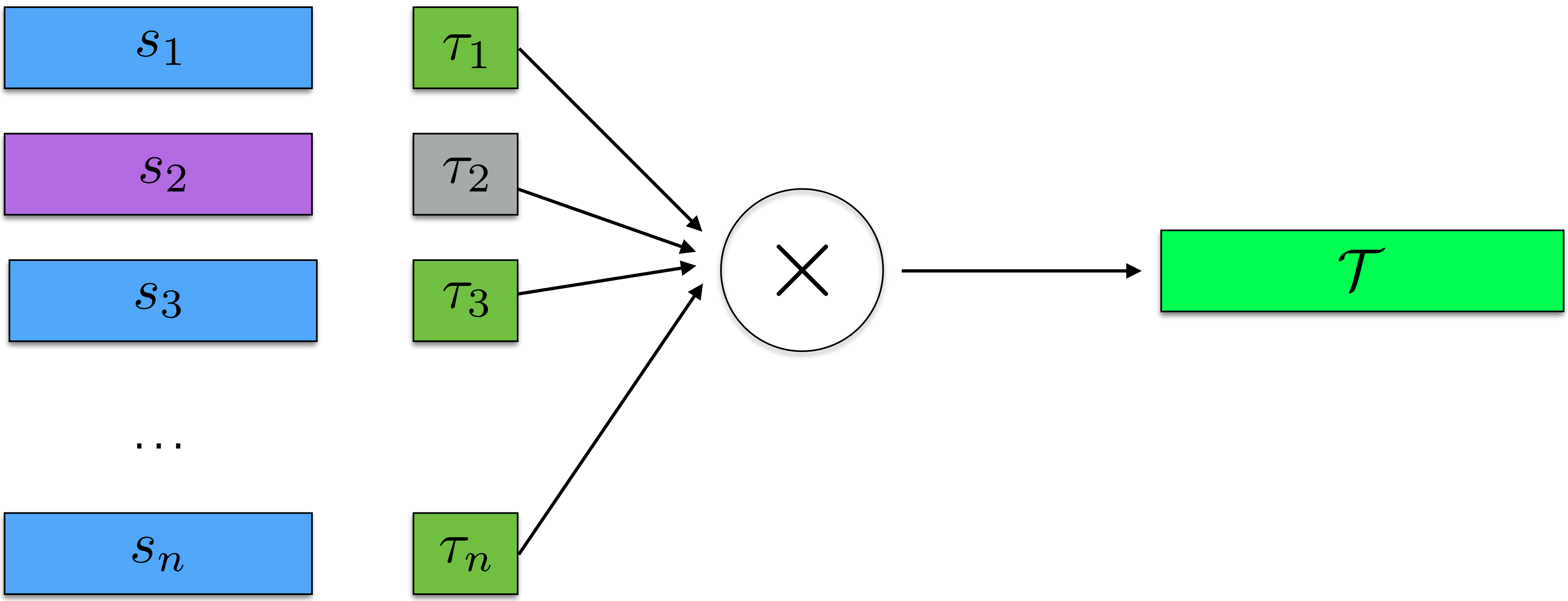
PPSS: Robust Gap Secret Sharing Scheme

Given $\mathsf{SSInfo} = \{ \boxed{\mathcal{S}} , N \}$

s_1	τ_1
s_2	τ_2
s_3	τ_3
$\ldots$	
s_n	τ_n

PPSS: Robust Gap Secret Sharing Scheme

Given $\mathsf{SSInfo} = \{ \ \mathcal{S} \ , N \ \}$



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Given $\mathsf{SSInfo} = \{\ \mathcal{S}\ , N\ \}$

$$\gamma = \frac{\mathcal{T}}{\mathcal{S}}$$

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$$\gamma = \frac{\boxed{\mathcal{T}}}{\boxed{\mathcal{S}}} = \frac{\boxed{\tau_1}\ \boxed{\tau_2}\ \boxed{\tau_3}\ \ldots\ \boxed{\tau_n}}{\boxed{\sigma_1}\ \boxed{\sigma_2}\ \boxed{\sigma_3}\ \ldots\ \boxed{\sigma_n}}$$

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$$|\gcd(\tau_1, \mathcal{T}')| \approx 1$$

PPSS: Robust Gap Secret Sharing Scheme

Given $\mathsf{SSInfo} = \{\ \boxed{\mathcal{S}}\ , N\ \}$

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$$|\gcd(\boxed{\tau_1}, \boxed{\mathcal{T}'})| \approx 1$$

Correct fingerprint!

PPSS: Robust Gap Secret Sharing Scheme

Given SSInfo $= \{ \boxed{\mathcal{S}} , N \}$

$$\gamma = \frac{\boxed{\mathcal{T}}}{\boxed{\mathcal{S}}} = \frac{\boxed{\tau_1}\ \boxed{\tau_2}\ \boxed{\tau_3}\ \ldots\ \boxed{\tau_n}}{\boxed{\sigma_1}\ \boxed{\sigma_2}\ \boxed{\sigma_3}\ \ldots\ \boxed{\sigma_n}} = \frac{\boxed{\mathcal{T}'}}{\boxed{\mathcal{S}'}}$$

$|\gcd(\boxed{\tau_1}, \boxed{\mathcal{T}'})| \approx 1$ Correct fingerprint!

$|\gcd(\boxed{\tau_2}, \boxed{\mathcal{T}'})| \approx k$ Incorrect fingerprint!

PPSS: Robust Gap Secret Sharing Scheme

Given $\mathsf{SSInfo} = \{ \boxed{\mathcal{S}}, N \}$

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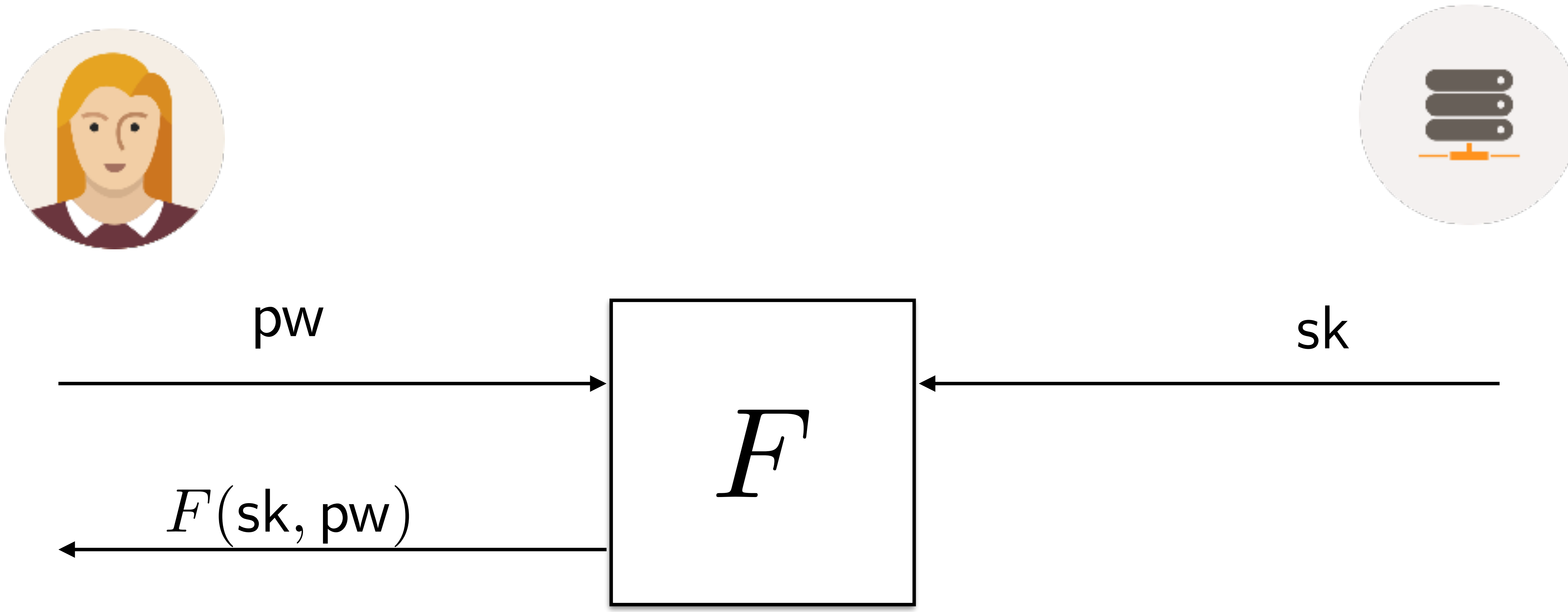
$|\gcd(\boxed{\tau_2}, \boxed{\mathcal{T}'})| \approx k$ Incorrect fingerprint!

$|\gcd(\boxed{\tau_3}, \boxed{\mathcal{T}'})| \approx 1$ Correct fingerprint!

PPSS: Oblivious PRF



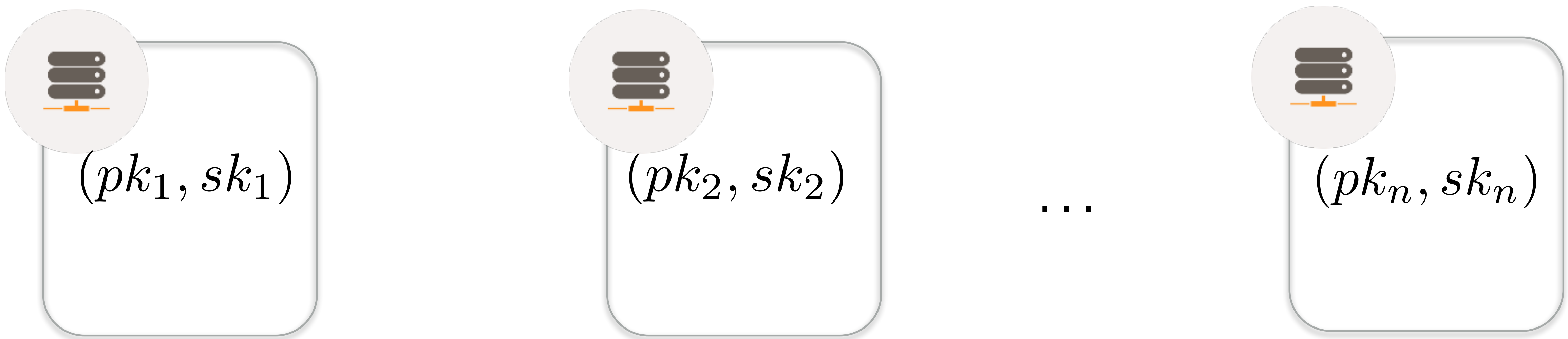
- The output is indistinguishable from random
- The server learns nothing

PPSS: Password-Protected Secret Sharing

Initialization phase

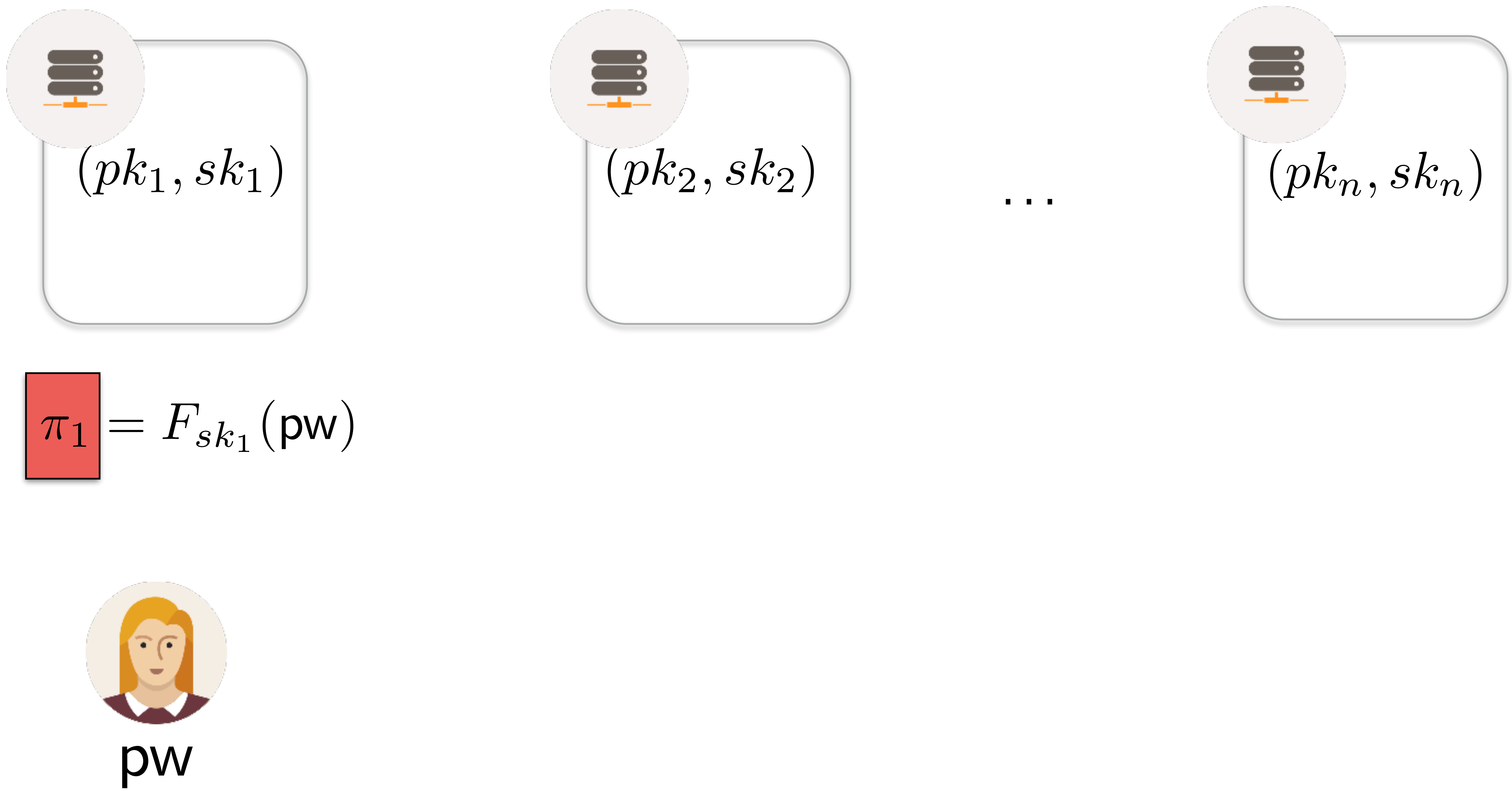
PPSS: Initialization

The user interacts with n servers to obliviously evaluate the PRF



PPSS: Initialization

The user interacts with n servers to obliviously evaluate the PRF



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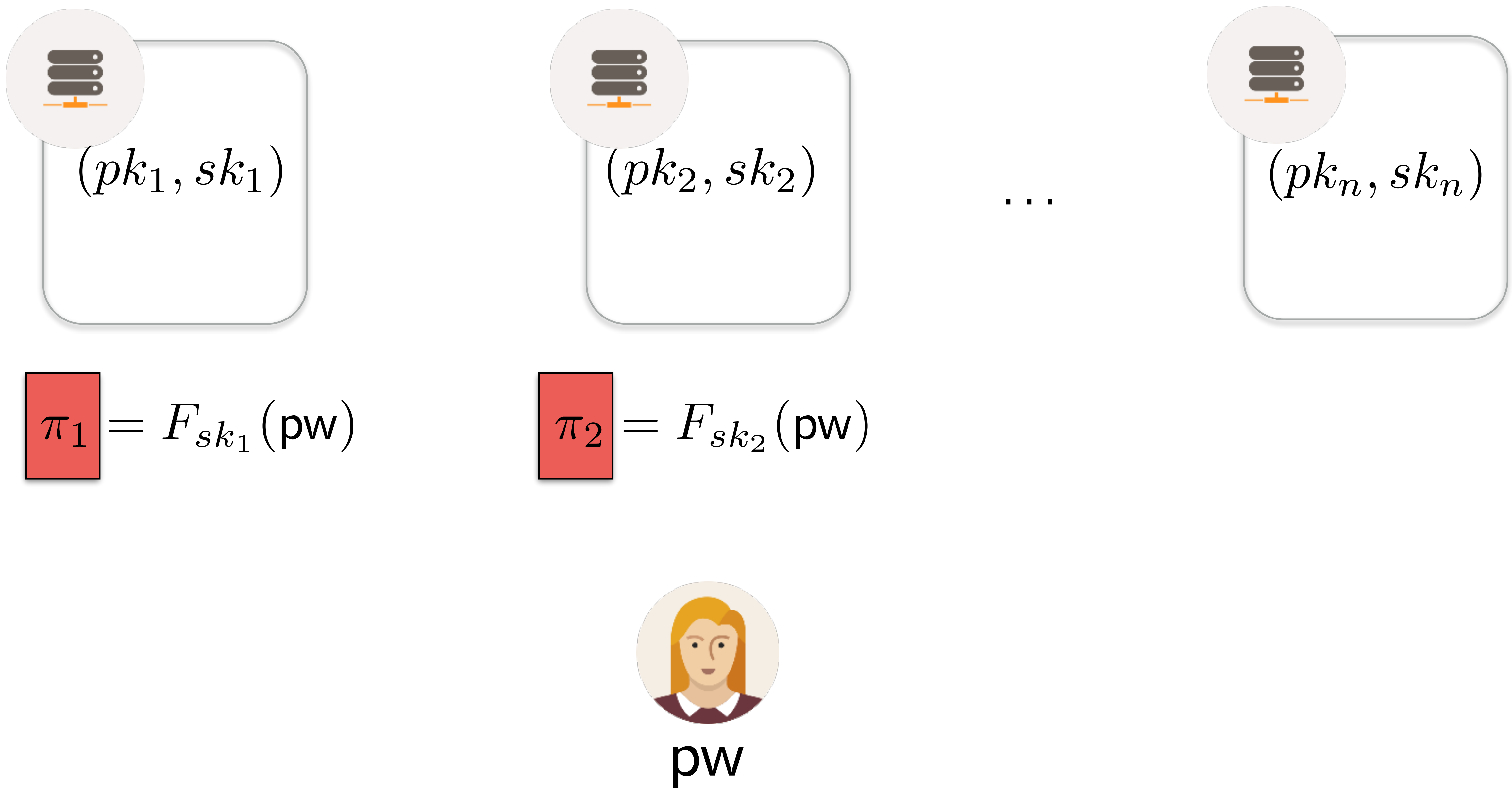

PPSS: Initialization

The user interacts with n servers to obliviously evaluate the PRF

(pk_1, sk_1) (pk_2, sk_2) $\dots$ (pk_n, sk_n)

$\pi_1 = F_{sk_1}(\mathsf{pw})$ $\pi_2 = F_{sk_2}(\mathsf{pw})$ $\pi_n = F_{sk_n}(\mathsf{pw})$

pw

PPSS: Initialization

Each share is encrypted using the each PRF evaluation

(pk_1, sk_1)

(pk_2, sk_2)

$\dots$

(pk_n, sk_n)

$\boxed{\pi_1} = F_{sk_1}(\mathsf{pw})$

$\boxed{\pi_2} = F_{sk_2}(\mathsf{pw})$

$\boxed{\pi_n} = F_{sk_n}(\mathsf{pw})$

$$R = K || r$$

$\mathsf{pw} \quad \{\pi_k\}_n \quad \{pk_k\}_n$

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$\ldots$

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$$R = K || r$$

$$(s_1, \ldots, s_n, \mathsf{SSInfo}) \leftarrow \mathsf{ShareGen}(R)$$

pw $\{\pi_k\}_n$ $\{pk_k\}_n$

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$$R = K || r$$

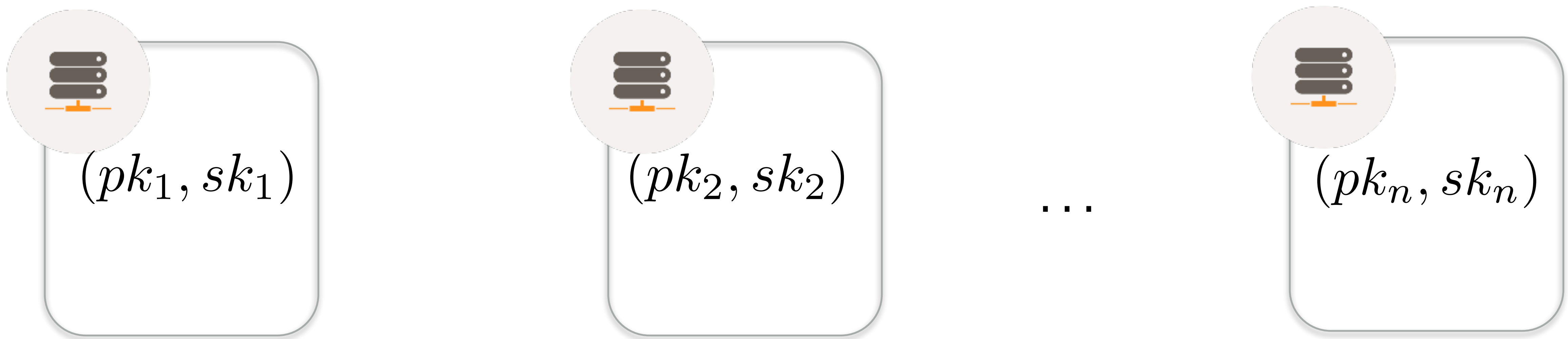
$$(s_1, \ldots, s_n, \mathsf{SSInfo}) \leftarrow \mathsf{ShareGen}(R)$$

$$\sigma_k = \pi_k \oplus s_k$$

pw $\{\pi_k\}_n$ $\{pk_k\}_n$

PPSS: Initialization

The user computes a commitment



$$C = \mathsf{Commit}(pw, H(\{pk_k\}_n, \{\sigma_k\}_n, \mathsf{SSInfo}, K); r)$$

pw $\{\pi_k\}_n$ $\{pk_k\}_n$ K r SSInfo $\{\sigma_k\}_n$

PPSS: Initialization

The user uploads the encrypted data

(pk_1, sk_1)

PInfo

(pk_2, sk_2)

PInfo

$\dots$

(pk_n, sk_n)

PInfo

$$\text{PInfo} = (\boxed{\{pk_k\}_n}, \boxed{\{\sigma_k\}_n}, \boxed{\text{SSInfo}}, C)$$

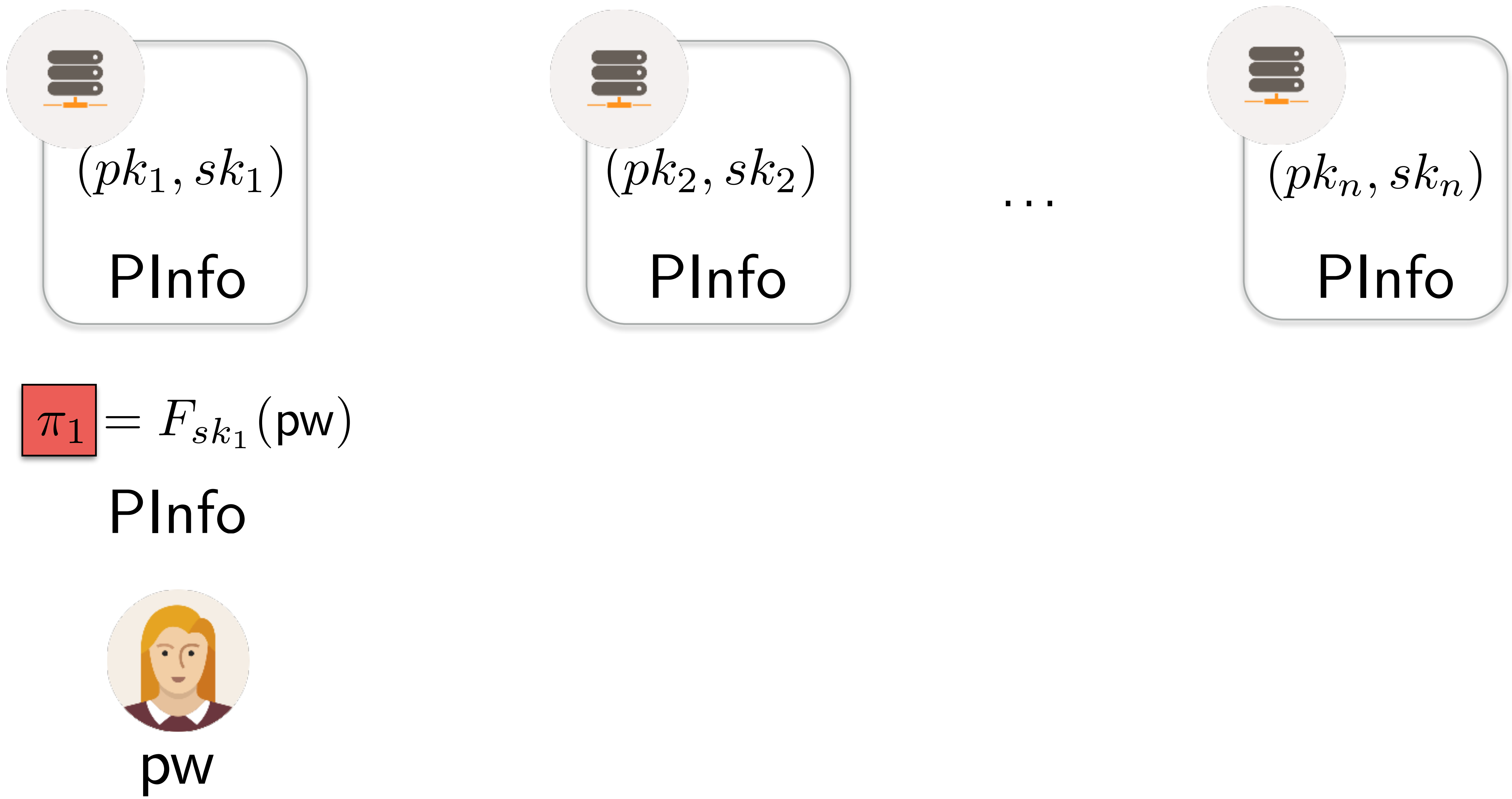
pw $\{\pi_k\}_n$ $\{pk_k\}_n$ K r SSInfo $\{\sigma_k\}_n$ C

PPSS: Password-Protected Secret Sharing

Reconstruction phase

PPSS: Reconstruction

The user interacts with the server



PPSS: Reconstruction

The user interacts with the server

(pk_1, sk_1)

PInfo

(pk_2, sk_2)

PInfo

$\ldots$

(pk_n, sk_n)

PInfo

$\boxed{\pi_1} = F_{sk_1}(\mathsf{pw})$

$\boxed{\pi_2} = F_{sk_2}(\mathsf{pw})$

PInfo

pw

PPSS: Reconstruction

The user interacts with the server

(pk_1, sk_1) PInfo

(pk_2, sk_2) PInfo

$\ldots$

(pk_n, sk_n) PInfo

$\boxed{\pi_1} = F_{sk_1}(\text{pw})$

$\boxed{\pi_2} = F_{sk_2}(\text{pw})$

$\boxed{\pi_n} = F_{sk_n}(\text{pw})$

PInfo

pw

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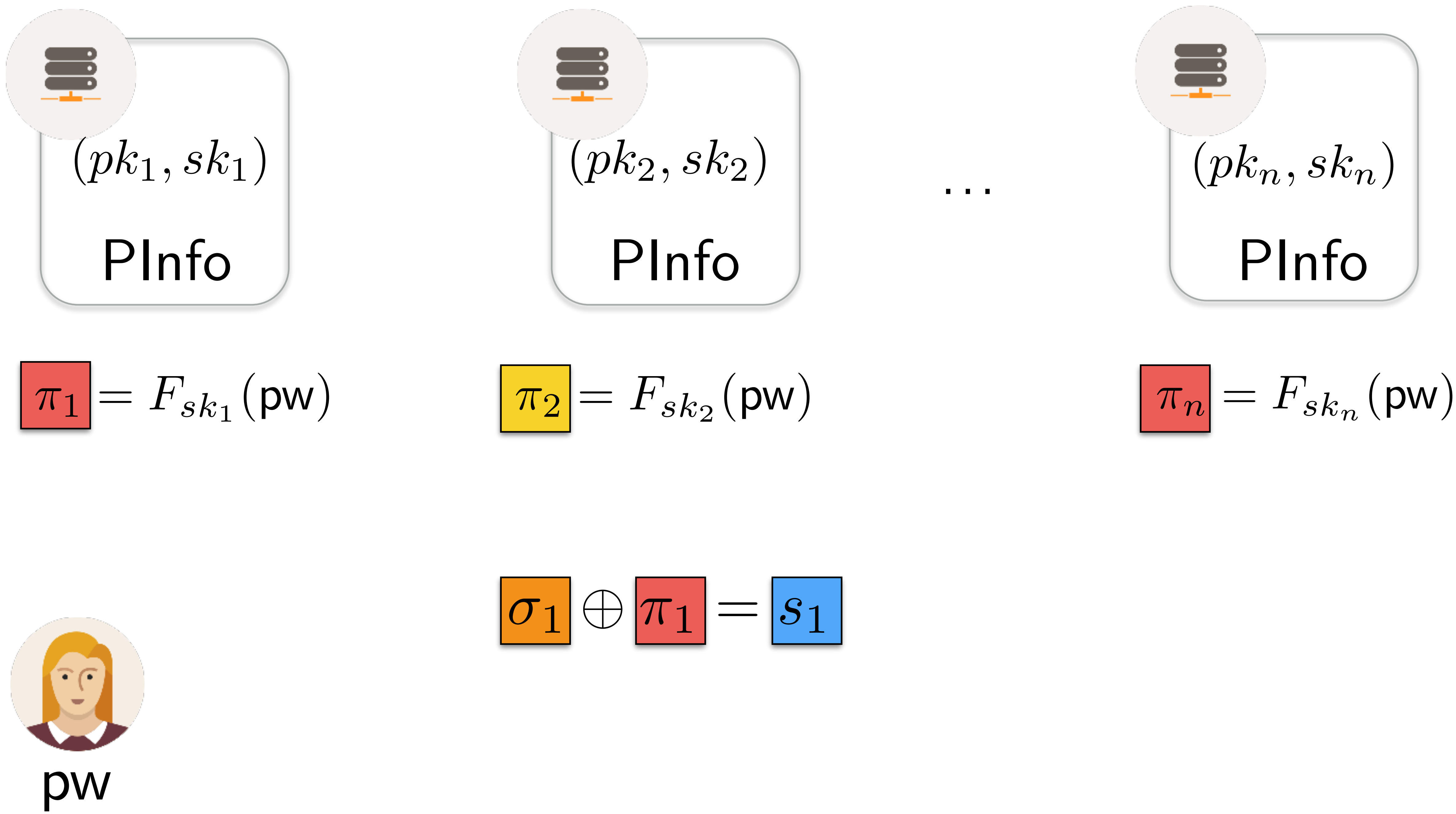

PPSS: Reconstruction

The user interacts with the server

(pk_1, sk_1)

PInfo

(pk_2, sk_2)

PInfo

$\cdots$

(pk_n, sk_n)

PInfo

$\pi_1 = F_{sk_1}(\text{pw})$

$\pi_2 = F_{sk_2}(\text{pw})$

$\pi_n = F_{sk_n}(\text{pw})$

$\sigma_1 \oplus \pi_1 = s_1$

$\sigma_2 \oplus \pi_2 = s_2$

pw

PPSS: Reconstruction

The user interacts with the server

(pk_1, sk_1)

PInfo

(pk_2, sk_2)

PInfo

$\ldots$

(pk_n, sk_n)

PInfo

$\pi_1 = F_{sk_1}(\text{pw})$

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pw

$\sigma_1 \oplus \pi_1 = s_1$

$\sigma_2 \oplus \pi_2 = s_2$

$\sigma_3 \oplus \pi_3 = s_3$

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The user interacts with the server

(pk_1, sk_1)

PInfo

(pk_2, sk_2)

PInfo

$\dots$

(pk_n, sk_n)

PInfo

$\pi_1 = F_{sk_1}(\text{pw})$

$\pi_2 = F_{sk_2}(\text{pw})$

$\pi_n = F_{sk_n}(\text{pw})$

pw

$$\sigma_1 \oplus \pi_1 = s_1$$

$$\sigma_2 \oplus \pi_2 = s_2$$

$$\sigma_3 \oplus \pi_3 = s_3$$

$$\sigma_n \oplus \pi_n = s_n$$

PPSS: Reconstruction

The user interacts with the server

(pk_1, sk_1)

PInfo

(pk_2, sk_2)

PInfo

$\dots$

(pk_n, sk_n)

PInfo

$\boxed{\pi_1} = F_{sk_1}(\mathsf{pw})$

$\boxed{\pi_2} = F_{sk_2}(\mathsf{pw})$

$\boxed{\pi_n} = F_{sk_n}(\mathsf{pw})$

pw

$$\text{Reconstruct}\left(\begin{array}{c} \boxed{s_1} \\ \boxed{s_2} \\ \boxed{s_3} \\ \boxed{s_n} \end{array} \right) \stackrel{?}{=} R = K||r$$

PPSS: Reconstruction

The user interacts with the server

(pk_1, sk_1)

PInfo

(pk_2, sk_2)

PInfo

$\ldots$

(pk_n, sk_n)

PInfo

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$\boxed{\pi_2} = F_{sk_2}(\mathsf{pw})$

$\boxed{\pi_n} = F_{sk_n}(\mathsf{pw})$

pw

$$\mathsf{Commit}(pw, H(\boxed{\{pk_k\}_n}, \boxed{\{\sigma_k\}_n}, \boxed{\mathsf{SSInfo}}, K); r) \stackrel{?}{=} C$$

PPSS: Proof [Sketch]

We build simulators for each PRFs

Adversary's probability is bounded by:

PPSS: Proof [Sketch]

We build simulators for each PRFs

Adversary's probability is bounded by:

Probability of guessing pw

$$\Pr[\mathsf{PWinC}] = \frac{q_u}{\#Dict}$$

PPSS: Proof [Sketch]

We build simulators for each PRFs

Adversary’s probability is bounded by:

Probability of breaking the OPRF

$$\Pr[\mathsf{PWinF}] = \varepsilon$$

PPSS: Comparison

- By using our robust threshold secret sharing we avoid the verifiability requirements for the OPRF.

- We reduce the communication to the half, because of the simplification of the OPRF.

- Our communication and computation complexities are asymptotically equivalent to [JKK14], in real life they are twice better.

Robust Password-Protected Secret Sharing

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PPSS: Experimental Results

Given SSInfo $= \{ \boxed{\mathcal{S}}, N \}$

$$\gamma = \frac{\boxed{\mathcal{T}}}{\boxed{\mathcal{S}}} = \frac{\boxed{\tau_1}\ \boxed{\tau_2}\ \boxed{\tau_3}\ \ldots\ \boxed{\tau_n}}{\boxed{\sigma_1}\ \boxed{\sigma_2}\ \boxed{\sigma_3}\ \ldots\ \boxed{\sigma_n}} = \frac{\boxed{\mathcal{T}'}}{\boxed{\mathcal{S}'}}$$

PPSS: Experimental Results

Given $\mathsf{SSInfo} = \{ \boxed{\mathcal{S}}, N \}$

$$\gamma = \frac{\boxed{\mathcal{T}}}{\boxed{\mathcal{S}}} = \frac{\boxed{\tau_1} \; \boxed{\tau_2} \; \boxed{\tau_3} \; \dots \; \boxed{\tau_n}}{\boxed{\sigma_1} \; \boxed{\sigma_2} \; \boxed{\sigma_3} \; \dots \; \boxed{\sigma_n}} = \frac{\boxed{\mathcal{T}''}}{\boxed{\mathcal{S}''}}$$

$$|\gcd(\boxed{\tau_1}, \boxed{\mathcal{T}''})| = 1$$

Correct fingerprint!

PPSS: Experimental Results

Given SSInfo $= \{ \boxed{\mathcal{S}}, N \}$

$$\gamma = \frac{\boxed{\mathcal{T}}}{\boxed{\mathcal{S}}} = \frac{\boxed{\tau_1}\ \boxed{\tau_2}\ \boxed{\tau_3}\ \ldots\ \boxed{\tau_n}}{\boxed{\sigma_1}\ \boxed{\sigma_2}\ \boxed{\sigma_3}\ \ldots\ \boxed{\sigma_n}} = \frac{\boxed{\mathcal{T}''}}{\boxed{\mathcal{S}''}}$$

$|\,\mathrm{gcd}(\boxed{\tau_1}, \boxed{\mathcal{T}''})| = 1$ Correct fingerprint!

$|\,\mathrm{gcd}(\boxed{\tau_1}, \boxed{\mathcal{T}''})| = 2$ Correct fingerprint!

$|\,\mathrm{gcd}(\boxed{\tau_1}, \boxed{\mathcal{T}''})| = 3$ Correct fingerprint!

PPSS: Experimental Results

$$|\gcd(\boxed{\tau_2}, \boxed{\mathcal{T}''})| = k$$ Incorrect fingerprint!

$$|\gcd(\boxed{\tau_2}, \boxed{\mathcal{T}''})| = k - 1$$ Incorrect fingerprint!

$$|\gcd(\boxed{\tau_2}, \boxed{\mathcal{T}''})| = k - 2$$ Incorrect fingerprint!

(a) $\gcd(\mathcal{T}'', \tau_i)$-bitlength for valid τ_i. (b) $\gcd(\mathcal{T}'', \tau_i)$-bitlength for invalid τ_i.

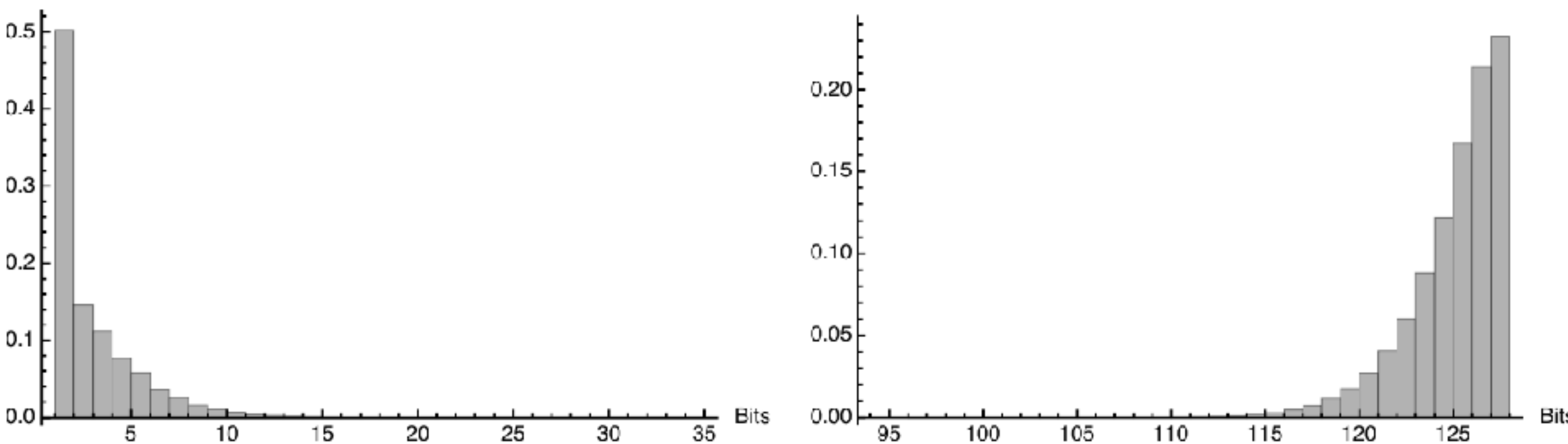

Fig. 1: Length in bits of $\gcd(\mathcal{T}'', \tau_i)$ for a fingerprint of size 128-bits and 32 shares

PPSS: CDH-based PRF (One-More Gap DH)

pw

$\mathsf{pk} = g^{\mathsf{sk}}$

sk

$\alpha \leftarrow \mathbb{Z}^*$

$A \leftarrow H_1(\mathsf{pw})^{\alpha}$

$\xrightarrow{\quad A \quad}$

$B \leftarrow A^{sk}$

$\xleftarrow{\quad B \quad}$

$C \leftarrow B^{1/\alpha} = H_1(\mathsf{pw})^{\mathsf{sk}}$

$F_{sk}(pw) = H_2(\mathsf{pw}, \mathsf{pk}, C)$

PPSS: DDH-based PRF

pk, $\{c_i = \mathsf{Enc}_{\mathsf{pk}}(a_i)\}$

$x = (x_1, x_2, \ldots, x_\ell) \in \{0,1\}^\ell$

$\alpha \leftarrow \mathbb{G}_s$

$C \leftarrow \mathsf{Enc}_{\mathsf{pk}}(\alpha \times a_0 \prod {a_i}^{x_i})$

sk $\in \mathbb{Z}_s$

$$\xrightarrow{\quad C \quad}$$

$$\longleftrightarrow$$

$$\xleftrightarrow{\mathsf{Proof}(\alpha, x_i)}$$

$$\xleftarrow{\quad G \quad}$$

$D \leftarrow \mathsf{Dec}_{\mathsf{sk}}(C)$

$G \leftarrow g^D$

$R \leftarrow G^{1/\alpha}$