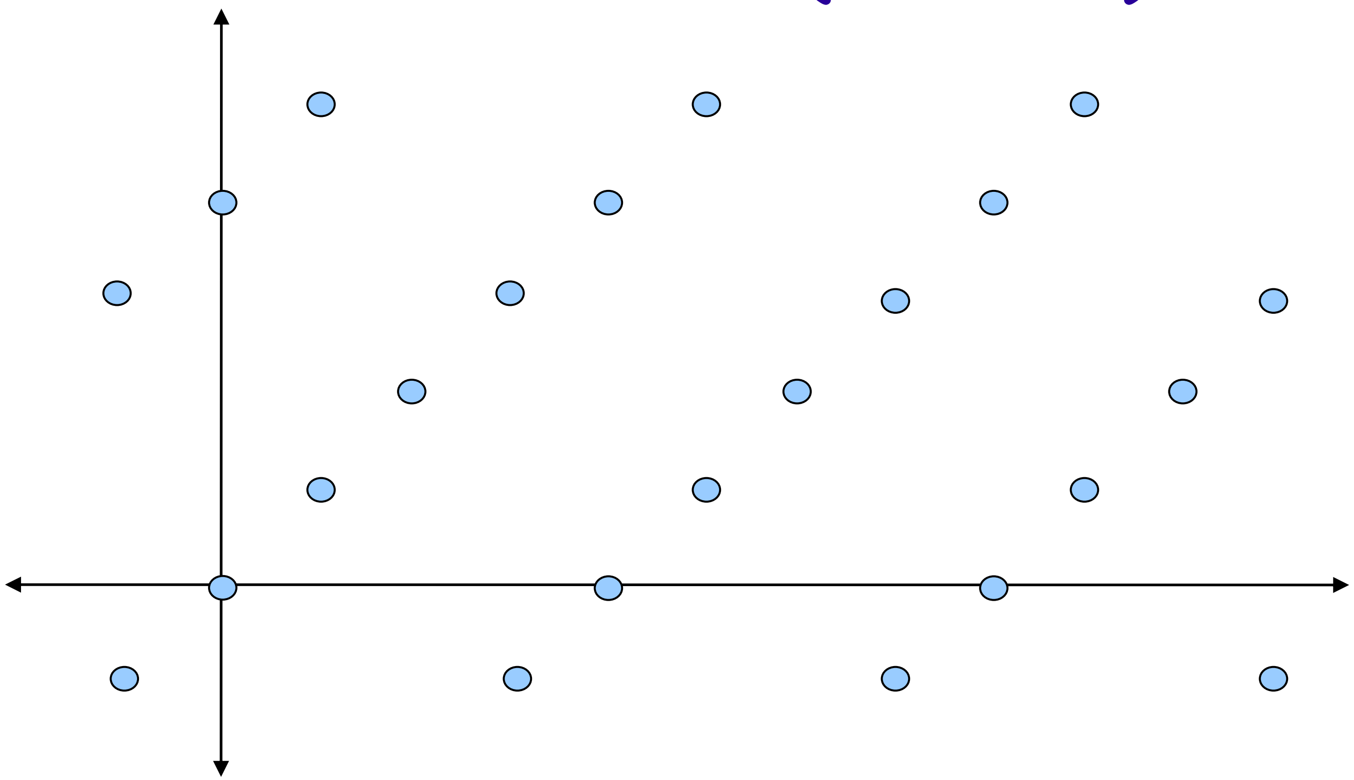
Shortest Independent Vector Problem (SIVP)



Find n short linearly independent vectors

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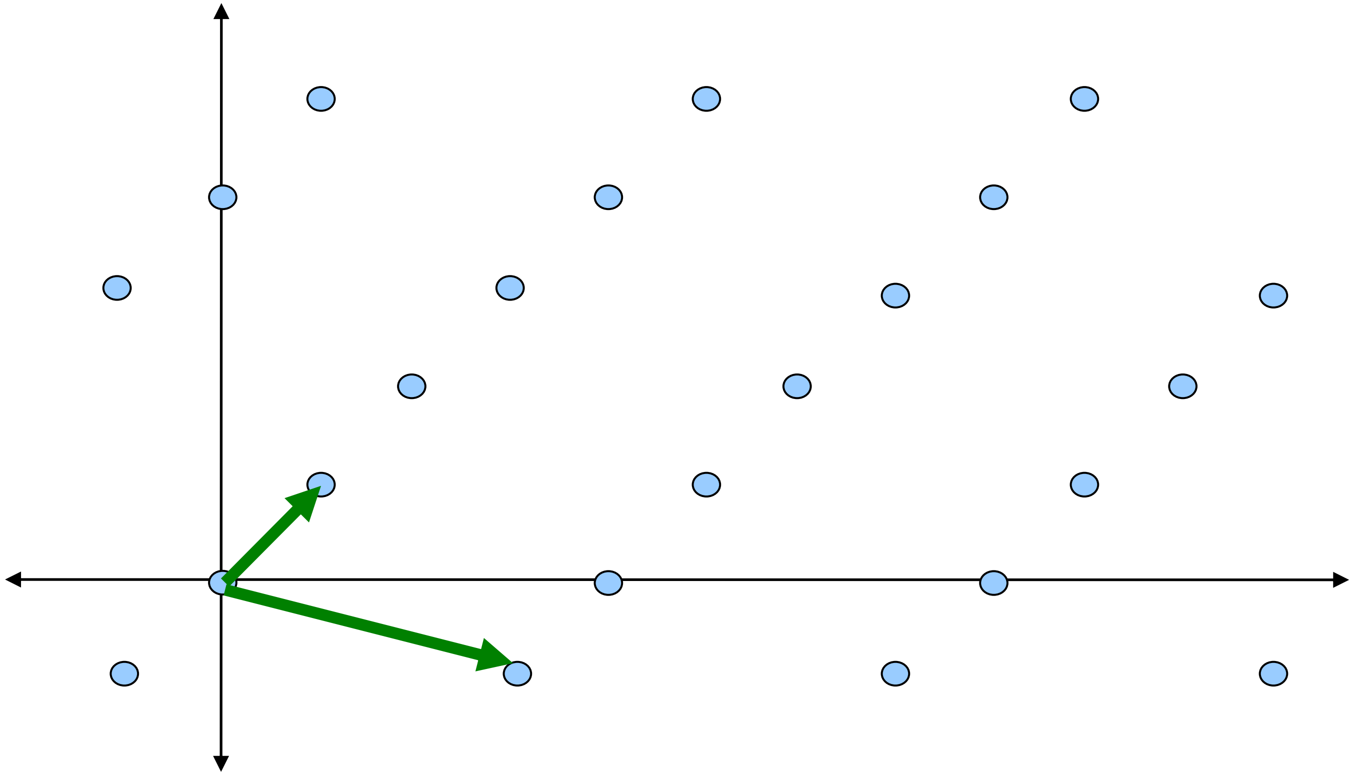
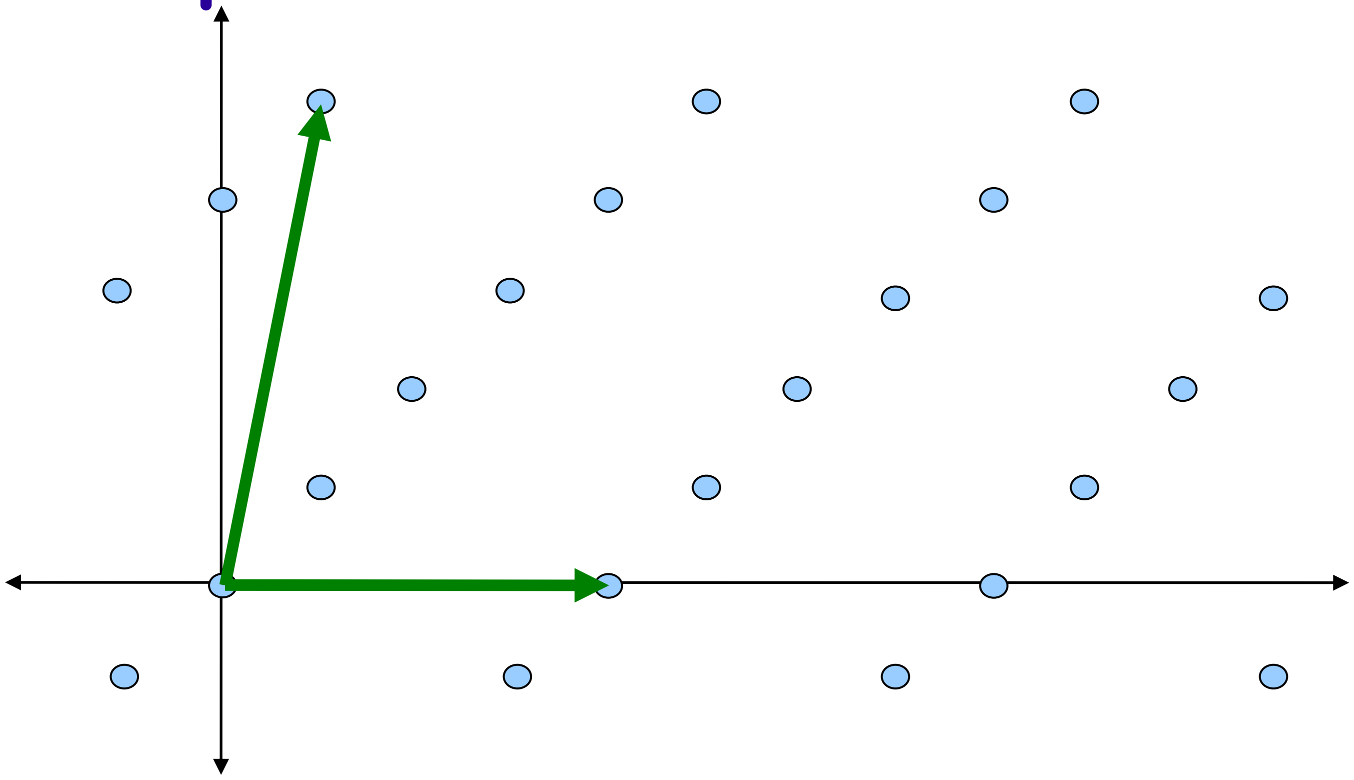
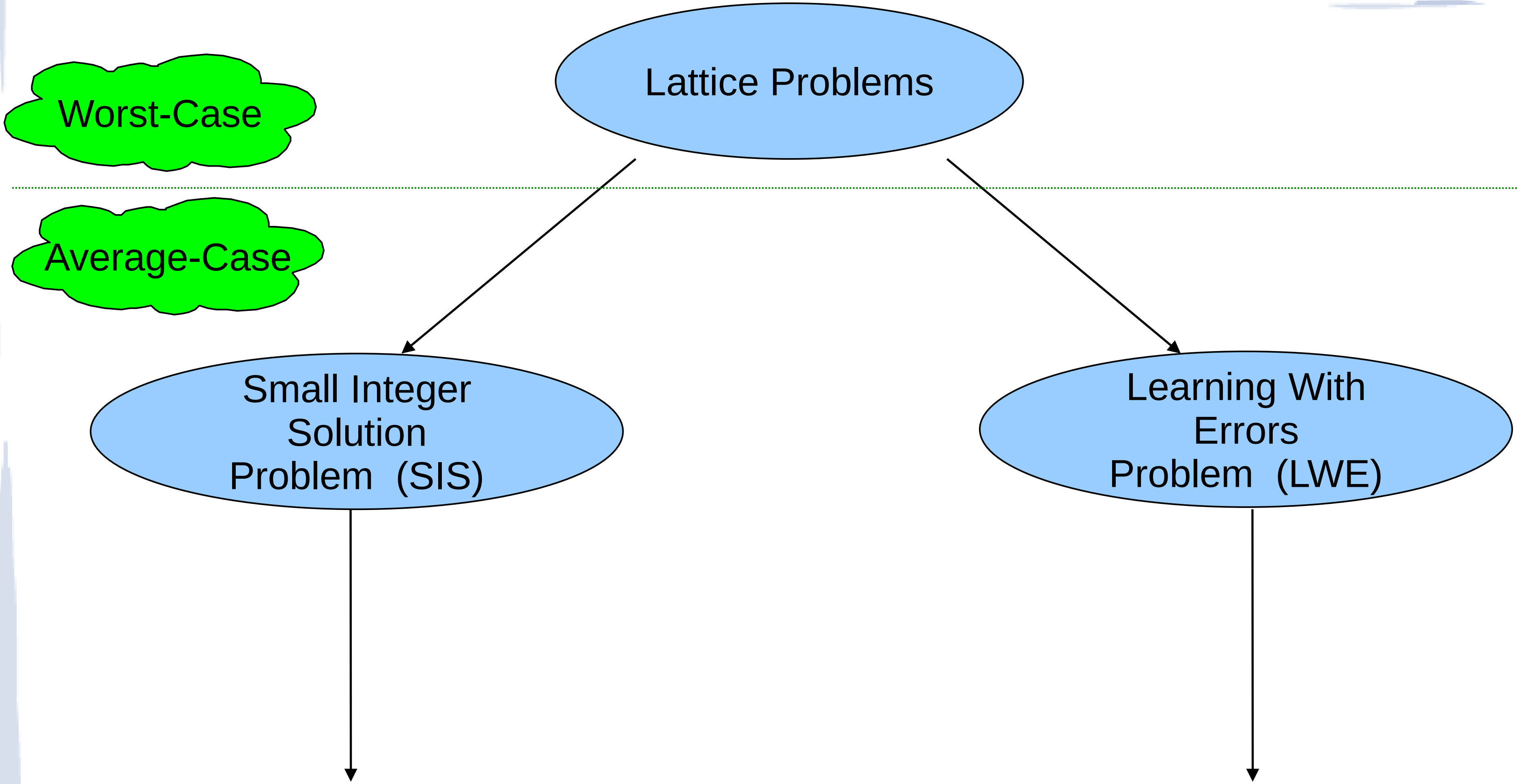


Find n short linearly independent vectors

Approximate Shortest Independent Vector Problem



Find n *pretty* short linearly independent vectors

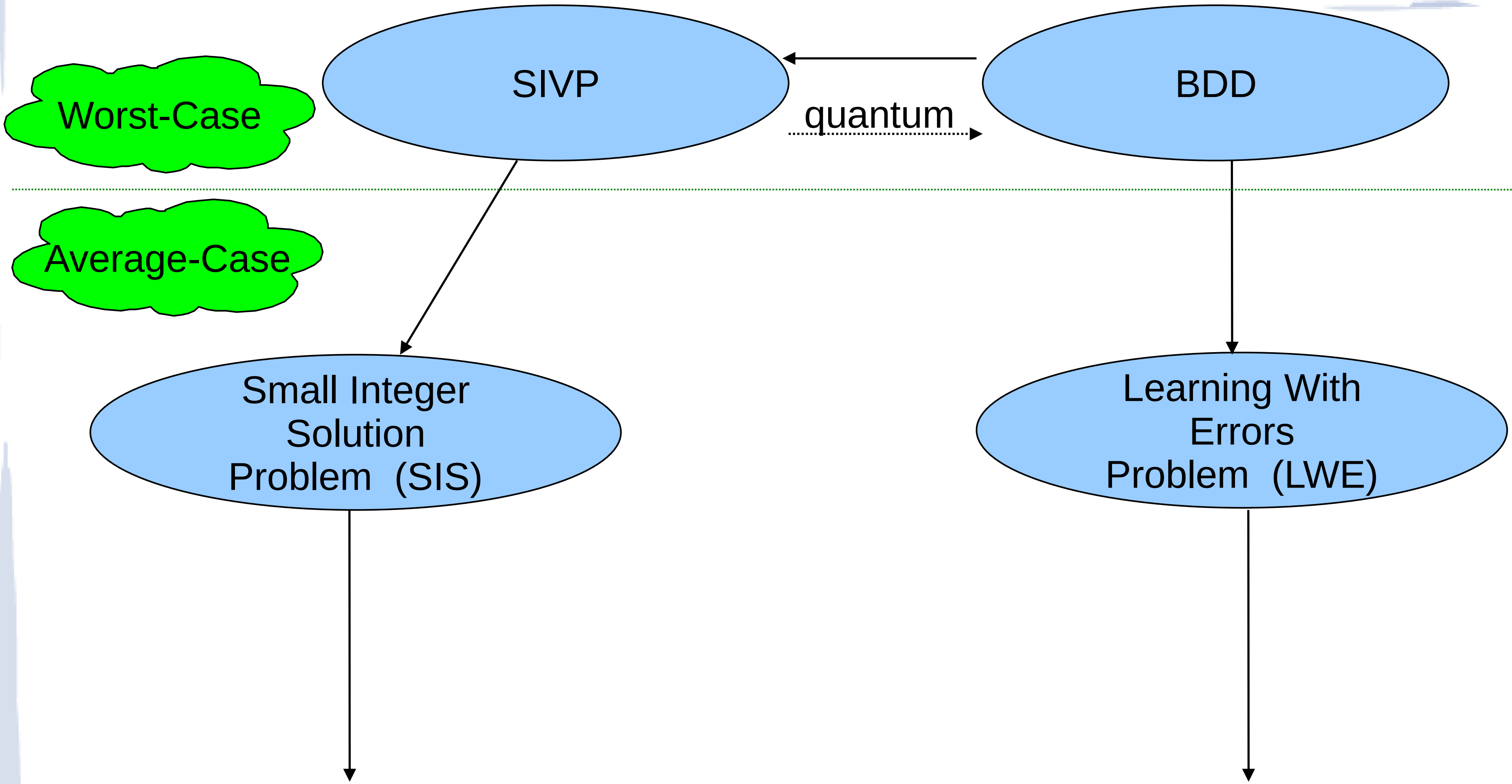


One-Way Functions
Collision-Resistant Hash Functions
Digital Signatures
Identification Schemes

(Minicrypt)

Public Key Encryption
Oblivious Transfer
Identity-Based Encryption
Hierarchical Identity-Based Encryption

(Cryptomania)



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Small Integer Solution Problem

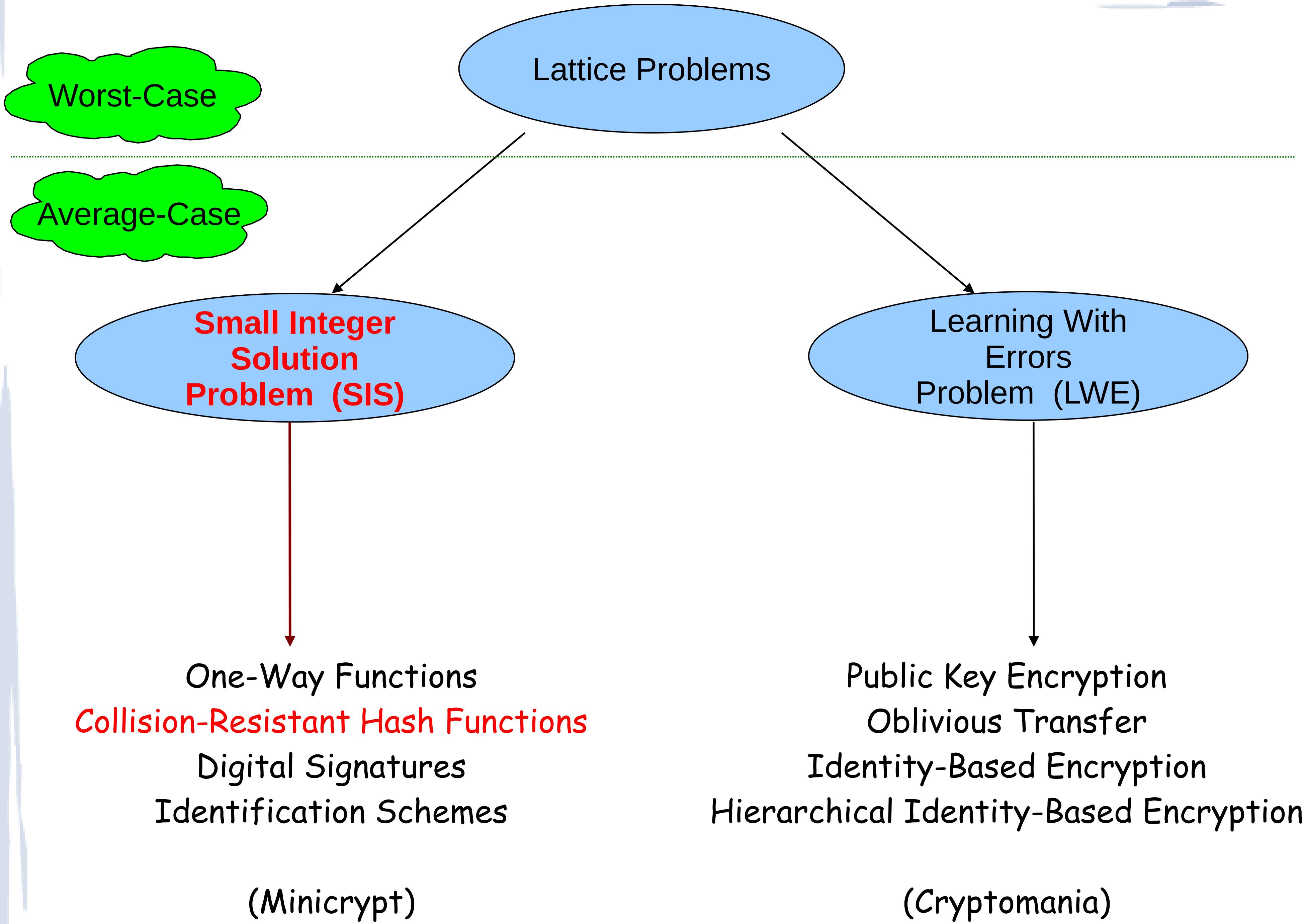
Given: Random vectors $a_1,\ldots,a_m$ in $\mathbf{Z}_q^n$

Find: non-trivial solution $z_1,\ldots,z_m$ in {-1,0,1} such that:

$$z_1 \, a_1 + z_2 \, a_2 + \ldots + z_m \, a_m = 0 \text{ in } \mathbf{Z}_q^n$$

Observations:

- If size of z_i is not restricted, then the problem is trivial
- Immediately implies a collision-resistant hash function


Lattice Problems
Worst-Case
Average-Case
Small Integer Solution Problem (SIS)
Learning With Errors Problem (LWE)
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Collision-Resistant Hash Function

Given: Random vectors $a_1,\ldots,a_m$ in $\mathbf{Z}_q^n$

Find: non-trivial solution $z_1,\ldots,z_m$ in {-1,0,1} such that:

$$z_1 \, a_1 + z_2 \, a_2 + \ldots + z_m \, a_m = 0 \text{ in } \mathbf{Z}_q^n$$

$A=(a_1,\ldots,a_m)$ Define $h_A: \{0,1\}^m \rightarrow \mathbf{Z}_q^n$ where

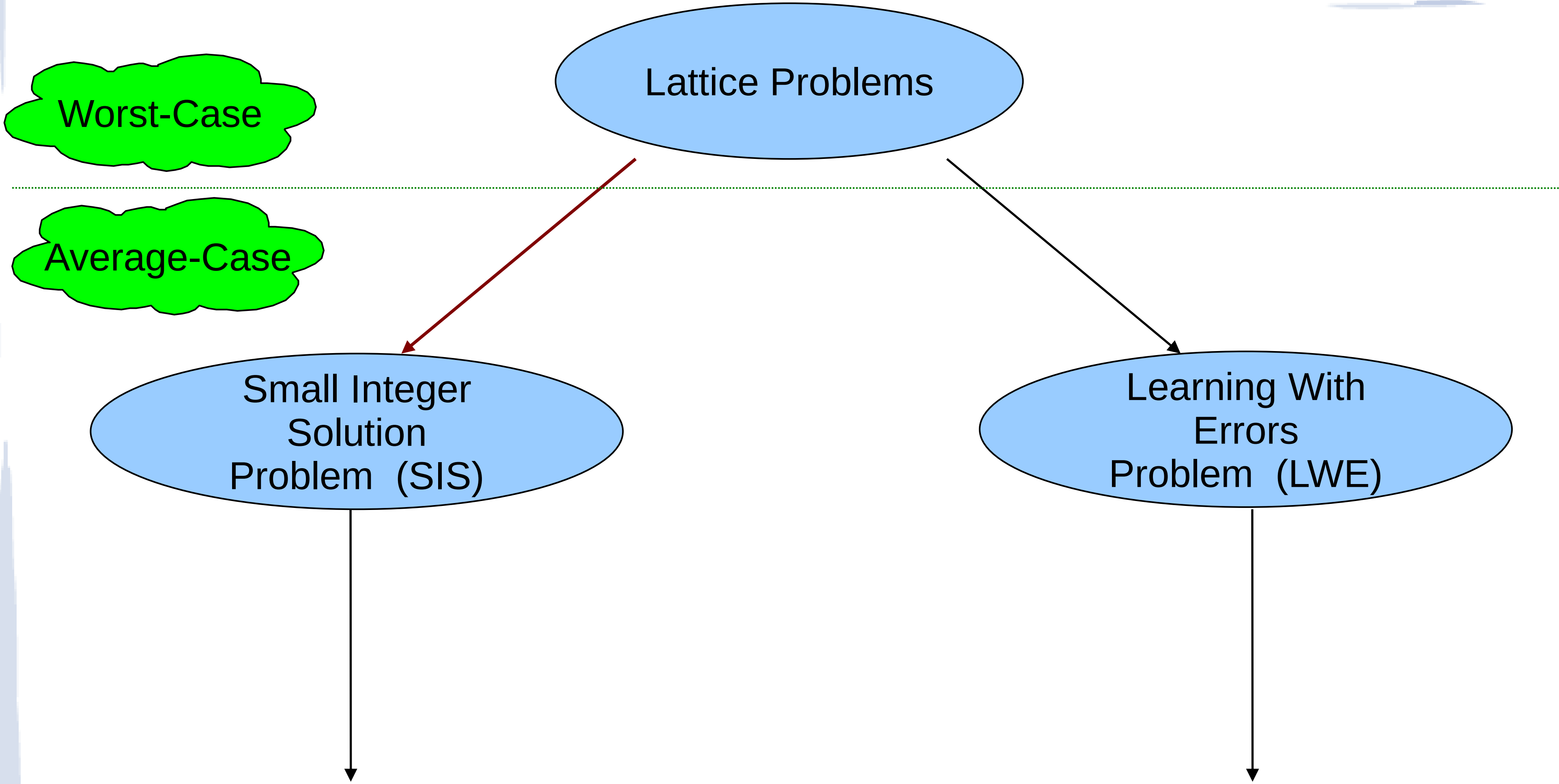
$h_A(z_1,\ldots,z_m)=a_1z_1 + \ldots + a_mz_m$

Domain of h = $\{0,1\}^m$ (size = 2^m) Range of h = $\mathbf{Z}_q^n$ (size = q^n)

Set $m>n\log q$ to get compression

Collision: $a_1z_1 + \ldots + a_mz_m = a_1y_1 + \ldots + a_my_m$

So, $a_1(z_1-y_1) + \ldots + a_m(z_m-y_m) = 0$ and z_i-y_i are in {-1,0,1}



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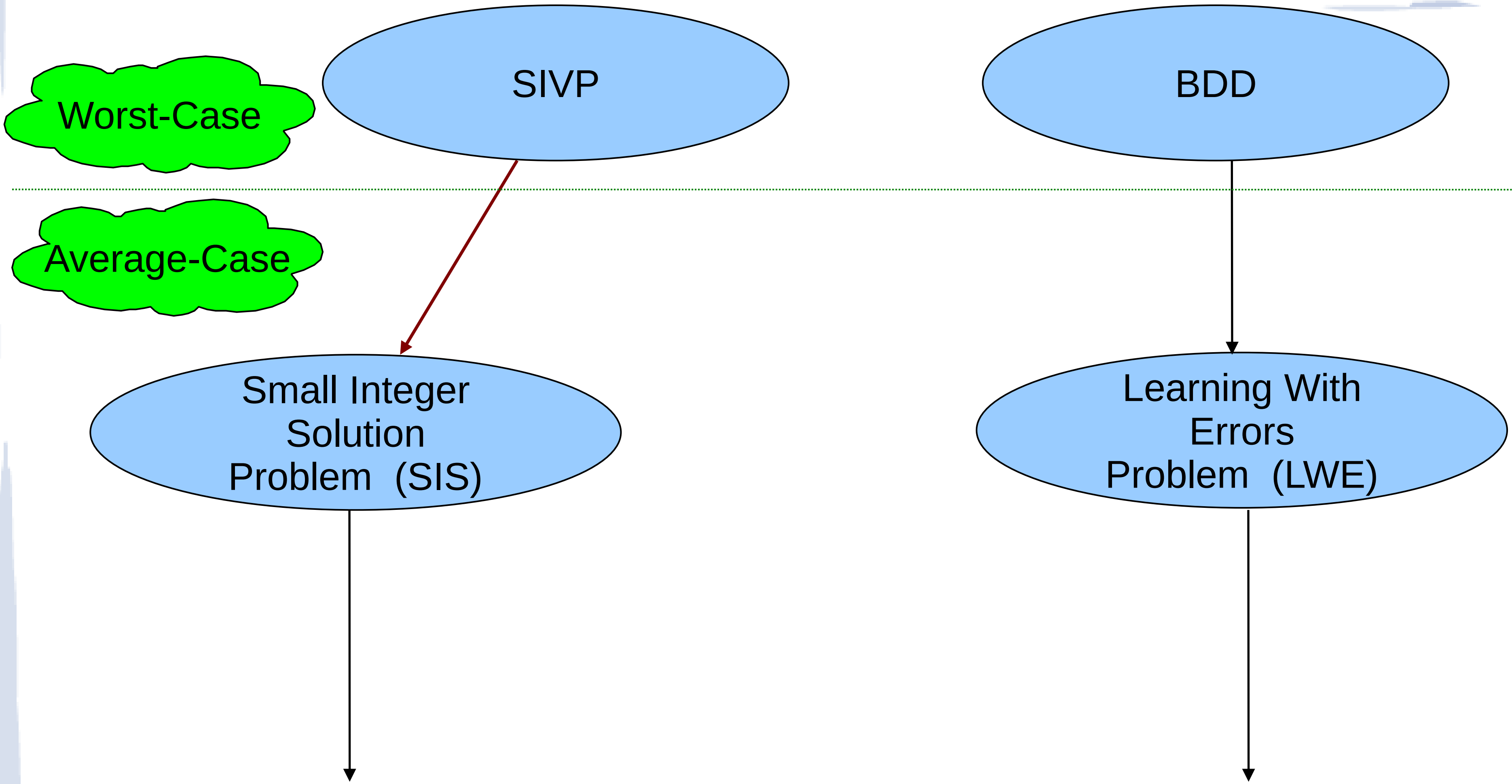

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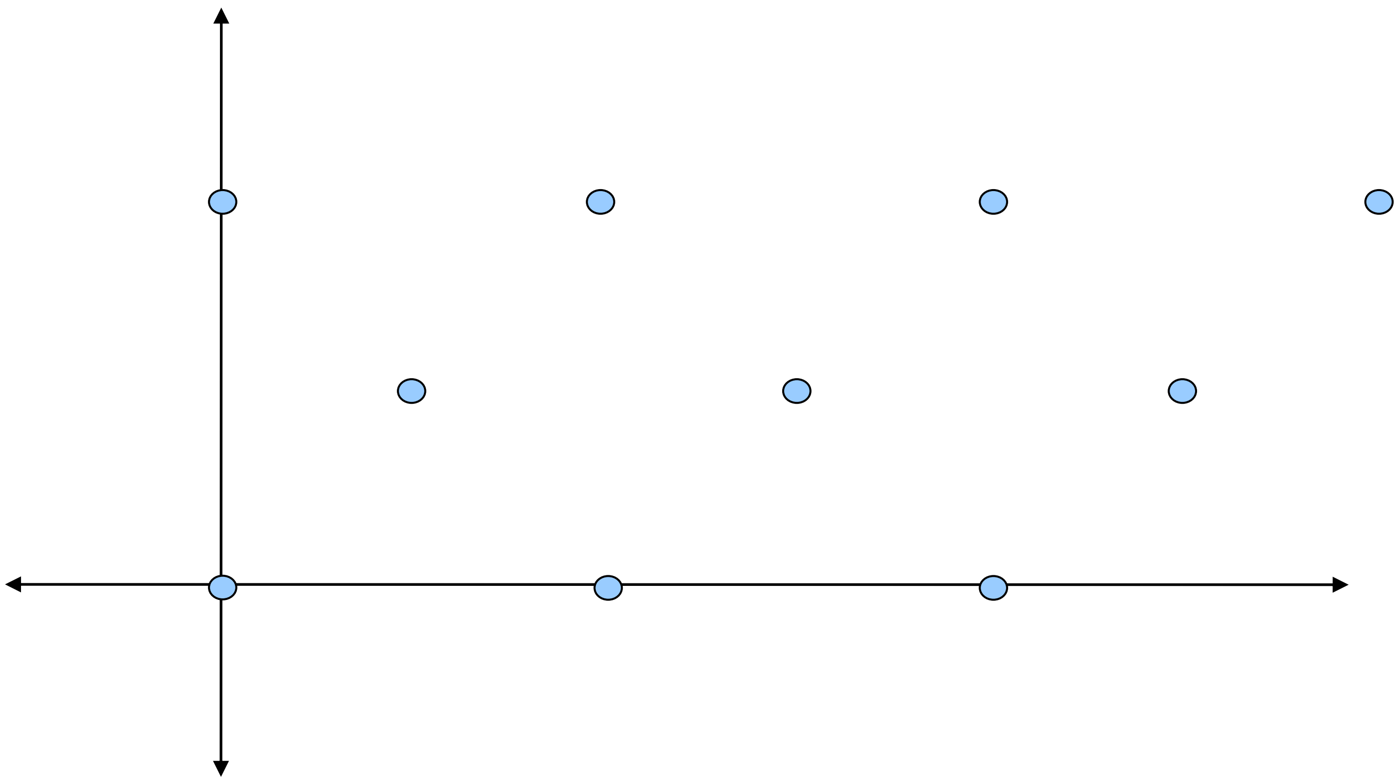
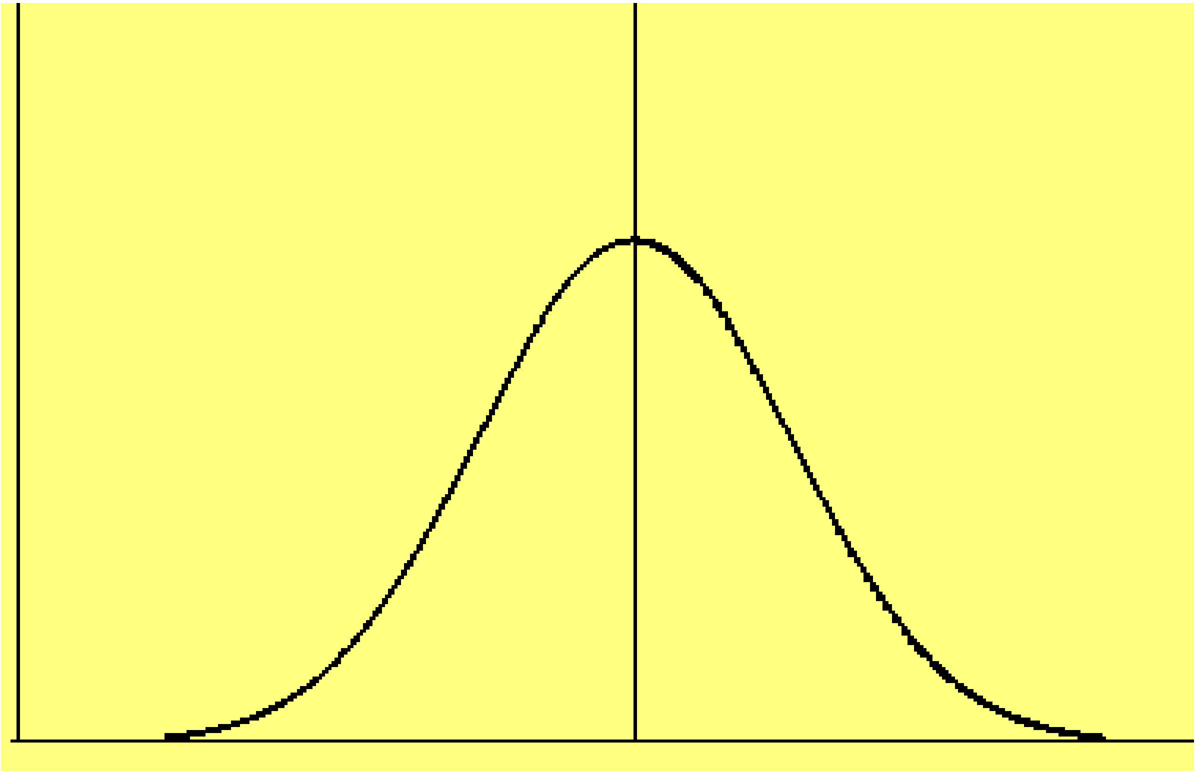
For Any Lattice ...



Consider the distribution obtained by:

1. Pick a uniformly random lattice point
2. Sample from a Gaussian distribution centered at the lattice point

One-Dimensional Gaussian Distribution



Two-Dimensional Gaussian Distribution

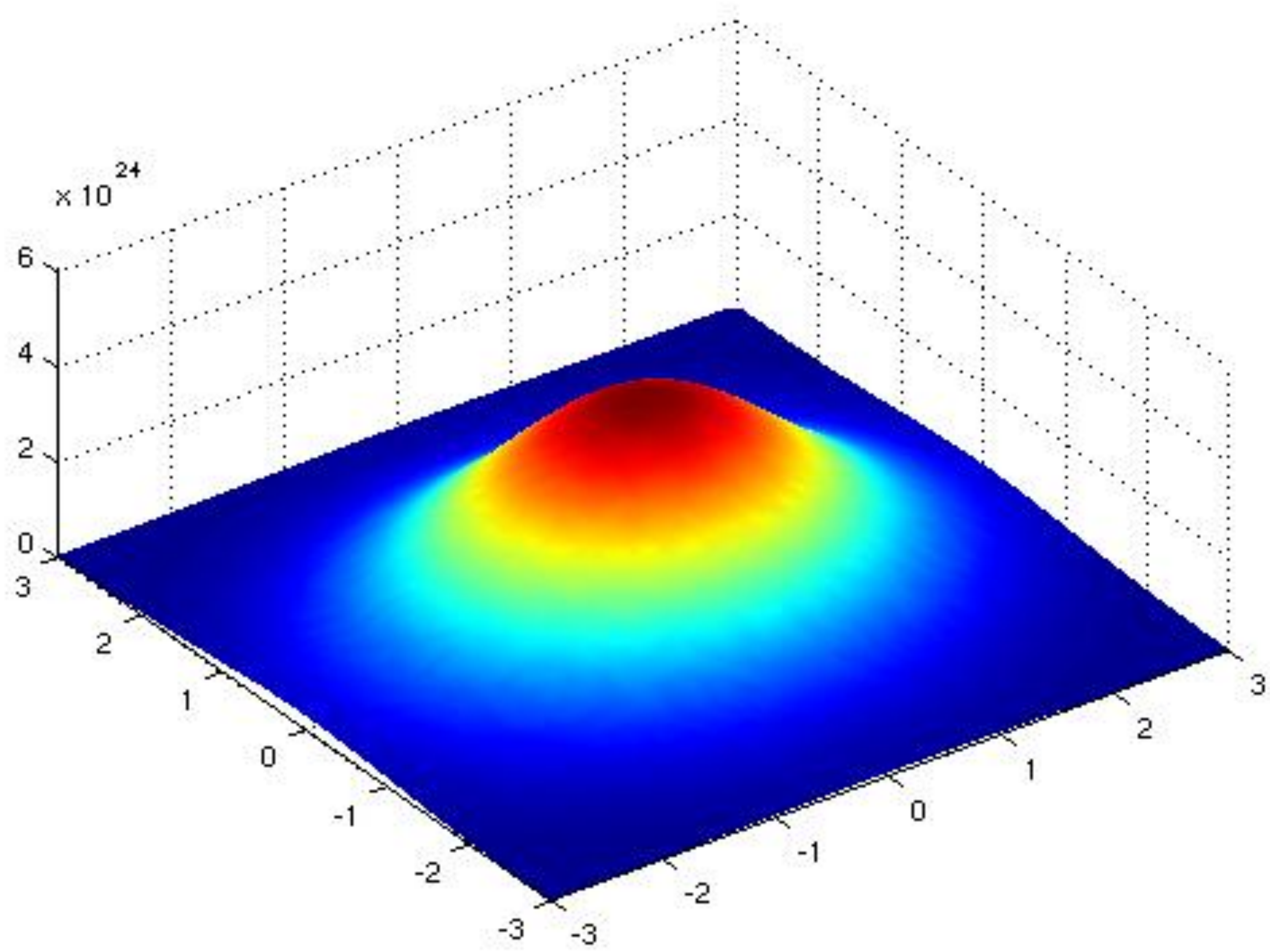


Image courtesy of wikipedia

Gaussians on Lattice Points

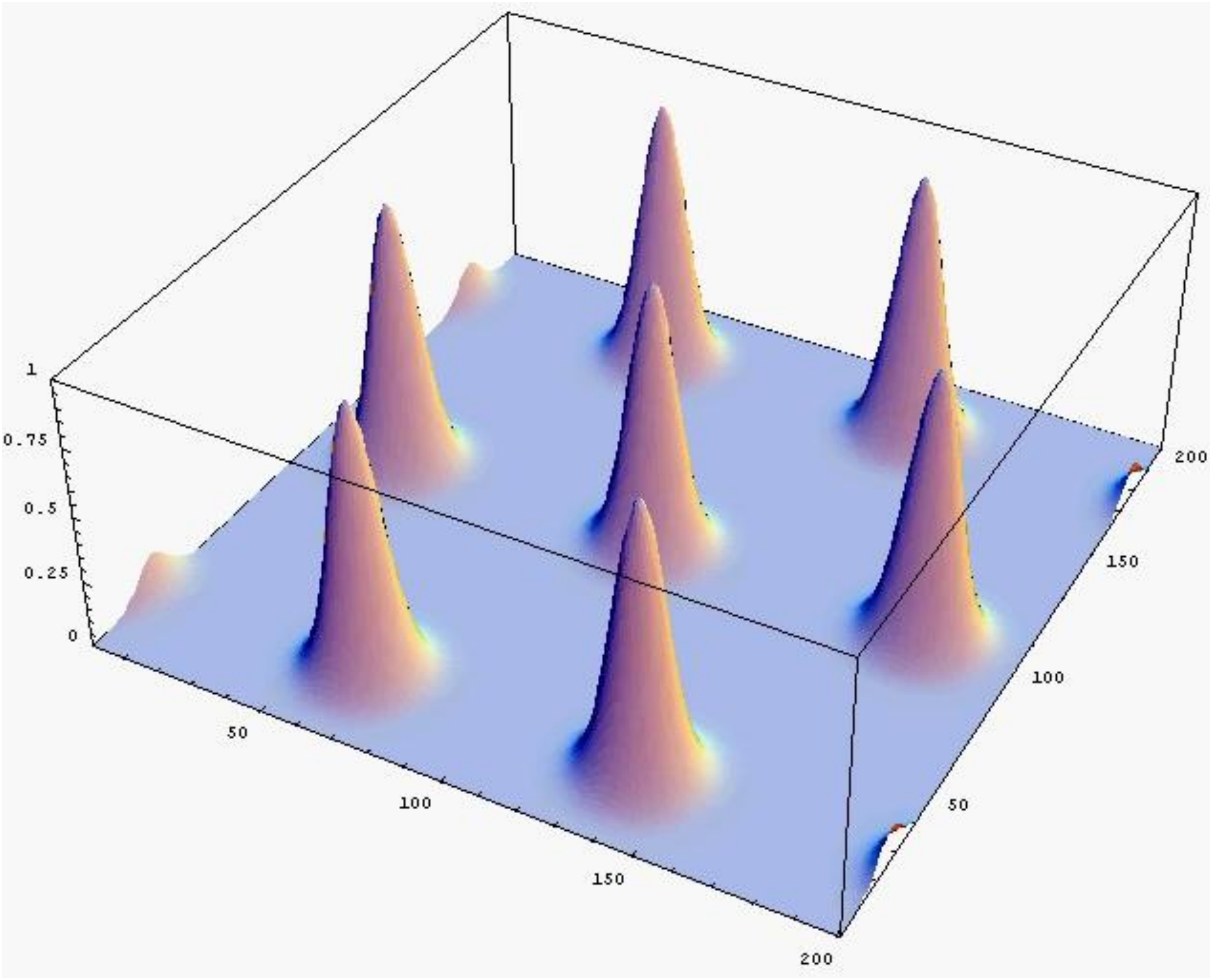
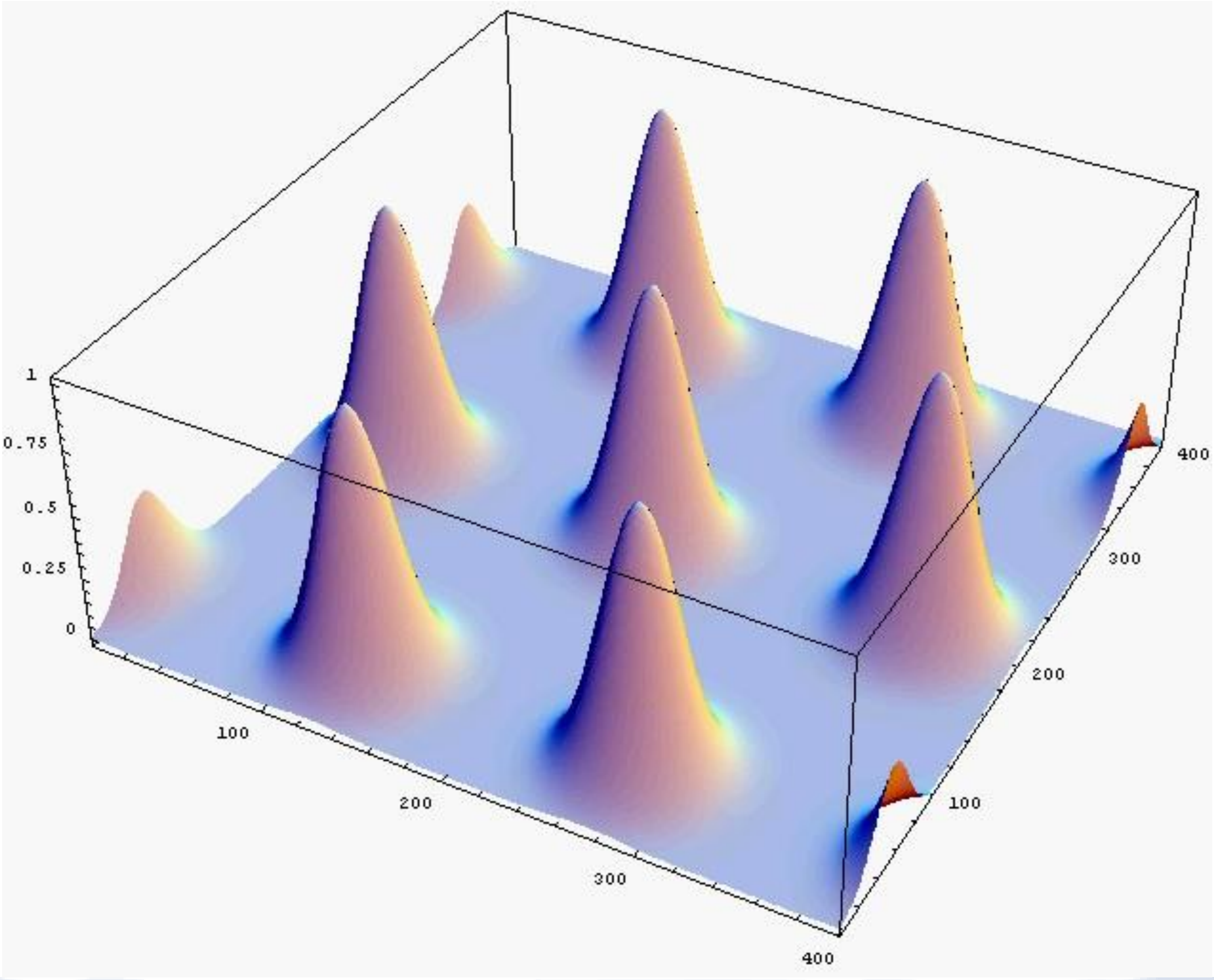


Image courtesy of Oded Regev

Gaussians on Lattice Points



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Gaussians on Lattice Points

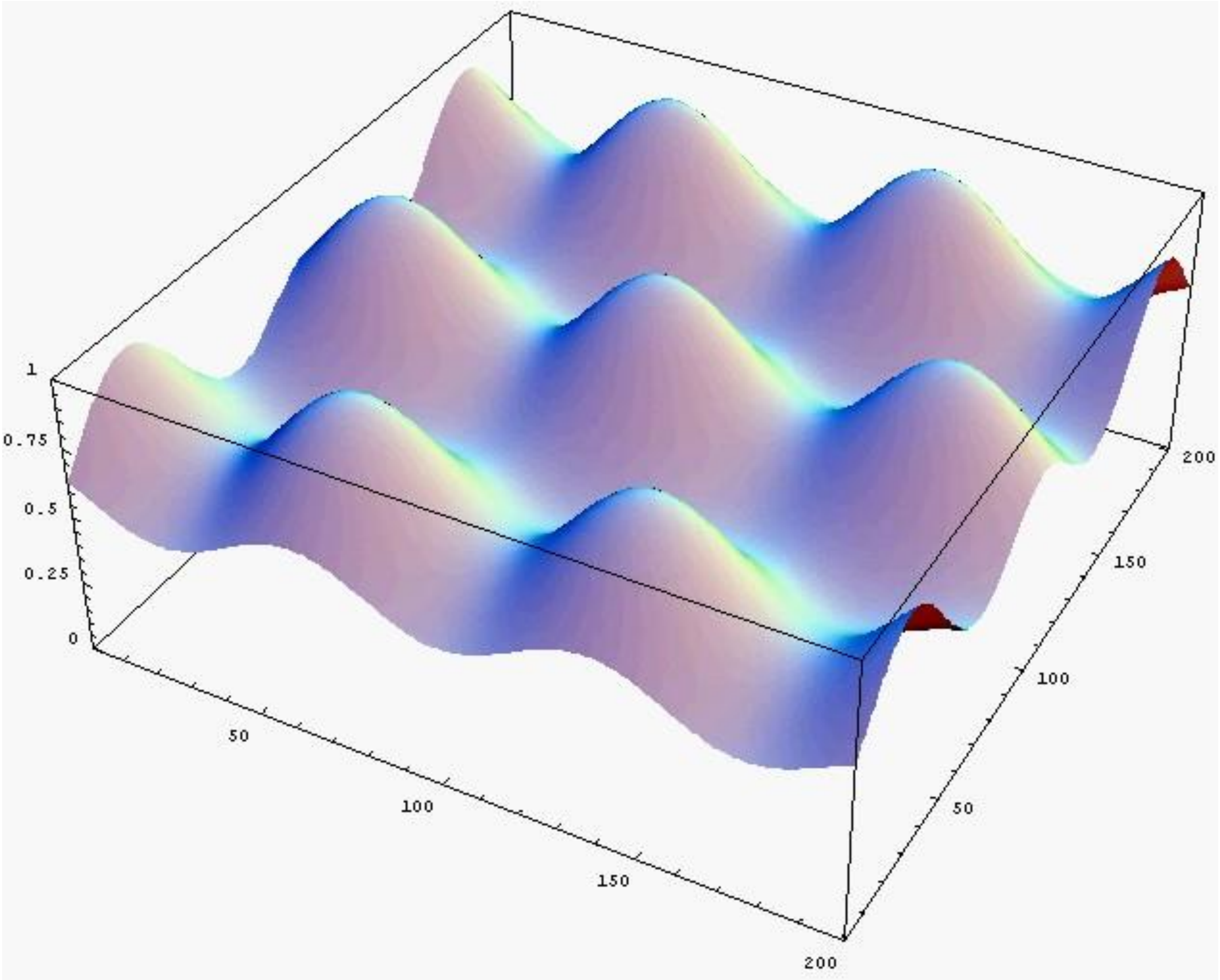
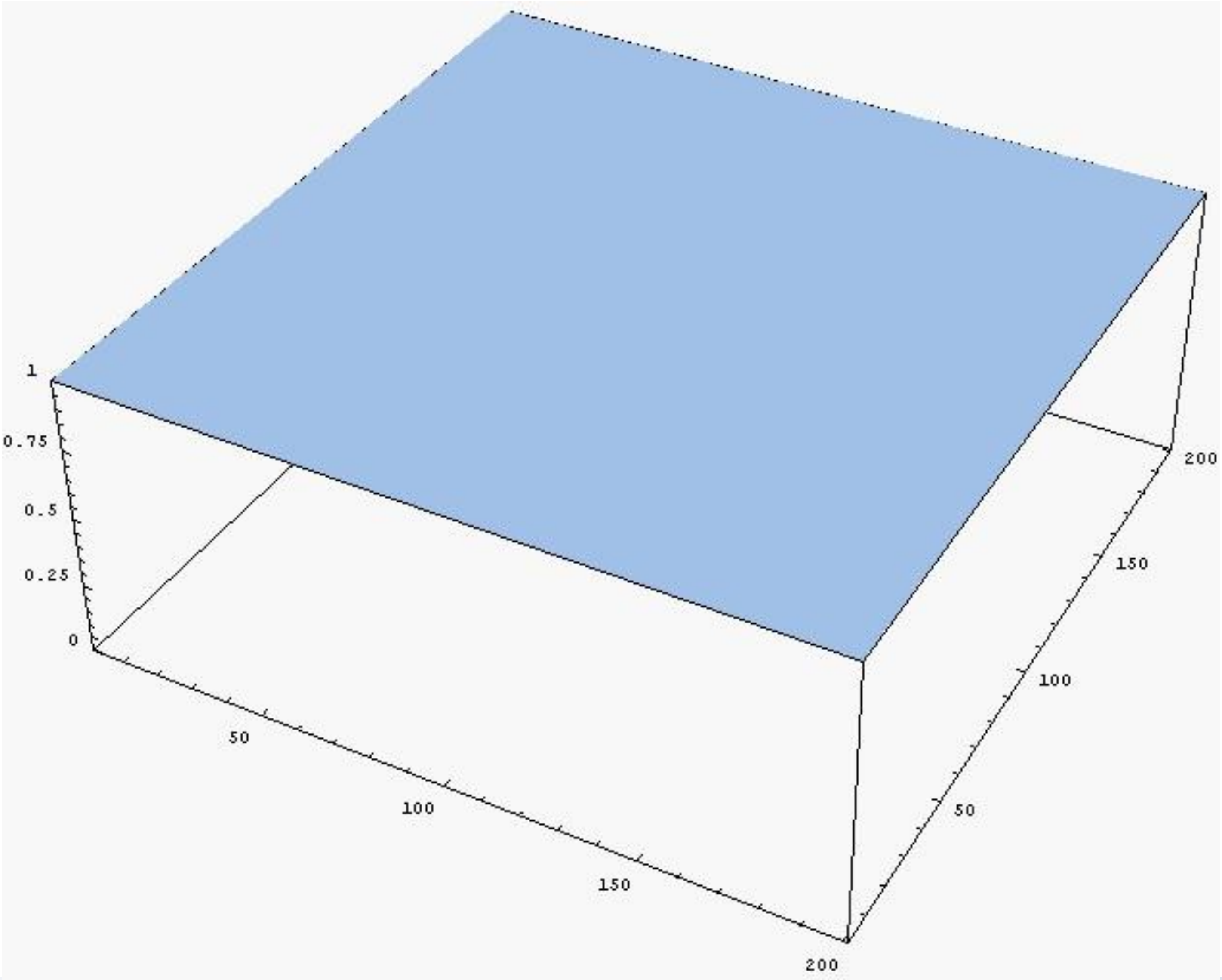


Image courtesy of Oded Regev

Gaussians on Lattice Points



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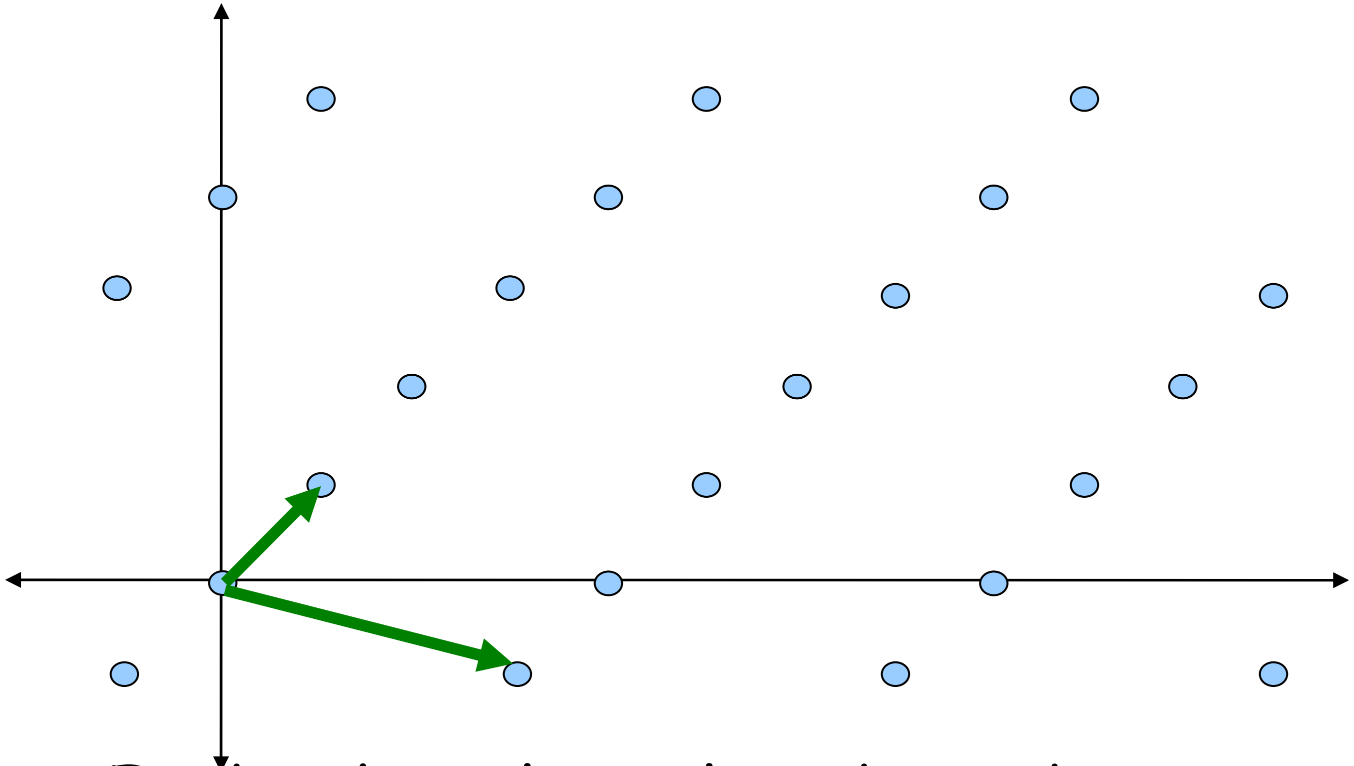
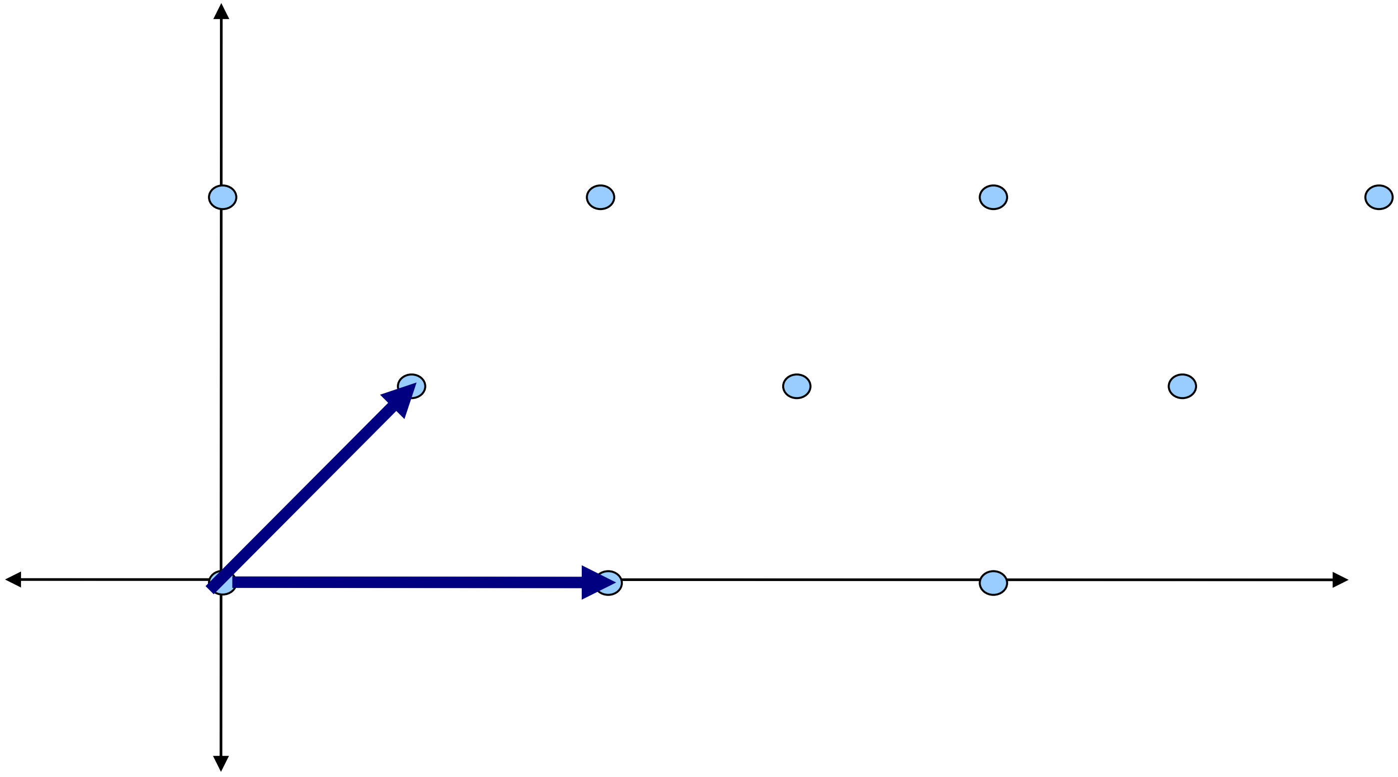
Shortest Independent Vector Problem (SIVP)



Find n short linearly independent vectors

Standard deviation of Gaussian that leads to the uniform distribution is related to the length of the longest vector in SIVP solution

Worst-Case to Average-Case Reduction



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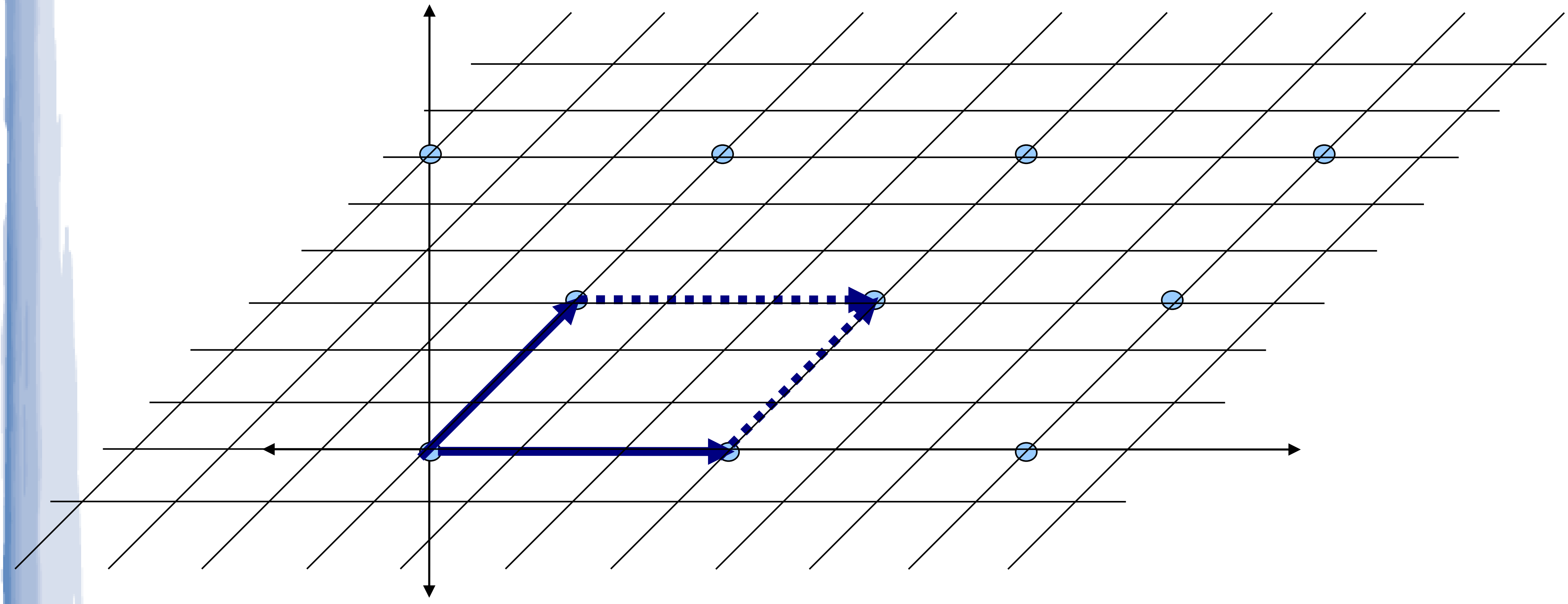
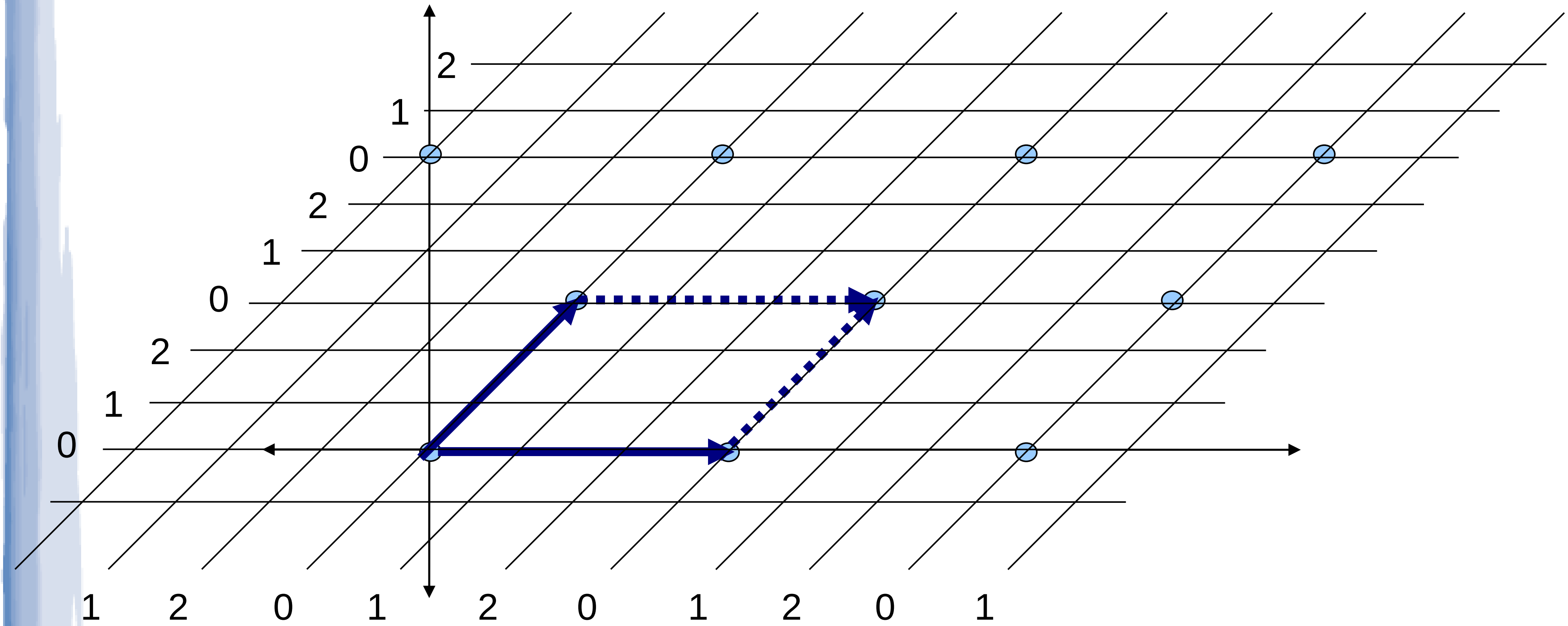


Worst-Case to Average-Case Reduction

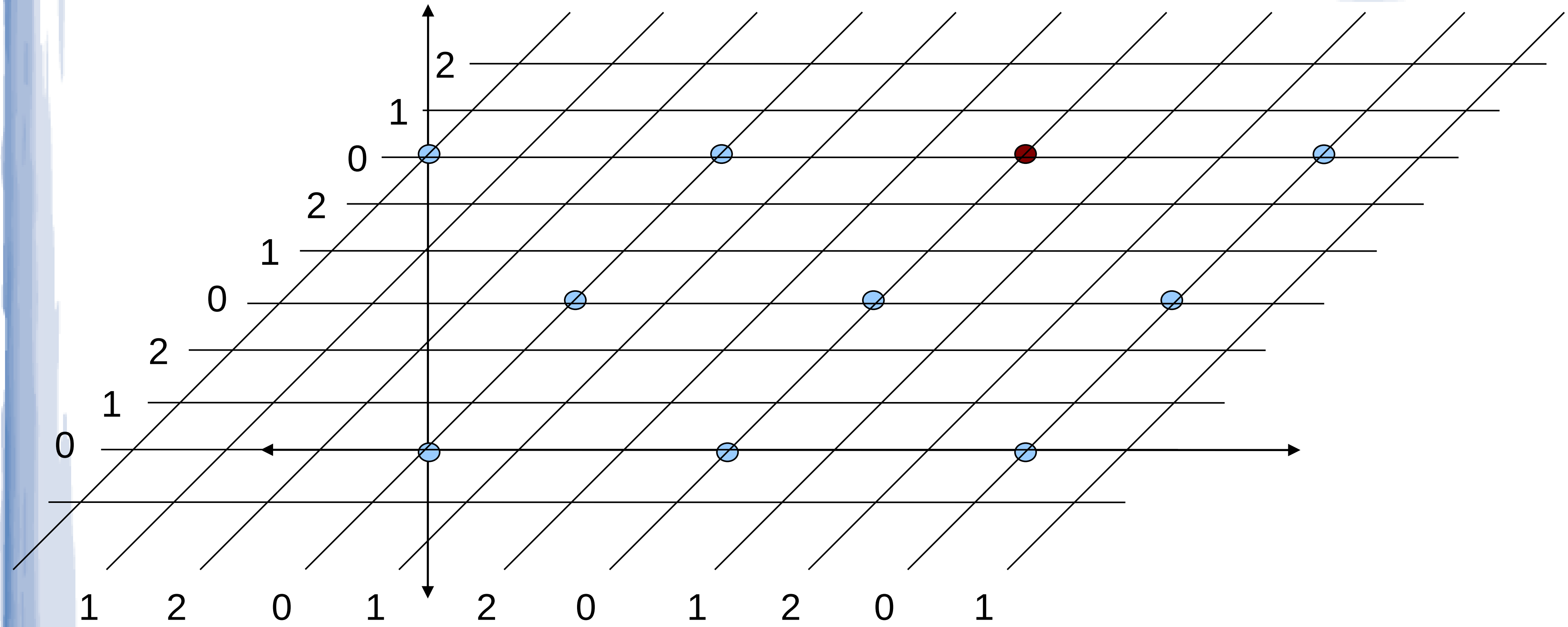


Important: All lattice points have label (0,0)

and

All points labeled (0,0) are lattice points

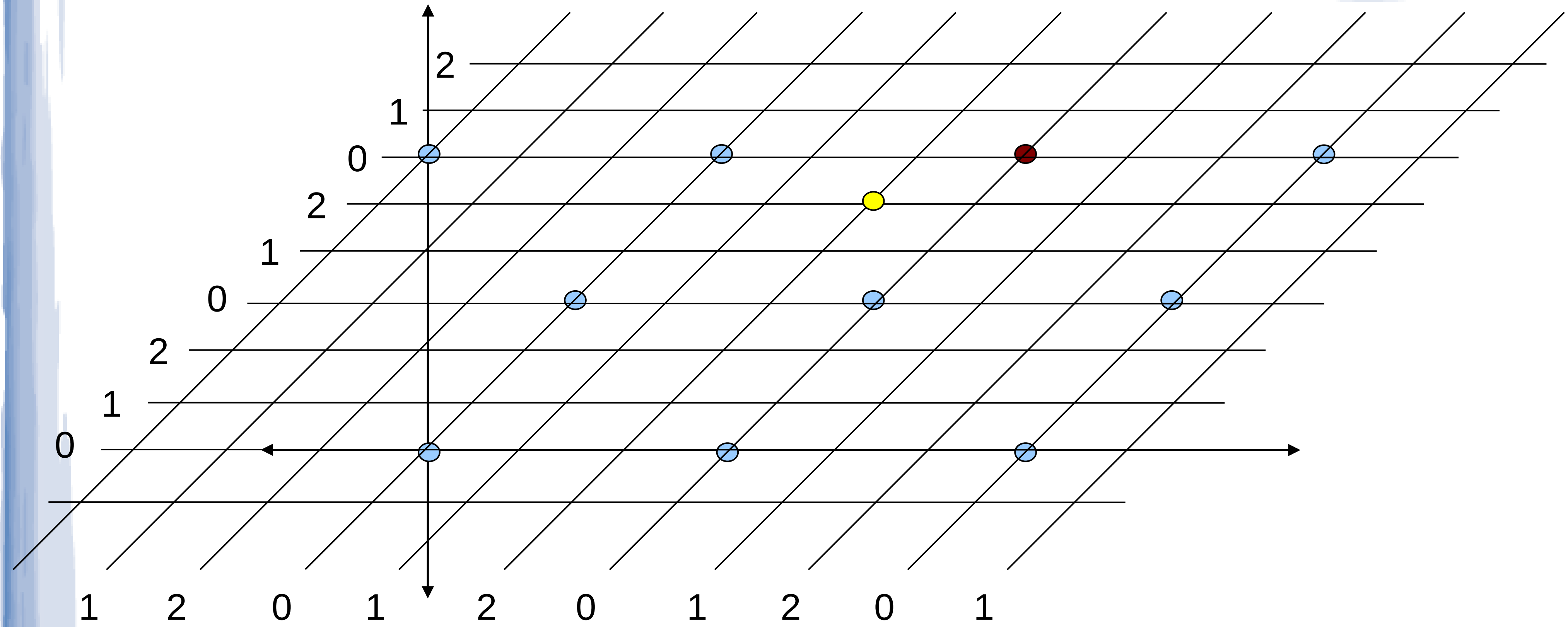
(0^n in n dimensional lattices)



<u>How to use the SIS oracle to find a short vector in any lattice:</u>

Repeat m times:

Pick a random lattice point

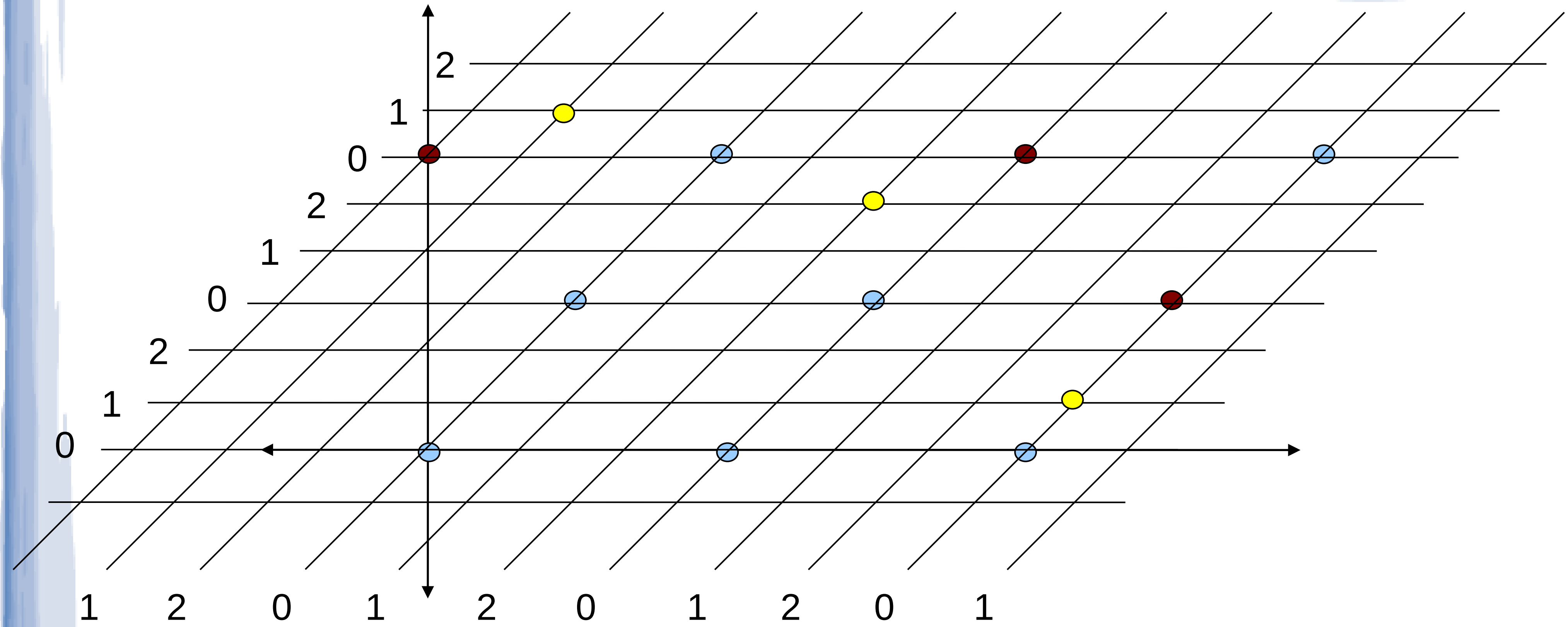


How to use the SIS oracle to find a short vector in any lattice:

Repeat m times:

Pick a random lattice point

Gaussian sample a point around the lattice point



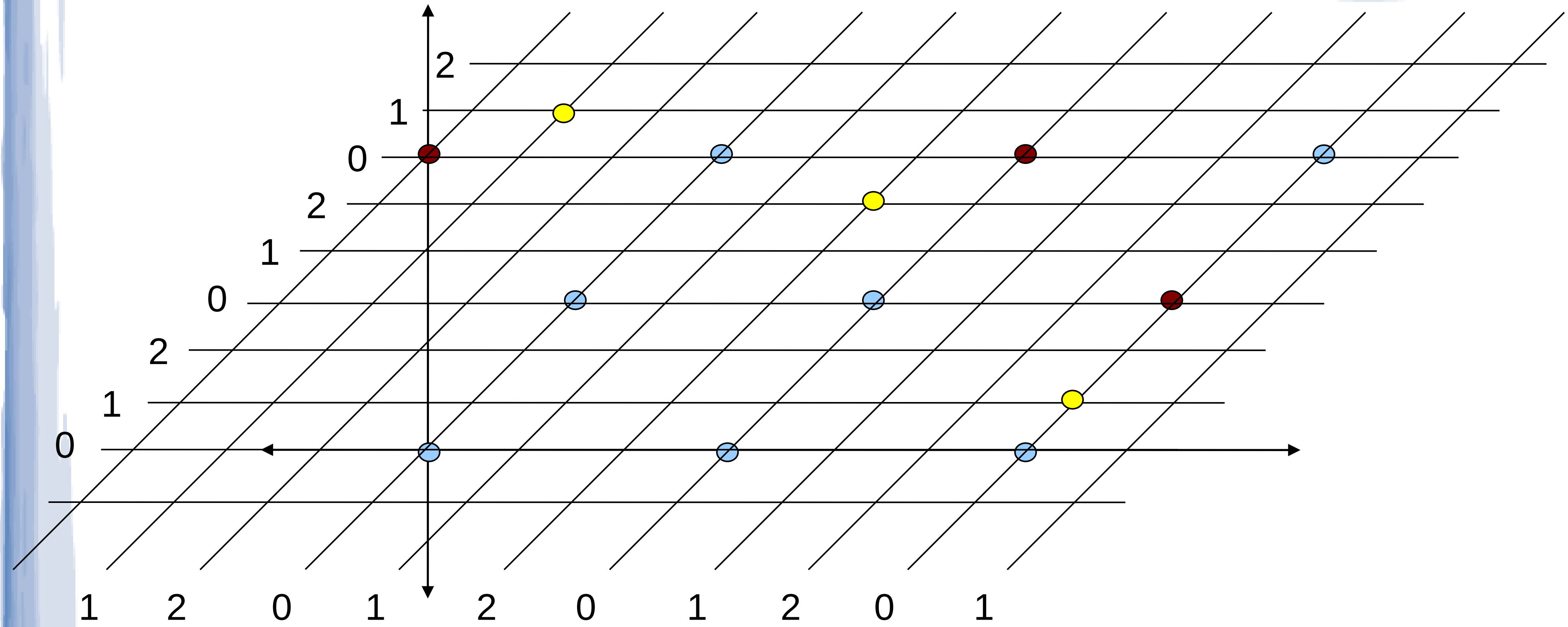
How to use the SIS oracle to find a short vector in any lattice:

Repeat m times:

Pick a random lattice point

Gaussian sample a point around the lattice point

All the samples are uniform in $\mathbf{Z}_q^n$



How to use the SIS oracle to find a short vector in any lattice:

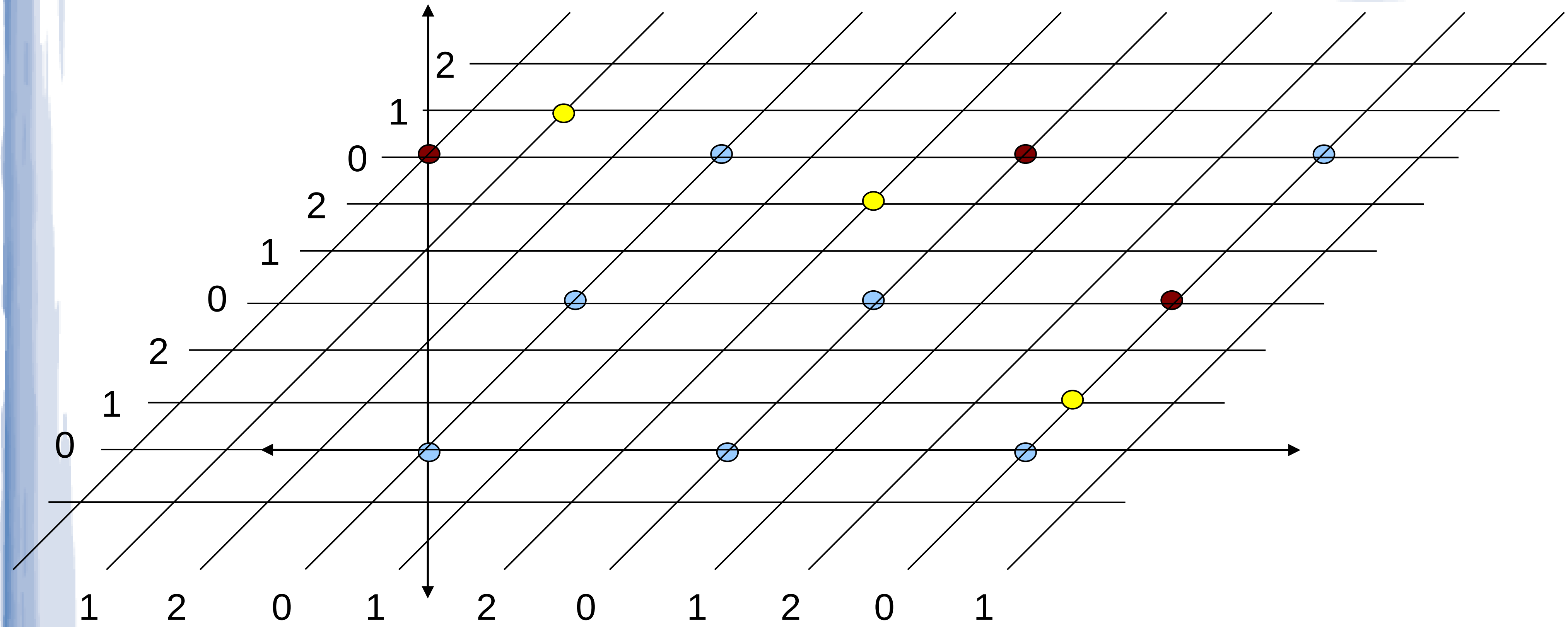
Repeat m times:

Pick a random lattice point

Gaussian sample a point around the lattice point

Give the m "$\mathbf{Z}_q^n$ samples" $a_1,\ldots,a_m$ to the SIS oracle

Oracle outputs $z_1,\ldots,z_m$ in {-1,0,1} such that $a_1z_1 + \ldots + a_mz_m = 0$



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Oracle outputs $z_1,\ldots,z_m$ in {-1,0,1} such that $a_1z_1 + \ldots + a_mz_m = 0$

● $= v_i$

● $= s_i$

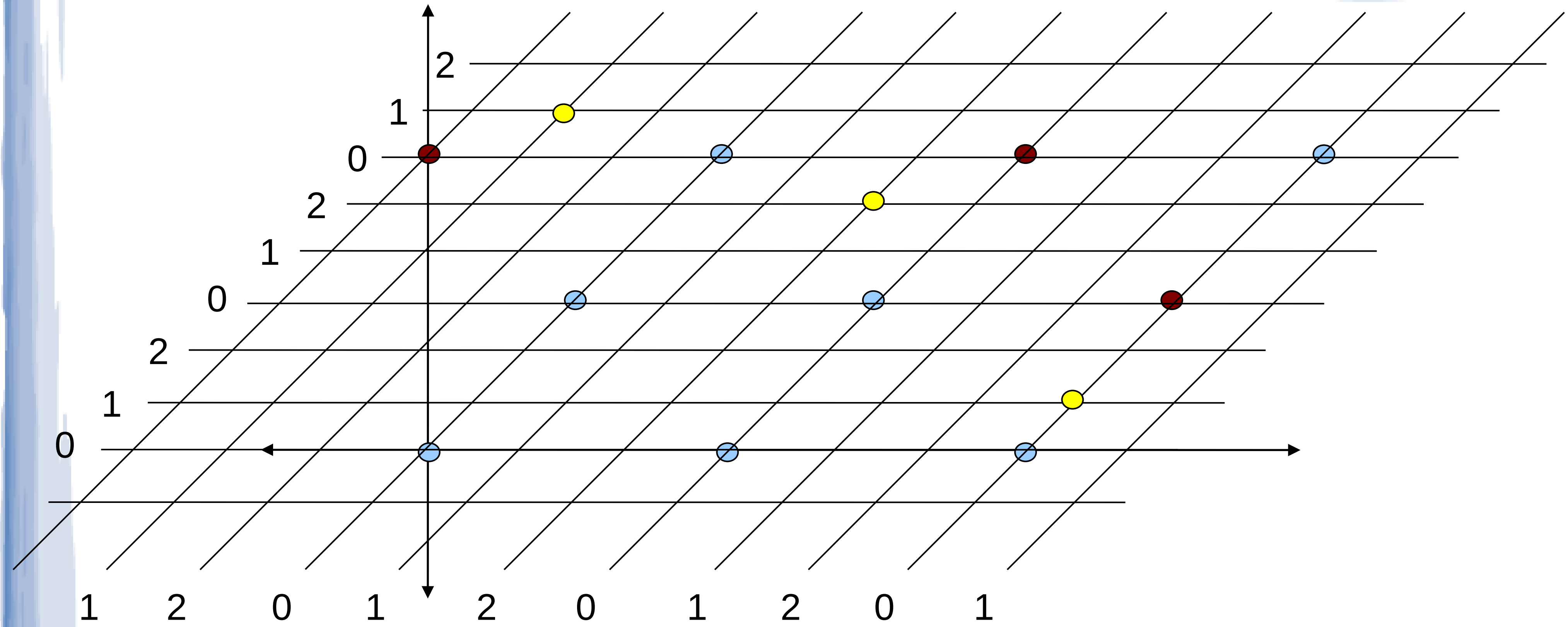
$v_i + r_i = s_i$

$s_1z_1+\ldots+s_mz_m$ is a lattice vector

$(v_1+r_1)z_1+\ldots+(v_m+r_m)z_m$ is a lattice vector

$(v_1z_1+\ldots+v_mz_m) + (r_1z_1+\ldots+r_mz_m)$ is a lattice vector

So $r_1z_1+\ldots+r_mz_m$ is a lattice vector



Give the m "Z_q^n samples" $a_1,\ldots,a_m$ to the SIS oracle

Oracle outputs $z_1,\ldots,z_m$ in $\{-1,0,1\}$ such that $a_1z_1 + \ldots + a_mz_m = 0$

● $= v_i$

● $= s_i$

$v_i + r_i = s_i$

So $r_1z_1+\ldots+r_mz_m$ is a lattice vector

r_i are short vectors, z_i are in $\{-1,0,1\}$

So $r_1z_1+\ldots+r_mz_m$ is a **short** lattice vector

Some Technicalities

- You can't sample a "uniformly random" lattice point
  - In the proofs, we work with $\mathbf{R}^n$ / L rather than $\mathbf{R}^n$
  - So you don't need to sample a random point lattice point
- What if $r_1z_1+\ldots+r_mz_m$ is 0?
  - Can show that with high probability it isn't
  - Given an s_i, there are multiple possible r_i
- Gaussian sampling doesn't give us points on the grid
  - You can round to a grid point
  - Must be careful to bound the "rounding distance"