Master 2 MPRI course 2-19 Biochemical Programming
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Written examination
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Abstract

We study the notion of counters in Kappa. The motivation is to extend the language with a third kind of interaction sites, which can bear integer values. The rules may perform simple arithmetical operations over them, such as incrementation and decrementation by 1, or comparison with respect to a constant integer. The main goal is to translate the extension of Kappa with counters into pure Kappa. Several such encodings are studied.

According to the choice of encoding, the potential values of counters may be constrained by a minimal value and/or a maximal value.

To evaluate the different encodings, for each operation (incrementation/decrementation, comparison test), the number and the size (in term of number of agents and of number of interaction sites) of the rules/patterns that are used for encoding this operation will be bounded by the value of an expression that may depend on the current value of counters, possible bounds on the potential values of counters, and bounds on the values againt which the values of counter may be tested in the rules.

1 Limit conditions

Question 1.1 *We assume that we are given a version of Kappa with counters supporting natural number values (non-negative numbers). Patterns can be used to check whether the value of a counter is equal, greater than or less than another natural number, while rules may be used to increment/decrement the value of a counter (under specific conditions specified by the left hand sides of these rules).*

Suggest two different mechanisms to ensure that the values of counters always remain between the bounds m and M.

2 Counters as strings

In that section, we consider that the values of counters may vary between two integer values m and M. We propose to encode the values of counters as string, such as "0", "1", "2", and so on. We assume that no additional structure is provided with these strings (this means that there is no notion of arithmetics).

In Fig. 1 is given a pattern in which the site c of an occurrence of the agent A carries a counter with the value 3.

Question 2.1 *Propose a rule (or a set of rules) to increment by* 1 *the value of the counter carried by the site c of an occurrence of the agent A (whenever that value is less than M).*

Question 2.2 *Propose a rule (or a set of rules) to decrement by* 1 *the value of the counter carried by the site c of an occurrence of the agent A (whenever that value is greater than m).*

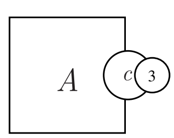

Figure 1: The site c of an occurrence of the agent A has the internal state 3.

Question 2.3 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site c of an occurrence of the agent A is equal to the natural value i (between m and M).*

Question 2.4 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site c of an occurrence of the agent A is less or equal to the natural value i (between m and M).*

Question 2.5 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site c of an occurrence of the agent A is greater or equal to the natural value i (between m and M).*

Question 2.6 *Conclude.*

3 Counters as chains of fictitious agents

In this section, we consider counters carrying natural number values. We propose to encode the value of each counter as a chain of occurrences of a fictitious agent, attached to the site which stands for this counter (the value of the counter being the length of the corresponding chain of occurrences of the fictitious agent minus 1).

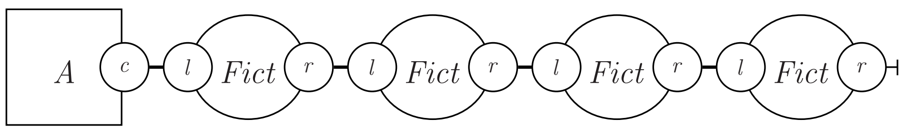

Figure 2: The site c of an occurrence of the agent A is bound to a chain of 4 occurrences of the fictitious agent, which, according to the current encoding, means that this is a counter and that its value is equal to 3 (4-1).

In Fig. 2 is given a pattern in which the site c of an occurrence of the agent A carries a counter with the value 3.

Question 3.1 *Propose a rule (or a set of rules) to increment by* 1 *the value of the counter carried by the site c of an occurrence of the agent A.*

Question 3.2 *Propose a rule (or a set of rules) to decrement by* 1 *the value of the counter carried by the site c of an occurrence of the agent A whenever that value is greater than* 0*.*

Question 3.3 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site c of an occurrence of the agent A is equal to the natural value i (in $\mathbb{N}$).*

Question 3.4 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site c of an occurrence of the agent A is less or equal to the natural value i (in $\mathbb{N}$).*

Question 3.5 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site c of an occurrence of the agent A is greater or equal to the natural value i (in $\mathbb{N}$).*

Question 3.6 *Conclude.*

4 Efficient encoding of $\leqslant$-comparison tests

In this section, we assume that we have an extension of Kappa with counters, where the values of counters range over natural numbers with support for incrementation, decrementation, and $\geqslant$-comparison tests. The goal of this section is to define an encoding of counters with values between two numerical bounds m and M, with support for $\leqslant$-comparison tests as well.

Question 4.1 *Propose an encoding for counters with values bounded between two integers m and M.*

Question 4.2 *Propose a rule (or a set of rules) to increment by* 1 *the value of the counter carried by the site c of an occurrence of the agent A whenever that value is less than M.*

Question 4.3 *Propose a rule (or a set of rules) to decrement by* 1 *the value of the counter carried by the site c of an occurrence of the agent A whenever that value is greater than m.*

Question 4.4 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site c of an occurrence of the agent A is equal to the natural value i (between m and M).*

Question 4.5 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site c of an occurrence of the agent A is less or equal to the natural value i (between m and M).*

Question 4.6 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site c of an occurrence of the agent A is greater or equal to the natural value i (between m and M).*

Question 4.7 *Conclude.*

5 Counters as fixed sequences of fictitious agents

In this section, we consider that the potential values of counters range between two integers m and M. We suggest to encode each counter by a chain of $M-m+1$ occurrences of a fictitious agent among which exactly one occurrence is linked to the site carrying the counter (the value of the counter corresponds to the position of the occurrence of the fictitious agent that is bound to the site standing for the counter).

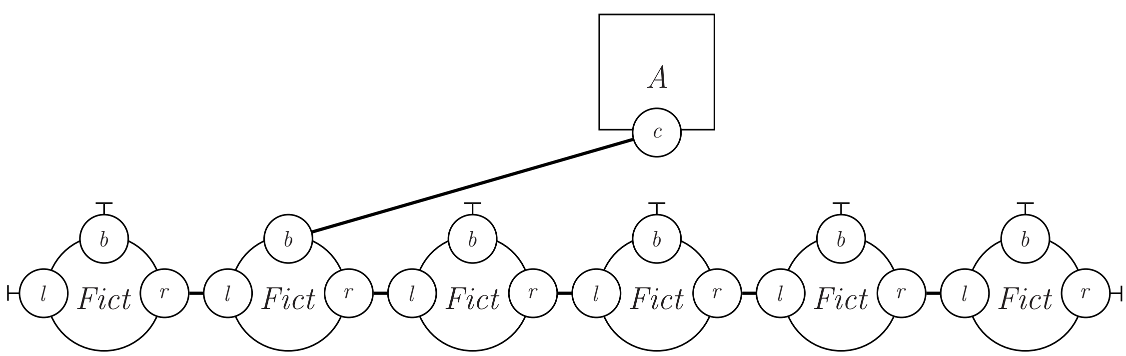

Figure 3: A sequence of 6 occurrences of a fictitious agent is used to encode values from 2 to 7. The first occurrence stands for the value 2, the second for the value 3, ..., and the last one for the value 7. The site c of an occurrence of the agent A is bound to the site b of the second occurrence of the fictitious agent, which means that this is a counter and that its value is equal to 3.

In Fig. 3 is given a pattern in which the site c of an occurrence of the agent A carries a counter with the value 3.

Question 5.1 *Propose a rule (or a set of rules) to increment by* 1 *the value of the counter carried by the site* c *of an occurrence of the agent* A *whenever that value is less than* M*.*

Question 5.2 *Propose a rule (or a set of rules) to decrement by* 1 *the value of the counter carried by the site* c *of an occurrence of the agent* A *whenever that value is greater than* m*.*

Question 5.3 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site* c *of an occurrence of the agent* A *is equal to the natural value* i *(between* m *and* M*).*

Question 5.4 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site* c *of an occurrence of the agent* A *is less or equal to the natural value* i *(between* m *and* M*).*

Question 5.5 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site* c *of an occurrence of the agent* A *is greater or equal to the natural value* i *(between* m *and* M*).*

Question 5.6 *Conclude.*

6 Efficient counters with unbounded values

In that section, we consider a model mades of a finite set of rules in Kappa with counters. The values of counters are not assumed to be bounded. The goal is to provide an encoding, inspired by the one of Sect. 5, to efficient implement incrementation, decrementation, and comparison tests.

Question 6.1 *Prove that there exists two integers* m *and* M *such that any integer against which the value of a counter is tested in a rule of the model ranges between* m *and* M*.*

Question 6.2 *Propose an efficient encoding for counters with unbounded values, for the specific model used in this section.*

Question 6.3 *Propose a rule (or a set of rules) to increment by* 1 *the value of the counter carried by the site* c *of an occurrence of the agent* A*.*

Question 6.4 *Propose a rule (or a set of rules) to decrement by* 1 *the value of the counter carried by the site* c *of an occurrence of the agent* A*.*

Question 6.5 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site* c *of an occurrence of the agent* A *is equal to the natural value* i *(between* m *and* M*).*

Question 6.6 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site* c *of an occurrence of the agent* A *is less or equal to the natural value* i *(between* m *and* M*).*

Question 6.7 *Propose a pattern (or a set of patterns) to match whether the value of the counter carried by the site* c *of an occurrence of the agent* A *is greater or equal to the natural value* i *(between* m *and* M*).*

Question 6.8 *Conclude.*

7 Ad lib

Question 7.1 *Propose another encoding for counters with bounded values, that provides another trade-off between the number of occurrences of ficitious agents in patterns and the number of sites per fictitious agents.*