Automorphic Signatures in Bilinear Groups

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UCL, 23.03.2010

1. Motivation: Anonymous Proxy Signatures

2. Groth-Sahai Witness-Indistinguishable Proofs

3. Automorphic Signatures

Anonymous Consecutive Delegation of Signing Rights

F, Pointcheval: Anonymous Proxy Signatures [SCN'08]

Delegation	A delegator delegates his signing rights to a proxy signer (or delegatee) who can then sign on the delegator's behalf
Consecutiveness	A delegatee may re-delegate the received signing rights $\Rightarrow$ intermediate delegators
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After verifying a proxy signature one knows that someone entitled signed but nothing more.

Application: GRID computing

User authenticates herself and starts process which needs to authenticate to resources / start subprocesses

$\Rightarrow$ Delegation and re-delegation of signing rights

No need to know that it was not the user herself to be authenticated

Relation to Other Primitives

Anonymous proxy signatures are a generalization of

- Proxy signatures (consecutive delegation) formalized by [BPW03]
- (Dynamic) group signatures (anonymity) formalized by [BSZ05]

and satisfy the respective security notions.

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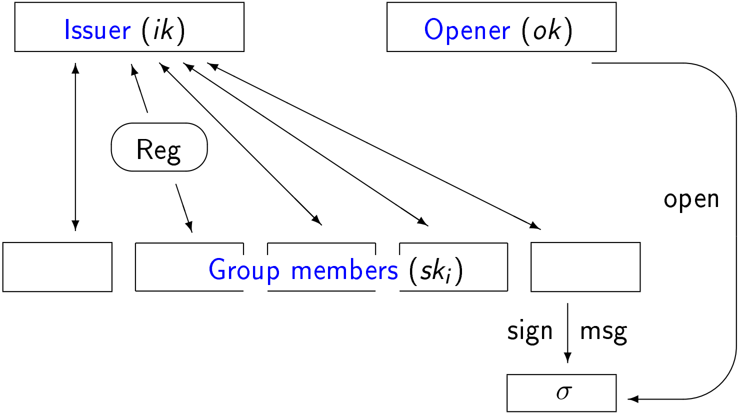
- more recently: Delegatable Anonymous Credentials [BCCKLS09]

(Dynamic) Group Signatures

Group public key: pk



Verification: $\text{Verify}(pk, \text{msg}, \sigma) = 1$

Delegator (pk_D)

delegate

Delegatee/Signer

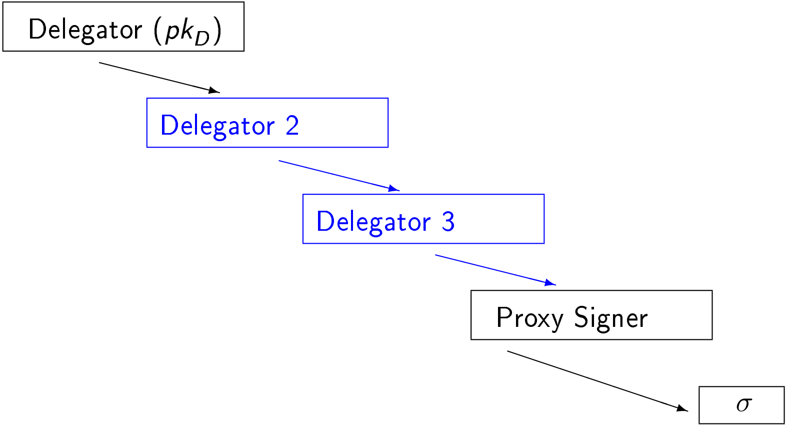
sign

msg

σ

Verify(pk_D, msg, σ)

Proxy Signatures, Consecutive Delegations



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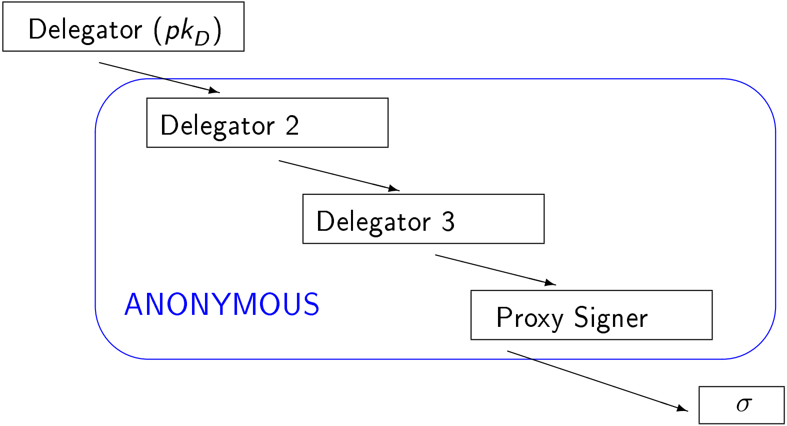

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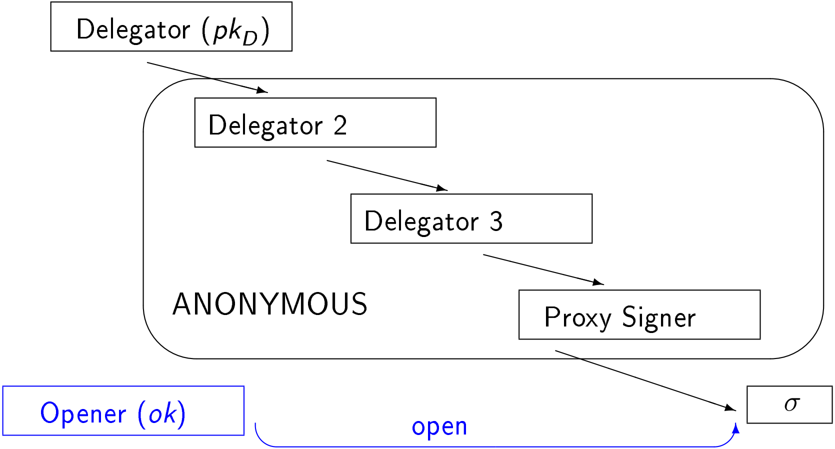

Algorithms of Anonymous Proxy Signature Scheme

$$1^\lambda \quad \rightarrow \quad \text{Setup} \quad \rightarrow \quad pp, ik, ok$$

Algorithms of Anonymous Proxy Signature Scheme

$$\boxed{\text{Issuer } (ik)} \quad \overset{\cdots}{\longleftrightarrow} \atop \underset{pk}{\longleftarrow} \quad \text{Reg} \quad \overset{\cdots}{\longleftrightarrow} \atop \underset{pk, sk}{\longrightarrow} \quad \boxed{\text{User}}$$

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ok, M, \sigma & \rightarrow & \text{Open} & \rightarrow & \text{a list of users or } \perp \text{ (failure)}
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Security for Anonymous Proxy Signatures

Security

Anonymity	intermediate delegators and proxy signer remain anonymous
Traceability	every valid signature can be traced to its intermediate delegators and proxy signer
Non-Frameability	no one can produce a signature that, when opened, wrongfully reveals a delegator or signer

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Generic Construction: Ingredients

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- Public-key encryption (IND-CPA)
- Non-interactive zero-knowledge proofs

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(Existence follows from trapdoor permutations)

Generic Construction: Overview

Setup Generates decryption key for opening authority; signing key for issuer
Parameters: resp. public keys, *crs* for NIZK

Register Issuer signs user's public key $\longrightarrow$ *certificate*

Delegate Sign delegatee's public key $\longrightarrow$ *warrant*
Re-delegate: additionally forward received warrants

Proxy-Sign Sign message, encrypt

- interm. delegators' verification keys and certificates
- warrants
- signature on message

Output

- ciphertext
- NIZK proof that plaintext contains valid signatures

Verify Verify NIZK proof

Open Decrypt ciphertext

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Instantiation

F, Pointcheval: Proofs on Encrypted Values in Bilinear Groups and an Application to Anonymity of Signatures. [PAIRING '09]

- Encryption and proofs based on a generalization of techniques of Boyen-Waters Group Signatures [PKC'07]
  based on *Subgroup Decision Assumption*
- Signature scheme inefficient due to bit-by-bit techniques

Non-Interactive Witness-Indistinguishable Proofs

An NP language $\mathcal{L}$ is defined by relation R as $\mathcal{L} := \{x \mid \exists w : (x, w) \in R\}$.

A NIWI for $\mathcal{L}$ consists of Setup, Prove and Verify.

- Setup outputs a common reference string *crs*
- Prove(crs, x, w) outputs a proof π
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It satisfies

- completeness
- soundness
- witness indistinguishability

Groth-Sahai I

Bilinear Groups and the Decision Linear Assumption [BBS04]

- Bilinear group $(p, \mathbb{G}, \mathbb{G}_T, e, G)$
  - $(\mathbb{G}, +)$ and $(\mathbb{G}_T, \cdot)$ cyclic groups of prime order p
  - $e\colon \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$ bilinear, i.e. $\forall X, Y \in \mathbb{G}, \forall a, b \in \mathbb{Z}$:
    $e(aX, bY) = e(X, Y)^{ab}$
  - $\mathbb{G} = \langle G \rangle$, $\mathbb{G}_T = \langle e(G, G) \rangle$

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- Given $(U, V, G, \alpha U, \beta V, \gamma G)$ it is hard to decide whether $\gamma = \alpha + \beta$.

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PPE

A *pairing-product equation* is an equation over variables $X_1, \ldots, X_n \in \mathbb{G}$ of the form

$$\prod_{i=1}^{n} e(A_i, X_i) \prod_{i=1}^{n} \prod_{j=1}^{n} e(X_i, X_j)^{\gamma_{i,j}} = t_T \ , \qquad \text{(E)}$$

determined by $A_i \in \mathbb{G}$, $\gamma_{i,j} \in \mathbb{Z}_p$ and $t_T \in \mathbb{G}_T$, for $1 \leq i, j \leq n$.

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Groth, Sahai: NIWI proof of *satisfiability* of PPE

Groth-Sahai II

- **Setup** on input the bilinear group output a commitment key ck
- **Com** on input ck, $X \in \mathbb{G}$, randomness ρ output commitment $\mathbf{c}_X$ to X
- **Prove** on input ck, $(X_i, \rho_i)_{i=1}^n$, equation E output a proof ϕ
- **Verify** on input ck, $\vec{\mathbf{c}}$, E, ϕ, output 0 or 1

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Correctness Honestly generated proofs are accepted by Verify

Soundness ExtSetup outputs (ck, ek) s.t.
given $\vec{\mathbf{c}}$ and ϕ s.t. Verify$(ck, \vec{\mathbf{c}}, E, \phi) = 1$ then Extract$(ek, \vec{\mathbf{c}})$ returns $\vec{X}$ that satisfies E

Witness-Indistinguishability WISetup outputs ck^* indist. from ck s.t.

- Com$(ck^*, \cdot, \cdot)$ produces statistically hiding commitments i.e.
$$\forall \mathbf{c}\ \forall X\ \exists \rho : \mathrm{Com}(ck^*, X, \rho) = \mathbf{c}$$
- Given $(X_i, \rho_i)_i$, $(X'_i, \rho'_i)_i$ s.t. $\mathbf{c}_i = \mathrm{Com}(ck^*, X_i, \rho_i) = \mathrm{Com}(ck^*, X'_i, \rho'_i)$ and $(X_i)_i$ and $(X'_i)_i$ satisfy E then
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Motivation

- Groth-Sahai proofs allow us to
  - commit to (encrypt) group elements and to
  - prove that they satisfy PPEs

  Opener's public and decryption key: $(ck, ek) \leftarrow \mathsf{ExtSetup}$

- To instantiate generic construction, we need signature scheme s.t.
  - signatures are group elements
  - verification by PPE
  - able to sign public keys
  - EUF-CMA

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Automorphic Signatures

Boneh-Boyen Signatures

The q-Strong Diffie-Hellman Problem (SDH) [BB04]

Given $(G, xG, x^2 G, \ldots, x^q G) \in \mathbb{G}^{q+1}$ for $x \leftarrow \mathbb{Z}_p^*$, output $(\frac{1}{x+c} G, c) \in \mathbb{G} \times \mathbb{Z}_p$.

Boneh-Boyen *Weak* Signatures

Given $G, xG \in \mathbb{G}$ and $q-1$ distinct pairs $(\frac{1}{x+c_i} G, c_i) \in \mathbb{G} \times \mathbb{Z}_p$, output a *new* pair $(\frac{1}{x+c} G, c) \in \mathbb{G} \times \mathbb{Z}_p$.

Boneh-Boyen Short Signatures

- Secret key $(x, y) \in \mathbb{Z}_p^2$, public key $X = xG,\ Y = yG$
- Sign $m \in \mathbb{Z}_p$: choose $r \leftarrow \mathbb{Z}_p$; signature: $(A = \frac{1}{x+m+ry} G, r)$
- Verify (A, r) on m under (X, Y) by checking
  $e(A, X + mG + rY) = e(G, G)$

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Boneh-Boyen *Weak* Signatures

Given $G, xG \in \mathbb{G}$ and $q-1$ distinct pairs $(\frac{1}{x+c_i}G, c_i) \in \mathbb{G} \times \mathbb{Z}_p$, output a *new* pair $(\frac{1}{x+c}G, c) \in \mathbb{G} \times \mathbb{Z}_p$.

Boneh-Boyen Short Signatures

- Secret key $(x, y) \in \mathbb{Z}_p^2$, public key $X = xG$, $Y = yG$
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- Verify (A, r) on m under (X, Y) by checking
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Variants of Boneh-Boyen

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Variants of Boneh-Boyen

The Hidden SDH [BW07]

Given $G, H, X := xG \in \mathbb{G}$ and $q-1$ distinct triples $(\frac{1}{x+c_i}G, c_iG, c_iH) \in \mathbb{G}^3$, output a *new* triple $(\frac{1}{x+c}G, cG, cH) \in \mathbb{G}^3$.

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The Hidden SDH [BW07]

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- All components are group elements
- Validity of a triple (A, C, D) is verifiable by PPEs:

$$e(A, X + C) = e(G, G)$$
$$e(C, H) = e(G, D)$$

Assumptions I

F, Pointcheval, Vergnaud: Transferable Constant-Size Fair E-Cash [CANS'09]

SDH implies hardness of the following:

Given $G, K, X := xG \in \mathbb{G}$ and $q-1$ triples $\left(\frac{1}{x+c_i}(K+v_iG), c_i, v_i\right) \in \mathbb{G} \times \mathbb{Z}_p{}^2$, output a *new* triple $\left(\frac{1}{x+c}(K+vG), c, v\right) \in \mathbb{G} \times \mathbb{Z}_p{}^2$.

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Asymm. Double Hidden SDH (ADHSDH)

Given $G, K, F, H, X := xG, Y := xH \in \mathbb{G}$ and $q-1$ tuples $\left(\frac{1}{x+c_i}(K+v_i G), c_i F, c_i H, v_i G, v_i H\right)$, output a *new* tuple $\left(\frac{1}{x+c}(K+vG), cF, cH, vG, vH\right)$.

Assumptions II

Verification

(A, C, D, V, W) satisfies

- $e(A, Y + D) = e(\frac{1}{x+c}(K + vG), xH + cH) = e(K + V, H)$,
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(Weak) Flexible CDH (WFCDH)

Given $(G, aG, bG) \in \mathbb{G}^3$, output $(R, aR, bR, abR) \in \mathbb{G}^4$ with $R \neq 0$.

F: Automorphic Signatures in Bilinear Groups
http://eprint.iacr.org/2009/320

Automorphic Signature

- Parameters: $(G, K, F, H, T) \leftarrow \mathbb{G}^5$, which define the message space as $\mathcal{DH} := \{(mG, mH) \,|\, m \in \mathbb{Z}_p\}$,
- KeyGen: secret key $x \leftarrow \mathbb{Z}_p$, public key $(X := xG, Y := yH)$
- Sign $(M, N) \in \mathcal{DH}$: choose $c, r \leftarrow \mathbb{Z}_p$, set

$$\big(A := \tfrac{1}{x+c}(K + rT + M), C := cF, D := cH, R := rG, S := rH\big)$$

- A signature on a message $(M, N) \in \mathcal{DH}$ is valid iff

$$e(A, Y + D) = e(K + M, H)\, e(T, S) \qquad \begin{array}{l} e(C, H) = e(F, D) \\ e(R, H) = e(G, S) \end{array}$$

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The above scheme is EUF-CMA under ADHSDH and WFCDH.

Applications

Efficiency

- Messages and public keys in $\mathbb{G}^2$, signatures in $\mathbb{G}^5$
- Verification: 7 pairing evaluations
- Also instantiable in *asymmetric* bilinear groups

In combination with Groth-Sahai proofs, automorphic signatures enable efficient instantiations of generic concepts.

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In combination with Groth-Sahai proofs, automorphic signatures enable efficient instantiations of generic concepts.

- Round-Optimal Blind Signatures
- Group Signatures
- Anonymous Proxy Signatures with new features:
  - Delegator anonymity (by randomizing Groth-Sahai proofs)
  - Blind delegation (using blind signatures)

Thank you! ☺