Random access protocols for channel access

Markov chains and their stability

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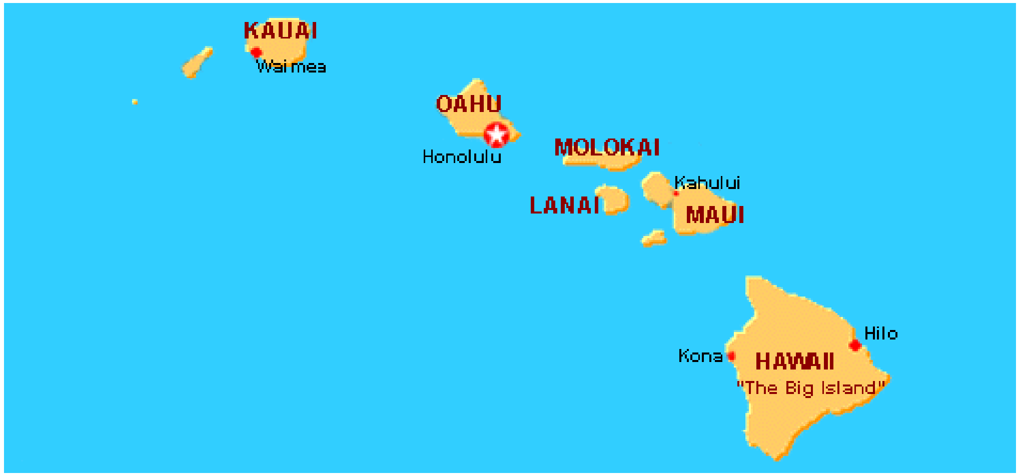
Aloha: the first random access protocol for channel access [Abramson, Hawaii 70]



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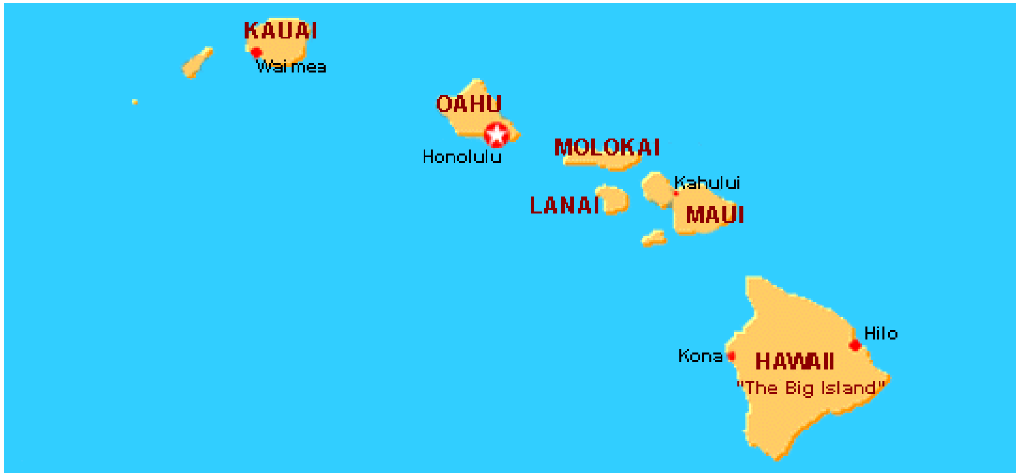

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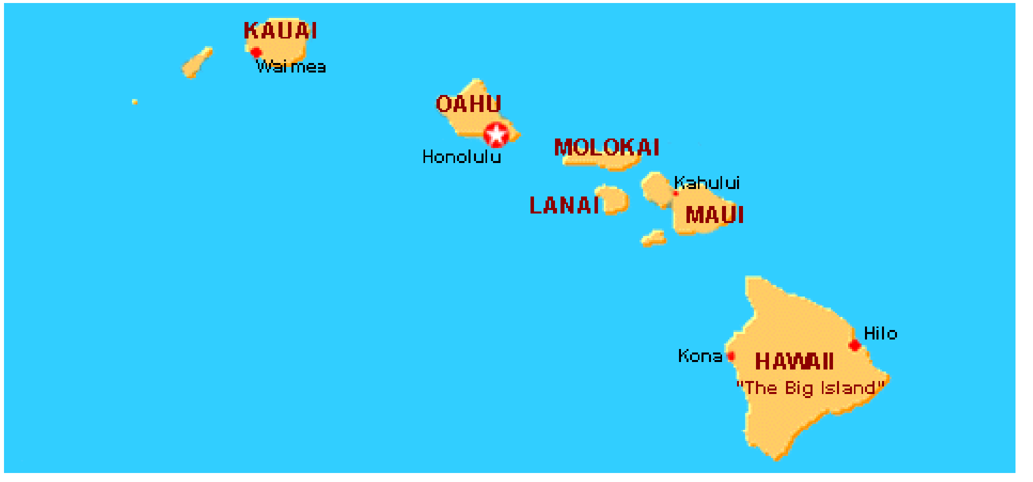

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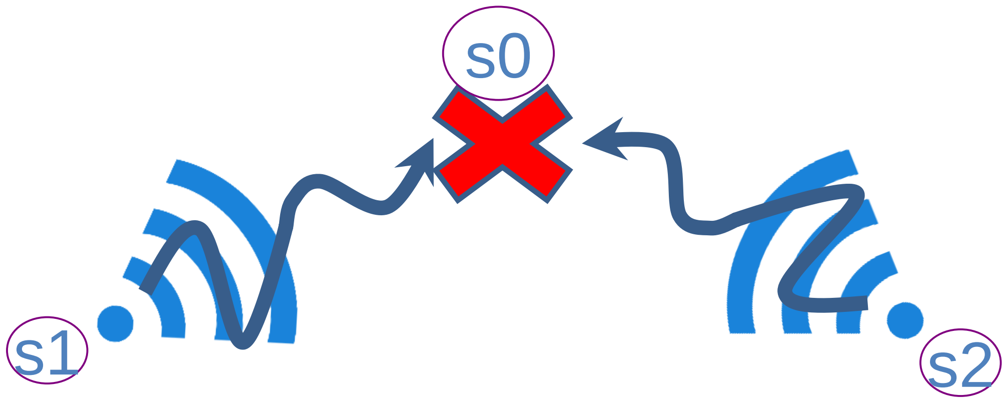
→ A randomized, distributed algorithm

Aloha's principle



Slotted time: fixed transmission intervals

Station with message to send: emits it with probability p

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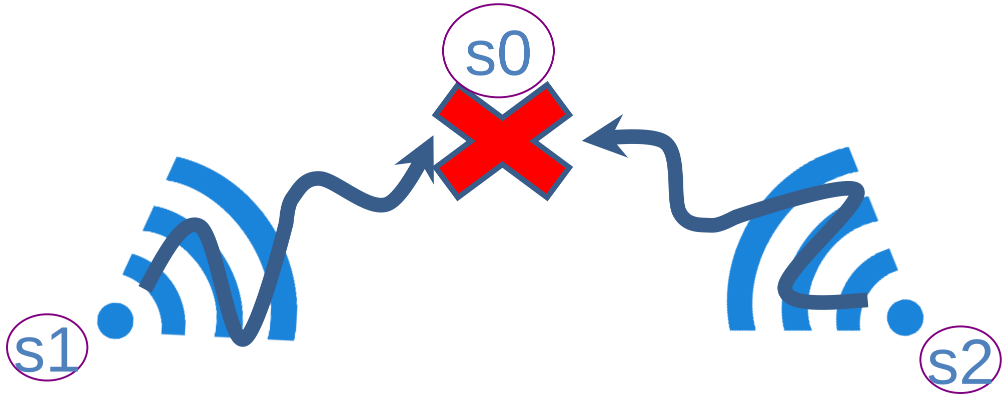

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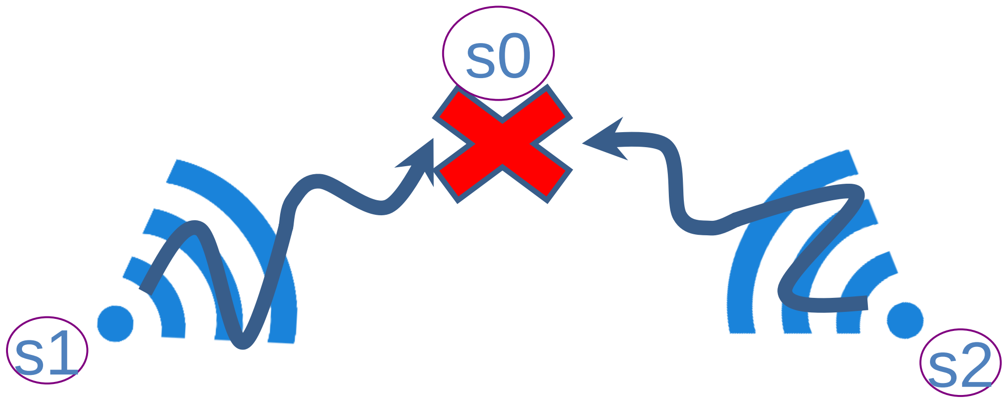

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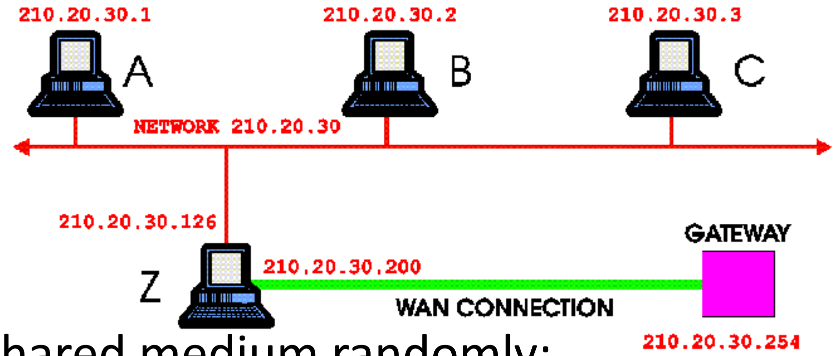
→ Minimal feedback (only listen for ack after having emitted)
→implicit coordination by receiver's acknowledgement

Ethernet principles [Metcalfe, Xerox Parc 73]



Machine emits on shared medium randomly:

After k failed attempts, waits before retransmitting for random number of slots picked uniformy from $\{1,2,\ldots,2^k\}$ (so-called contention window)

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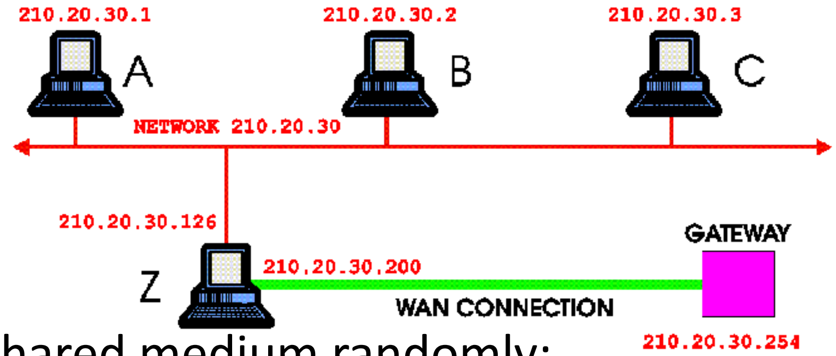

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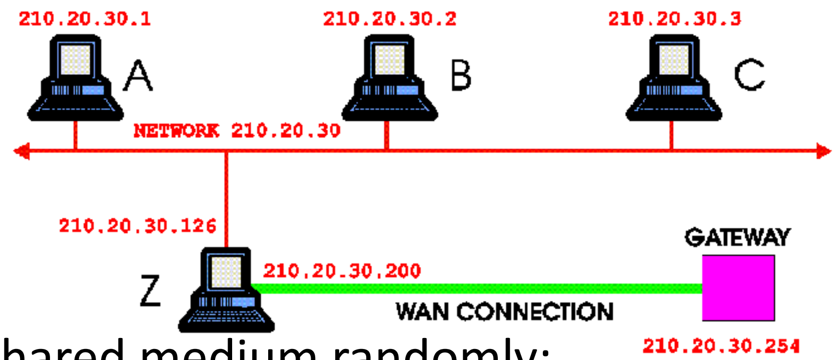

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Other refinement: sense channel before transmitting (allows to compete by random access only during small fraction of total time)

Principles underly 802.11x (Wi-Fi) protocols

Goals

Understand performance of random access protocols

→for given traffic, or demand, or workload offered to system (=process of message request arrivals),

Does system transmit them all?

Does it reach some steady state behaviour?

How long do transmissions take?

Outline

- Introduction to Markov chain theory
  - Fundamental notions (recurrence, irreducibility, ergodicity, transience)
  - Criteria for ergodicity or transience

- Performance of Random Access Protocols
  - Aloha with finitely many stations
  - Aloha with an infinite number of stations
  - Results for Ethernet and other variants

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- E a countable set (e.g., $\mathbb{N}$ or $[n] = \{1, \ldots, n\}$)

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- Definition: $\{X_n\}_{n \in \mathbb{N}}$ Markov chain with transition matrix P iff $\forall n > 0, \forall x_0^n = \{x_0, \ldots, x_n\} \in E^{n+1}$,

  $\mathbb{P}(X_n = x_n | X_0^{n-1} = x_0^{n-1}) = \mathbb{P}(X_n = x_n | X_{n-1} = x_{n-1}) = p_{x_n x_{n-1}}$

  where $\forall x, y \in E,\ p_{xy} \geq 0$ and $\sum_{z \in E} p_{xz} = 1$
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  (i.e. P is a *stochastic matrix*)
- Canonical example
  X_0 independent of $\{Y_n\}_{n\geq 0}$ an i.i.d. sequence, $Y_n \in E'$
  For some function $f : E \times E' \to E$,
  $$\forall n \geq 0,\ X_{n+1} = f(X_n, Y_n)$$

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- Illustration: reflected Random Walk on $\mathbb{N}$: $X_{n+1} = \max(0, X_n + Y_n)$

Basic properties

- By induction $\mathbb{P}(X_n^{n+m} = x_n^{n+m}) = \mathbb{P}(X_n = x_n) \prod_{i=n+1}^{n+m} p_{x_{i-1}x_i}$

$$\Rightarrow \mathbb{P}(X_0^{n+m} = x_0^{n+m} | X_n = x_n) = \begin{array}{l} \mathbb{P}(X_0^{n-1} = x_0^{n-1} | X_n = x_n) \times \cdots \\ \cdots \times \mathbb{P}(X_{n+1}^{n+m} = x_{n+1}^{n+m} | X_n = x_n) \end{array}$$

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- Noting $p_{x,y}^n = \mathbb{P}(X_n = y | X_0 = x)$, semi-group property:

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- Linear algebra interpretation
  For finite E (e.g. $E = [k]$), Matrix p^n = n-th power of P

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- Def: $T \in \mathbb{N} \cup \{+\infty\}$ **stopping time** iff
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  $\exists \phi_n : E^{n+1} \to \{0, 1\}$ such that $\mathbb{I}_{T=n} = \phi_n(X_0^n)$
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- **Strong Markov property**
  Markov chain X_0^∞ with transition matrix P, stopping time T
  Then conditionally on $T < +\infty$ and $X_T = x$,
  X_0^T and X_T^∞ independent with $X_T^\infty \sim \mathbb{P}_x$

Positive recurrence, null recurrence, transience, periodicity

State x is

- **recurrent** if $\mathbb{P}_x(T_x < +\infty) = 1$
- **positive recurrent** if $\mathbb{E}_x(T_x) < +\infty$
- **null recurrent** if $\mathbb{P}_x(T_x < +\infty) = 1$ & $\mathbb{E}_x(T_x) = +\infty$
- **transient** if not recurrent, i.e. $\mathbb{P}_x(T_x < +\infty) < 1$
- d**-periodic** if $d = \mathrm{GCD}(n \geq 0 : p_{xx}^n > 0)$

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ILLUSTRATION: reflected random walk on $\mathbb{N}$,
$S_{n+1} = \max(0, S_n + Y_n)$
State 0 is

- **positive recurrent** if $\mathbb{E}(Y_n) < 0$
- **transient** if $\mathbb{E}(Y_n) > 0$
- **null recurrent** if $\mathbb{E}(Y_n) = 0$ & $0 < \mathrm{Var}(Y_n) < +\infty$

Decomposition in cycles of recurrent chains

Fix a state x that is recurrent ($\mathbb{P}_x(T_x < +\infty) = 1$),

Let $T_{x,k}$ = instant of k-th visit to state x

$\Rightarrow$ Trajectory X_1^∞: concatenation of cycles
$C_k := \{X_n\}_{T_{x,k} < n \leq T_{x,k+1}}$

Strong Markov property $\Rightarrow$ cycles C_k are i.i.d.

Irreducibility

Markov chain is **irreducible** iff $\forall x, y \in E$,
$\exists n \in \mathbb{N}, x_0^n \in E^{n+1} \mid x_0 = x,\ x_n = y \ \& \prod_{i=1}^n p_{x_{i-1}x_i} > 0$

i.e., graph on E with directed edge (x, y) iff $p_{xy} > 0$ **strongly connected**

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Standard random walk on graph G irreducible iff G connected

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Proposition

For irreducible chain, if one state x is transient (resp. null recurrent, positive recurrent, d-periodic) then all are

Stationary measures

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Notation: $\mathbb{P}_\nu := \sum_{x \in E} \nu_x \mathbb{P}_x$ chain's distribution when $X_0 \sim \nu$

$\Rightarrow$ For stationary probability distribution π,
$\forall n > 0, \mathbb{P}_\pi(X_n^\infty \in \cdot) = \mathbb{P}_\pi(X_0^\infty \in \cdot)$

Limit theorems 1

Recurrence and stationary measures

Irreducible recurrent chain admits a stationary measure, unique up to multiplicative factor $\forall y \in E,\ \pi_y = \mathbb{E}_x \sum_{n=1}^{T_x} \mathbb{I}_{X_n = y}$

Irreducible chain admits a stationary probability distribution iff it is positive recurrent

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Ergodic theorem

Irreducible, positive recurrent chain satisfies almost sure convergence

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} f(X_n) = \sum_{x\in E} \pi_x f(x)$$

for all π-integrable f, where $\pi =$ unique stationary distribution

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Such chains are called ergodic

Limit theorems 2

Convergence in distribution

Ergodic, aperiodic chain satisfies $\forall x \in E, \lim_{n\to\infty} \mathbb{P}(X_n = x) = \pi_x$
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"Converse"

Irreducible, non-ergodic chain satisfies $\forall x \in E, \lim_{n\to\infty} \mathbb{P}(X_n = x) = 0$

Foster-Lyapunov criterion for ergodicity

Theorem

An irreducible chain such that there exist $V : E \to \mathbb{R}_+$, a finite set $K \subset E$ and $\epsilon, b > 0$ satisfying

$$\mathbb{E}(V(X_{n+1}) - V(X_n) | X_n = x) \leq \begin{cases} -\epsilon, & x \notin K, \\ b - \epsilon, & x \in K, \end{cases}$$

is then ergodic.

Aloha with finitely many stations

Stations $s \in \mathcal{S}$, $|\mathcal{S}| < \infty$

- New arrivals at station s in slot n: $A_{n,s} \in \mathbb{N}$, $\{A_{n,s}\}_{n \geq 0}$ i.i.d.
- Probability of transmission by s if message in queue: p_s
- Source of randomness: $\{B_{n,s}\}_{n \geq 0}$ i.i.d., Bernoulli(p_s)
- Transmits iff $B'_{n,s} = 0$ where $B'_{n,s} = B_{n,s} \mathbb{I}_{L_{n,s} > 0}$

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Queue dynamics

$$L_{n+1,s} = L_{n,s} + A_{n,s} - B'_{n,s} \prod_{s' \neq s} (1 - B'_{n,s'})$$

Aloha with finitely many stations

Assume $\forall s,\ 0 < \mathbb{P}(A_{n,s} = 0) < 1$
Then chain is irreducible and aperiodic

Sufficient condition for ergodicity

$$\forall s, \lambda_s := \mathbb{E}(A_{n,s}) < p_s \prod_{s' \neq s} (1 - p_{s'})$$

Sufficient condition for transience

$$\forall s, \lambda_s > p_s \prod_{s' \neq s} (1 - p_{s'})$$

Aloha with finitely many stations

Symmetric case $\lambda_s = \lambda/|\mathcal{S}|$, $p_s \equiv p$:

Recurrence if $\lambda < |\mathcal{S}|p(1-p)^{|\mathcal{S}|-1}$

Transience if $\lambda > |\mathcal{S}|p(1-p)^{|\mathcal{S}|-1}$

$\Rightarrow$ To achieve stability (ergodicity) for fixed λ, need $p \to 0$ as $|\mathcal{S}| \to \infty$

Impractical! (Collisions take forever to be resolved)

Aloha with infinitely many stations

Many stations, very rarely active (just one message)

- A_n new messages in interval n, $\{A_n\}_{n\geq 0}$ i.i.d.
- Source of randomness $\{B_{n,i}\}_{n,i\geq 0}$ i.i.d., Bernoulli (p)
- Queue evolution

$$L_{n+1} = L_n + A_n - \mathbb{I}_{\sum_{i=1}^{L_n} B_{n,i}=1}$$

- Assumption $0 < \mathbb{P}(A_n = 0) < 1$ ensures irreducibility (and aperiodicity)

Aloha with infinitely many stations

ABRAMSON'S HEURISTIC ARGUMENT

For $A_n \sim \text{Poisson}(\lambda)$, Nb of attempts per slot $\approx \text{Poisson}(G)$ for unknown G

Hence successful transmission with probability Ge^{-G} per slot

Solution to $\lambda = Ge^{-G}$ exists for all $\lambda < 1/e$

Hence "Aloha should be stable (ergodic) whenever $\lambda < 1/e$"

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Theorem: Instability of Aloha

With probability 1, channel jammed forever ($\sum_{i=1}^{L_n} B_{n,i} > 1$) after finite time. Hence only finite number of messages ever transmitted.

Fixing Aloha: richer feedback

Assumption: L_n known

Backlog-dependent retransmission probability $p_n = 1/L_n$
Then system ergodic if $\lambda := \mathbb{E}(A_n) < \frac{1}{e} \approx 0.368$

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Weaker assumption: channel state J_n heard by all stations

Backlog-dependent retransmission probability $p_n = 1/\hat{L}_n$, where estimate $\hat{L}_n$ computed by

$$\hat{L}_{n+1} = \max(1, \hat{L}_n + \alpha \mathbb{I}_{J_n = *} - \beta \mathbb{I}_{J_n = 0})$$

renders Markov chain $(L_n, \hat{L}_n)_{n \geq 0}$ ergodic for suitable $\alpha, \beta > 0$ if $\lambda := \mathbb{E}(A_n) < \frac{1}{e} \approx 0.368$

Fixing Aloha: richer feedback

With same ternary feedback $J_n = \{0, 1, *\}$, can stability hold for $\lambda > 1/e$?

Yes: rather intricate protocols have been invented and shown to achieve stability up to $\lambda = 0.487$

Largest λ for which some protocol based on this feedback is stable? Unknown (only bounds)

Ethernet and variants

Return to Acknowledgement-based feedback (only listen channel's state after transmission)

Variant of exponential backoff: transmit with probability 2^{-k} after k collisions

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Unsolved conjecture

No acknowledgement-based scheme can induce a stable (ergodic) system for any $\lambda > 0$.

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- Time to instability could be huge ("metastable" behavior)
- Only small fraction of channel time used for random access collision resolution:
  Once station "wins" channel access, others wait till its transmission is over

$\rightarrow$ Alternative protocols based on ternary feedback have not been used

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- Simple questions on performance of Random Access yet unsolved, and results more negative than positive
- Still, analysis of Aloha has led to new insights and new designs such as Ethernet