Commuting signatures and verifiable encryption

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Results

New primitive: Commuting signatures (and verifiable encryption)

- New functionality
- Efficient instantiation in pairing groups

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- Significant efficiency improvements

Other results: Groth-Sahai proofs

- Properties of proofs
- Stronger notion of simulation

Outline of this talk

1. **Commuting signatures**
2. **Delegatable anonymous credentials**
3. **Instantiating commuting signatures**

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3. Instantiating commuting signatures

Commuting signatures and verifiable encryption I

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Proof adaptation:

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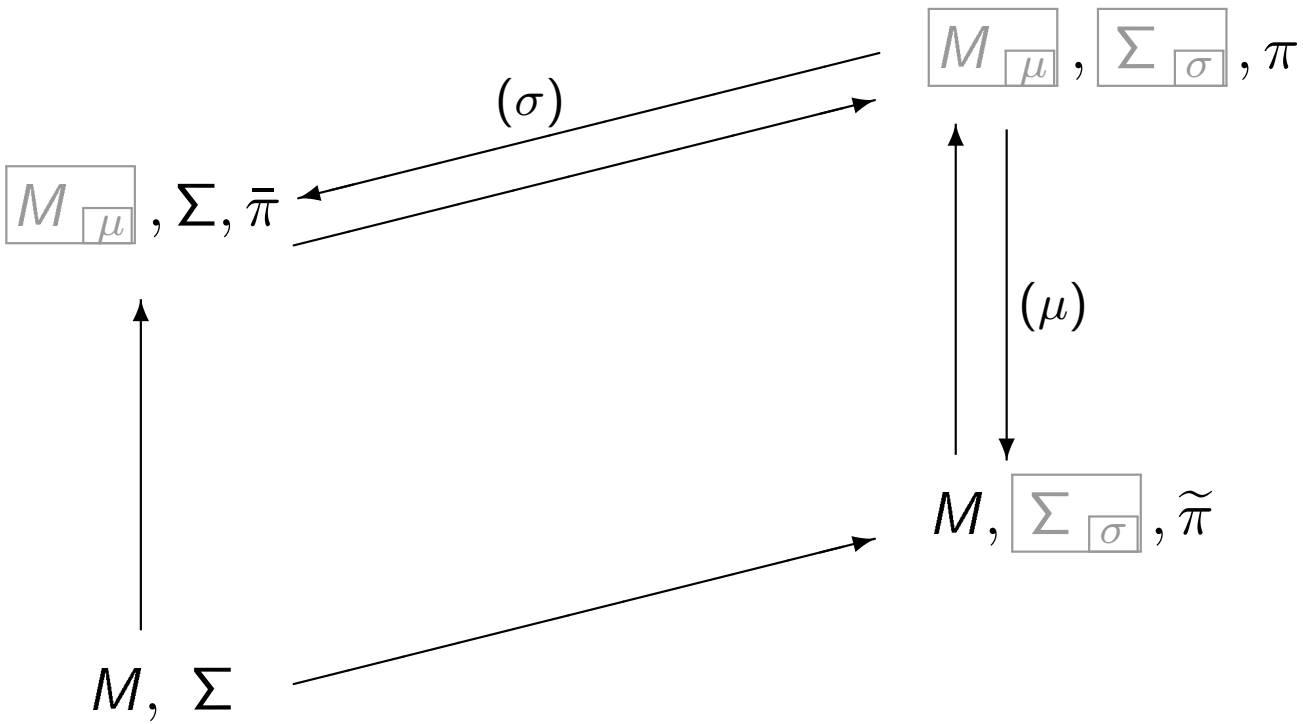
- Commuting signature and verifiable encryption

Sign plaintext then encrypt $\iff$ encrypt then sign plaintext

Sign M given $\boxed{M}$: $\boxed{M} \xrightarrow{sk} \boxed{\Sigma}, \pi$ Verification: $vk, \boxed{M}, \boxed{\Sigma}, \pi$

Commuting signatures and verifiable encryption II



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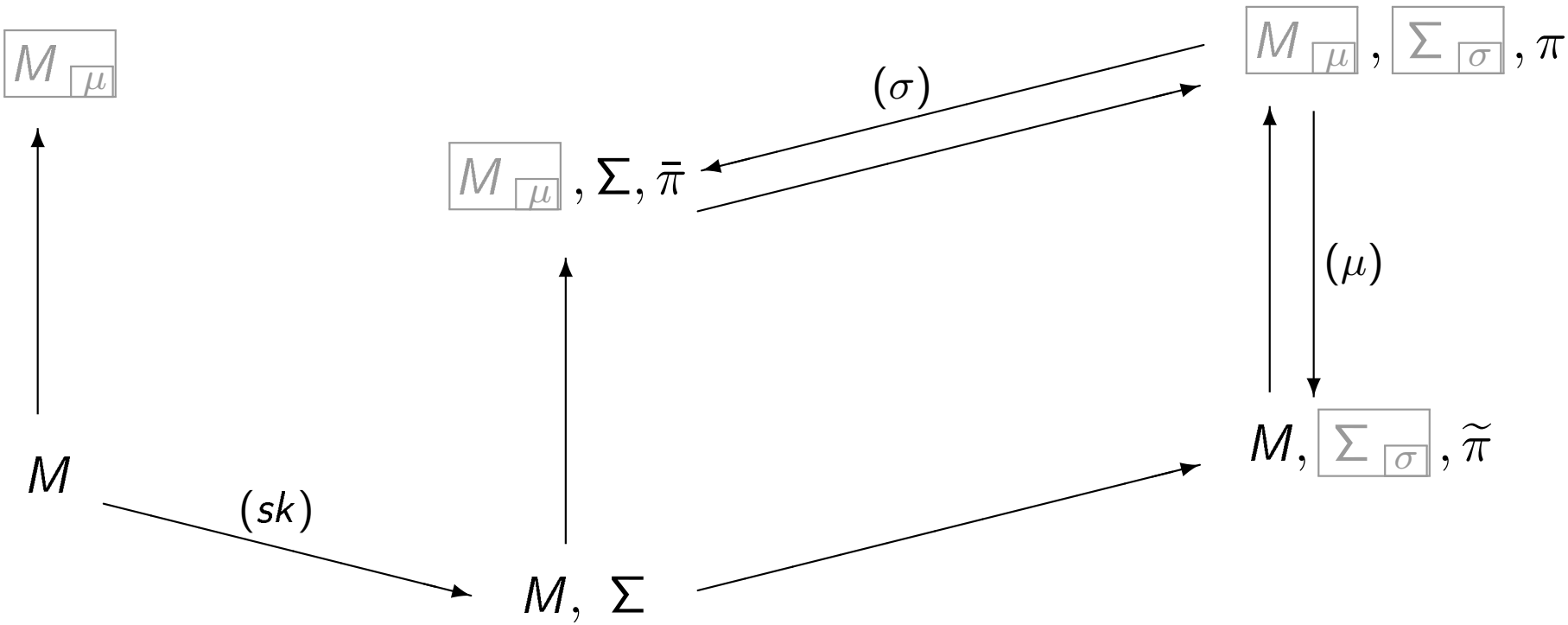

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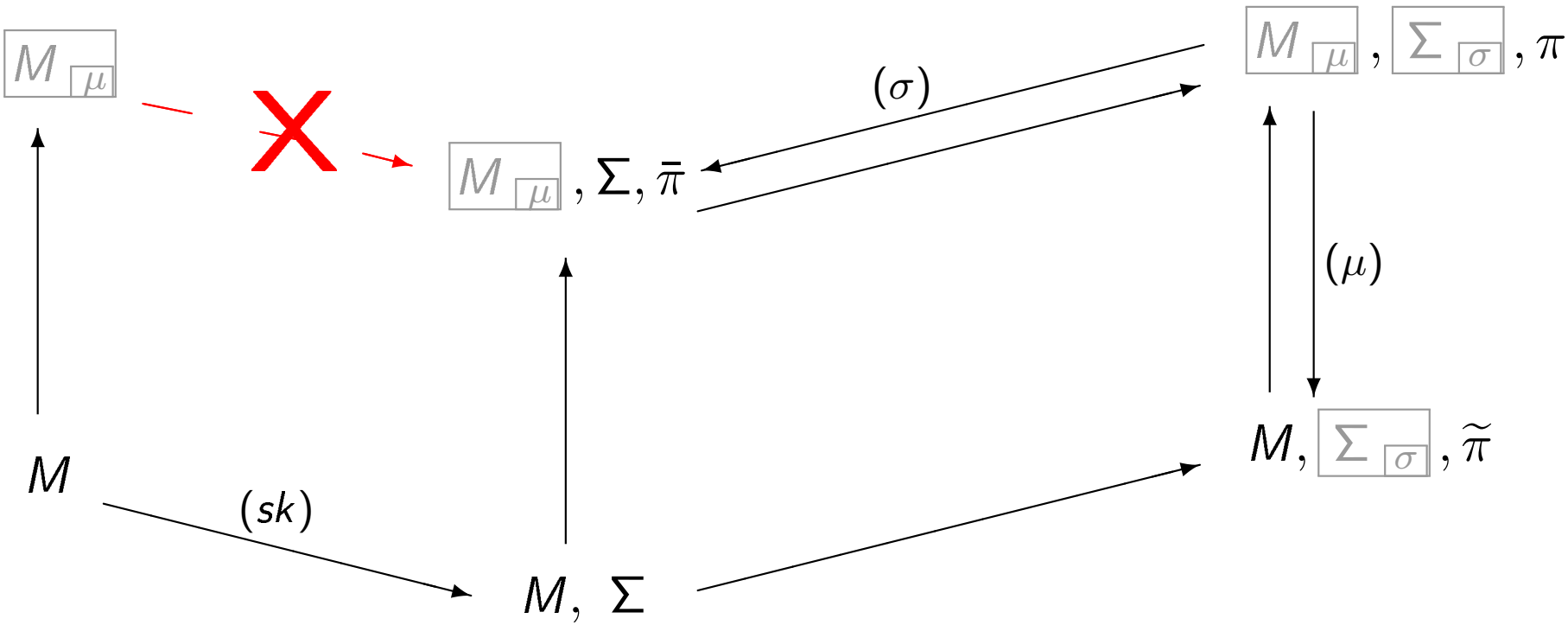

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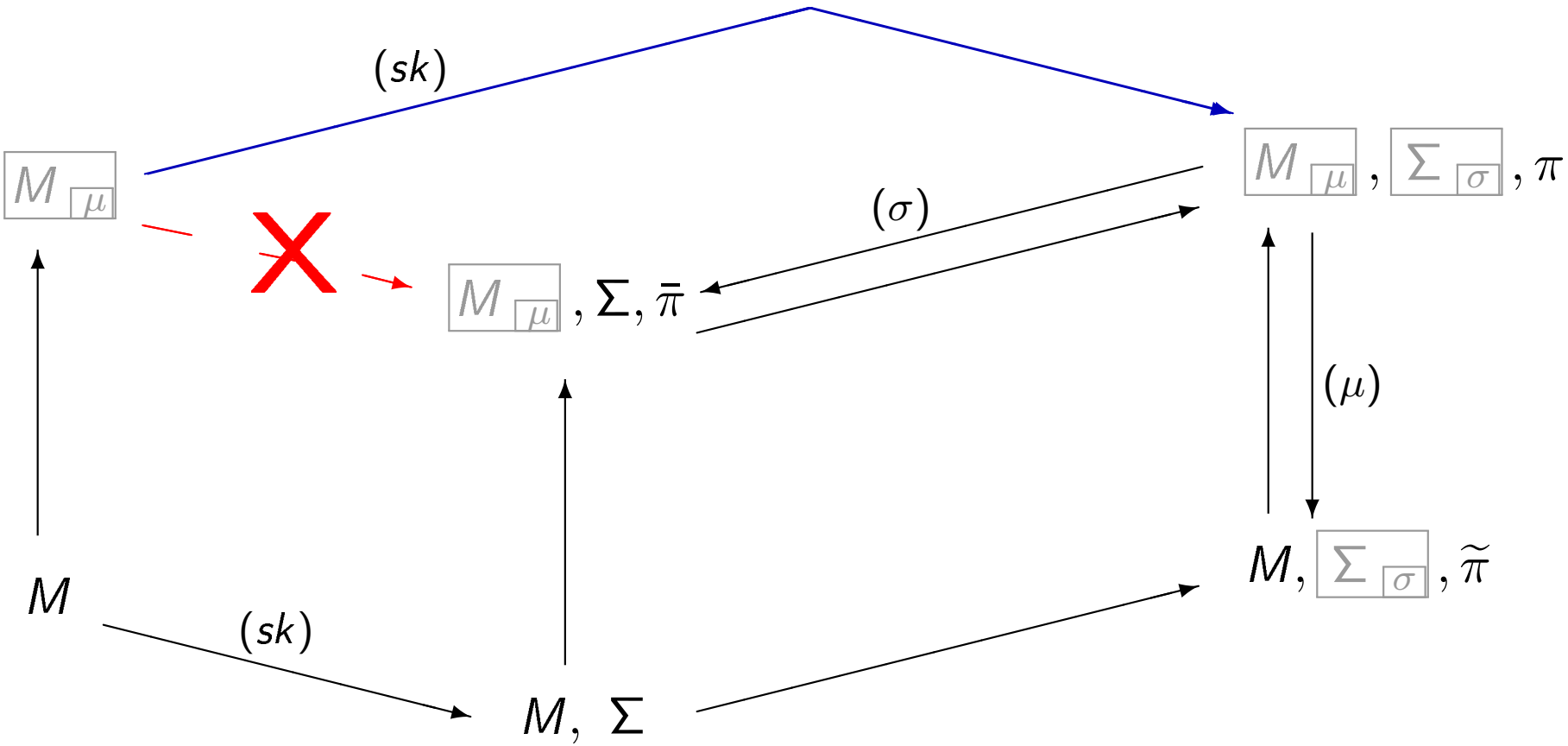

1. Commuting signatures

2. **Delegatable anonymous credentials**

3. Instantiating commuting signatures

Delegatable anonymous credentials [BCCKLS09]

- Users can *prove* to hold credential w/o revealing their identity
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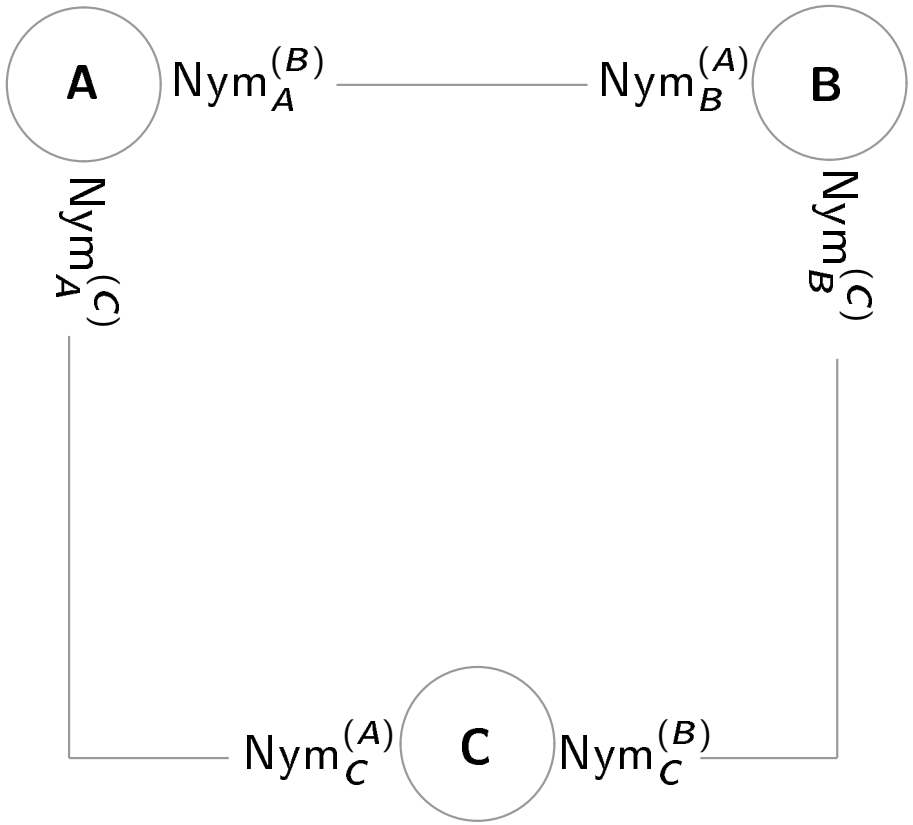
- . . . can publish pseudonym as *public key* for a credential
- . . . run *interactive protocol* to issue/delegate credentials to other users
- . . . prove to hold credentials for every pseudonym

Non-interactively delegatable anonymous credentials



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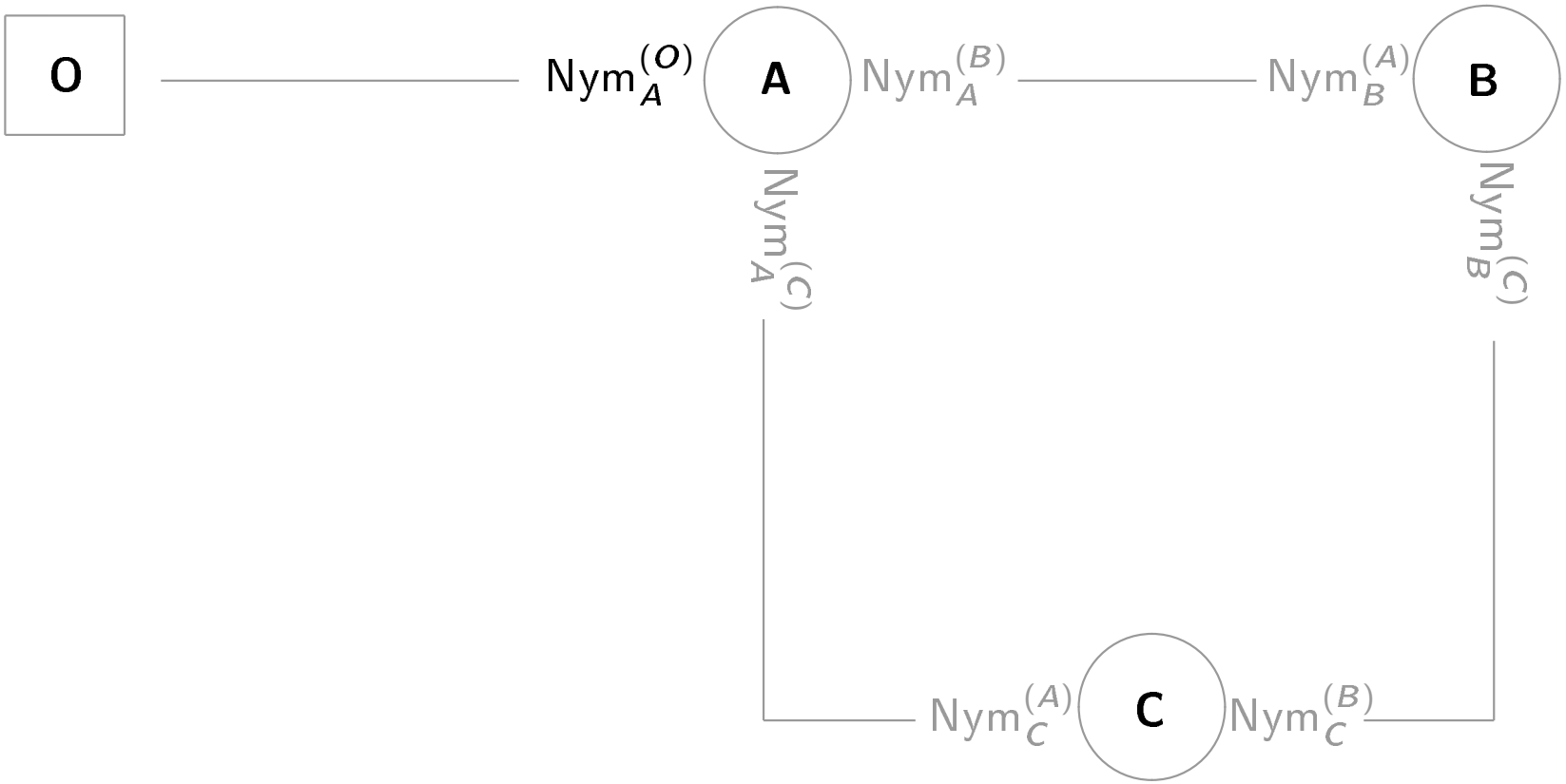

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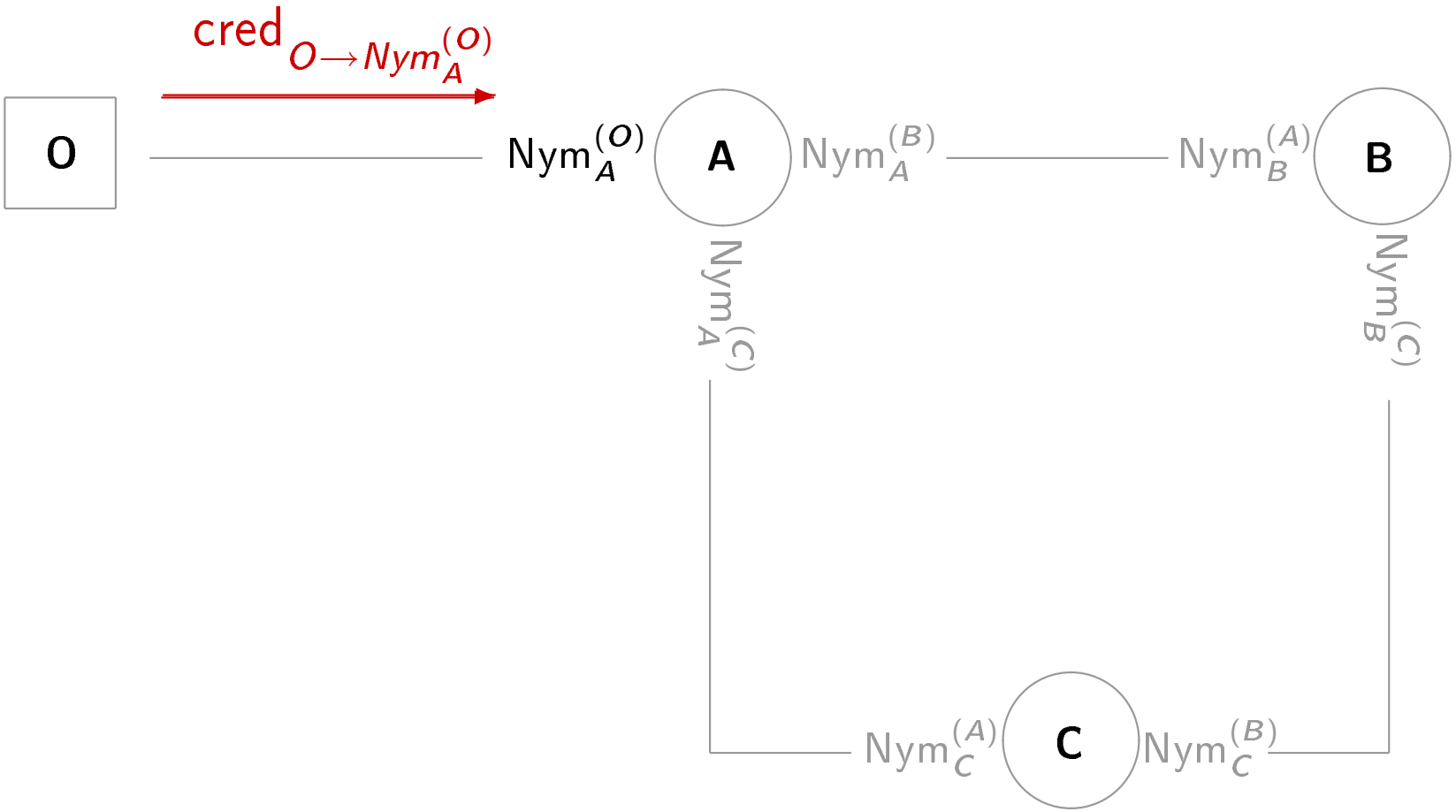

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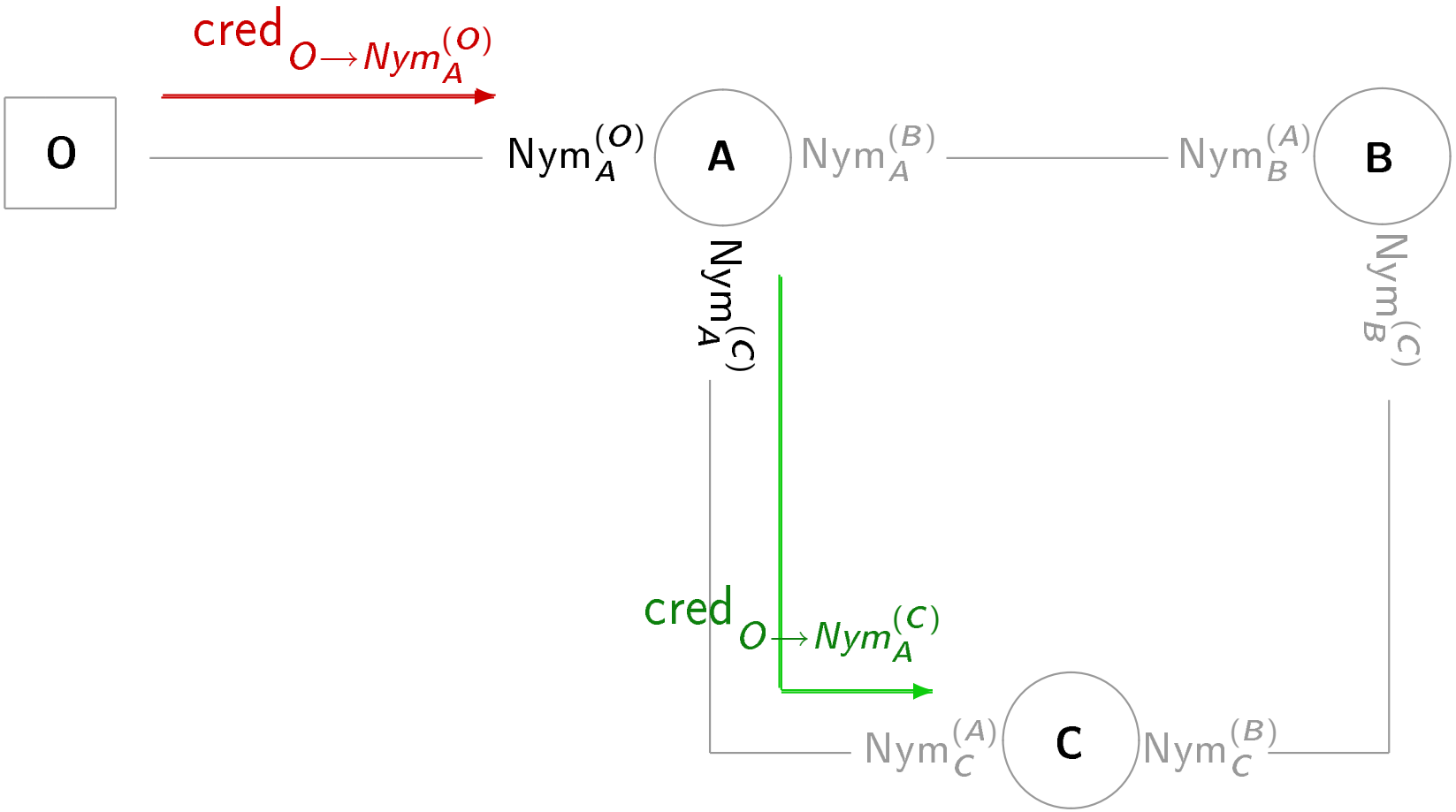

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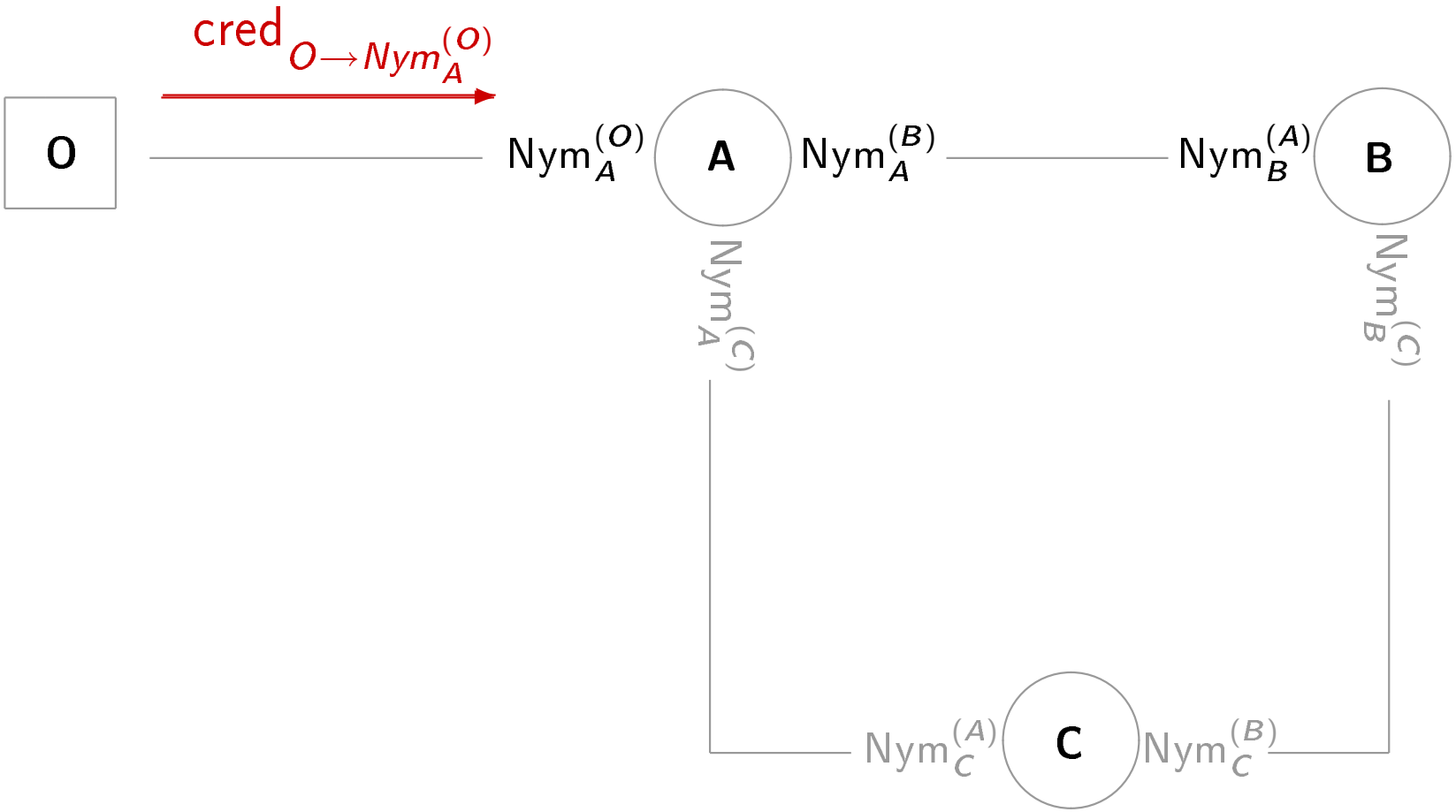

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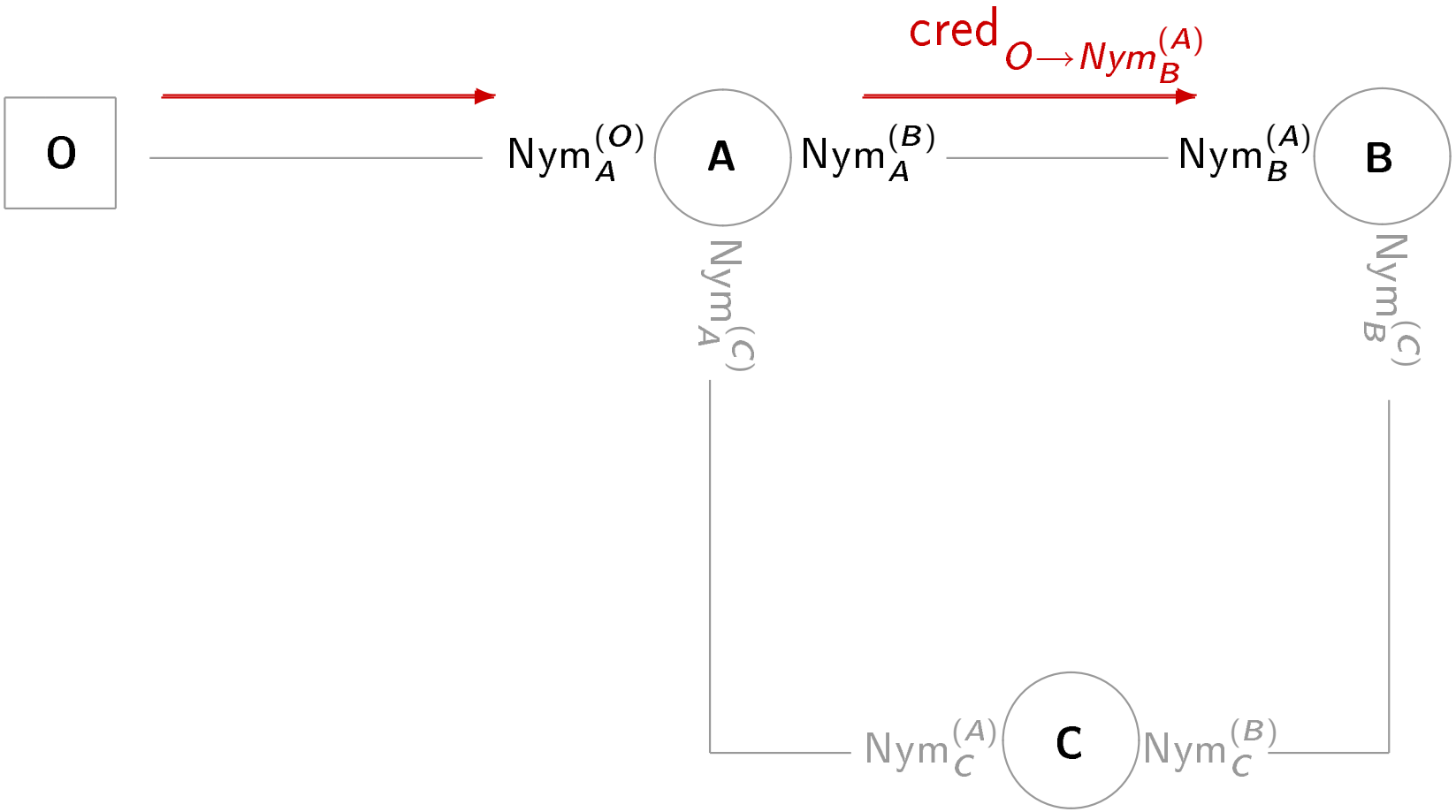

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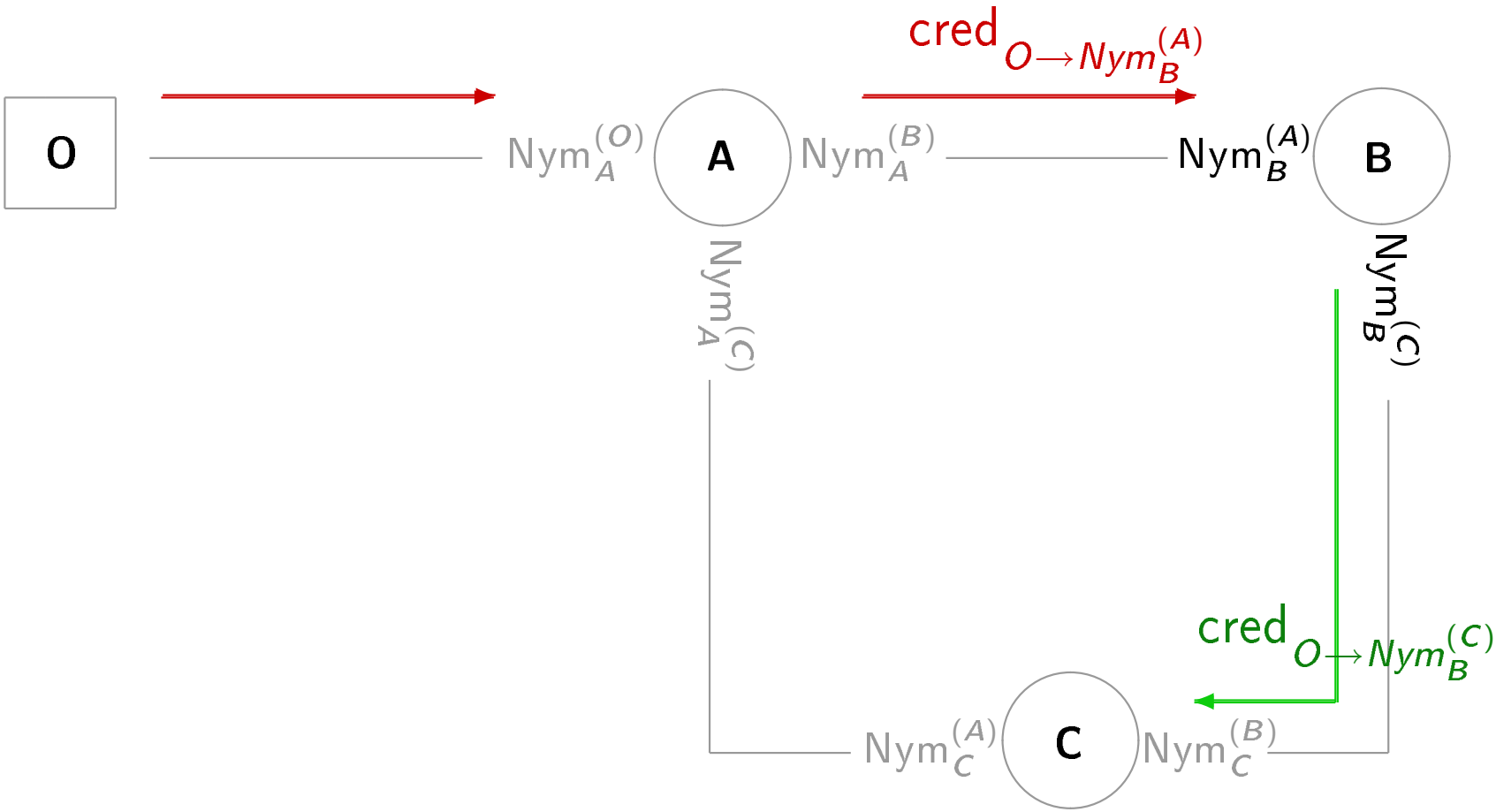

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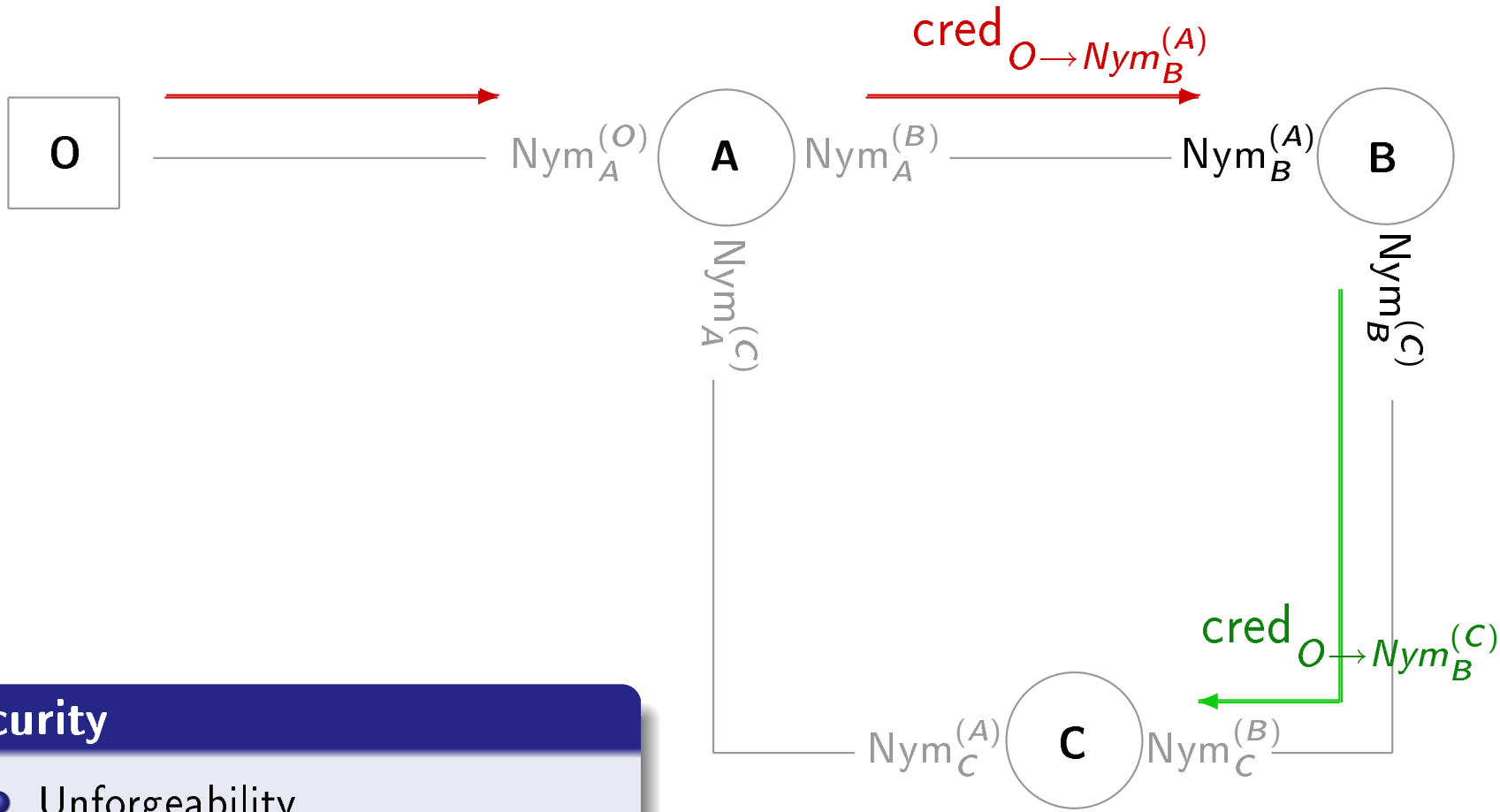

Security

- Unforgeability
- Anonymity (simulation-based)

Black-box instantiation of NIDAC

In a nutshell

- Pseudonym: encryption of user verification key
- Credential: verifiably encrypted signature
- Non-interactive delegation: commuting signature

Black-box instantiation of NIDAC

- Delegation of signing rights

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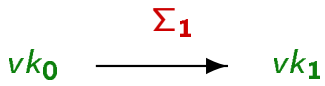

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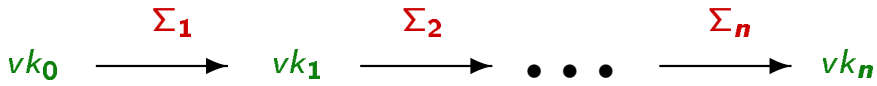

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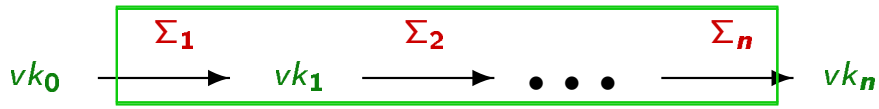

Signatures

credential

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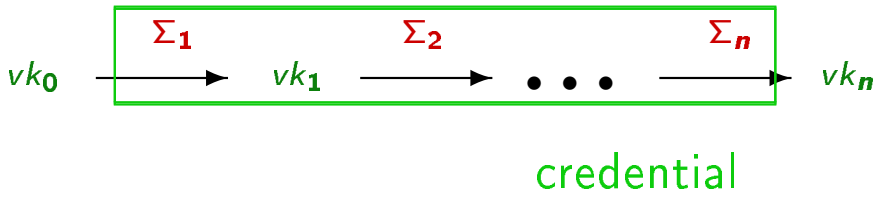
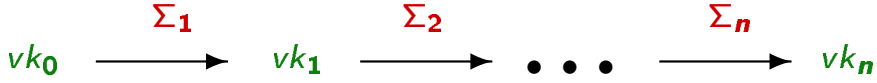


- Anonymous show



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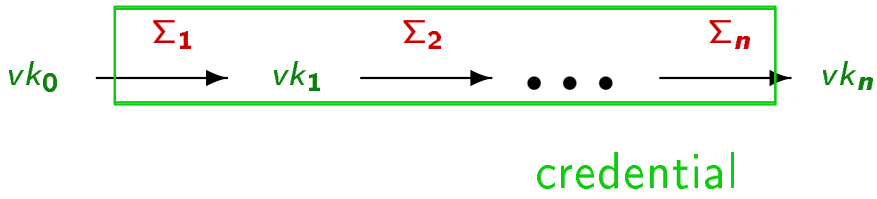

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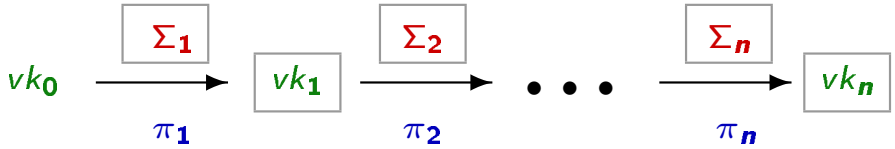

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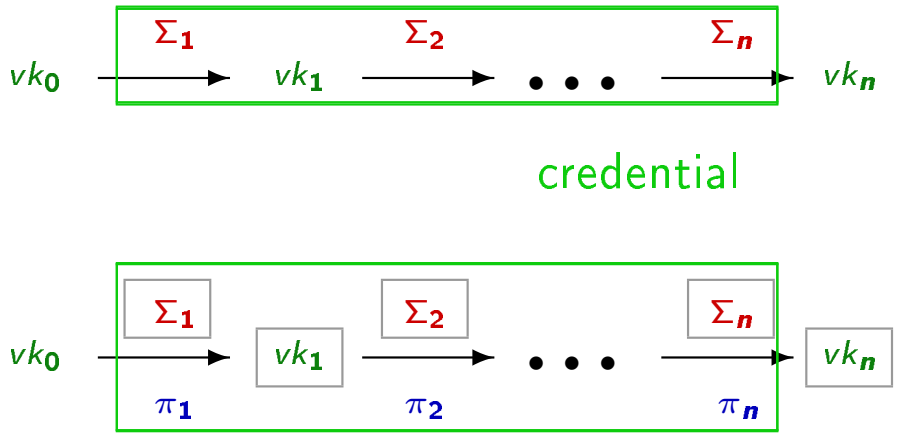
  Verifiable encryption



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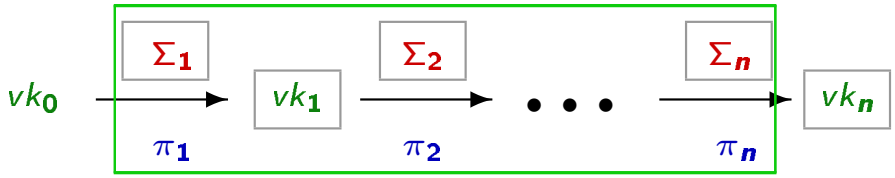

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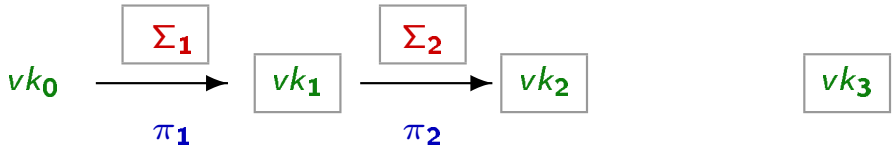

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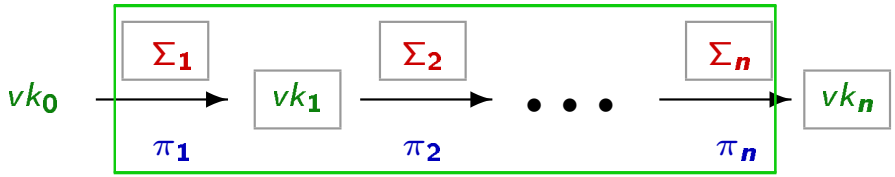

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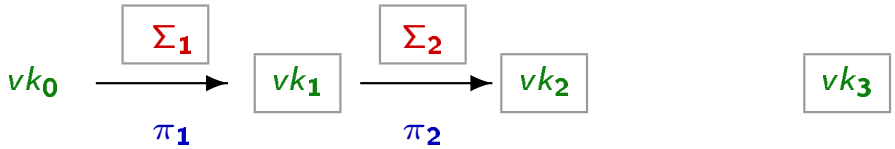

Commuting signatures

- Sign encrypted value $\boxed{vk_3}$ $\Rightarrow (vk_2, \boxed{\Sigma_3}, \boxed{vk_3}, \pi'_3)$

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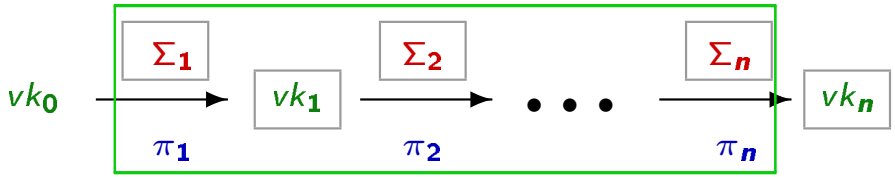

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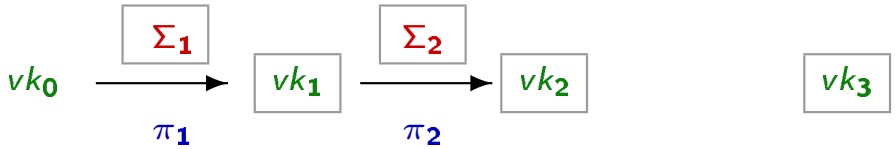

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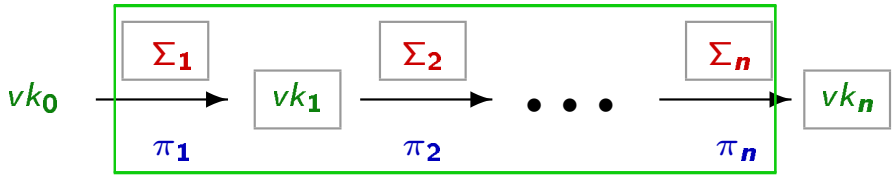

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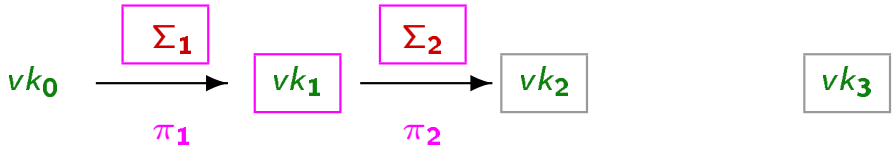

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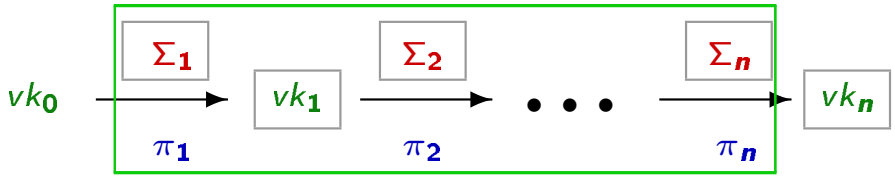

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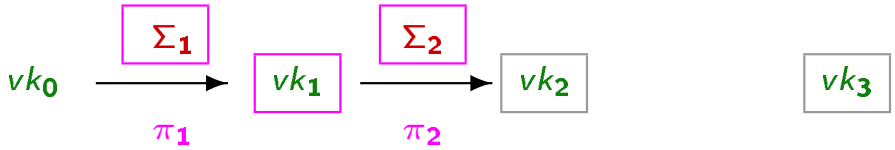

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Send credential $(\boxed{\Sigma_1}, \pi_1, \boxed{vk_1}, \boxed{\Sigma_2}, \pi_2, \boxed{vk_2}, \boxed{\Sigma_3}, \pi_3)$

1. Commuting signatures
2. Delegatable anonymous credentials
3. **Instantiating commuting signatures**

Building blocks

Bilinear group: $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, G, H)$ with

Pairing: $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ bilinear

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Pairing-product equation (PPE)

over variables $X_1, \ldots, X_m \in \mathbb{G}_1$, $Y_1, \ldots, Y_n \in \mathbb{G}_2$

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Building blocks

Bilinear group: $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, G, H)$ with

Pairing: $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ bilinear

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- Messages and signatures are group elements
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Groth-Sahai proofs + structure-pres. signatures
= verifiably encrypted signatures

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Properties of Groth-Sahai proofs

$$\prod_{i=j}^{n} e(A_j, Y_j) \prod_{i=1}^{m} e(X_i, B_i) \prod_{i=1}^{m} \prod_{j=1}^{n} e(X_i, Y_j)^{\gamma_{i,j}} = \mathbf{t} \qquad \text{(E)}$$

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Proofs can be adapted when constants are turned into variables or vice versa

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Then $\pi \cdot \pi'$ is a proof for $\mathsf{E} \cdot \mathsf{E}'$ and $(\mathbf{c}_1, \ldots, \mathbf{c}_m, \mathbf{c}'_1, \ldots, \mathbf{c}'_{m'}, \mathbf{d}_1, \ldots, \mathbf{d}'_{n'})$

$$\prod e(A_j, Y_j) \prod e(A'_j, Y'_j) \prod e(X_i, B_i) \prod e(X'_i, B'_i) \prod\prod e(X_i, Y_j)^{\gamma_{i,j}} \prod\prod e(X'_i, Y'_j)^{\gamma'_{i,j}} = \mathbf{t} \cdot \mathbf{t}' \qquad (\mathsf{E} \cdot \mathsf{E}')$$

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Stronger notion of simulatability

- [GS08]: NIZK proof of satisfiability:
  Given E, simulator can produce $(\mathbf{c}_1, \ldots, \mathbf{c}_m, \mathbf{d}_1, \ldots, \mathbf{d}_n)$ and π

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Application: Given pseudonyms of delegator and delegatee $\Rightarrow$ simulate credential

Instantiating commuting signatures

Goal: $\boxed{M} \xrightarrow{sk} \boxed{\Sigma}, \pi$

Round-optimal blind signature

Protocol to sign M from [AFGHO10]:

- **User** sends
  - *Randomization* $\widetilde{M}$ of M
  - *Encryptions* $\boxed{M}$, $\boxed{R}$
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Conclusion

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- CL signatures and P-signatures

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Updated full version in June: `eprint.iacr.org/2010/233`

Thank you! ☺