Masking the GLP Lattice-Based Signature Scheme at any Order

Gilles Barthe (IMDEA Software Institute)
Sonia Belaïd (CryptoExperts)
Thomas Espitau (UPMC)
Pierre-Alain Fouque (Univ. Rennes I and IUF)
Benjamin Grégoire (INRIA Sophia Antipolis)
Mélissa Rossi (ENS Paris and Thales)
Mehdi Tibouchi (NTT Secure Platform Laboratories)

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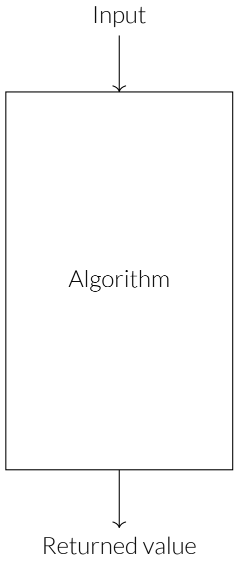
Masking a post-quantum signature

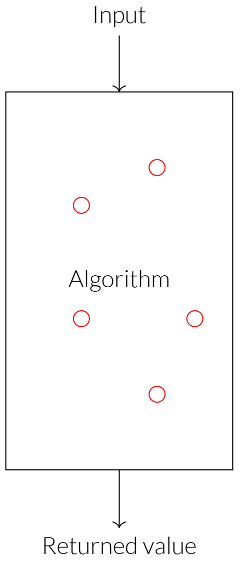
- Numerous side channel attacks against lattice-based schemes (Gaussian distributions, rejection sampling)
- Few countermeasures exist, especially on signatures
- Call for concrete implementations of post-quantum cryptography

Strong countermeasures needed

Leakage models and masking



Leakage models and masking



Ishai, Sahai and Wagner model [ISW03] :
The attacker can access the exact values of at most d intermediate values

Proof-Friendly

Leakage models and masking

Input

Algorithm

Returned value

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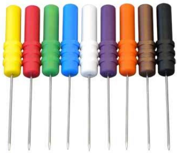
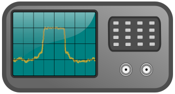


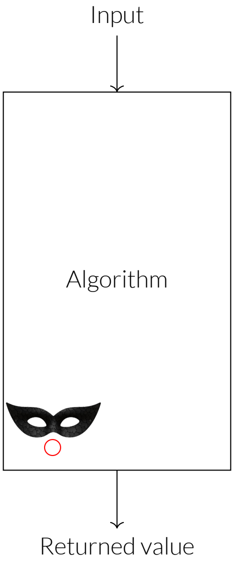
Proof-Friendly

Noisy leakage model [CJRR99, PR13]:
The attacker can access the **noisy** values of all the intermediate values

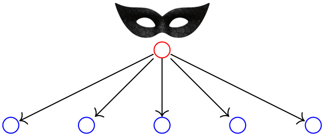


Realistic

Leakage models and masking



Security in the ISW model: d order **masking**
Each sensitive value is replaced by $d+1$ shares.



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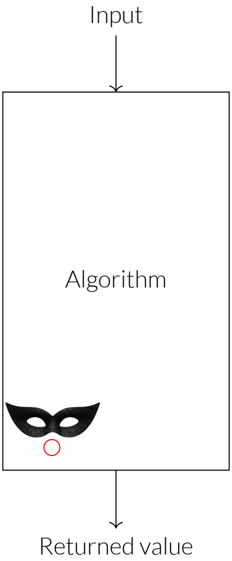

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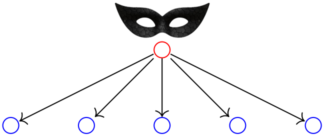

Such that it is impossible to recover the value without having all $d+1$ shares

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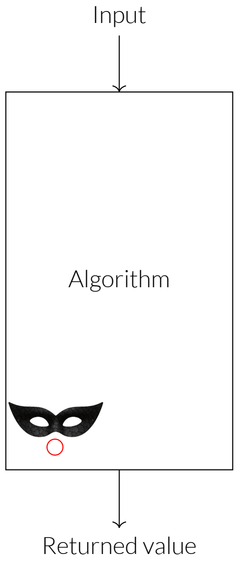

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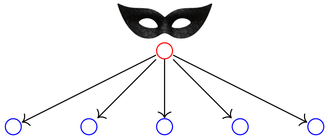

Such that it is impossible to recover the value without having all $d+1$ shares

Any strict subset of at most d shares is independant from the sensitive value

Our contribution

The first **provable** masked implementation of a lattice-based signature scheme **at any order**

- New techniques for masking lattice-based Fiat–Shamir with abort signatures
- New proofs for masking probabilistic algorithms

Why GLP signature scheme ?

Introduced in [Lyu09, Lyu12]
Implemented by **Güneysu, Lyubashevsky and Pöppelmann** in [GLP12]

- Ancestor of BLISS and Dilithium
- No Gaussians, only uniform distributions

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But still some new difficulties

- Probabilistic algorithm
- Reliance on rejection sampling

GLP Key derivation

$$\mathcal{R} = \frac{\mathbb{Z}_p[x]}{(x^n+1)}$$

$\mathcal{R}_k$: coefficients in the range $[-k, k]$

Algorithm 1 GLP key derivation

Ensure: Signing key $\boldsymbol{sk}$, verification key $\boldsymbol{pk}$

1: $\mathrm{s}_1, \mathrm{s}_2 \xleftarrow{\$} \mathcal{R}_1$ //s_1 and s_2 have coefficients in $\{-1, 0, 1\}$
2: $\mathrm{a} \xleftarrow{\$} \mathcal{R}$
3: $\mathrm{t} \leftarrow \mathrm{a}\mathrm{s}_1 + \mathrm{s}_2$
4: $\boldsymbol{sk} \leftarrow (\mathrm{s}_1, \mathrm{s}_2)$
5: $\boldsymbol{pk} \leftarrow (\mathrm{a}, \mathrm{t})$

- Based on the Decisional Compact Knapsack problem

GLP signature

- Fiat–Shamir with abort signature

Algorithm 2 GLP sign

Require: $m, pk = (a, t), sk = (s_1, s_2)$
Ensure: Signature σ

1: $y_1, y_2 \xleftarrow{\$} \mathcal{R}_k$ — Random generation
2: $c \leftarrow H(r = ay_1 + y_2, m)$ — Commitment and challenge
3: $z_1 \leftarrow s_1 c + y_1$
4: $z_2 \leftarrow s_2 c + y_2$
5: if z_1 or $z_2 \notin \mathcal{R}_{k-\alpha}$ then restart — Rejection Sampling
6: return $\sigma = (z_1, z_2, c)$

$$k = 2^{14} \quad \alpha = 16 \quad n = 512 \quad p = 8383489$$

Verification : $z_1, z_2 \in \mathcal{R}_{k-\alpha}$ and $c = H(az_1 + z_2 - tc, m)$

Structure of the countermeasure and its proof

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Every set of at most d intermediate variables can be perfectly simulated with at most d shares of each input.

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Non interferent with public outputs
Every set of at most d intermediate variables can be perfectly simulated **with the public outputs and** at most d shares of each input.

New

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We give some values (called outputs) to the attacker and prove that the countermeasure **does not leak more** than the outputs.

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3. A composition proof combines all the securities to the whole scheme

Masking GLP key generation

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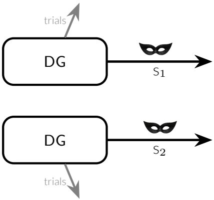

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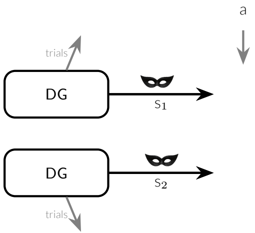

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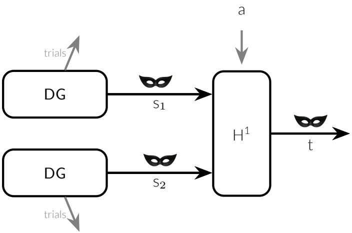

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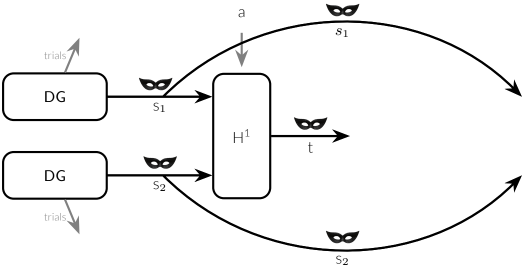

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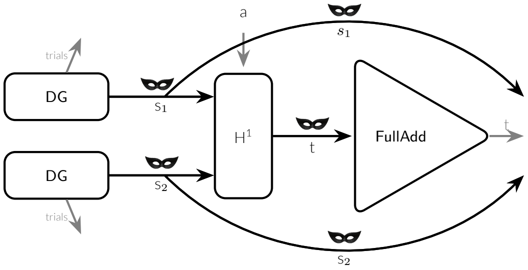

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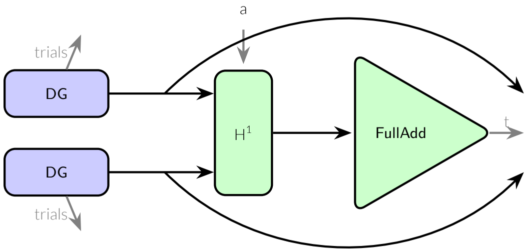

Not masked

Non interferent

Non interferent with public output $trials$

Masking the signature

Algorithm 2 GLP sign

Require: $\mathsf{m}, pk = (\mathsf{a}, \mathsf{t}), sk = (\mathsf{s}_1, \mathsf{s}_2)$
Ensure: Signature σ

1: $\mathsf{y}_1, \mathsf{y}_2 \xleftarrow{\$} \mathcal{R}_k$
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Masking the commitment : unnecessary

Distinguishing (c, r) pairs from uniform is heuristically[1] a hard problem even for rejected executions.

[1] Thanks' to V. Lyubashevsky, we also provided a non heuristic approach which requires somes changes in the algorithm

Masking the signature

Algorithm 3 Tweaked GLP sign

Require: $\mathrm{m}, \boldsymbol{pk} = (\mathrm{a}, \mathrm{t}), \boldsymbol{sk} = (\mathrm{s}_1, \mathrm{s}_2)$
Ensure: Signature $\boldsymbol{\sigma}$

$\mathrm{y}_1, \mathrm{y}_2 \xleftarrow{\$} \mathcal{R}_k$
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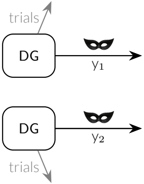

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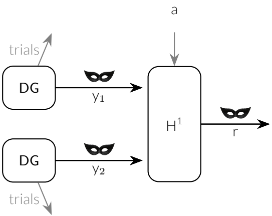

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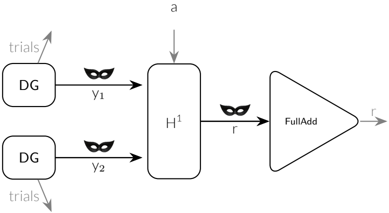

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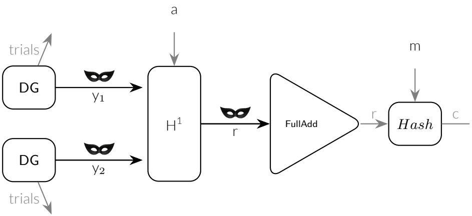

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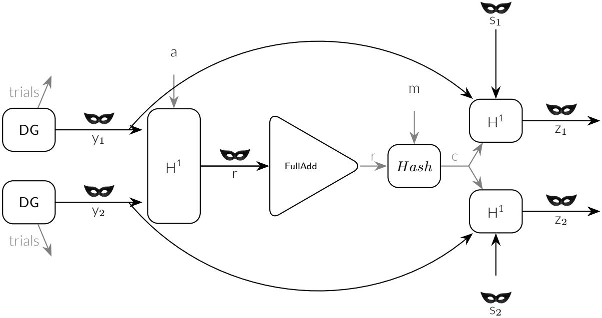

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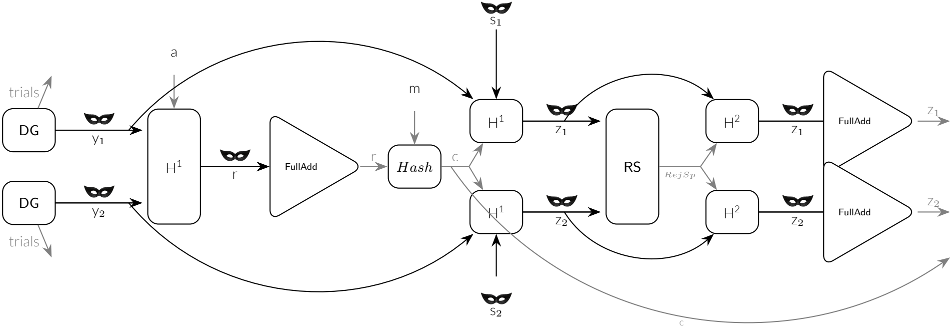
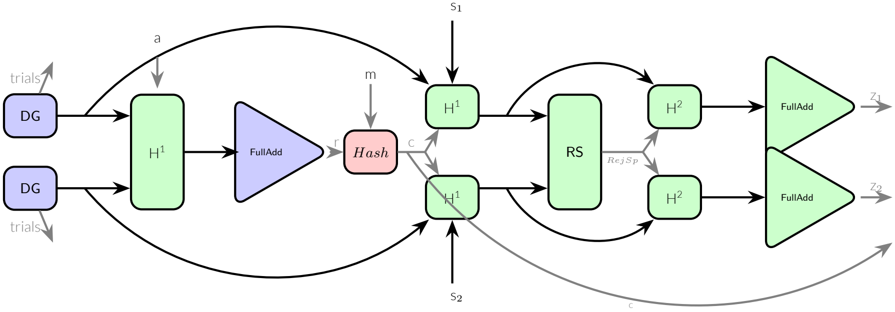


Composition



Not masked | Non interferent | Non interferent with public outputs $trials$ and r

Conversions Boolean to arithmetic

Proving the non interference of certain blocks (Rejection Sampling, Data Generation) was challenging

Algorithm 2 GLP signature

Require: $\mathsf{m}, \boldsymbol{pk}, \boldsymbol{sk}$
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$$\sum_{i=0}^{i=d} z_{1,i} \mod p \quad \leq \quad k - \alpha? \tag{1}$$

We had to adapt **arithmetic to Boolean conversions** from Coron, Großschädl and Vadnala in [CGV14].

$$\sum_{i=0}^{i=d} z_{1,i} \mod p \quad \rightarrow \quad \bigoplus_{i=0}^{i=d} z'_{1,i} \tag{2}$$

Performances

Table 1: Performances

Number of shares $(d+1)$	Unprotected	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{4}$	$\mathbf{5}$	$\mathbf{6}$
Total CPU time (s)	0.540	8.15	16.4	39.5	62.1	111
Penalty factor	—	$\times 15$	$\times 30$	$\times 73$	$\times 115$	$\times 206$

Timings are provided for $\mathbf{100}$ executions of the signing algorithm, on one core of an Intel Core i7-3770 CPU-based desktop machine.

- The code will be published soon
- Quite promising in view of the lack of optimization

Future work

In a nutshell,

- Provable masked implementation of **GLP** signature scheme
- New security notions adapted to Fiat–Shamir framework.

➳ Can be applied directly to **Dilithium** (implementation in progress, Vincent Migliore)

BLISS and **Dilithium-G**

➳ Gaussians

➳ Not sure the Hash function can be unmasked

Conclusion

Thank you for your attention

Questions ?

Blog article on the RISQ project webpage : http://risq.fr/?page_id=365&lang=en

Eprint : https://eprint.iacr.org/2018/381

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