Diffusions et cascades dans les graphes aléatoires

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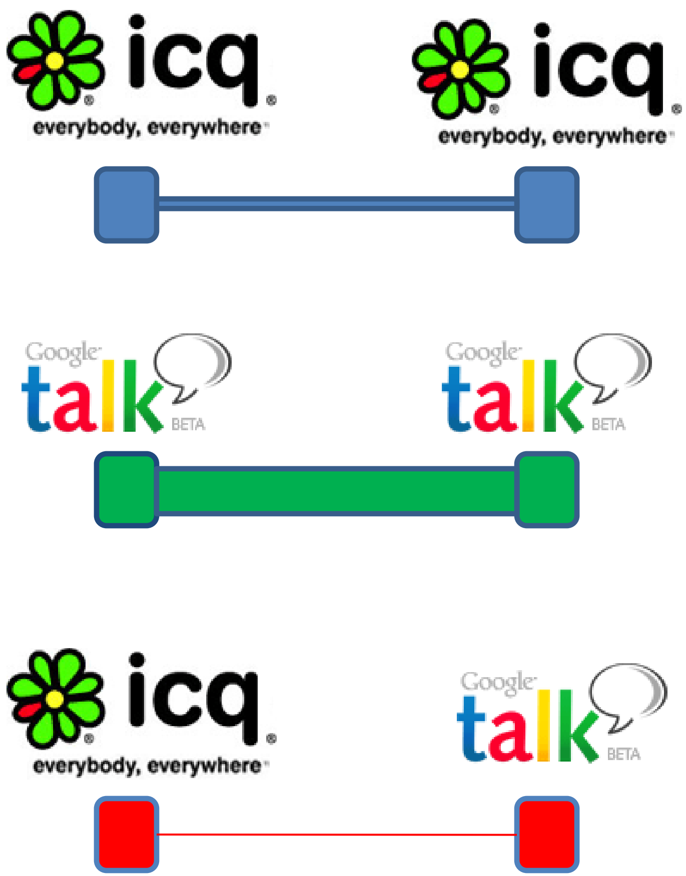
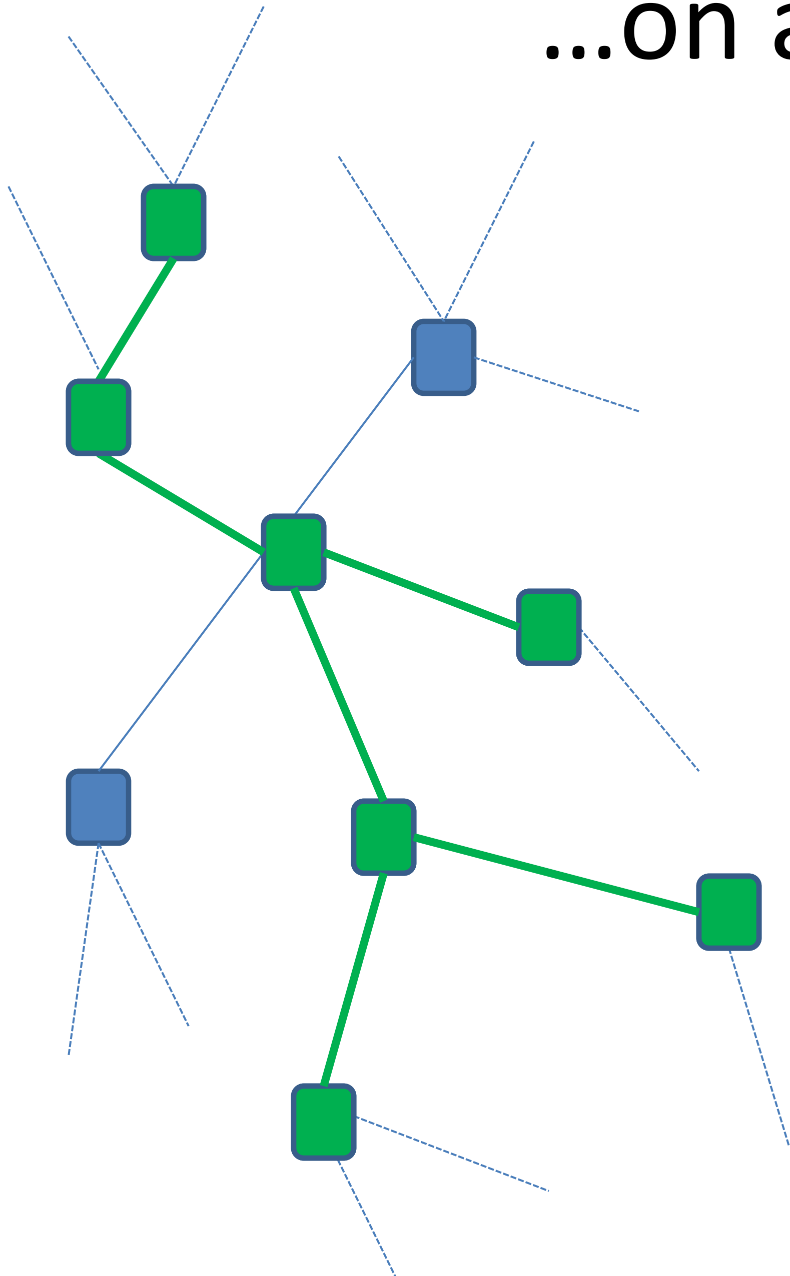
Game-theoretic diffusion model…



- Both receive payoff q.
- Both receive payoff $1-q>q$.
- Both receive nothing.

Morris (2000)

…on a network.



- Everybody start with icq
- Total payoff = sum of the payoffs with each neighbor.
- A fraction of the population is forced to Google talk
- If $2(1-q)>3q$, i.e.

  $2>5q$

Threshold Model

- State of agent i is represented by

$$X_i = \begin{cases} 0 & \text{if icq} \\ 1 & \text{if Google talk} \end{cases}$$

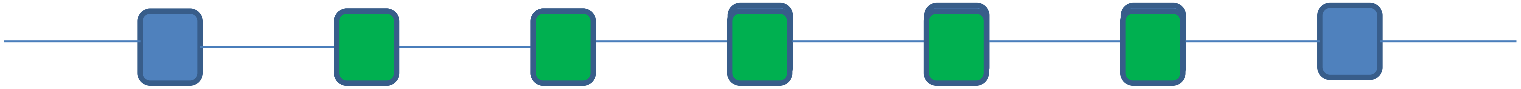
- Switch from icq everybody, everywhere to Google talk BETA if:

$$\sum_{j \sim i} X_j \geq q d_i$$

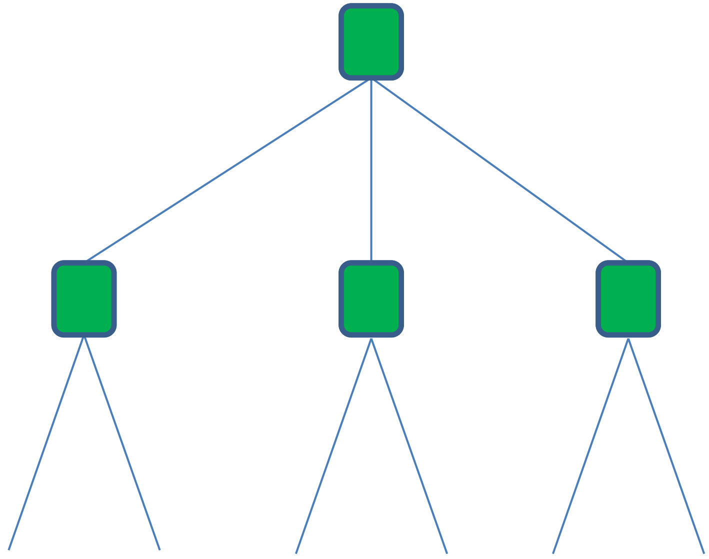
Morris, Contagion (2000)

- Does there exist a finite groupe of players such that their action under best response dynamics spreads contagiously everywhere?
- Contagion threshold: q_c = largest q for which contagious dynamics are possible.
- Example: interaction on the line

$$q_c = \frac{1}{2}$$



Another example: d-regular trees



$$q_c = \frac{1}{d}$$

What happens for random graphs?

- Random graphs with given degree sequence introduced by Molloy and Reed (1995).
- Examples:
  - Random regular graphs. $D = d$
  - Erdös-Réyni graphs, G(n,λ/n). $D \stackrel{d}{=} Poi(\lambda)$
- We are interested in large population asymptotics.
- D is the asymptotic degree distribution.

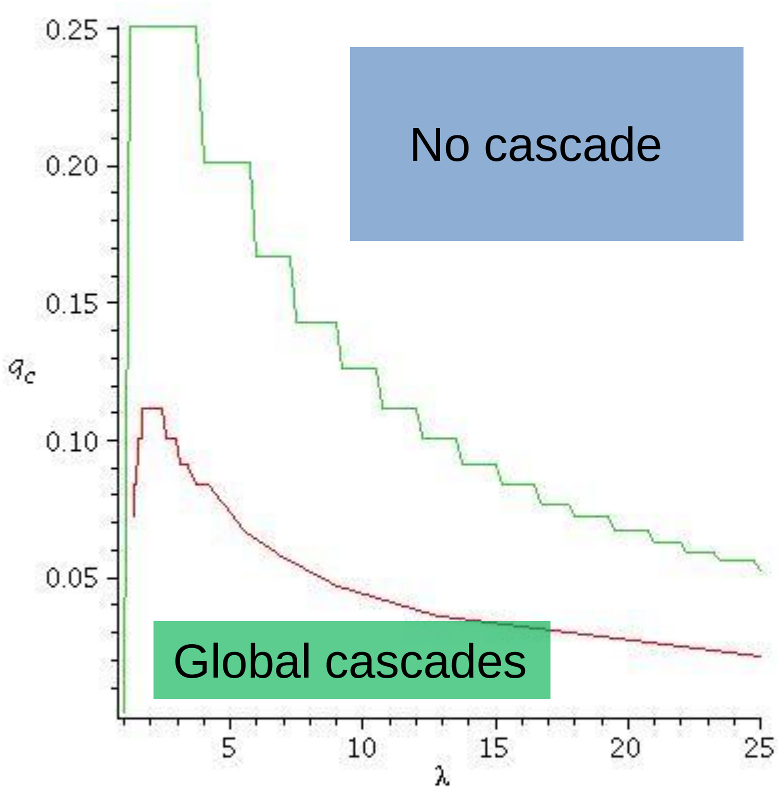
What happens for random graphs?

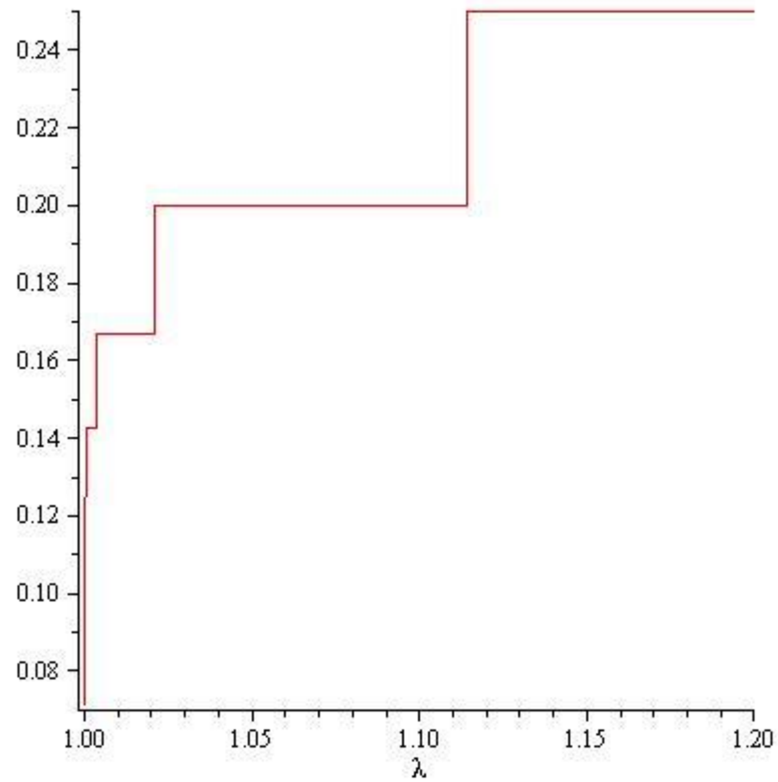
- Random graphs are locally tree-like.

$$q_c = \mathsf{sup}\left\{q :\ \mathbb{E}\left[D(D-1)\mathbb{1}\left(D < q^{-1}\right)\right] > \mathbb{E}[D]\right\}$$

- Random d-regular graph: $q_c = \frac{1}{d}$

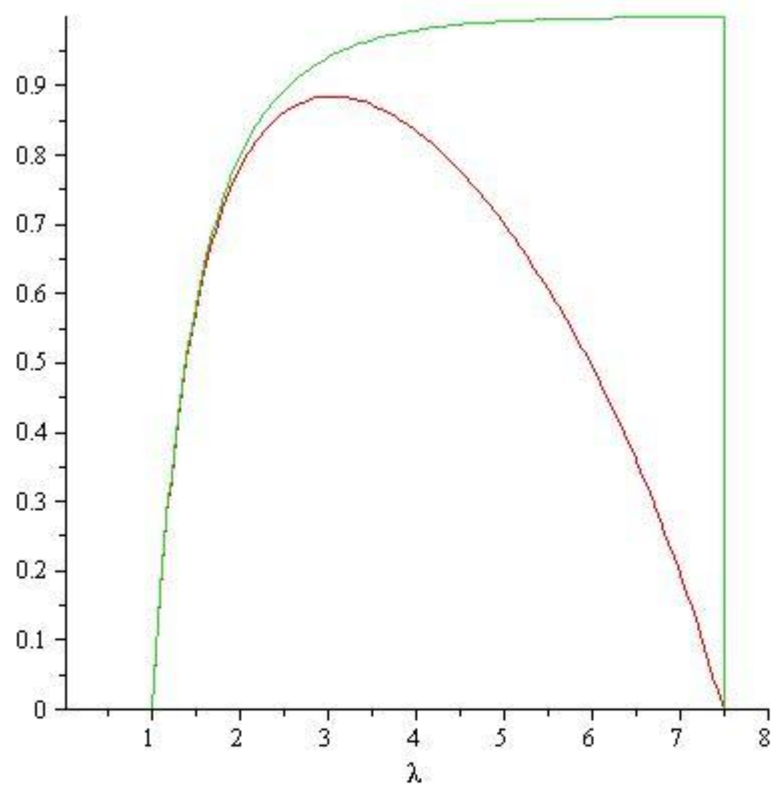
Erdös-Réyni graphs G(n,λ/n)





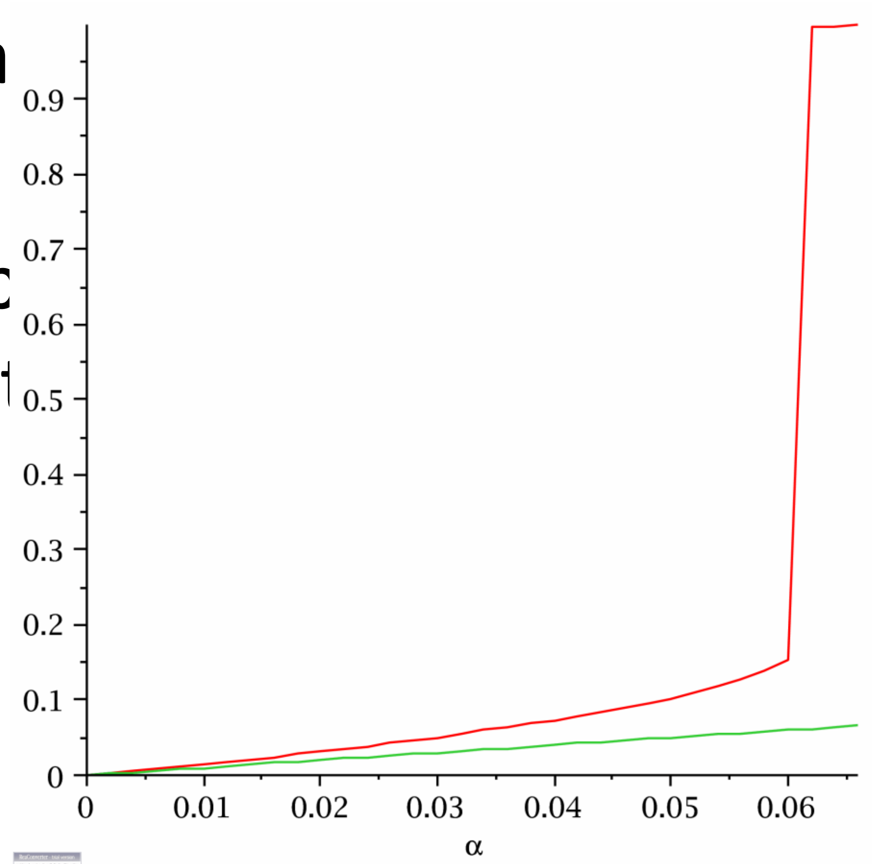
Erdös-Réyni graphs G(n,λ/n)

- Pivotal players: giant component of the subgraph with degrees $\leq q^{-1}$

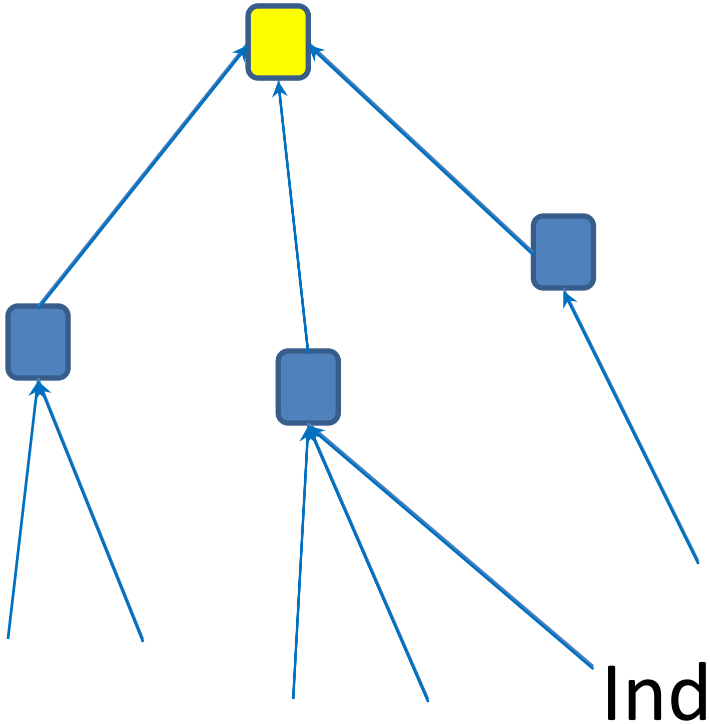


Phase transition continued

- What happens when q is bigger than the cascade capacity?
- If the seed pla
  monotone.
- In a finite grap ble
  final state for t



Locally tree-like



Independent computations on trees

Branching Process Approximation

- Local structure of G = random tree
- Recursive Distributional Equation (RDE) or:

$$Y_i = \begin{cases} 1 & \text{if infected from 'below'} \\ 0 & \text{otherwise.} \end{cases}$$

$$Y_i = 1 - (1 - \sigma_i) \mathbb{1} \left(\sum_{\ell \to i} Y_\ell \leq q d_i \right)$$

Solving the RDE

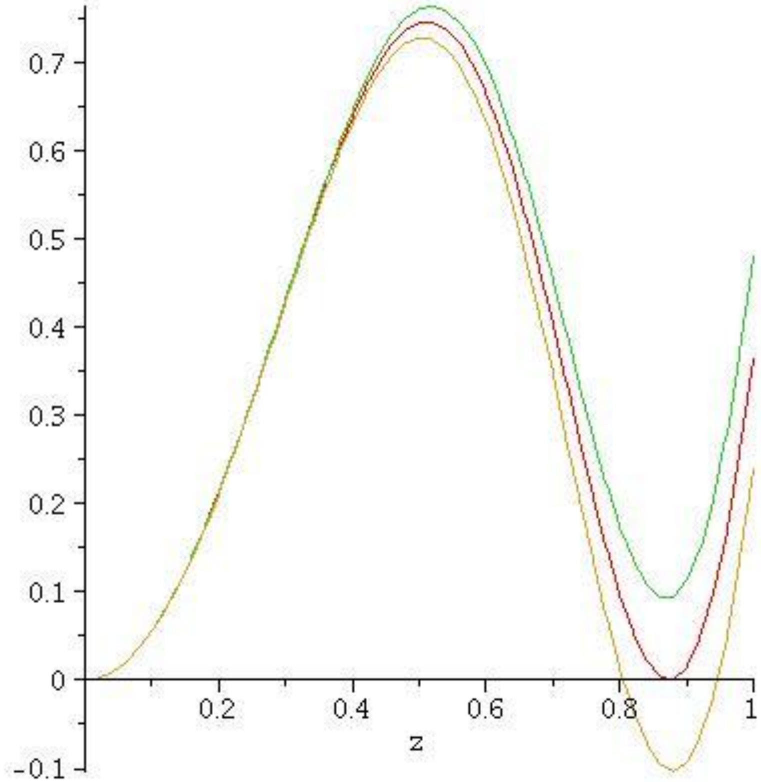
$$Y \stackrel{d}{=} 1 - (1-\sigma)\mathbb{1}\left(\sum_{\ell=1}^{\widehat{D}-1} Y_\ell \le q\widehat{D}\right)$$

$$z = \mathbb{P}(Y = 0)$$

$$\lambda z^2 = (1-\alpha)h(z)$$

$$h(z) = \sum_{s,r \ge s - \lfloor qs \rfloor} r p_s \binom{s}{r} z^r (1-z)^{s-r}$$

Phase transition in one picture



$$z^* = \max\{z \in [0,1],\ \lambda z^2 - (1-\alpha)h(z) = 0\}$$

Conclusion

- The locally tree-like structure gives the intuition for the solution…
- … but not the proof!
- Configuration model + results of Janson and Luczak.
- Generic epidemic model which allows to retrieve basic results for random graphs…
- …. and new ones: contagion threshold, phase transitions.

Merci!

- Diffusion and Cascading Behavior in Random Networks. Available at http://www.di.ens.fr/~lelarge