Proofs on Encrypted Values in Bilinear Groups and an Application to Anonymity of Signatures

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Anonymous Consecutive Delegation of Signing Rights

Fuchsbauer, Pointcheval: Anonymous Proxy Signatures [SCN'08]

Delegation A delegator delegates his signing rights to a proxy signer (or delegatee) who can then sign on the delegator's behalf

Consecutiveness A delegatee may re-delegate the received signing rights
$\Rightarrow$ intermediate delegators

Anonymity All intermediate delegators and the proxy signer remain anonymous

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After verifying a proxy signature one knows that someone entitled signed but nothing more.

Application: GRID computing

User authenticates herself and starts process which needs to authenticate to resources / start subprocesses

$\Rightarrow$ Delegation and re-delegation of signing rights

No need to know that it was not the user herself to be authenticated

Relation to Other Primitives

Anonymous proxy signatures are a generalization of

- Proxy signatures (consecutive delegation)
  formalized by [BPW03]
- (Dynamic) Group signatures (anonymity)
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and satisfy the respective security notions.

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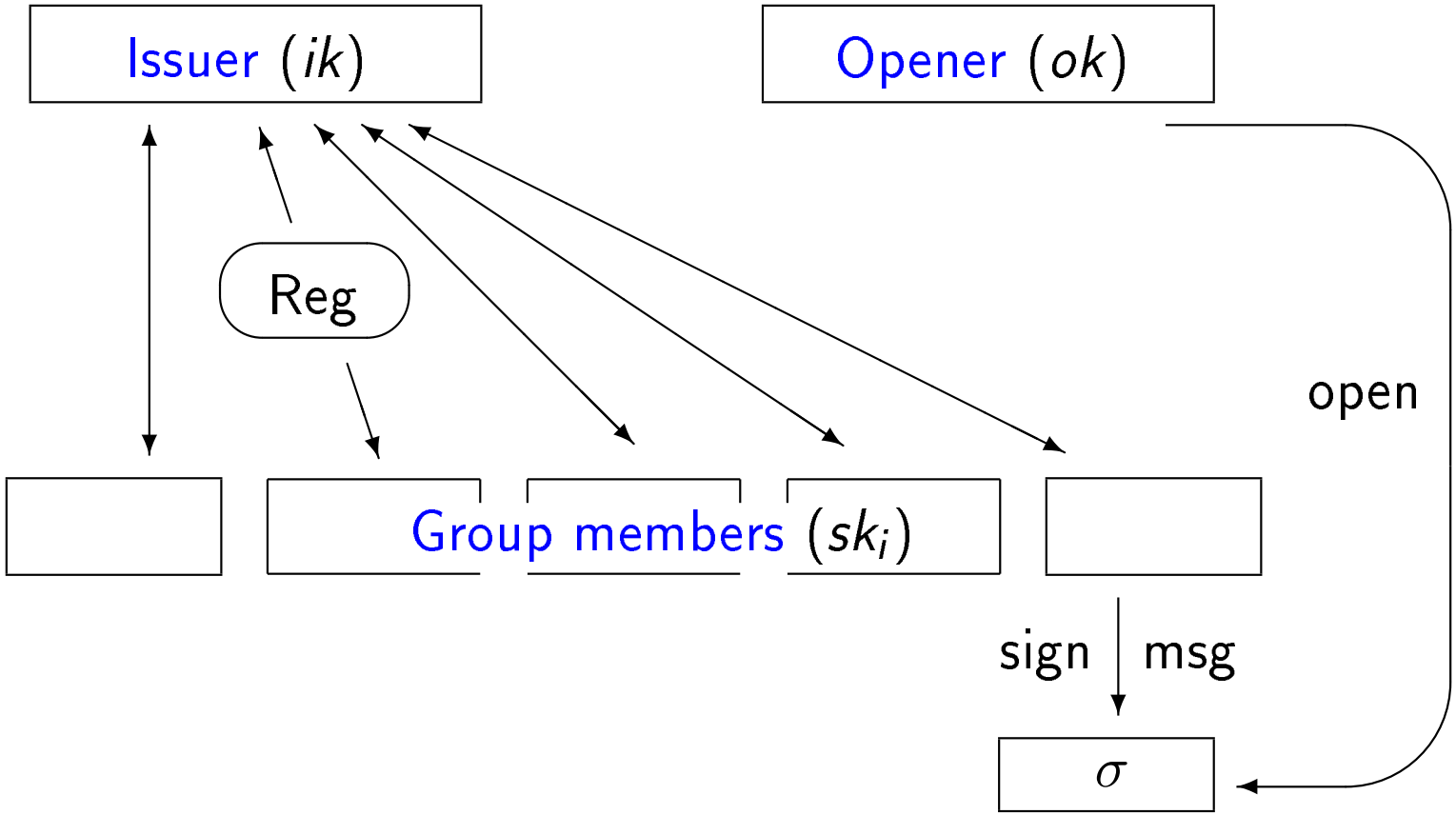
- recently: Delegatable Anonymous Credentials [BCCKLS09]

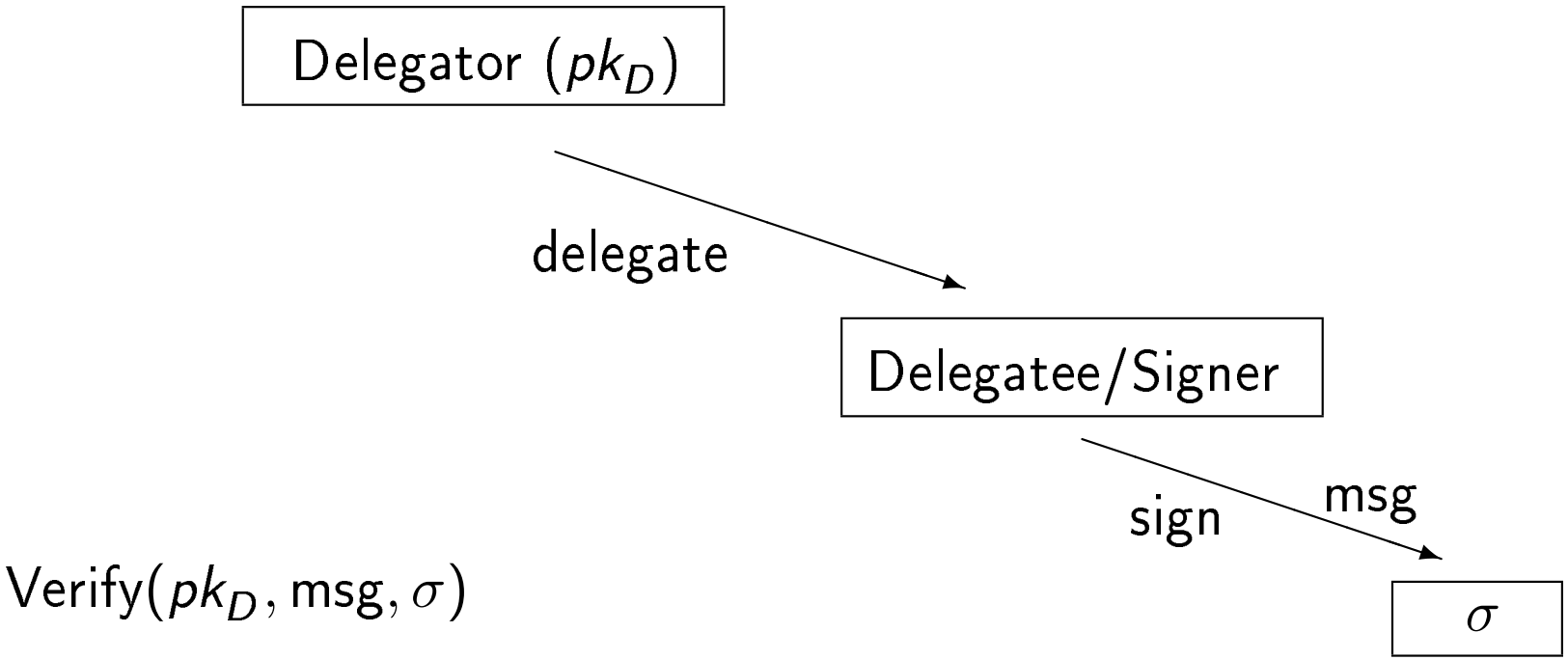
(Dynamic) Group Signatures

Group public key: pk



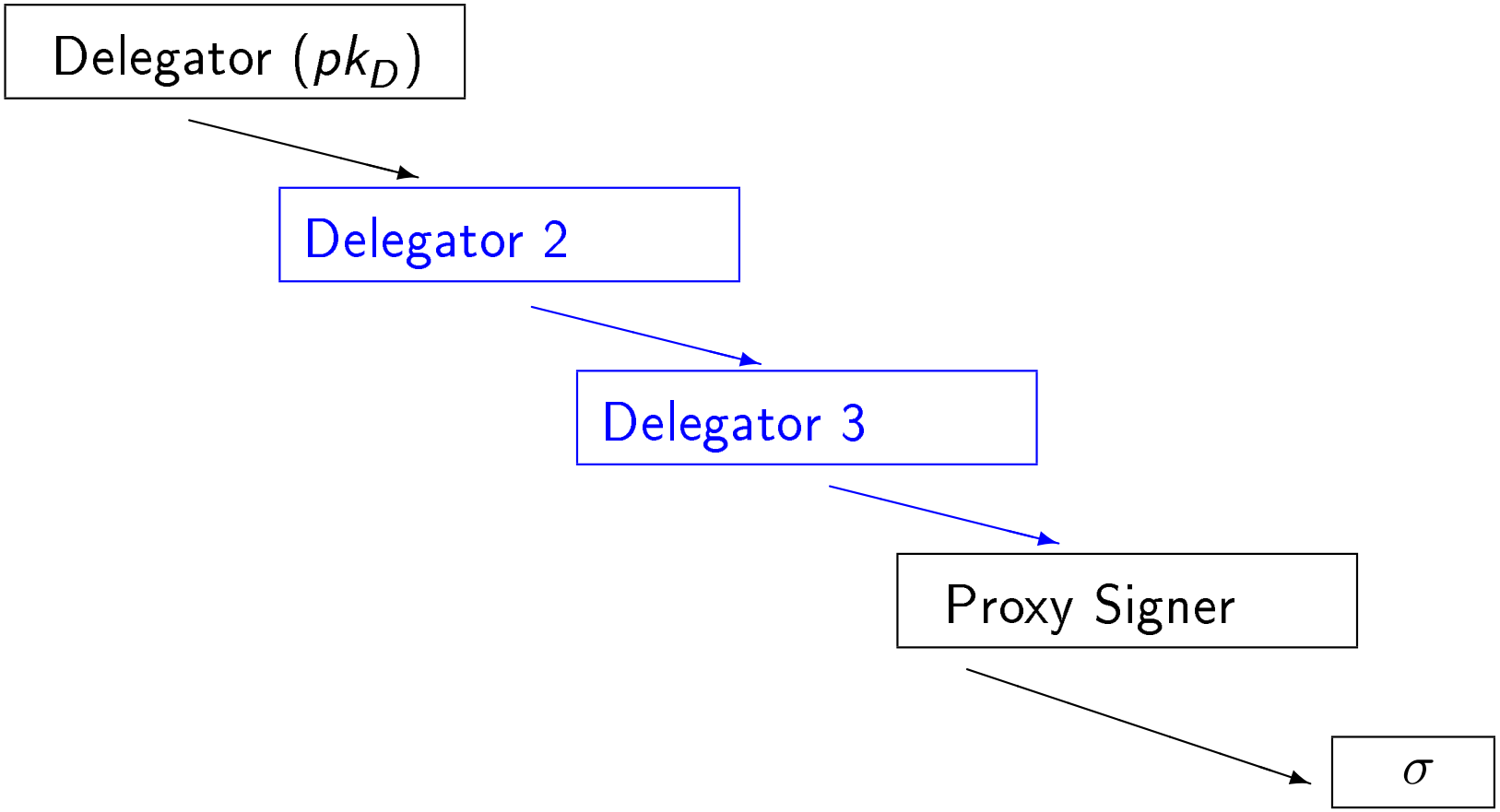
Verification: $\mathsf{Verify}(pk, \mathsf{msg}, \sigma) = 1$

Proxy Signatures



$\text{Verify}(pk_D, \text{msg}, \sigma)$

Proxy Signatures, Consecutive Delegations



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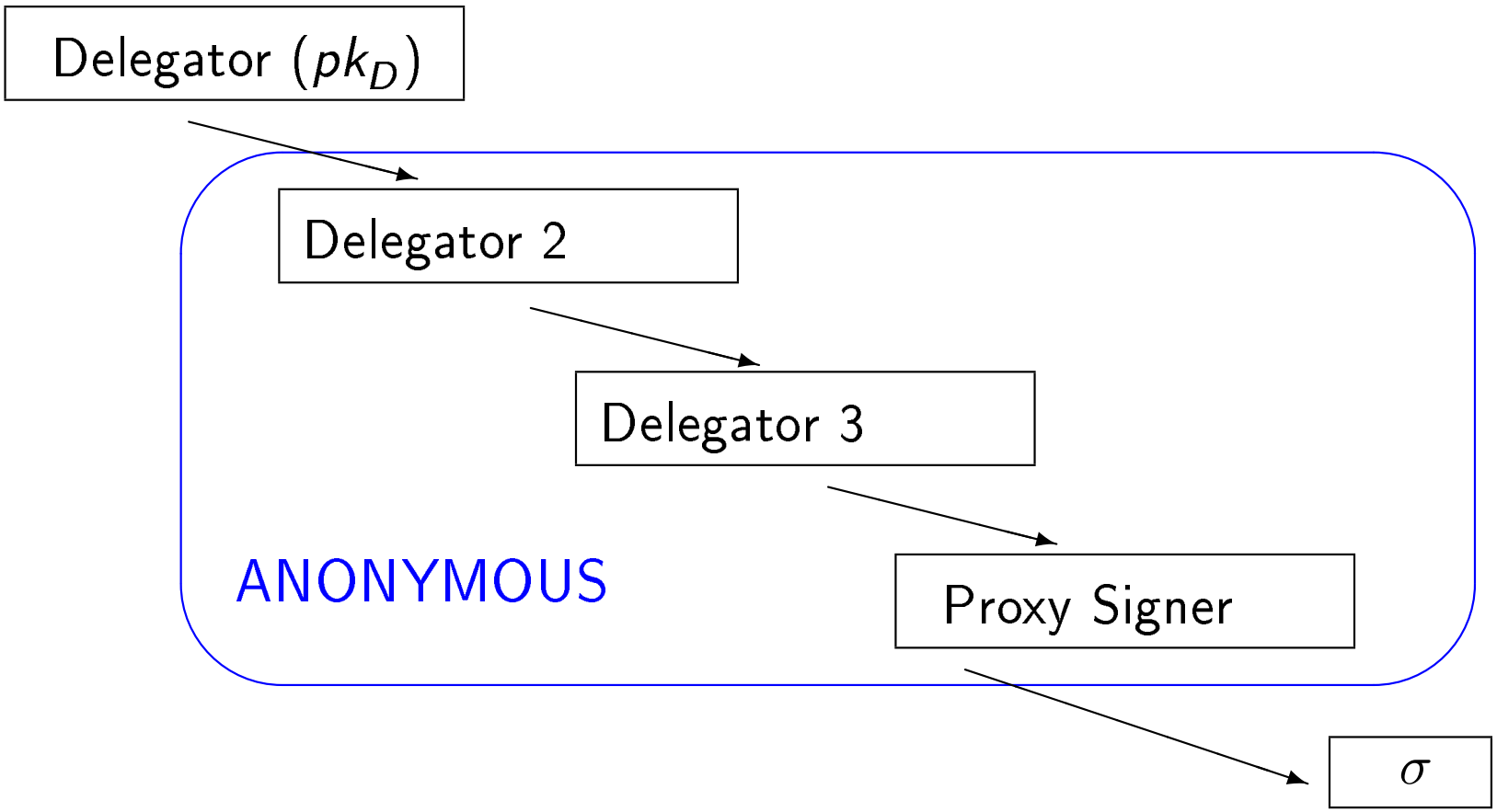

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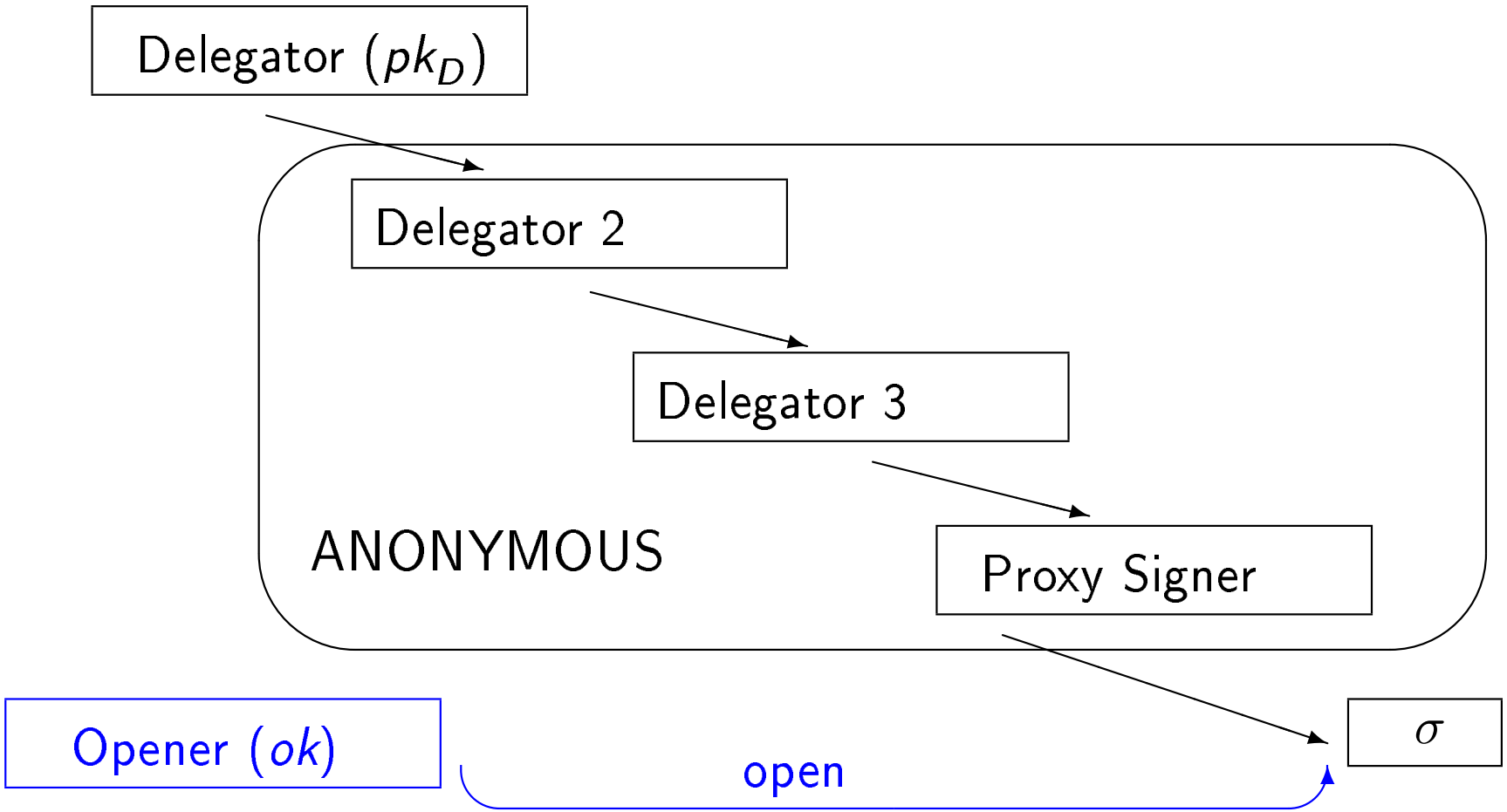

Algorithms of Anonymous Proxy Signature Scheme

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$$ok, M, \sigma \longrightarrow \text{Open} \longrightarrow \text{a list of users or } \perp \text{ (failure)}$$

Security for Anonymous Proxy Signatures

Anonymity	intermediate delegators and proxy signer remain anonymous
Traceability	every valid signature can be traced to its intermediate delegators and proxy signer
Non-Frameability	no one can produce a signature that, when opened, wrongfully reveals a delegator or signer

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- Public-key encryption (IND-CPA)
- Non-interactive zero-knowledge proofs

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(existence follows from trapdoor permutations)

Generic Construction, Overview

Setup Generates decryption key for opening authority; signing key for issuer
Parameters: resp. public keys, *crs* for NIZK

Register Issuer signs user's public key $\longrightarrow$ *certificate*

Delegate Sign delegatee's public key $\longrightarrow$ *warrant*
Re-delegate: additionally forward received warrants

Proxy-Sign Sign message, encrypt

- delegators' verification keys and certificates
- warrants
- signature on message

NIZK that plaintext contains valid signatures

Verify Verify NIZK

Open Decrypt ciphertext

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The Subgroup Decision Assumption and BGN Encryption

Let $\mathbf{G} = (n, \mathbb{G}, \mathbb{G}_T, e, g)$ be a bilinear group of order $|\mathbb{G}| = n = pq$, where p, q prime.

Subgroup Decision Assumption (SD)

No p.p.t. adversary can distinguish a random element of $\mathbb{G}$ from a random element of $\mathbb{G}_q$, the subgroup of order q.

Boneh-Goh-Nissim (BGN) Encryption [TCC'05]

KeyGen Generate bilinear group $\mathbf{G}$ with $|\mathbb{G}| = n = pq$, choose
$h \leftarrow \mathbb{G}_q$.
$sk = q$.

Encrypt
- Choose $r \leftarrow \mathbb{Z}_n$
- Set $C := g^m h^r$.

Decrypt $C^q = (g^m h^r)^q = (g^q)^m$, thus $\log_{g^q} C^q = m$.

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Boyen Waters Group Signatures [PKC'07] I

Hybrid Scheme

Setup

- Choose bilinear group $\mathbf{G}$
- Choose parameters for Waters signatures $(g, g_2, \vec{u}) \in \mathbb{G}^{m+3}$
- Choose $\omega, \alpha \leftarrow \mathbb{Z}_p$,
- Define $pp = (g, g_2, \vec{u}, \Omega = g^{\omega}, A = e(g,g)^{\alpha})$ $mk = (\omega, g^{\alpha})$

Extract

- Choose $s_i \leftarrow \mathbb{Z}_p$
- Return $K_i = ((g^{\alpha})^{\frac{1}{\omega + s_i}}, g^{s_i}, g_2^{s_i})$

Sign

- Choose $r \leftarrow \mathbb{Z}_p$ let $\mathcal{F}(M) := u_0 \prod_{i=1}^{m} u_i^{M_i}$
- Return $S = (K_1, K_2, K_3 \cdot \mathcal{F}(M)^r, g^{-r}) \in \mathbb{G}^4$

Verify

$$e(S_1, S_2\Omega) \cdot A^{-1} \stackrel{?}{=} 1$$

$$e(S_2, g_2)^{-1} \cdot e(g, S_3) \cdot e(S_4, \mathcal{F}(M)) \stackrel{?}{=} 1$$

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Boyen Waters Group Signatures [PKC'07] II

Group Signatures

Setup
- Choose a bilinear group $\mathbf{G}$ of order $n = pq$
- Choose keys pp and mk for hybrid scheme
- Publish an additional element $h \in \mathbb{G}_q$
- Set the tracing key as $tk = q \in \mathbb{Z}$

Enroll
- Choose $s_i \leftarrow \mathbb{Z}_p$, s.t. $\omega + s_i \in \mathbb{Z}_n^\times$
- Return $K_i = ((g^\alpha)^{\frac{1}{\omega+s_i}}, g^{s_i}, g_2^{s_i})$

Sign
- Make hybrid signature (S_1, S_2, S_3, S_4)
- BGN-encrypt components: $S_i' := S_i \cdot h^{\rho_i}$ for $\rho_i \leftarrow \mathbb{Z}_n$
- Add proofs π_1, π_2 for each verification relation

Verify

$$e(S_1', S_2'\Omega) \cdot A^{-1} \stackrel{?}{=} e(h, \pi_1)$$

$$e(S_2', g_2)^{-1} \cdot e(g, S_3') \cdot e(S_4', \mathcal{F}(M)) \stackrel{?}{=} e(h, \pi_2)$$

Trace for each s_i, check $(S_2')^q \stackrel{?}{=} (g^{s_i})^q$

Boyen Waters Group Signatures [PKC'07] III

Security of Group Signature

Anonymity.

- Replace h by a random element in $\mathbb{G}$ (indist. by SD).
- Lemma: If $h \leftarrow \mathbb{G}$ then signer id is statistically hidden, i.e. π_1 and π_2 do not leak information.

"Full Traceability". Reduction to unforgeability of hybrid signatures in $\mathbb{G}_p$

- Get parameters, build twin scheme in $\mathbb{G}_q$.
- Parameters of group signature are products of parameters of schemes in $\mathbb{G}_p$ and $\mathbb{G}_q$.
- Forgery (S_1', S_2', S_3', S_4') is projected to $\mathbb{G}_p$ by raising elements to the power of θ with $\theta \equiv 1 \pmod{p}$ and $\theta \equiv 0 \pmod{q}$.

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The Leak-Tightness Lemma I

Let $(n, \mathbb{G}, \mathbb{G}_T, e, g)$ be a bilinear group, and let $a_j, b_j \in \mathbb{G}$, $\delta_{j,i}, \varepsilon_{j,i} \in \mathbb{Z}_n$ for $1 \leq j \leq \ell$, $1 \leq i \leq m$. Let $(X_i)_{i=1}^m \in \mathbb{G}^m$ satisfy a **pairing-product equation** $E_{(a_j,b_j)_j}$ that is

$$E_{(a_j,b_j)_j}(X_1, \ldots, X_m): \quad \prod_{j=1}^{\ell} e\left(a_j \prod_{i=1}^{m} X_i^{\delta_{j,i}},\ b_j \prod_{i=1}^{m} X_i^{\varepsilon_{j,i}}\right) = 1\ .$$

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1. Form of a proof

The Leak-Tightness Lemma I

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1. Let $H \in \mathbb{G}$, $(\rho_i)_{i=1}^m \in {\mathbb{Z}_n}^m$. Then $\widetilde{X}_i := X_i H^{\rho_i}$ for $1 \leq i \leq m$ satisfy

$$\prod_j e\Big(a_j \prod_i \widetilde{X}_i^{\delta_{j,i}},\ b_j \prod_i \widetilde{X}_i^{\varepsilon_{j,i}}\Big) = e\Big(H,\ P_E\big((X_i),(\rho_i)\big)\Big)\ , \qquad (\tilde{E})$$

where $P_E\big((X_i),(\rho_i)\big) :=$

$$\prod_j \Big((a_j \prod_i X_i^{\delta_{j,i}})^{\sum \varepsilon_{j,i}\rho_i} (b_j \prod_i X_i^{\varepsilon_{j,i}})^{\sum \delta_{j,i}\rho_i} H^{(\sum \delta_{j,i}\rho_i)(\sum \varepsilon_{j,i}\rho_i)}\Big)\ .$$

The Leak-Tightness Lemma II

2. Proofs do not leak info on plaintexts

The Leak-Tightness Lemma II

2. Given (X_i) and (X'_i) both satisfying E, and (ρ_i), (ρ'_i), s.t. for all $1 \leq i \leq m$: $X_i H^{\rho_i} = X'_i H^{\rho'_i}$, then

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3. Simulation

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3. Let $|G| = pq$, let $a_j, b_j, X_i \in \mathbb{G}_p$; $c_j, d_j, Y_i \in \mathbb{G}_q$ for all i, j. If (X_i) satisfy $E_{(a_j,b_j)_j}$ and (Y_i) satisfy $E_{(c_j,d_j)_j}$, then $(X_i Y_i)$ satisfy $E_{(a_j c_j, b_j d_j)_j}$.

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4. Projection

The Leak-Tightness Lemma II

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$$P_E\big((X_i),(\rho_i)\big) = P_E\big((X'_i),(\rho'_i)\big) \ .$$

3. Let $|G| = pq$, let $a_j, b_j, X_i \in \mathbb{G}_p$; $c_j, d_j, Y_i \in \mathbb{G}_q$ for all i, j. If (X_i) satisfy $E_{(a_j,b_j)_j}$ and (Y_i) satisfy $E_{(c_j,d_j)_j}$, then $(X_i Y_i)$ satisfy $E_{(a_j c_j, b_j d_j)_j}$.

4. Let furthermore $H \in \mathbb{G}_q$ and $\theta \in \mathbb{N}$ be such that $\theta \equiv 1 \pmod{p}$ and $\theta \equiv 0 \pmod{q}$. If $(\widetilde{X}_i) \in \mathbb{G}$ satisfy $\tilde{E}_{(a_j c_j, b_j d_j)_j}$ for some P_E, then $(\widetilde{X}_i^{\theta})$ satisfy $E_{(a_j,b_j)_j}$.

Re-randomization

$$(X_i) \text{ satisfy } \quad \prod_j e\left(a_j \prod_i X_i^{\delta_{j,i}},\ b_j \prod_i X_i^{\varepsilon_{j,i}}\right) = 1$$

$$(\widetilde{X}_i) \text{ satisfy } \quad \prod_j e\left(a_j \prod_i \widetilde{X}_i^{\delta_{j,i}},\ b_j \prod_i \widetilde{X}_i^{\varepsilon_{j,i}}\right) = e\left(H,\ P_E\left((X_i),(\rho_i)\right)\right)$$

Re-randomization

$$(X_i) \text{ satisfy} \quad \prod_j e\left(a_j \prod_i X_i^{\delta_{j,i}},\ b_j \prod_i X_i^{\varepsilon_{j,i}}\right) = e(H, P')$$

$$(\widetilde{X}_i) \text{ satisfy} \quad \prod_j e\left(a_j \prod_i \widetilde{X}_i^{\delta_{j,i}},\ b_j \prod_i \widetilde{X}_i^{\varepsilon_{j,i}}\right) = e\left(H,\ P' \cdot P_E\left((X_i), (\rho_i)\right)\right)$$

Re-randomization

$$(X_i) \text{ satisfy} \quad \prod_j e\left(a_j \prod_i X_i^{\delta_{j,i}},\ b_j \prod_i X_i^{\varepsilon_{j,i}}\right) = e(H, P')$$

$$(\widetilde{X}_i) \text{ satisfy} \quad \prod_j e\left(a_j \prod_i \widetilde{X}_i^{\delta_{j,i}},\ b_j \prod_i \widetilde{X}_i^{\varepsilon_{j,i}}\right) = e\left(H,\ P' \cdot P_E((X_i), (\rho_i))\right)$$

So, given a proof P for $(\widetilde{X}_i)$ satisfying $\widetilde{E}$, one can re-randomize the $(\widetilde{X}_i)$:

$$\widetilde{\widetilde{X}}_i := \widetilde{X}_i H^{\rho'_i} \qquad \text{with } \rho'_i \leftarrow \mathbb{Z}_p$$

and adapt the proof (without knowledge of the plaintexts!):

$$P_{\text{new}} := P \cdot P_E((\widetilde{X}), (\rho'_i))$$

If $((\widetilde{X}_i), P)$ and $((\widetilde{Y}_i), P')$ both satisfy $\widetilde{E}$, then their re-randomizations are indistinguishable by SD and the Lemma.

Application of Re-randomization

Intermediate Delegator Anonymity

Even the delegatee cannot distinguish warrants from different previous delegators.

Compatible Signature Scheme

A signature scheme with the following properties:

- EUF-CMA secure
- verification keys lie in message space
- messages and signatures are $\mathbb{G}$-elements
- verification by checking pairing-product equations.
- (efficient?)

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- verification keys lie in message space
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This paper: not quite efficient, based on new kind of assumption.
Efficient implementation:

F: Automorphic Signatures in Bilinear Groups
`http://eprint.iacr.org/2009/320`

Messages and public keys in $\mathbb{G}^2$. Signatures in $\mathbb{G}^5$
Verification: 7 pairing evaluations.
q-SDH-style Assumption

Thank you!