FHE Circuit Privacy Almost For Free

Florian Bourse, Rafaël Del Pino, Michele Minelli,
and Hoeteck Wee

CNRS, École normale supérieure, INRIA, PSL, Paris, France

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Example: online diagnostic

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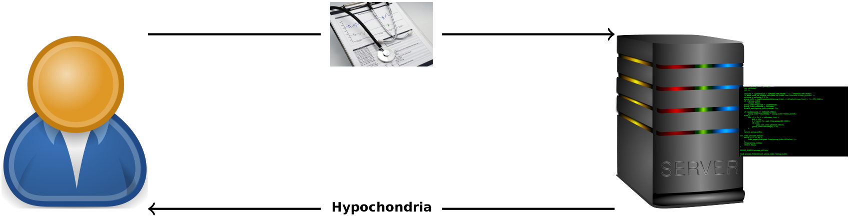
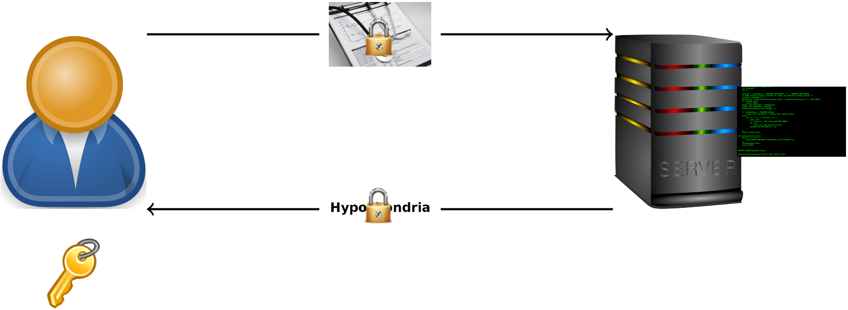


Data privacy: FHE



GSW encryption scheme [GenSahWat13]

$$\mathbf{G} = \mathbf{Id}_n \otimes \mathbf{g}, \quad \mathbf{g} = \left(1, 2, \ldots, 2^k\right)$$

$$\mathbf{C} = \mathrm{Enc}(\mu) = \begin{pmatrix} \mathbf{A} \\ \mathbf{s}\mathbf{A} + \mathbf{e} \end{pmatrix} + \mu\mathbf{G} \in \mathbb{Z}_q^{n \times m}$$

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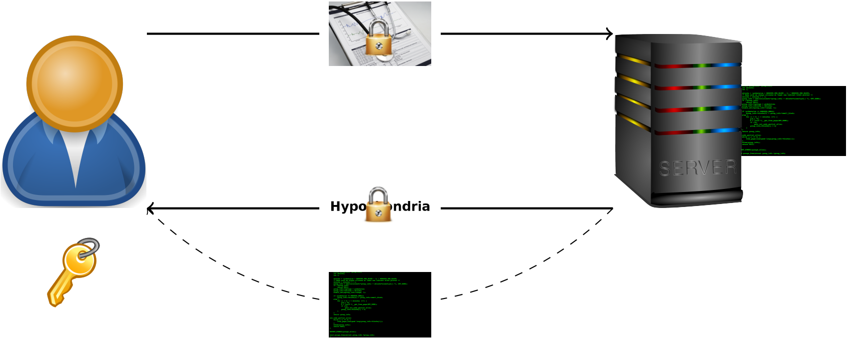
Sum $\mathrm{Enc}(\mu_1) + \mathrm{Enc}(\mu_2)$

Product $\mathrm{Enc}(\mu_1) \cdot \mathbf{G}^{-1}(\mathrm{Enc}(\mu_2))$

where $\forall\, \mathbf{v} \in \mathbb{Z}_q^n$, $\mathbf{G}^{-1}(\mathbf{v}) \in \mathbb{Z}_q^m$ is *small* and s.t. $\mathbf{G} \cdot \mathbf{G}^{-1}(\mathbf{v}) = \mathbf{v}$

Protecting the algorithm: circuit privacy



Leakage in the error term: toy example

Given $\mathbf{s}$, and 3 encryptions of 0:

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$\mathbf{C_i} + \mathbf{C_j}$ leaks i and j:
The error term is $\mathbf{e}_i + \mathbf{e}_j$!

Protecting the algorithm: circuit privacy

$\mathsf{Eval}(f, \mathbf{C}_1, \ldots, \mathbf{C}_\ell)$ should reveal nothing on f but $f(\mu_1, \ldots, \mu_\ell)$.

Noise flooding [Gen09]

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Pros Destroys all information contained in the noise

Cons Requires superpolynomial modulus, not multi-hop

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Pros Works with polynomial modulus, multi-hop

Cons Requires circular security (bootstrapping)

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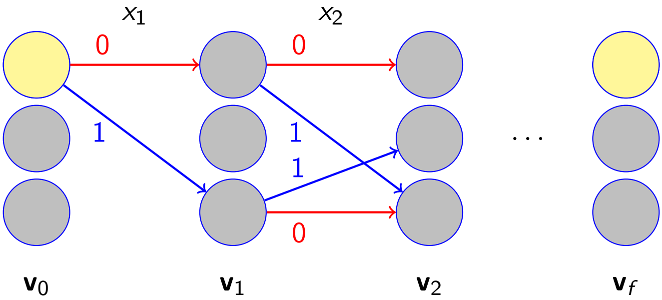
Pros Polynomial modulus, no circular security, multi-hop

Cons Only for NC^1 evaluations on GSW, leaks $|f|$

Branching programs



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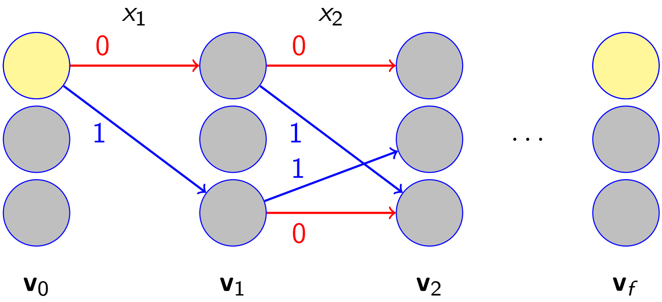

$$\mathbf{v}_t[i] = \begin{cases} \mathbf{v}_{t-1}[j] \text{ if } x_t = 1 \\ \mathbf{v}_{t-1}[k] \text{ if } x_t = 0 \end{cases}$$

$$= \mathbf{v}_{t-1}[MUX(x_t, j, k)]$$

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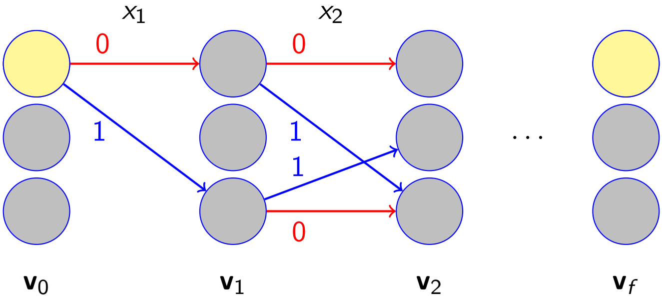

$$\mathbf{v}_t[i] = x_t \mathbf{v}_{t-1}[j] + (1 - x_t)\mathbf{v}_{t-1}[k]$$

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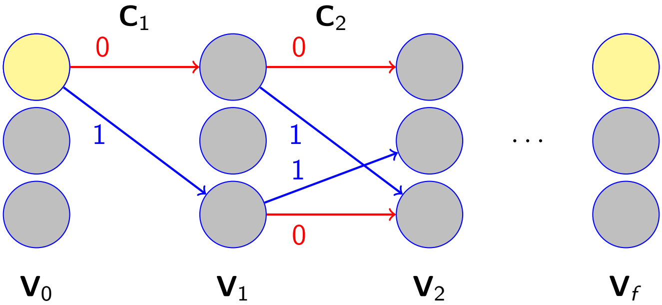

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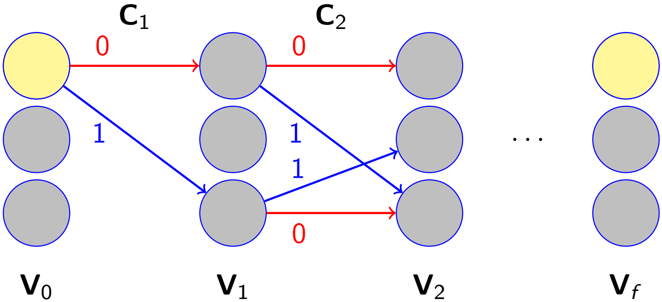

$$\mathbf{V}_t[i] = \mathbf{C}_t\mathbf{G}^{-1}(\mathbf{V}_{t-1}[j]) + (\mathbf{G} - \mathbf{C}_t)\mathbf{G}^{-1}(\mathbf{V}_{t-1}[k])$$

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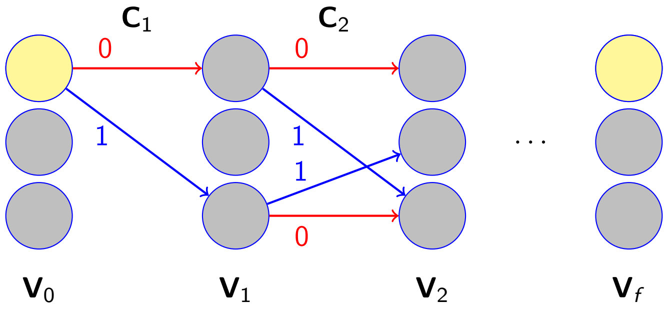
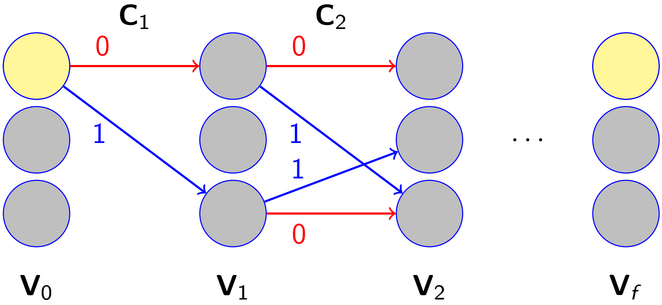


Let $\mathbf{C}_t = \mathbf{B}_t + x_t\mathbf{G}$

$$\mathbf{V}_t[i] = \mathbf{V}_{t-1}[MUX(x_t, j, k)] + \mathbf{B}_t\mathbf{G}^{-1}(\mathbf{V}_{t-1}[j] + \mathbf{V}_{t-1}[k])$$



Let $\mathbf{C}_t = \mathbf{B}_t + x_t\mathbf{G}$

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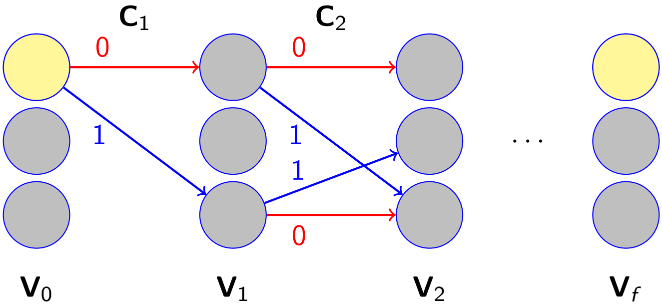

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The additional noise should not leak information about the branching program

Our core lemma: GSW rerandomization

Let $\mathbf{C}$ be an encryption of 0 with error $\mathbf{e}$. For any matrix $\mathbf{V}$:

$$\mathbf{C} \cdot \mathbf{G}^{-1}(\mathbf{V}) + \begin{pmatrix} \mathbf{O} \\ \mathbf{z} \end{pmatrix} \approx_s \mathbf{C}'$$

- $\mathbf{G}^{-1}$ Gaussian s.t $\mathbf{G} \cdot \mathbf{G}^{-1}(\mathbf{V}) = \mathbf{V}$
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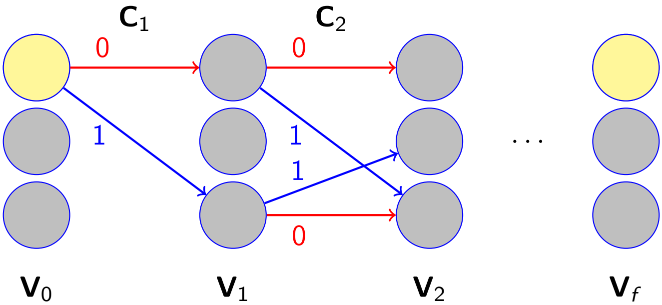
$\Rightarrow$ $\mathbf{C}'$ is independent of $\mathbf{V}$!

Modified evaluation



$$\mathbf{V}_t[i] = \mathbf{V}_{t-1}[MUX(x_t, j, k)] + \mathbf{B}_t \mathbf{G}^{-1}(\mathbf{V}_{t-1}[j] + \mathbf{V}_{t-1}[k]))$$

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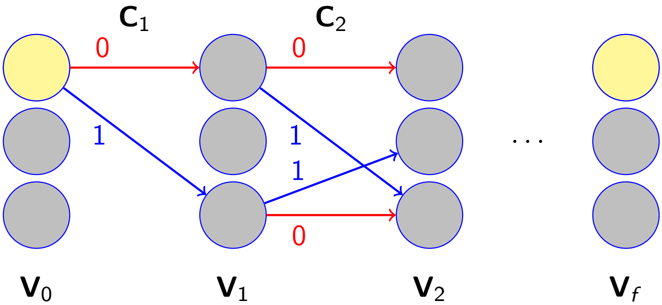

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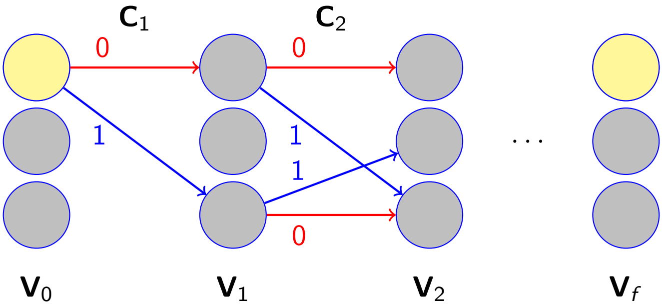

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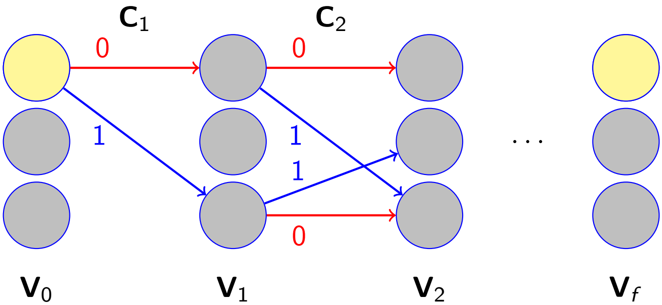

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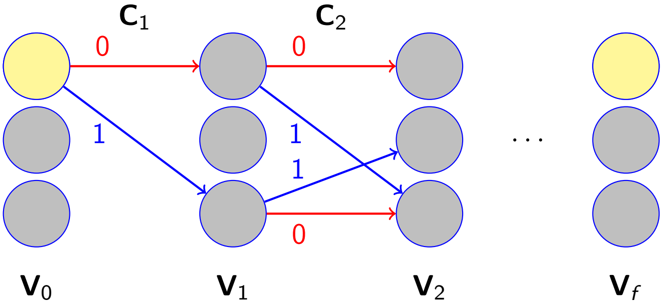

$$\mathbf{V}_t[i] = \mathbf{V}_{t-1}[MUX(x_t, j, k)] + \mathbf{C}$$

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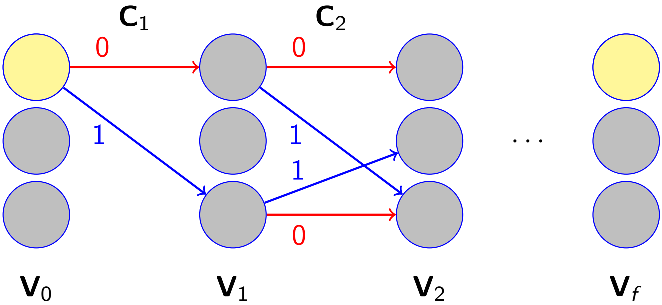
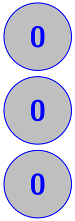


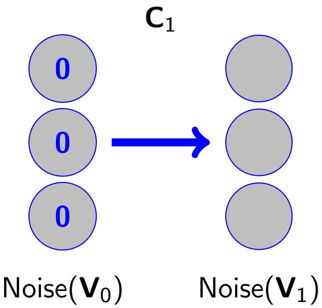
$$\mathbf{V}_t[i] = \mathbf{V}_{t-1}[MUX(x_t, j, k)] + \underbrace{\mathbf{C}}_{\text{depends only on } \mathbf{C}_t}$$

Circuit privacy by induction



Noise($\mathbf{V}_0$)

Circuit privacy by induction



$$\text{Noise}(\mathbf{V}_1[i]) = \text{Noise}(\mathbf{V}_0[MUX(x_1, j, k)]) + \mathbf{B}_t \mathbf{G}^{-1}(\mathbf{V}_0[j] + \mathbf{V}_0[k]) + \begin{pmatrix} \mathbf{O} \\ \mathbf{z} \end{pmatrix}$$

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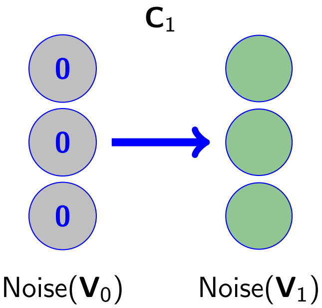

$$\text{Noise}(\mathbf{V}_1[i]) \approx_s \mathbf{O} + \mathbf{C}_1'$$

Where $\mathbf{C}_1'$ depends only on $\mathbf{C}_1$

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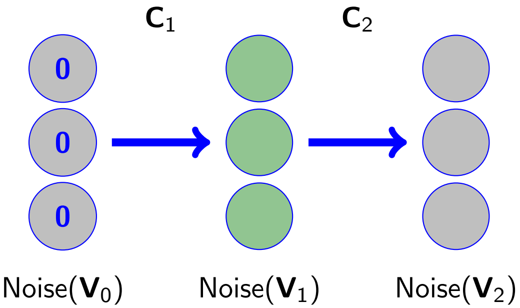

$$\begin{aligned}\mathrm{Noise}(\mathbf{V}_2[i]) = &\mathrm{Noise}(\mathbf{V}_1[MUX(x_2, j, k)]) \\ &+ \mathbf{B}_t \mathbf{G}^{-1}(\mathbf{V}_1[j] + \mathbf{V}_1[k]) + \begin{pmatrix} \mathbf{0} \\ \mathbf{z} \end{pmatrix}\end{aligned}$$

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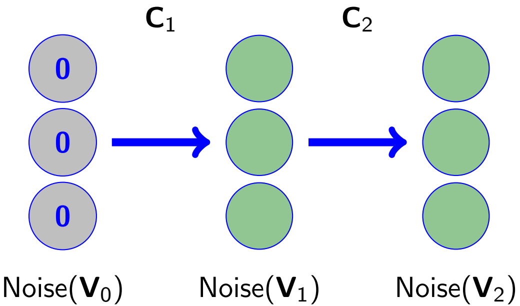
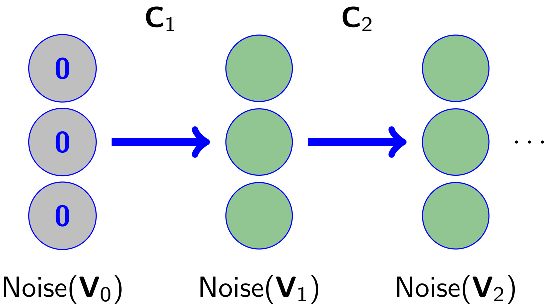


$$\text{Noise}(\mathbf{V}_1[i]) \approx_s \mathbf{C}'_1 + \mathbf{C}'_2$$

Where $\mathbf{C}'_2$ depends only on $\mathbf{C}_1$

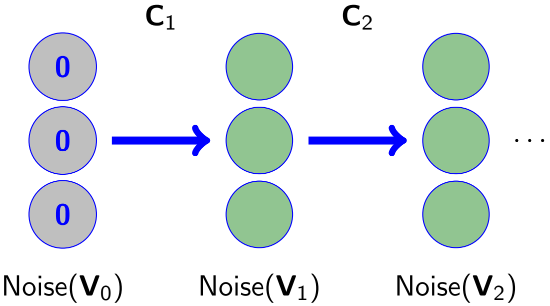
Circuit privacy by induction



$$\text{Noise}(\mathbf{V}_f[i]) \approx_s \sum_1^f \mathbf{C}'_k$$

The final noise depends only on the number of time each choice bit has been used

Circuit privacy by induction



$$\text{Noise}(\mathbf{V}_f[i]) \approx_s \sum_{1}^{f} \mathbf{C}'_k$$

Padding $\Rightarrow \text{Noise}(\mathbf{V}_f[i]) = \text{function of } |BP|$

Thank you!

Questions?