

A Semantics of Core Erlang With Handling of Signals

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Motivations and Context

Fault explanation in order to debug

- bugs caused by non determinism
- focus on evaluation order of messages

Needs

- a small step semantics
- describing symmetrical links and monitoring links
- handling signals

→ **Problem:** no existing semantics cover these mechanisms

Design Choices

Erlang system

- hierarchy of concurrent processes
- interactions only via asynchronous signals
- no nodes (simplification)
- no global variables (simplification)
- no continuous time (simplification)

Rules of thumb

- the official documentation and initial specification first
- if an information is needed but absent from the documentation, go to the implementation
- if documentation and implementation disagree, the documentation wins

Choices we made

- Core Erlang – less syntactic sugar
- a restored receive, but without the after clause
- no try_catch structure

Execution model

State of a system

$$\Pi \cup \{(pid, (e, \theta, \tau, m, stack, r), sig_{sent}, L)\}$$

- Π = set of the others processes

Execution model

State of a system

$$\Pi \cup \{(\text{pid}, (e, \theta, \tau, m, \text{stack}, r), \text{sig}_{\text{sent}}, L)\}$$

- pid = process identifier

Execution model

State of a system – execution task

$$\Pi \cup \{(pid, (\mathbf{e}, \theta, \tau, \mathbf{m}, \mathbf{stack}, \mathbf{r}), sig_{sent}, L)\}$$

- Expression evaluation:
 - e = current expression
 - θ = evaluation context
 - τ = functions table
 - m = current module
- Debugging info:
 - $stack$ = current call stack
 - r = ending reason

Execution model

State of a system – signals

$$\Pi \cup \{(pid, (e, \theta, \tau, m, stack, r), sig_{sent}, L)\}$$

- outbox: list of signals sent by pid, in the order of their emission
- two kinds of signals:
 - messages – treated during receive expression
 - others – treated as soon as they are received
- general form of a signal: $(flag_{type}, pid_{target}, content)$

Execution model

State of a system – signals

$$\Pi \cup \{(pid, (e, \theta, \tau, m, stack, r), \text{sig}_{\text{sent}}, L)\}$$

Guarantees on the order:

- if a process sends signals to another one, the reception order is the same as the sending order
- if more than one process send signals to a same process, no guarantee on the order of signals coming from different processes

Execution model

State of a system – links

$$\Pi \cup \{(pid, (e, \theta, \tau, m, stack, r), sig_{sent}, \mathbf{L})\}$$

- set of links
- two kinds of links:
 - monitor links
 - monitored side: $(monitored_by, pid_i, lid)$
 - monitoring side: $(monitoring, pid_i, lid)$
 - symmetrical link: $(link, pid_i, pid_j)$

Semantics – Running Example

In the file of the 'test' module

```
'go'/0 = fun() ->
  do call 'erlang': '+'(2,3)
     call 'erlang': '+'(4,4)
```

First instruction

```
do
  call 'erlang': 'monitor'(process,
    call 'erlang': 'spawn'('test', 'go', []))
)
receive
  _ when 'true' -> 'stop'
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parent:

- creates a child with first instruction
call test:go()
- creates a monitoring link with its child
- waits for a message

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child:

- evaluates
call erlang:+(2,3)
- receives the monitoring
signal from parent
- updates L
- evaluates
call erlang:+(4,4)
- terminates

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- receives the down signal from child
- turns the down signal into a message
- follows the clause of its receive
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parent:

- receives the down signal from child
- turns the down signal into a message
- follows the clause of its receive
- **terminates**

Semantics – End of a process I

Behavior

- Reasons of Termination:
 - normal evaluation of its first expression
 - a problem led to the end of the process
- Result:
 - sends a down signal for each monitoring or symmetrical link
 - terminates itself

Semantically

- Two special values:
 - EoP: the process is about to be terminated
 - ended: the process is dead
 - Two types of transitions:
 - $\xrightarrow{term}$: evaluation of expressions at top level
 - $\xrightarrow{aux}$: exploration and evaluation of sub-expressions recursively
- Necessary to handle signal of end of processes

Semantics – End of a process II

From *aux* to *term*

$$\begin{array}{c} \Pi \cup \{(pid, (e_1, \theta, \tau, m, \text{stack} \cdot (e_1, \theta, m), r), s_{sent}, L)\} \\ \xrightarrow{\text{aux}} \\ \Pi' \cup \{(pid, (e'_1, \theta', \tau', m', \text{stack}, r'), s'_{sent}, L')\} \\ \text{EXPR_TERM} \frac{}{\Pi \cup \{(pid, (e_1, \theta, \tau, m, \text{stack}, r), s_{sent}, L)\}} \\ \xrightarrow{\text{term}} \\ \Pi' \cup \{(pid, (e'_1, \theta, \tau', m', \text{stack}, r'), s'_{sent}, L')\} \end{array}$$

Semantics – End of a process III

Last evaluation: sending signals

$$\begin{aligned}
 J &= \{i \mid 1 \leq i \leq n \wedge fl_i \in \{\text{monitored_by}, \text{link}\}\} \\
 M &= \{(\text{not_msg}, pid_j, (\text{down}, fl_j, lid_j, \text{end_reason}(r, \text{normal}))) \mid j \in J\} \\
 sigs &\in \text{permutations}(M) \qquad v \neq \text{ended}
 \end{aligned}$$

VAL

$$\prod \cup \{ (pid, (v, \theta, \tau, m, stack, r), sig_{sent}, \{(fl_i, pid_i, lid_i) \mid 1 \leq i \leq n \wedge lid_i \in \{\mathcal{I}d, Ref_L\}\}) \}$$

term
→

$$\prod \cup \{ (pid, (\text{ended}, \theta, \tau, m, stack, r), sig_{sent} \cdot sigs, \{(fl_i, pid_i, lid_i) \mid 1 \leq i \leq n \wedge lid_i \in \{\mathcal{I}d, Ref_L\}\}) \}$$

Semantics – End of a process – Example

Child process: initial state

- born from call 'erlang': 'spawn'('test', 'go', [])
- first instruction: call 'test': 'go'()

$$\Pi \cup \{(\textcolor{red}{pid}_{child}, (\textcolor{red}{call} \textcolor{red{'test'}} : \textcolor{red}{go}(), [], \tau_{ini}, \textcolor{red}{test}, (), \perp), \textcolor{blue}{()}, \{\})\}$$

Semantics – End of a process – Example

Child process: initial state

- born from call `'erlang': 'spawn'('test', 'go', [])`
- first instruction: `call 'test': 'go'()`

$$\Pi \cup \{(\textcolor{red}{pid}_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, test, (), \perp), (), \{\})\}$$

pid of the child process

Semantics – End of a process – Example

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first instruction

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empty evaluation context θ and initial functions table τ

Semantics – End of a process – Example

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$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, \text{test}, (), \perp), (), \{\})\}$$

module: *test*, empty stack, reason of termination: $\perp$

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$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, test, (), \perp), (), \{\})\}$$

empty outbox

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- first instruction: `call 'test': 'go'()`

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, test, (), \perp), (), \{\})\}$$

empty set of links with other processes

Semantics – End of a process – Example

Child process: evaluation

- expression $\notin \text{Val}$
- need to use $\xrightarrow{\text{aux}}$

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, \text{test}, ((\text{call test : go}(), [], \text{test})), \perp), (), \{\})\}$$

$\xrightarrow{\text{aux}}$

?

EXPR_TERM

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, \text{test}, (), \perp), (), \{\})\}$$

$\xrightarrow{\text{term}}$

?

updated stack

Semantics – End of a process – Example

Child process: evaluation a few operations later

- expression $\notin \text{Val}$
- need to use $\xrightarrow{\text{aux}}$

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, \text{test}, ((\text{call test : go}(), [], \text{test})), \perp), (), \{\})\}$$

$$\xrightarrow{\text{aux}}$$

$$\text{EXPR_TERM} \frac{\Pi' \cup \{(pid, (8, [], \tau_{ini}, \text{test}, (), \perp), (), \{(\text{monitored_by}, pid_{parent}, lid)\})\}}{\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, \text{test}, (), \perp), (), \{\})\}}$$

$$\xrightarrow{\text{term}}$$

$$\Pi' \cup \{(pid, (8, [], \tau_{ini}, \text{test}, (), \perp), (), \{(\text{monitored_by}, pid_{parent}, lid)\})\}$$

Semantics – End of a process – Example

Child process: evaluation a few operations later

- expression $\notin \text{Val}$
- need to use $\xrightarrow{\text{aux}}$

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, test, ((call \text{ test : go}(), [], test)), \perp), (), \{\})\} \xrightarrow{\text{aux}}$$

$$\Pi' \cup \{(pid, (8, [], \tau_{ini}, test, (), \perp), (), \{(\text{monitored_by}, pid_{parent}, lid)\})\}$$

EXPR_TERM

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, test, (), \perp), (), \{\})\} \xrightarrow{\text{term}}$$

$$\Pi' \cup \{(pid, (\textcolor{red}{8}, [], \tau_{ini}, test, (), \perp), (), \{(\text{monitored_by}, pid_{parent}, lid)\})\}$$

resulting expression

Semantics – End of a process – Example

Child process: evaluation a few operations later

- expression $\notin \text{Val}$
- need to use $\xrightarrow{\text{aux}}$

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, \text{test}, ((\text{call test : go}(), [], \text{test})), \perp), (), \{\})\}$$

$$\xrightarrow{\text{aux}}$$

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$$\xrightarrow{\text{term}}$$

$$\Pi' \cup \{(pid, (8, [], \tau_{ini}, \text{test}, (), \perp), (), \{(\text{monitored_by}, pid_{parent}, lid)\})\}$$

normal termination: reason of termination = $\perp$

Semantics – End of a process – Example

Child process: evaluation a few operations later

- expression $\notin \text{Val}$
- need to use $\xrightarrow{\text{aux}}$

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, test, ((call \text{ test : go}(), [], test)), \perp), (), \{\})\} \xrightarrow{\text{aux}}$$

$$\Pi' \cup \{(pid, (8, [], \tau_{ini}, test, (), \perp), (), \{(\text{monitored_by}, pid_{parent}, lid)\})\}$$

EXPR_TERM

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, test, (), \perp), (), \{\})\} \xrightarrow{\text{term}}$$

$$\Pi' \cup \{(pid, (8, [], \tau_{ini}, test, (), \perp), (\text{!}), \{(\text{monitored_by}, pid_{parent}, lid)\})\}$$

no signal sent: empty outbox

Semantics – End of a process – Example

Child process: evaluation a few operations later

- expression $\notin \text{Val}$
- need to use $\xrightarrow{\text{aux}}$

$$\Pi \cup \{(pid_{child}, (\underline{\text{call}} \text{ 'test' : go}(), [], \tau_{ini}, test, ((call \text{ test : go}(), [], test)), \perp), (), \{\})\} \xrightarrow{\text{aux}}$$

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$$\Pi' \cup \{(pid, (8, [], \tau_{ini}, test, (), \perp), (), \{(\text{monitored_by}, pid_{parent}, lid)\})\}$$

monitor link with the parent process

Semantics – End of a process – Example

Child process: terminating

- expression $\in \text{Val}$
- can apply VAL

$$\begin{array}{c} \text{VAL} \quad \frac{M = \{(\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))\} \\ \text{signs} \in \text{permutations}(M) \quad v \neq \text{ended}}{\Pi' \cup \{(\text{pid}, (8, [], \tau_{\text{ini}}, \text{test}, (), \perp), (), \{(\text{monitored_by}, \text{pid}_{\text{parent}}, \text{lid})\})\}} \\ \xrightarrow{\text{term}} \\ \Pi' \cup \\ \{(\text{pid}, (\text{ended}, [], \tau_{\text{ini}}, \text{test}, (), \perp), ((\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))), \\ \{(\text{monitored_by}, \text{pid}_{\text{parent}}, \text{lid})\})\} \end{array}$$

progression toward *ended*

Semantics – End of a process – Example

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- expression $\in \text{Val}$
- can apply VAL

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 \text{VAL} \frac{}{\Pi' \cup \{(\text{pid}, (8, [], \tau_{\text{ini}}, \text{test}, (), \perp), (), \{(\text{monitored_by}, \text{pid}_{\text{parent}}, \text{lid})\})\}} \\
 \xrightarrow{\text{term}} \\
 \Pi' \cup \\
 \{(\text{pid}, (\text{ended}, [], \tau_{\text{ini}}, \text{test}, (), \perp), ((\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))), \\
 \{(\text{monitored_by}, \text{pid}_{\text{parent}}, \text{lid})\})\}
 \end{array}$$

one link flagged "monitored_by " $\rightarrow$ one signal sent

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- can apply VAL

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 \text{VAL} \quad \frac{
 \begin{array}{c}
 M = \{(\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))\} \\
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 }{
 \begin{array}{c}
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 \xrightarrow{\text{term}} \\
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 \end{array}
 }
 \end{array}$$

signal: information about the signal (nature, destination)

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 }{
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 \{(\text{monitored_by}, \text{pid}_{\text{parent}}, \text{lid})\})\}
 \end{array}
 }
 \end{array}$$

signal: information about the link (nature, identifier)

Semantics – End of a process – Example

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signal: reason of termination

Semantics – Handling signals I

Characteristics

- Asynchronous
- Automatic: handled by default, impossible to prevent it

Down signals – monitor link

$$\begin{array}{c}
 \text{SIG_EXIT_MONIT} \quad \frac{
 \begin{array}{l}
 \text{first_sig}(\text{sig}_{\text{sent},i},j) = k \quad i \neq j \\
 s_{i,k} = (\text{not_msg}, \text{pid}_j, (\text{down}, \text{monitored_by}, \text{Ref}, r)) \\
 (\text{monitoring}, \text{pid}_i, \text{ref}) \in L_j \quad e_j \notin \{\text{EoP}, \text{ended}\}
 \end{array}
 }{
 \begin{array}{l}
 \Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, \mathbf{m}_j, \text{stack}_j, \mathbf{r}_j), \text{sig}_{\text{sent},j}, L_j); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, \mathbf{m}_i, \text{stack}_i, \mathbf{r}_i), (s_{i,1}, \dots, s_{i,k}, \dots, s_{i,n_i}), L_i)\} \\
 \xrightarrow{\text{aux}} \\
 \Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, \mathbf{m}_j, \text{stack}_j, \mathbf{r}_j), \text{sig}_{\text{sent},j}, L_j); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, \mathbf{m}_i, \text{stack}_i, \mathbf{r}_i), (s_{i,1}, \dots, s_{i,k-1}, s_{i,k+1}, \dots, s_{i,n_i})) \cdot \\
 (\text{msg}, \text{pid}_j, (\{ \text{'DOWN'} \}, \text{ref}, \text{process}, \text{pid}_i, r)), L_i)\}
 \end{array}
 }
 \end{array}$$

Semantics – Handling signals II

Down signals – monitor link

$$\begin{array}{c}
 \text{SIG_EXIT_MONIT} \quad \frac{
 \begin{array}{l}
 \text{first_sig}(\text{sig}_{\text{sent},i},j) = k \quad i \neq j \\
 s_{i,k} = (\text{not_msg}, \text{pid}_j, (\text{down}, \text{monitored_by}, \text{Ref}, r)) \\
 (\text{monitoring}, \text{pid}_i, \text{ref}) \in L_j \quad e_j \notin \{\text{EoP}, \text{ended}\}
 \end{array}
 }{
 \begin{array}{l}
 \Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, \mathfrak{m}_j, \text{stack}_j, \mathfrak{r}_j), \text{sig}_{\text{sent},j}, L_j); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, \mathfrak{m}_i, \text{stack}_i, \mathfrak{r}_i), (s_{i,1}, \dots, s_{i,k}, \dots, s_{i,n_i}), L_i)\} \\
 \xrightarrow{\text{aux}} \\
 \Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, \mathfrak{m}_j, \text{stack}_j, \mathfrak{r}_j), \text{sig}_{\text{sent},j}, L_j); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, \mathfrak{m}_i, \text{stack}_i, \mathfrak{r}_i), (s_{i,1}, \dots, s_{i,k-1}, s_{i,k+1}, \dots, s_{i,n_i}), \\
 (\text{msg}, \text{pid}_j, (\{\text{'DOWN'\}}, \text{ref}, \text{process}, \text{pid}_i, r))), L_i)\}
 \end{array}
 }
 \end{array}$$

two processes

Semantics – Handling signals II

Down signals – monitor link

$$\begin{array}{c}
 \text{first_sig}(\text{sig}_{\text{sent},i,j}) = k \quad i \neq j \\
 s_{i,k} = (\text{not_msg}, \text{pid}_j, (\text{down}, \text{monitored_by}, \text{Ref}, r)) \\
 (\text{monitoring}, \text{pid}_i, \text{ref}) \in L_j \quad e_j \notin \{\text{EoP}, \text{ended}\} \\
 \hline
 \text{SIG_EXIT_MONIT} \quad \Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, \mathfrak{m}_j, \text{stack}_j, \mathfrak{r}_j), \text{sig}_{\text{sent},j}, L_j); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, \mathfrak{m}_i, \text{stack}_i, \mathfrak{r}_i), (s_{i,1}, \dots, s_{i,k}, \dots, s_{i,n_i}), L_i)\} \\
 \xrightarrow{\text{aux}} \\
 \Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, \mathfrak{m}_j, \text{stack}_j, \mathfrak{r}_j), \text{sig}_{\text{sent},j}, L_j); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, \mathfrak{m}_i, \text{stack}_i, \mathfrak{r}_i), (s_{i,1}, \dots, s_{i,k-1}, s_{i,k+1}, \dots, s_{i,n_i}) \cdot \\
 (\text{msg}, \text{pid}_j, (\{\text{'DOWN'}, ref, process, \text{pid}_i, r\}), L_i)\}
 \end{array}$$

the process j monitoring i has been sent a down signal

Semantics – Handling signals II

Down signals – monitor link

$$\begin{array}{c}
 \text{SIG_EXIT_MONIT} \quad \frac{
 \begin{array}{l}
 \text{first_sig}(\text{sig}_{\text{sent},i,j}) = k \quad i \neq j \\
 s_{i,k} = (\text{not_msg}, \text{pid}_j, (\text{down}, \text{monitored_by}, \text{Ref}, r)) \\
 (\text{monitoring}, \text{pid}_i, \text{ref}) \in L_j \quad e_j \notin \{\text{EoP}, \text{ended}\}
 \end{array}
 }{
 \begin{array}{l}
 \Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, m_j, \text{stack}_j, r_j), \text{sig}_{\text{sent},j}, L_j); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, m_i, \text{stack}_i, r_i), (s_{i,1}, \dots, s_{i,k}, \dots, s_{i,n_i}), L_i)\} \\
 \xrightarrow{\text{aux}} \\
 \Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, m_j, \text{stack}_j, r_j), \text{sig}_{\text{sent},j}, L_j); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, m_i, \text{stack}_i, r_i), (s_{i,1}, \dots, s_{i,k-1}, s_{i,k+1}, \dots, s_{i,n_i}) \cdot \\
 (\text{msg}, \text{pid}_j, (\{\text{'DOWN'}, ref, process, \text{pid}_i, r\})), L_i)\}
 \end{array}
 }
 \end{array}$$

signal deleted, corresponding message appended to the outbox

Semantics – Handling signals – Example

$\text{first_sig}(((\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))), \text{parent}) = 1$
 $s_{\text{child},1} = (\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))$
 $(\text{monitoring}, \text{pid}_{\text{child}}, \text{lid}) \in L_{\text{parent}} \quad e_{\text{parent}} \notin \{\text{EoP}, \text{ended}\}$

__EXIT__MONIT

$\{(\text{pid}_{\text{parent}}, (\text{receive } _ \text{ when 'true' } \rightarrow \text{'stop'}, [], \tau_{\text{parent}}, \text{erlang},$
 $\text{stack}_{\text{parent}}, \perp), (), \{(\text{monitoring}, \text{pid}_{\text{child}}, \text{lid})\});$

$(\text{pid}_{\text{child}}, (\text{ended}, [], \tau_{\text{ini}}, \text{test}, (), \perp), ((\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down},$
 $\text{monitored_by}, \text{lid}, \text{normal}))), \{(\text{monitored_by}, \text{pid}_{\text{parent}}, \text{lid})\})\}$

$\xrightarrow{\text{aux}}$

$\{(\text{pid}_{\text{parent}}, (\text{receive } _ \text{ when 'true' } \rightarrow \text{'stop'}, [], \tau_{\text{parent}}, \text{erlang},$
 $\text{stack}_{\text{parent}}, \perp), (), \{(\text{monitoring}, \text{pid}_{\text{child}}, \text{lid})\});$

$(\text{pid}_{\text{child}}, (\text{ended}, [], \tau_{\text{ini}}, \text{test}, (), \perp),$

$(\text{msg}, \text{pid}_{\text{parent}}, (\text{'DOWN'}, \text{lid}, \text{process}, \text{pid}_{\text{child}}, \text{normal})), L_{\text{child}})\}$

Semantics – Handling signals – Example

first_sig(((not_msg, pid_{parent}, (down, monitored_by, lid, normal))), parent) = 1
 s_{child,1} = (not_msg, pid_{parent}, (down, monitored_by, lid, normal))

(monitoring, pid_{child}, lid) ∈ L_{parent} e_{parent} ∉ {EoP, ended}

_EXIT__MONIT

{(**pid_{parent}**, (receive _ when 'true' -> 'stop', [], τ_{parent}, erlang,
 stack_{parent}, ⊥), ()), {(monitoring, pid_{child}, lid)});

(**pid_{child}**, (ended, [], τ_{ini}, test, (), ⊥), ((not_msg, pid_{parent}, (down,
 monitored_by, lid, normal))), {(monitored_by, pid_{parent}, lid)}))

$\xrightarrow{aux}$

{(pid_{parent}, (receive _ when 'true' -> 'stop', [], τ_{parent}, erlang,
 stack_{parent}, ⊥), ()), {(monitoring, pid_{child}, lid)});

(pid_{child}, (ended, [], τ_{ini}, test, (), ⊥),

(msg, pid_{parent}, ({'DOWN', lid, process, pid_{child}, normal})), L_{child}))

two processes, parent monitoring child

Semantics – Handling signals – Example

Down signals – monitor link

$\text{first_sig}(((\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))), \text{parent}) = 1$
 $s_{\text{child},1} = (\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))$
 $(\text{monitoring}, \text{pid}_{\text{child}}, \text{lid}) \in L_{\text{parent}} \quad e_{\text{parent}} \notin \{\text{EoP}, \text{ended}\}$

EXIT MONIT

$\{(\text{pid}_{\text{parent}}, (\text{receive } _ \text{ when 'true' } \rightarrow \text{'stop'}, [], \tau_{\text{parent}}, \text{erlang},$
 $\text{stack}_{\text{parent}}, \perp), (), \{(\text{monitoring}, \text{pid}_{\text{child}}, \text{lid})\});$

$(\text{pid}_{\text{child}}, (\text{ended}, [], \tau_{\text{ini}}, \text{test}, (), \perp), ((\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down},$
 $\text{monitored_by}, \text{lid}, \text{normal}))), \{(\text{monitored_by}, \text{pid}_{\text{parent}}, \text{lid})\})\}$

$\xrightarrow{\text{aux}}$

$\{(\text{pid}_{\text{parent}}, (\text{receive } _ \text{ when 'true' } \rightarrow \text{'stop'}, [], \tau_{\text{parent}}, \text{erlang},$
 $\text{stack}_{\text{parent}}, \perp), (), \{(\text{monitoring}, \text{pid}_{\text{child}}, \text{lid})\});$

$(\text{pid}_{\text{child}}, (\text{ended}, [], \tau_{\text{ini}}, \text{test}, (), \perp),$

$(\text{msg}, \text{pid}_{\text{parent}}, (\text{'DOWN'}, \text{lid}, \text{process}, \text{pid}_{\text{child}}, \text{normal})), L_{\text{child}})\}$

child sent a down signal to parent

Semantics – Handling signals – Example

Down signals – monitor link

$\text{first_sig}(((\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))), \text{parent}) = 1$
 $s_{\text{child},1} = (\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down}, \text{monitored_by}, \text{lid}, \text{normal}))$
 $(\text{monitoring}, \text{pid}_{\text{child}}, \text{lid}) \in L_{\text{parent}} \quad e_{\text{parent}} \notin \{\text{EoP}, \text{ended}\}$

EXIT MONIT

$\{(\text{pid}_{\text{parent}}, (\text{receive } _ \text{ when 'true' } \rightarrow \text{'stop'}, [], \tau_{\text{parent}}, \text{erlang},$
 $\text{stack}_{\text{parent}}, \perp), (), \{(\text{monitoring}, \text{pid}_{\text{child}}, \text{lid})\});$

$(\text{pid}_{\text{child}}, (\text{ended}, [], \tau_{\text{ini}}, \text{test}, (), \perp), ((\text{not_msg}, \text{pid}_{\text{parent}}, (\text{down},$
 $\text{monitored_by}, \text{lid}, \text{normal}))), \{(\text{monitored_by}, \text{pid}_{\text{parent}}, \text{lid})\})\}$

$\xrightarrow{\text{aux}}$

$\{(\text{pid}_{\text{parent}}, (\text{receive } _ \text{ when 'true' } \rightarrow \text{'stop'}, [], \tau_{\text{parent}}, \text{erlang},$
 $\text{stack}_{\text{parent}}, \perp), (), \{(\text{monitoring}, \text{pid}_{\text{child}}, \text{lid})\});$

$(\text{pid}_{\text{child}}, (\text{ended}, [], \tau_{\text{ini}}, \text{test}, (), \perp),$

$(\text{msg}, \text{pid}_{\text{parent}}, (\text{'DOWN', lid, process, pid}_{\text{child}}, \text{normal})), L_{\text{child}})\}$

signal deleted, corresponding message appended to the outbox

Conclusion

Contribution

Small step semantics of a subset of Core Erlang:

- handling of monitoring links and symmetrical links
- handling of signals

Coming ...

- implementation
- investigate the causal analysis of faults originating from non deterministic messages order

End

Thank you

Syntax

```
Module ::= module atom [Fname1, ..., Fnamek]  
         attributes [atom1 = Cst1, ..., atomm = Cstm]  
         Fdef1 ... Fdefn  
Fdef ::= Fname = Fun  
Fname ::= atom/integer  
Cst(c) ::= Lit | [ Cst1 | Cst2 ]  
         | { Cst1, ..., Cstn } | Fun  
Val(v) ::= Cst | < Cst1, ..., Cstk > | EoP | ended  
Lit ::= integer|float|atom|char|string| []  
Fun ::= fun ( var1, ..., varn ) - > Exprs  
Exprs ::= Expr | < Expr1, ..., Exprn >  
Expr ::= var | Fname | Lit | Fun  
         | [ Expr1, ..., Exprn ] | { Expr1, ..., Exprn }  
         | let Vars = Exprs1 in Exprs2  
         | do Exprs1 Exprs2  
         | letrec Fdef1 ... Fdefn in Exprs  
         | apply Exprs0 ( Exprs1, ..., Exprsn )  
         | call Exprs1 : Exprs2 ( Exprs3, ..., Exprsn+2 )  
         | primop atom ( Exprs1, ..., Exprsn )  
         | case Exprs of Cls1 ... Clsn end  
         | receive Cls1 ... Clsn after Exprs1 - > Exprs2  
Vars(x) ::= var | < var1, ..., varn >  
Cls(cl) ::= Pats when Exprs1 - > Exprs2  
Pats ::= Pat | < Pat1, ..., Patn >  
Pat(p) ::= var | Lit | [ Pat1 | Pat2 ]  
         | { Pat1, ..., Patn } | var = Pat
```

Syntax – receive

```
letrec 'recv$^0'/0 =  
  fun() ->  
    let <var1, var2> = primop 'recv_peck_message'()  
      in case var1 of  
        <'true'> when 'true' ->  
          case var2 of  
            Cls1 ... Clsn  
              (<Other> when 'true' ->  
                do primop 'recv_next'()  
                  (apply 'recv$^0'/0())  
              )  
            end  
          (<'false'> when 'true' ->  
            let var3 = primop 'recv_wait_timeout'(Expr1)  
              in case var3 of  
                <'true'> when 'true' -> Expr2  
                (<'false'> when 'true' -> (apply 'recv$^0'/0 ()))  
              end)  
            end  
          in (apply 'recv$^0'/0 ()))
```


Semantics – Recursive calls I

Choice

- Documentation: the arguments of a function can be evaluated in any order
- Current implementation: left to right, depth first
- Past implementation: right to left, depth first

→ we are following the documentation

$$\begin{array}{c} \text{SUBEXPR_OK} \frac{\begin{array}{c} e_{eval} \in Expr \setminus Val \wedge e'_{eval} \neq EoP \\ \Pi \cup \{(pid, (e_{eval}, \theta, \tau, m, \text{stack} \cdot (e_{eval}, \theta, m), r), s_{sent}, L))\} \\ \xrightarrow{aux} \\ \Pi' \cup \{(pid, (e'_{eval}, \theta', \tau', m', \text{stack}', r'), s'_{sent}, L')\} \end{array}}{\begin{array}{c} \Pi \cup \{(pid, (c[e_{eval}], \theta, \tau, m, \text{stack}, r), s_{sent}, L))\} \\ \xrightarrow{aux} \\ \Pi' \cup \{(pid, (c[e'_{eval}], \theta, \tau', m, \text{stack}, r'), s'_{sent}, L')\} \end{array}} \end{array}$$

Semantics – Recursive calls II

Choice

- Documentation: the arguments of a function can be evaluated in any order
- Current implementation: left to right, depth first
- Past implementation: right to left, depth first

→ we are following the documentation

$$\begin{array}{c} \text{SUBEXPR_KO} \quad \frac{\begin{array}{c} e_{eval} \in Expr \setminus Val \\ \Pi \cup \{(pid, (e_{eval}, \theta, \tau, m, \text{stack} \cdot (e_{eval}, \theta, m), r), s_{sent}, L))\} \\ \xrightarrow{aux} \\ \Pi' \cup \{(pid, (EoP, \theta', \tau', m', \text{stack}', r'), s'_{sent}, L')\} \end{array}}{\begin{array}{c} \Pi \cup \{(pid, (c[e_{eval}], \theta, \tau, m, \text{stack}, r), s_{sent}, L))\} \\ \xrightarrow{aux} \\ \Pi' \cup \{(pid, (EoP, \theta, \tau', m, \text{stack}, r'), s'_{sent}, L')\} \end{array}} \end{array}$$

Spawning a process

$$\begin{array}{c} \text{RSPAWN} \quad \frac{\text{fresh_pid}(\Pi) = \text{pid}_{\text{fils}} \quad Mname, v_1, \dots, v_n \in Val}{\Pi \cup \{(\text{pid}_{\text{pere}}, (\text{call } 'erlang': 'spawn' \\ (Mname, Fname, [v_1, \dots, v_n]), \theta, \tau, m), \text{sig}_{\text{sent}}, L)\}} \\ \xrightarrow{\text{aux}} \\ \Pi \cup \{(\text{pid}_{\text{pere}}, (\text{pid}_{\text{fils}}, \theta, \tau, m), \text{sig}_{\text{sent}}, L); \\ (\text{pid}_{\text{fils}}, (\text{call } Mname : Fname(v_1, \dots, v_n), [], \tau_{\text{stat}}, Mname), [], [])\} \end{array}$$

Semantics – Handling signals – down link

Characteristics

- Asynchronous
- Automatic: handled by default, impossible to prevent it

Down signals – symetrical link

$$\text{first_sig}((s_{i,1}, \dots, s_{i,k}, \dots, s_{i,n_i}), j) = k \quad i \neq j$$

$$s_{i,k} = (\text{not_msg}, \text{pid}_j, (\text{down}, \text{link}, \text{pid}_i, r))$$

$$(\text{link}, \text{pid}_i, \text{pid}_j) \in L_j \quad e_j \notin \{\text{EoP}, \text{ended}\}$$

SIG_EXIT_LINK

$$\Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, m_j, \text{stack}_j, r_j), \text{sig}_{\text{sent},j}, L_j);$$
$$(\text{pid}_i, (e_i, \theta_i, \tau_i, m_i, \text{stack}_i, r_i), (s_{i,1}, \dots, s_{i,k}, \dots, s_{i,n_i}), L_i)\}$$

$\xrightarrow{\text{aux}}$

$$\Pi \cup \{(\text{pid}_j, (\text{EoP}, \theta_j, \tau_j, m_j, \text{stack}_j, \text{end_reason}(r_j, r)), \text{sig}_{\text{sent},j}, L_j);$$
$$(\text{pid}_i, (e_i, \theta_i, \tau_i, m_i, \text{stack}_i, r_i), (s_{i,1}, \dots, s_{i,k-1}, s_{i,k+1}, \dots, s_{i,n_i}), L_i)\}$$

Semantics – Monitor I

First step

$$\begin{array}{c}
 \text{MONITOR} \quad \frac{\text{fresh_lid}(L_1) = \text{lid}}{\Pi \cup \{(\text{pid}_1, (\text{call } \text{erlang} : \text{monitor}(\text{process}, \text{pid}_2), \theta, \tau, \mathbf{m}, \text{stack}, \mathbf{r}), \text{sig}_{\text{sent}}, L)\}} \\
 \xrightarrow{\text{aux}} \\
 \Pi \cup \{(\text{pid}_1, (\text{lid}, \theta, \tau, \text{erlang}, \mathbf{m}, \text{stack}, \mathbf{r}), \\
 \text{sig}_{\text{sent}} \cdot (\text{not_msg}, \text{pid}_2, (\text{monitored_by}, \text{pid}_1, \text{lid})), L_1 \cup \{(\text{monitoring}, \text{pid}_2, \text{lid})\})\}
 \end{array}$$

Update on the other side

$$\begin{array}{c}
 \text{MONITORED_BY} \quad \frac{\begin{array}{l} \text{first_sig}((s_{i,1}, \dots, s_{i,k}, \dots, s_{i,n_i}), j) = k \quad e_j \notin \{\text{EoP}, \text{ended}\} \\ s_{i,k} = (\text{not_msg}, \text{pid}_j, (\text{monitored_by}, \text{pid}_i, \text{lid})) \quad i \neq j \end{array}}{\Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, \mathbf{m}_j, \text{stack}_j, \mathbf{r}_j), \text{sig}_{\text{sent},j}, L_j); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, \mathbf{m}_i, \text{stack}_i, \mathbf{r}_i), (s_{i,1}, \dots, s_{i,k}, \dots, s_{i,n_i}), L_i)\}} \\
 \xrightarrow{\text{aux}} \\
 \Pi \cup \{(\text{pid}_j, (e_j, \theta_j, \tau_j, \mathbf{m}_j, \text{stack}_j, \mathbf{r}_j), \text{sig}_{\text{sent},j}, L_j \cup \{(\text{monitored_by}, \text{pid}_i, \text{lid})\}); \\
 (\text{pid}_i, (e_i, \theta_i, \tau_i, \mathbf{m}_i, \text{stack}_i, \mathbf{r}_i), (s_{i,1}, \dots, s_{i,k-1}, s_{i,k+1}, \dots, s_{i,n_i}), L_i)\}
 \end{array}$$