Functional Encryption for Inner-Product Evaluations

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École normale supérieure, CNRS, INRIA, PSL, Paris, France

PhD Defense — Paris
Wednesday, December 13

Public Key Encryption

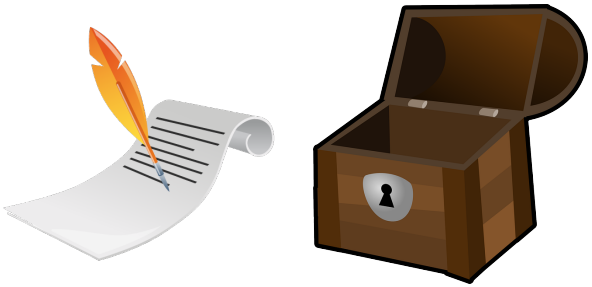
Bob

Alice

Public Key Encryption

Public Key Encryption

Bob



Alice

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Bob

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Inner-Product Functional Encryption

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Applications:

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- evaluate statistics on encrypted data;

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- evaluate statistics on encrypted data;
- building block for more general Functional Encryption[AR17];

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- evaluate statistics on encrypted data;
- building block for more general Functional Encryption[AR17];
- traitor tracing[ABP+17];
- ...

Inner-Product Functional Encryption[ABDP15]

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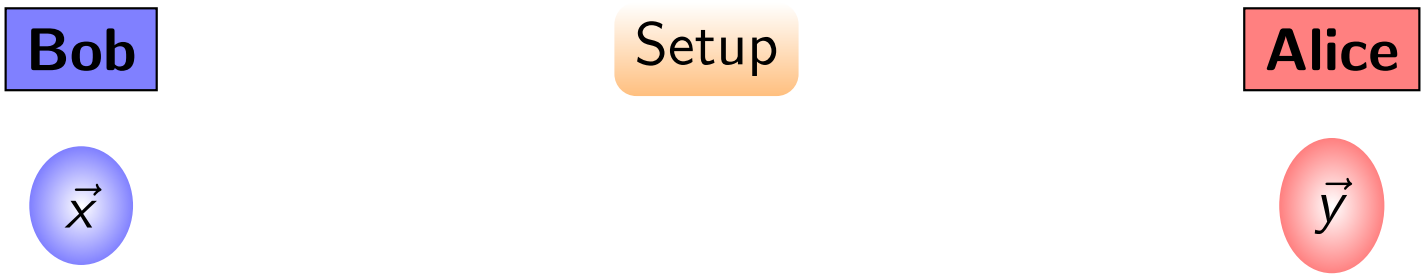
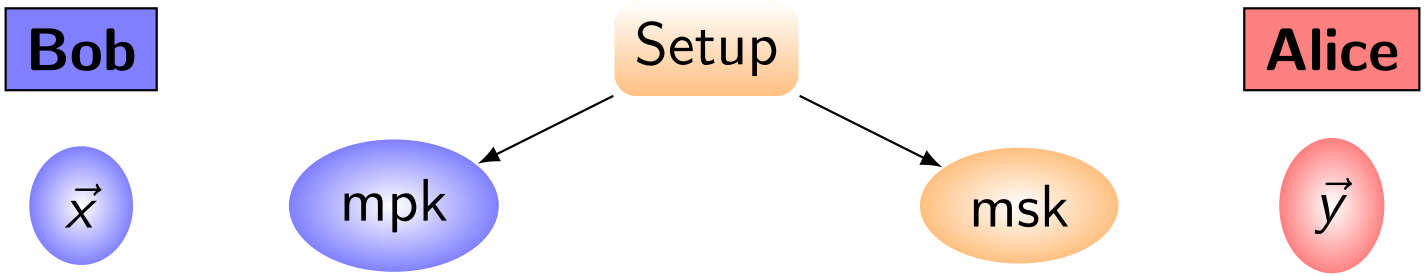


Inner-Product Functional Encryption[ABDP15]



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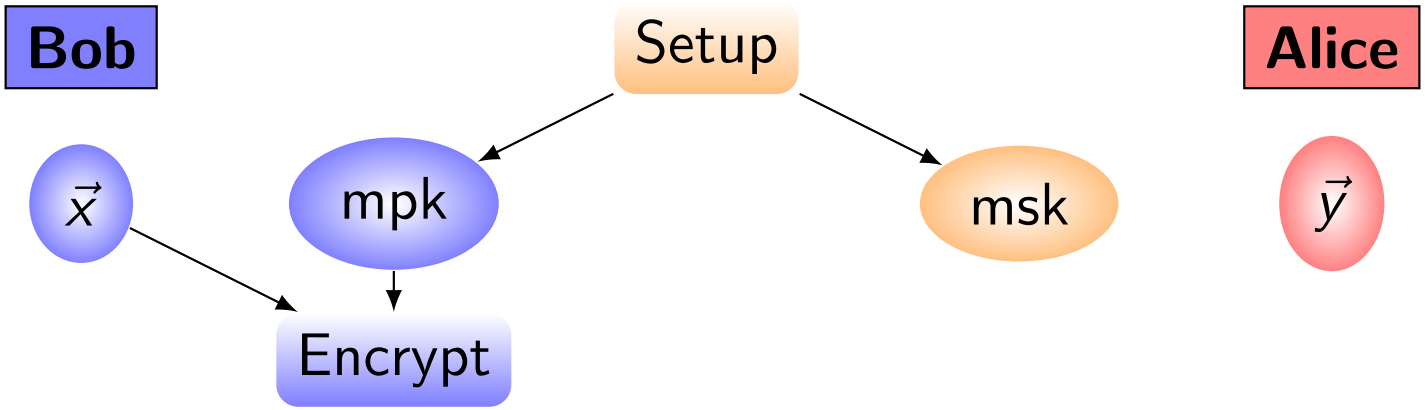

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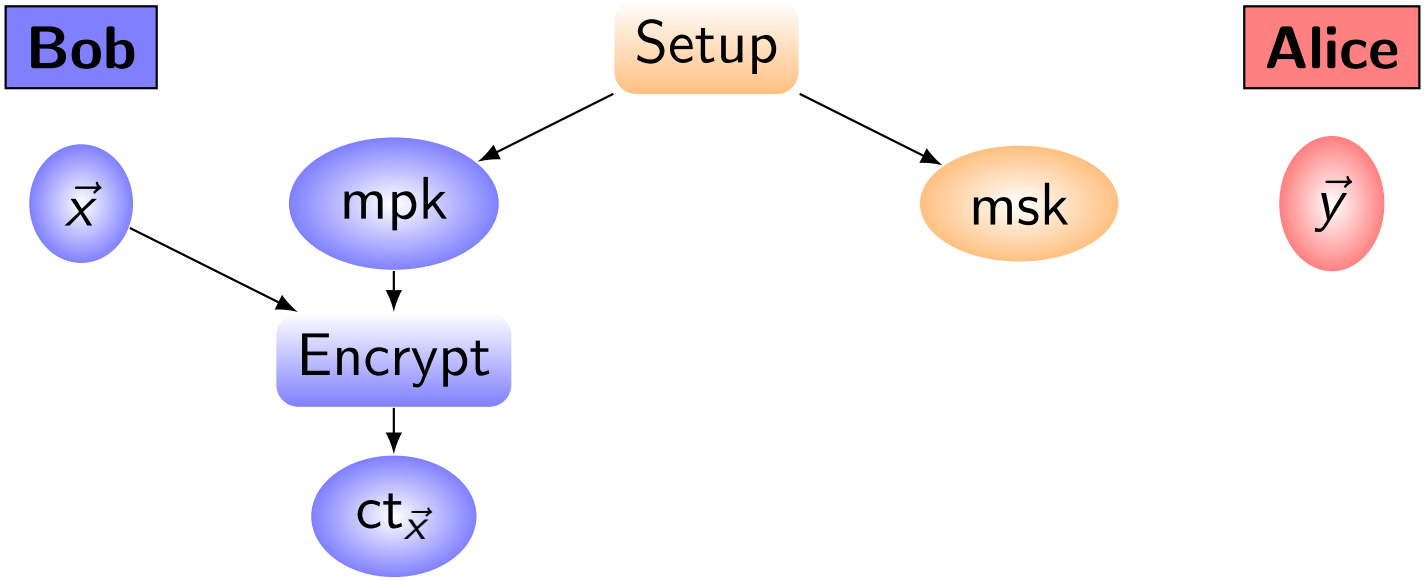

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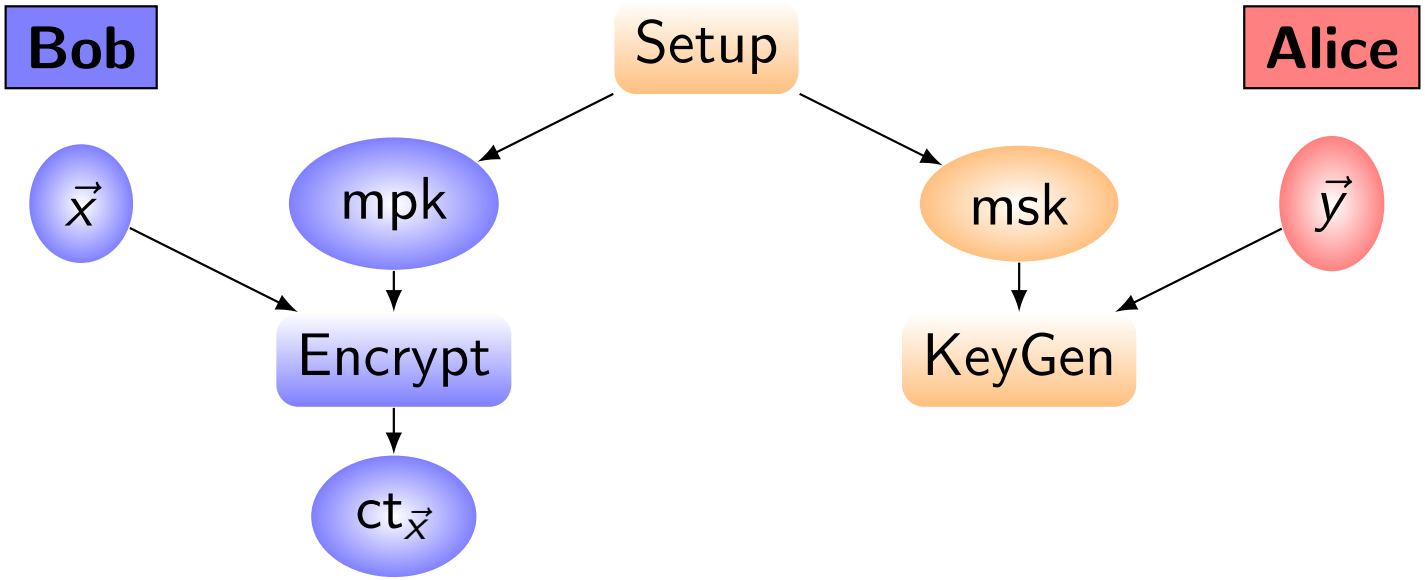

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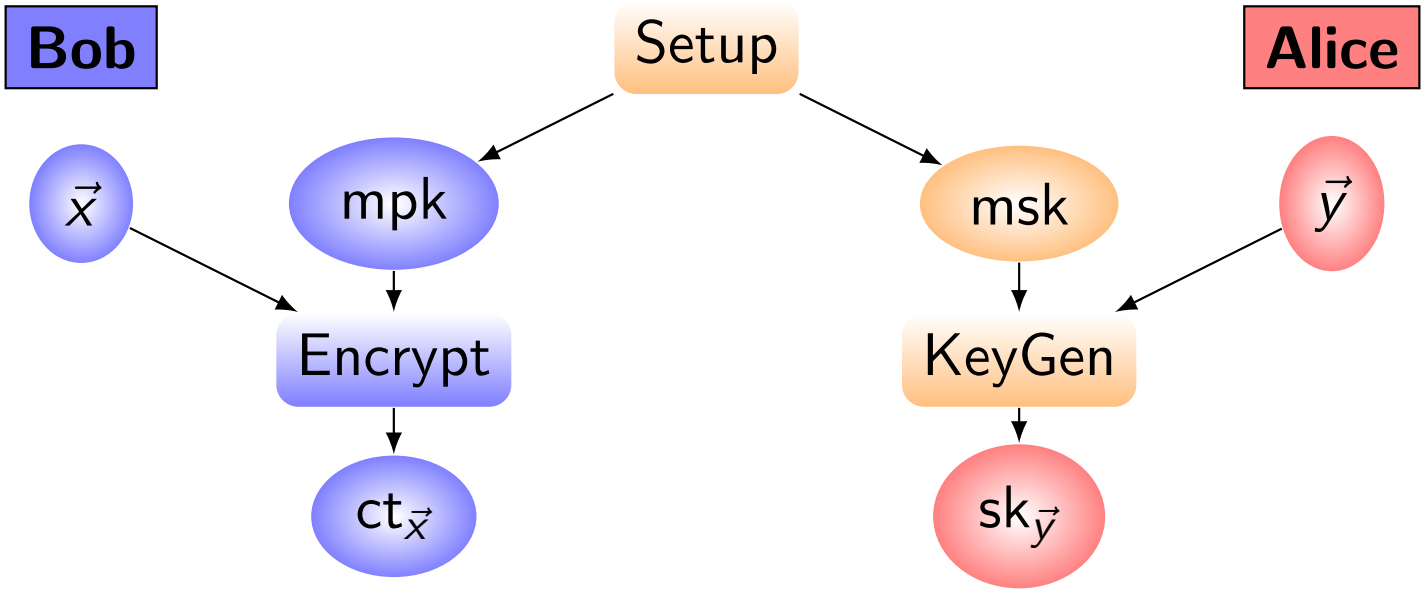

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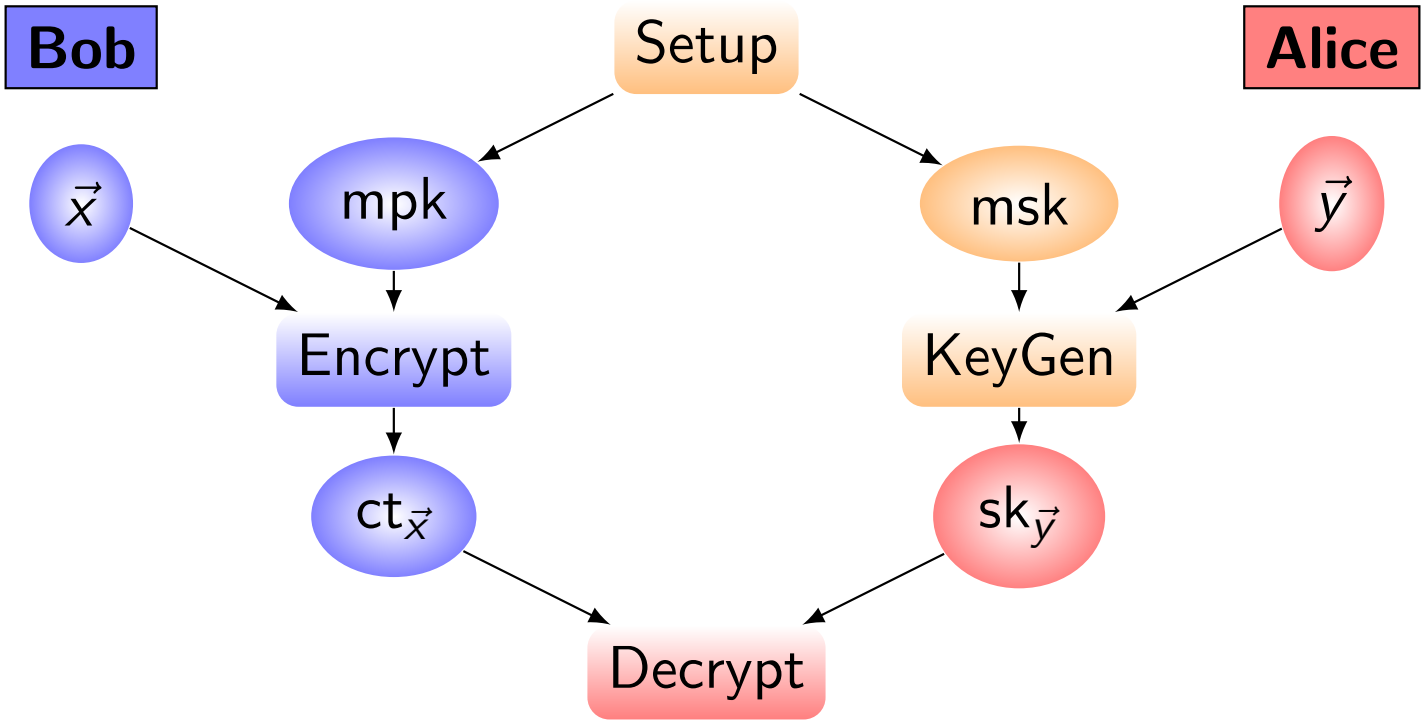

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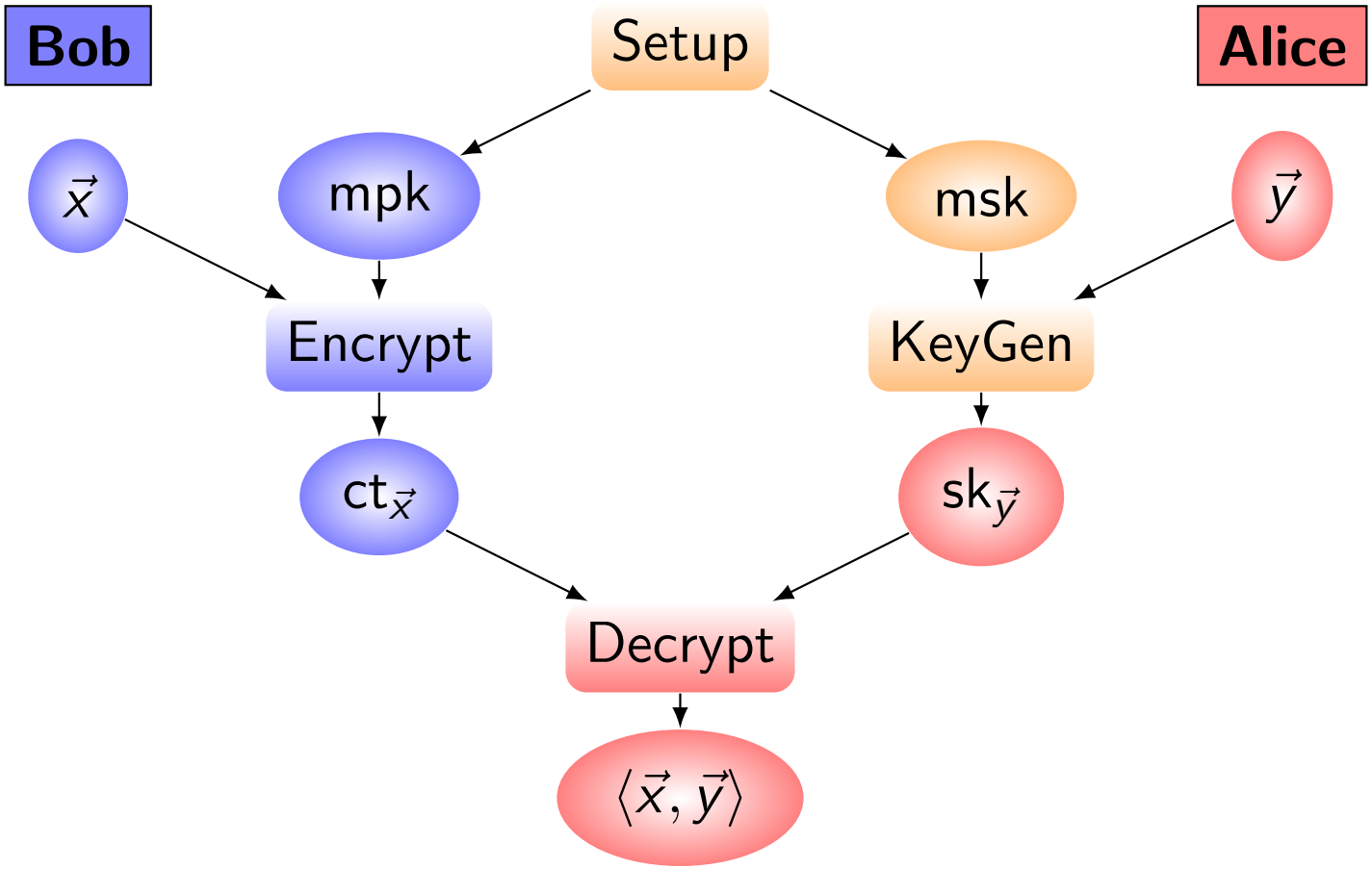
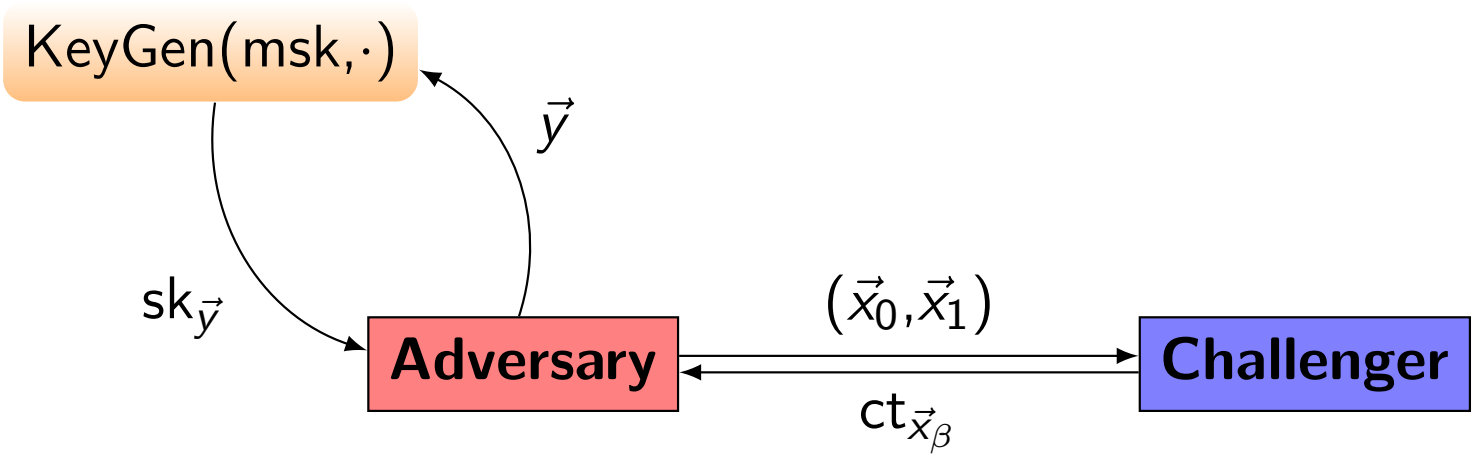
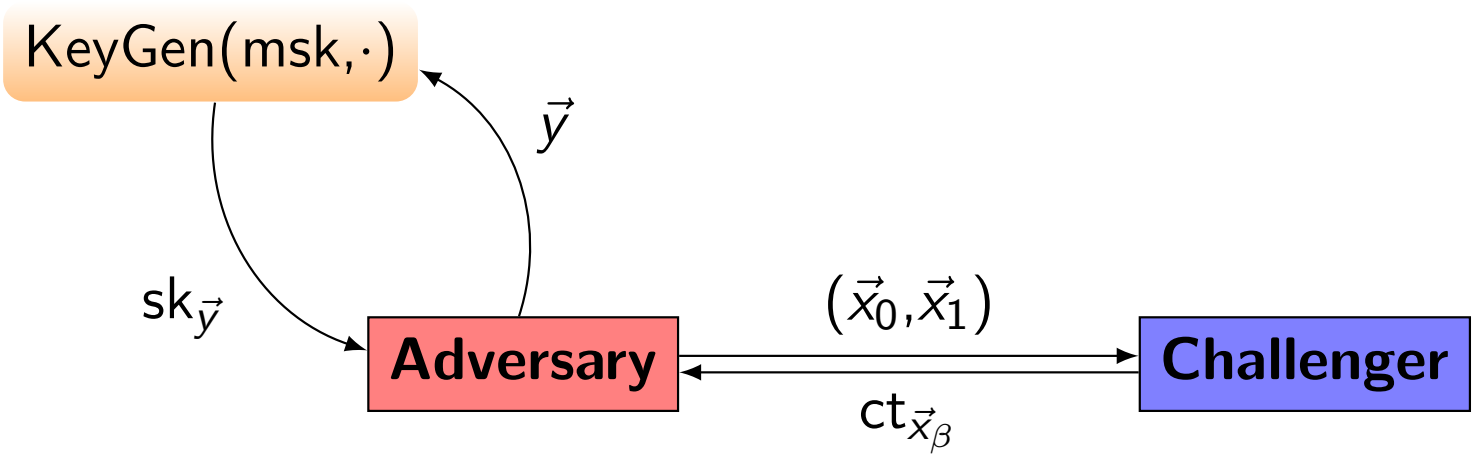


IND-CPA security



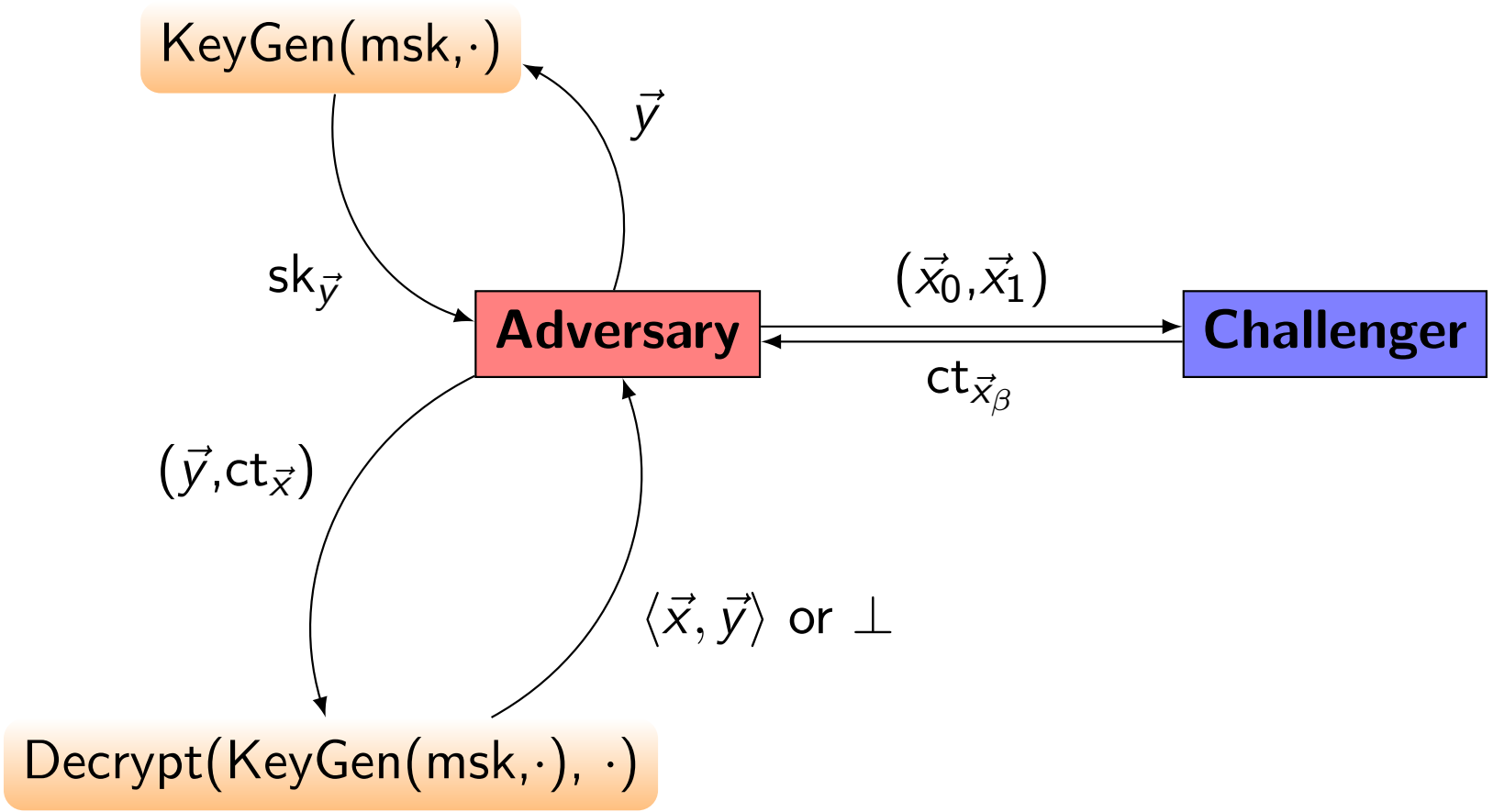
Adversary wins if $\beta' = \beta \wedge \forall \vec{y}, \langle \vec{x}_0, \vec{y} \rangle = \langle \vec{x}_1, \vec{y} \rangle$

IND-CPA security



Adversary wins if $\beta' = \beta \wedge \forall \vec{y}, \vec{y} \in (\vec{x}_1 - \vec{x}_0)^\perp$

IND-CCA security



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NP language

Let $\mathcal{L} = \{\mathbf{b}/\exists w \text{ s.t. } \mathcal{R}(w, \mathbf{b}) = 1\}$ be an NP language.

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Example: DDH, fix $(g, h) \in \mathbb{G}^2$

$$\mathcal{L} = \{(g^r, h^r)\}_{r \in \mathbb{Z}_p}$$

Projective Hash Function: $(w, \mathbf{b}) \in \mathcal{L}$

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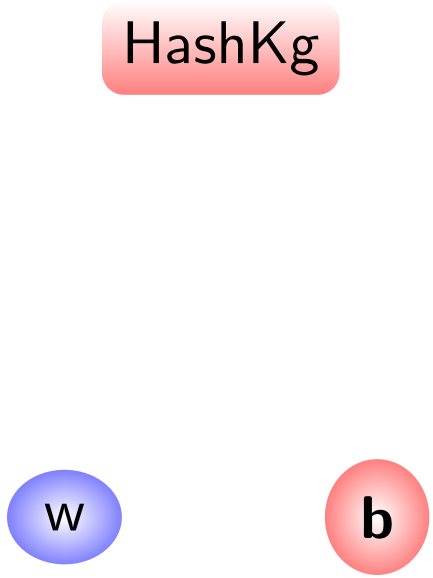

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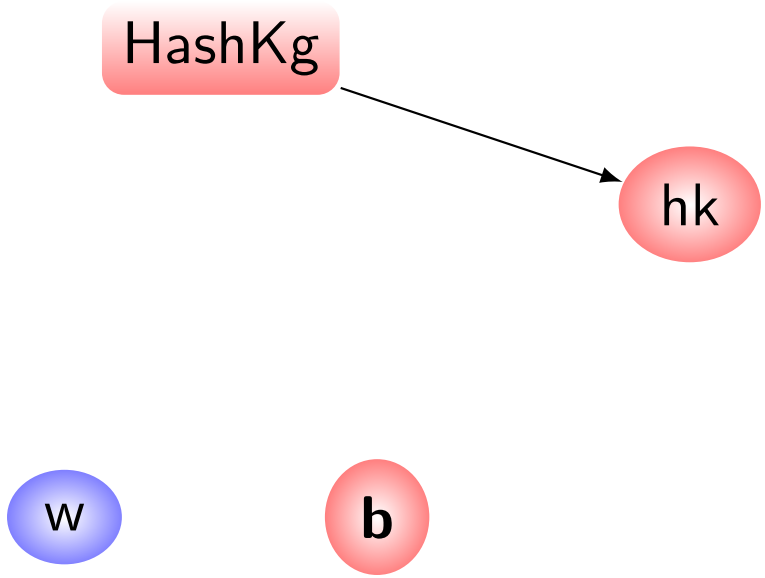

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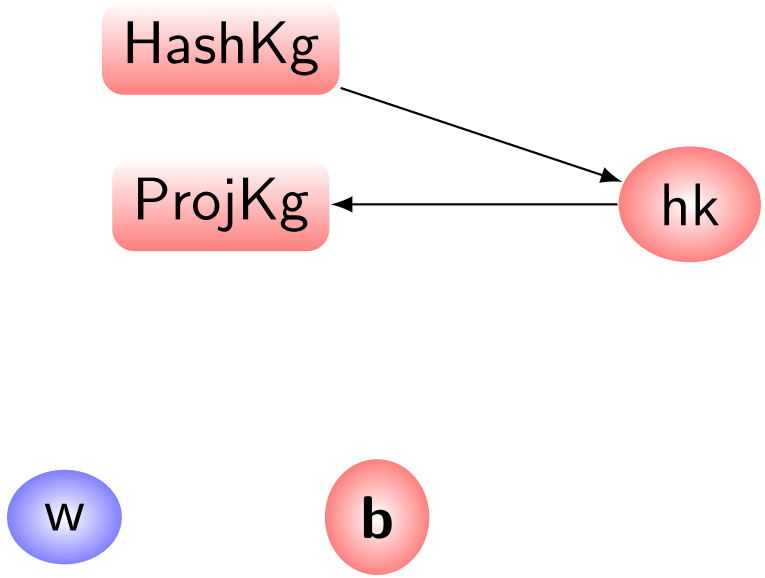

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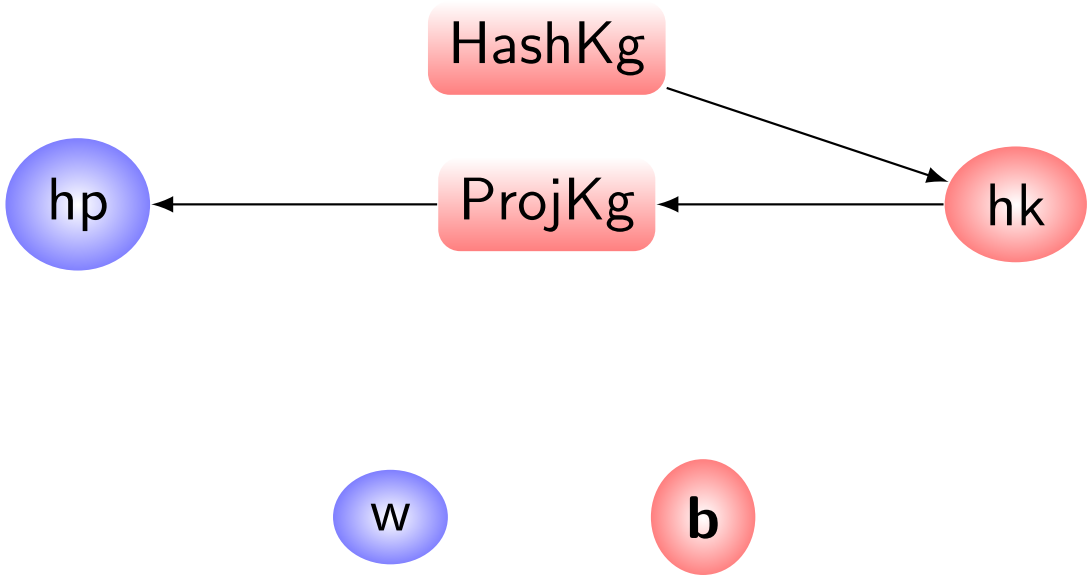

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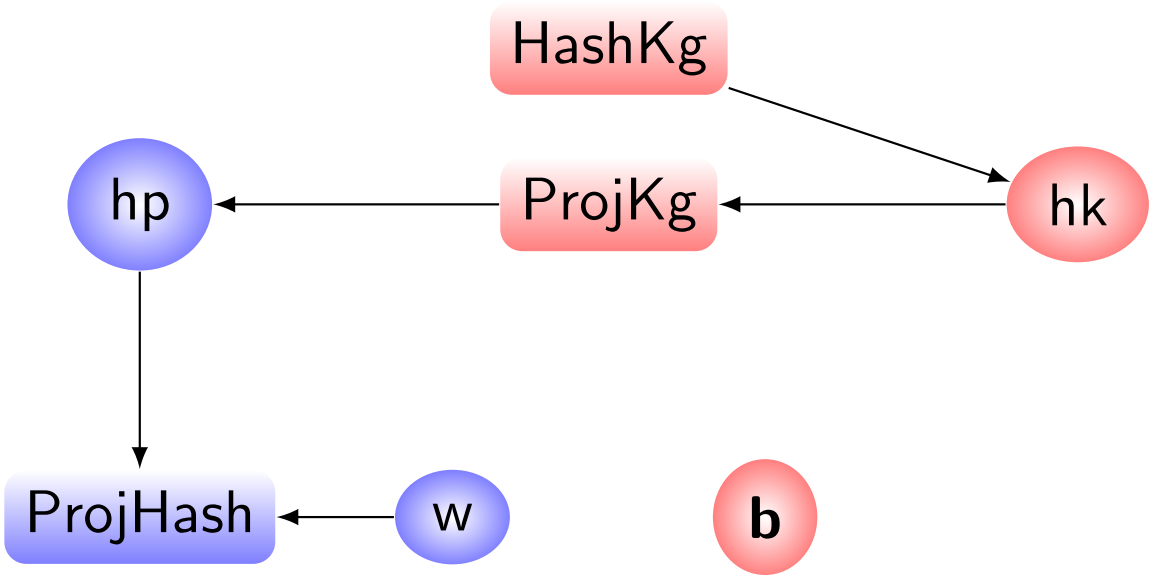

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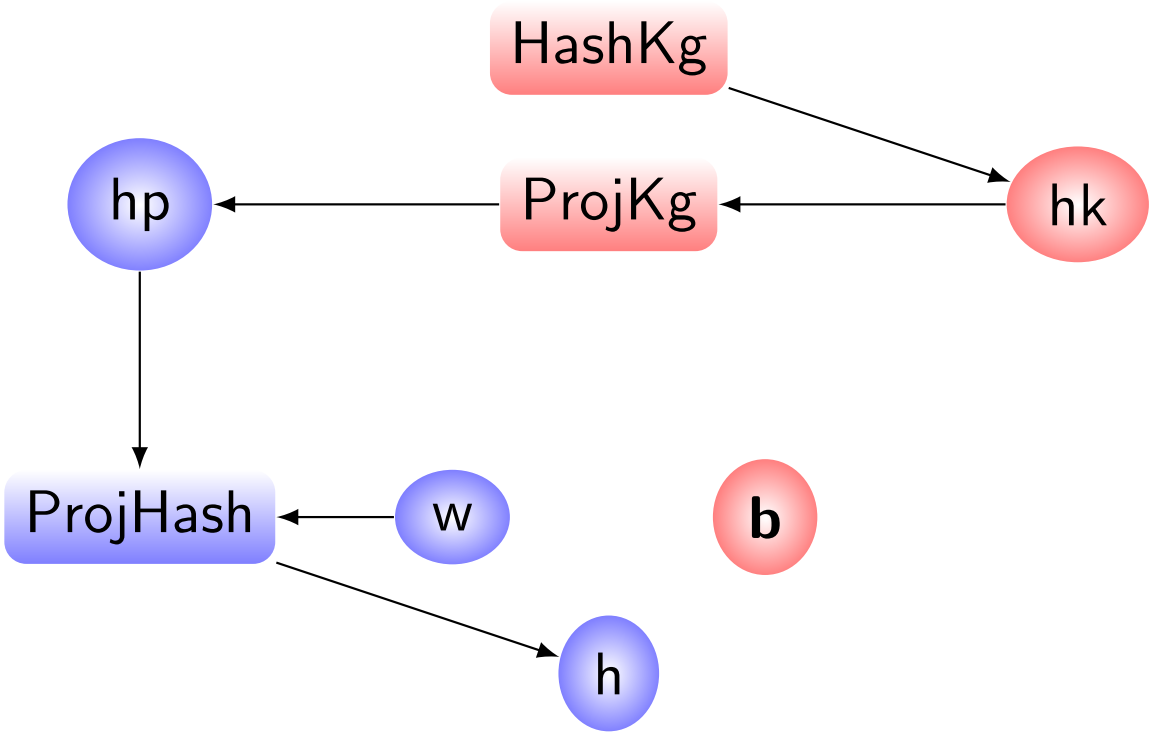

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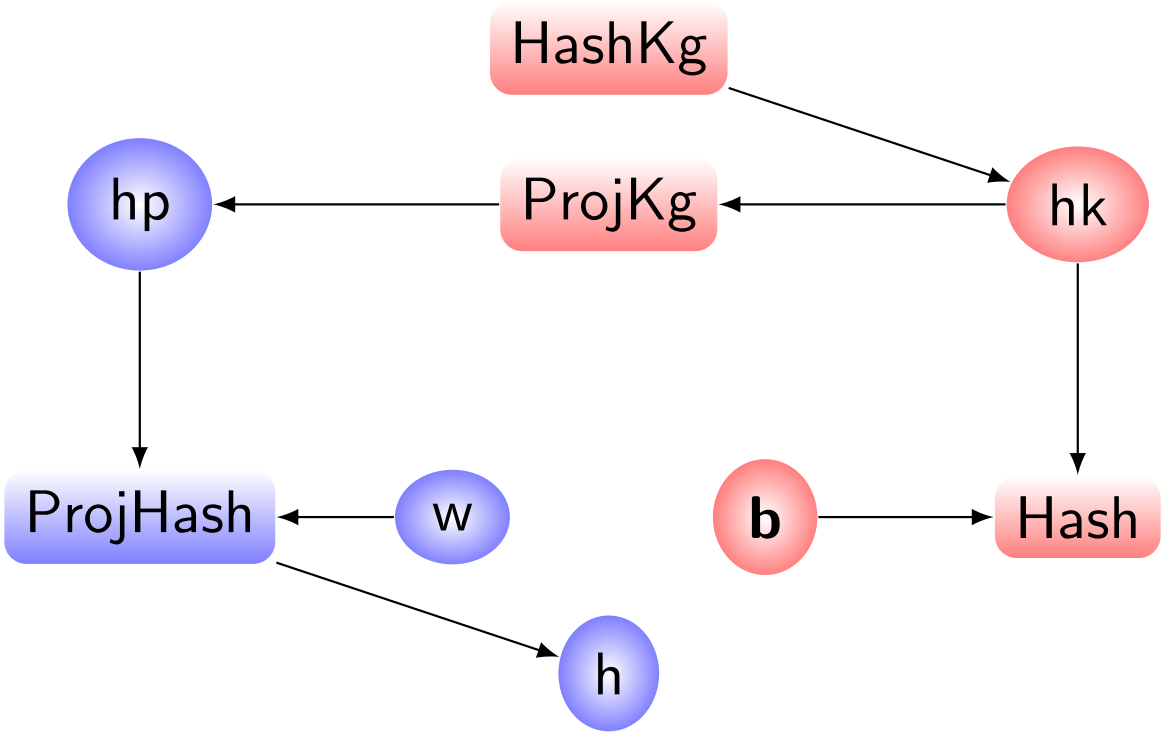

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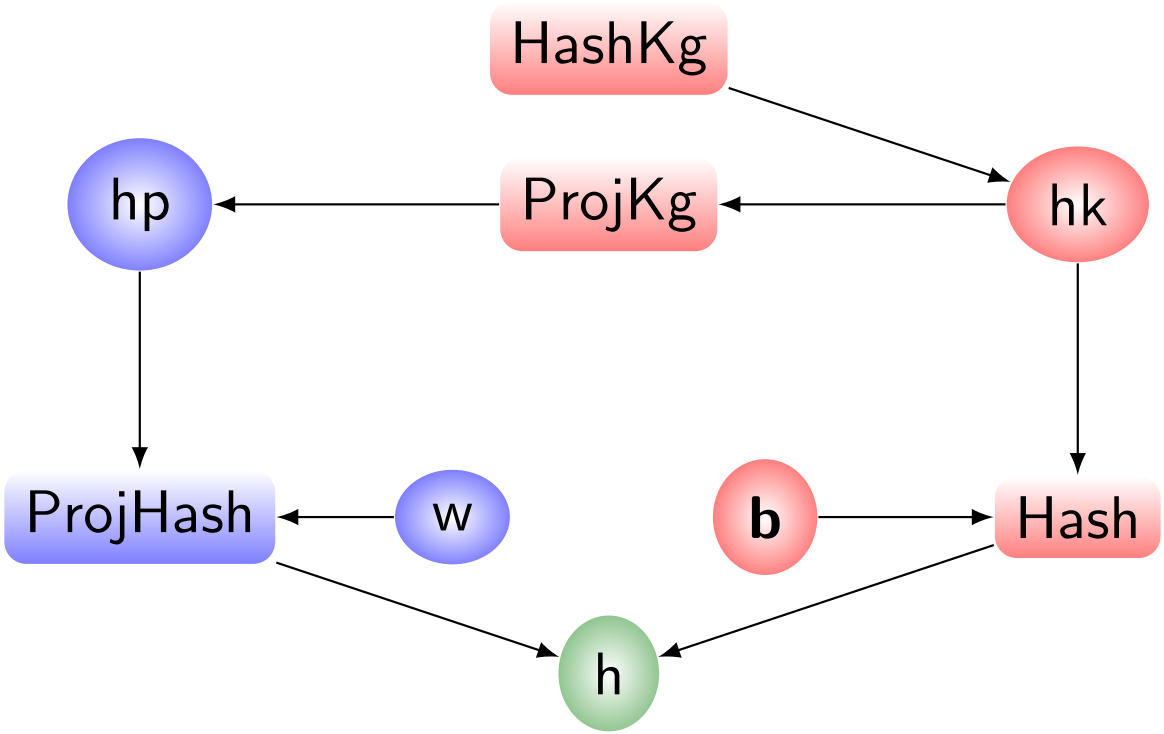
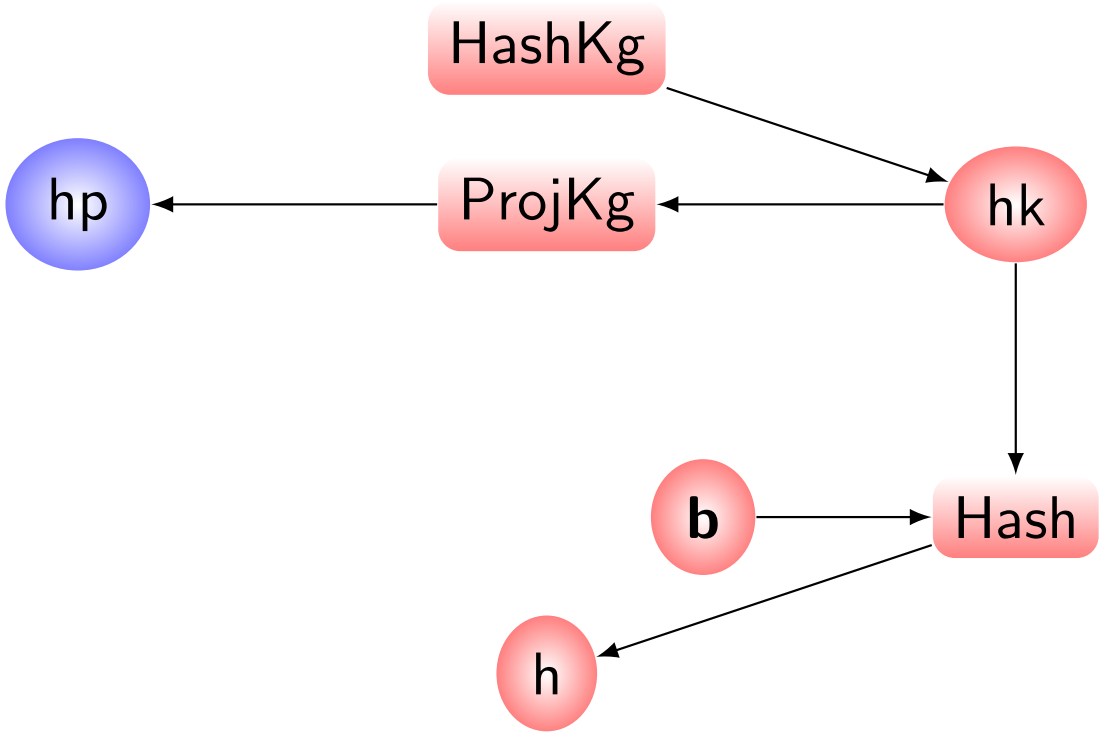
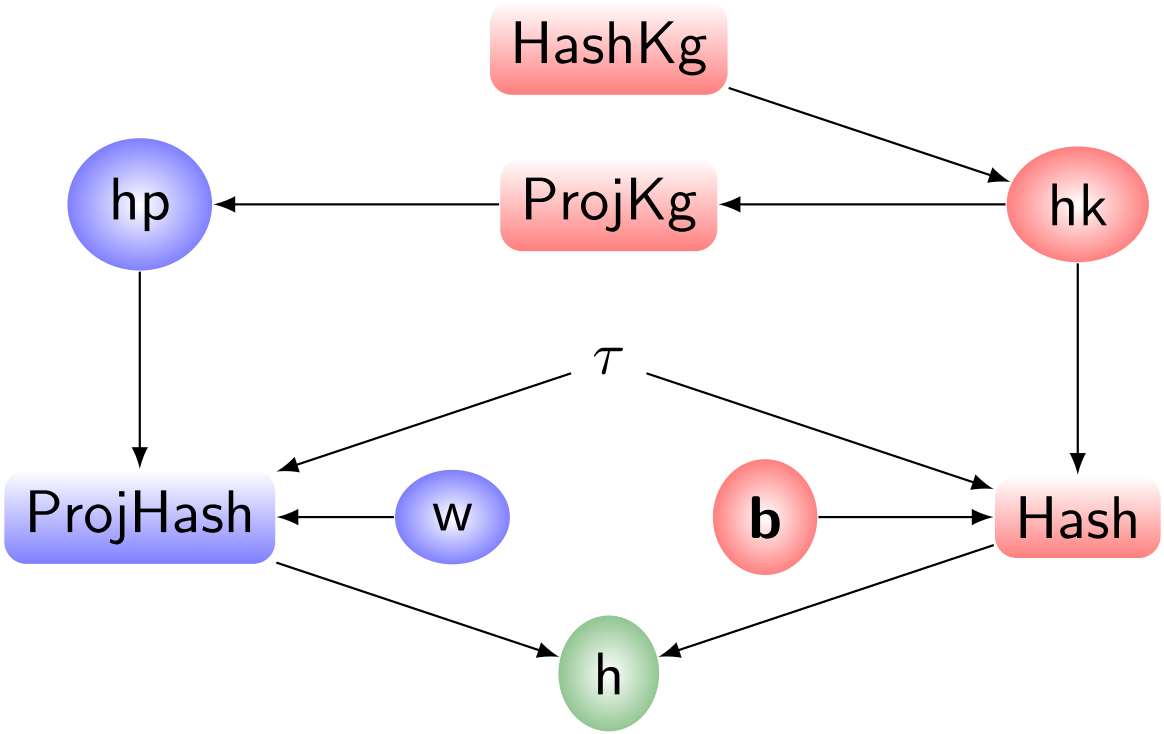


Projective Hash Function: $\mathbf{b} \notin \mathcal{L}$



Tag-Based PHF



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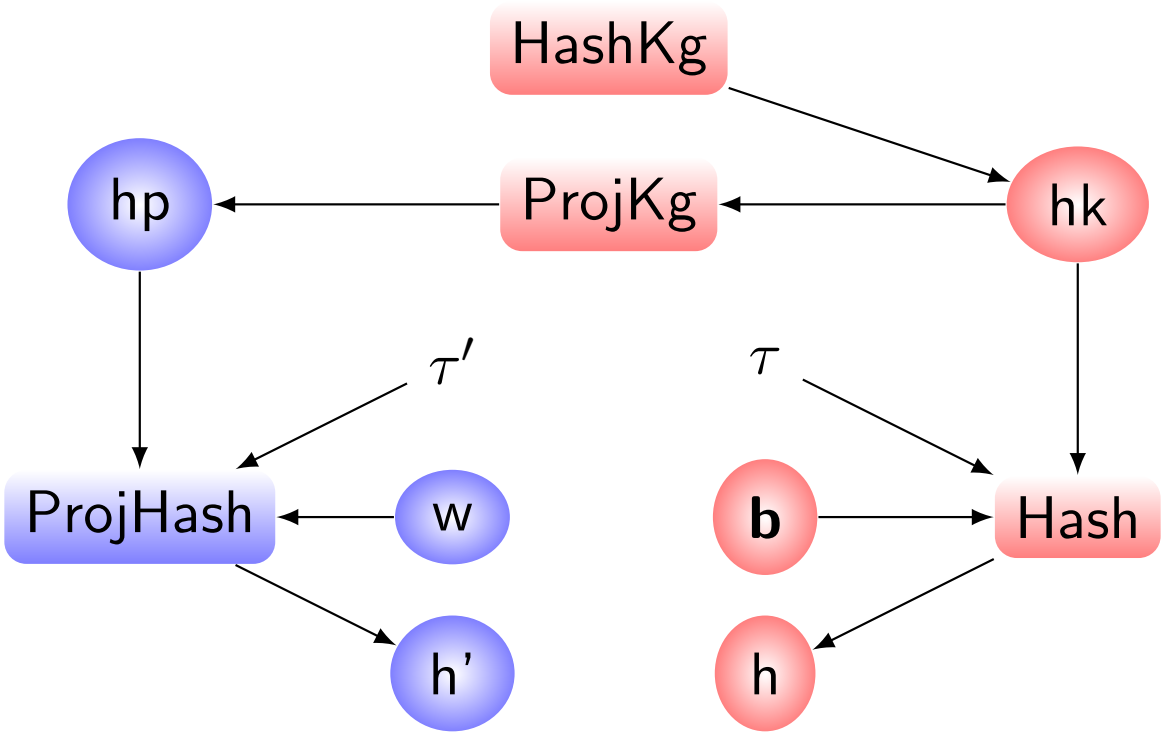
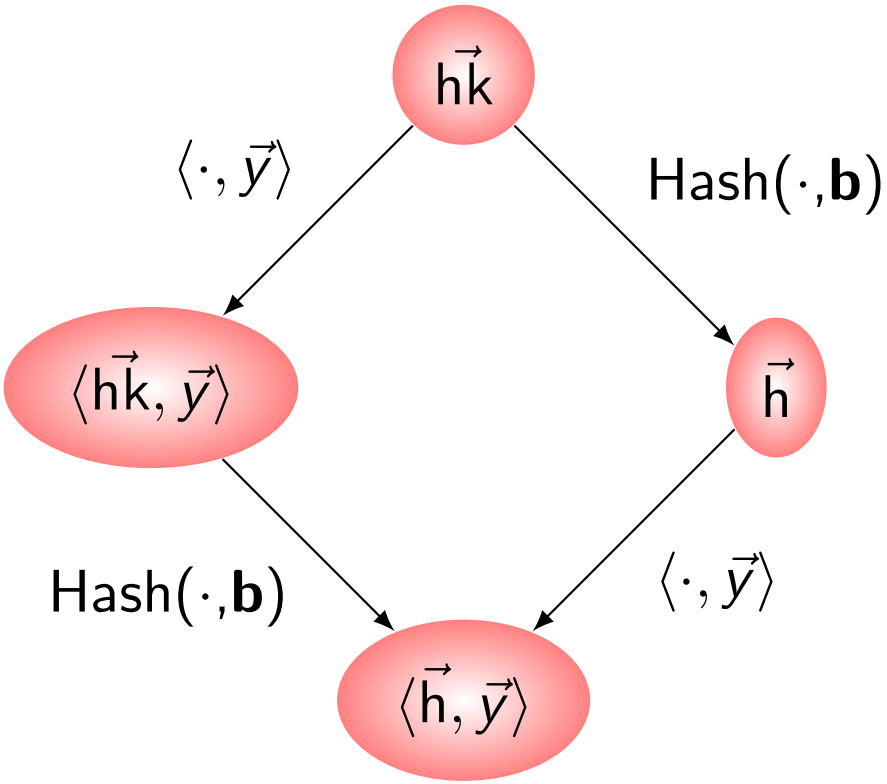


Hash is an Additive Homomorphism



IPFE from PHF

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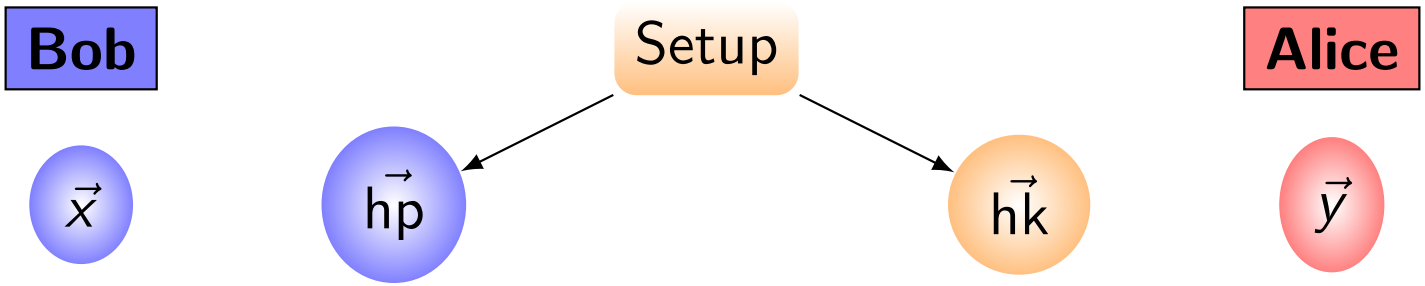

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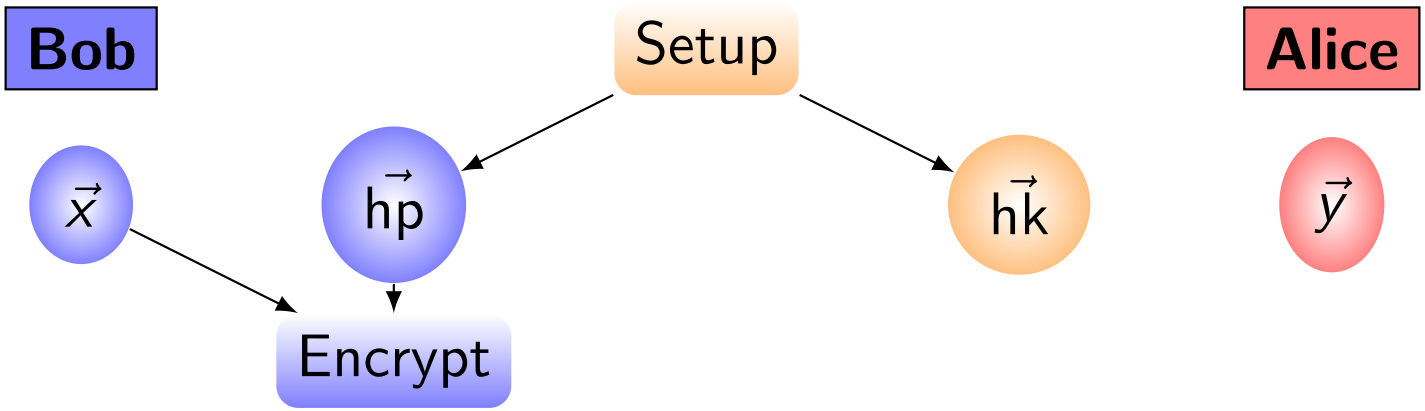

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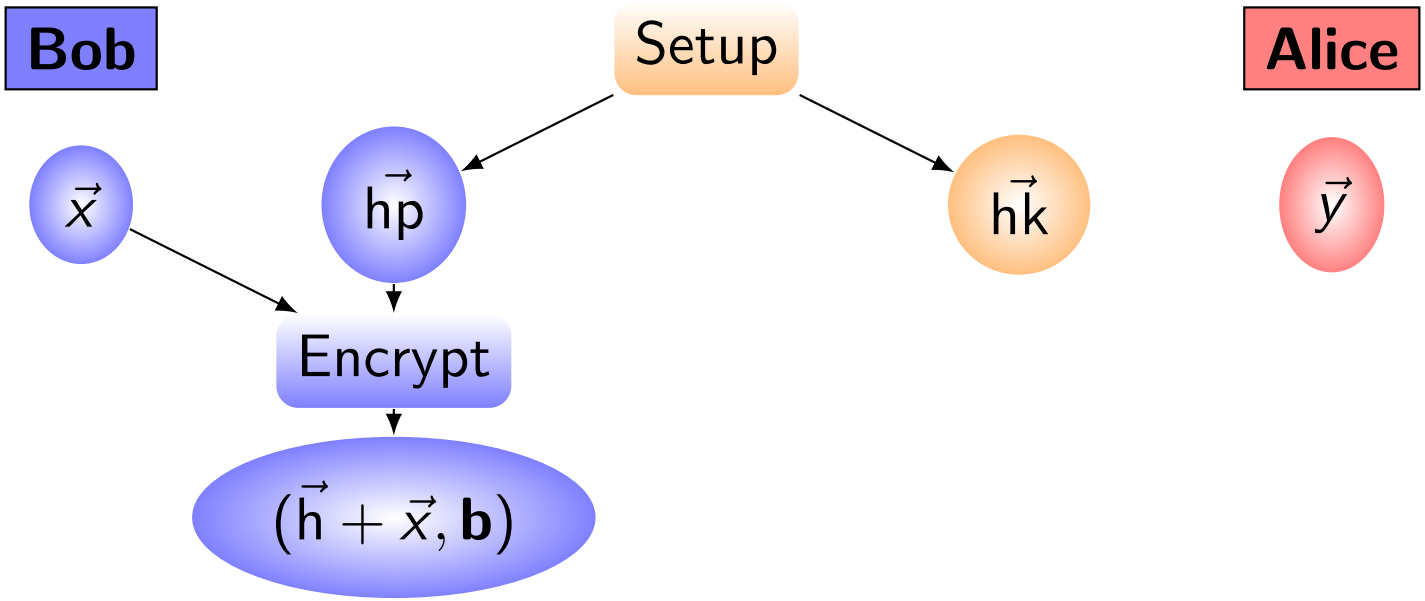

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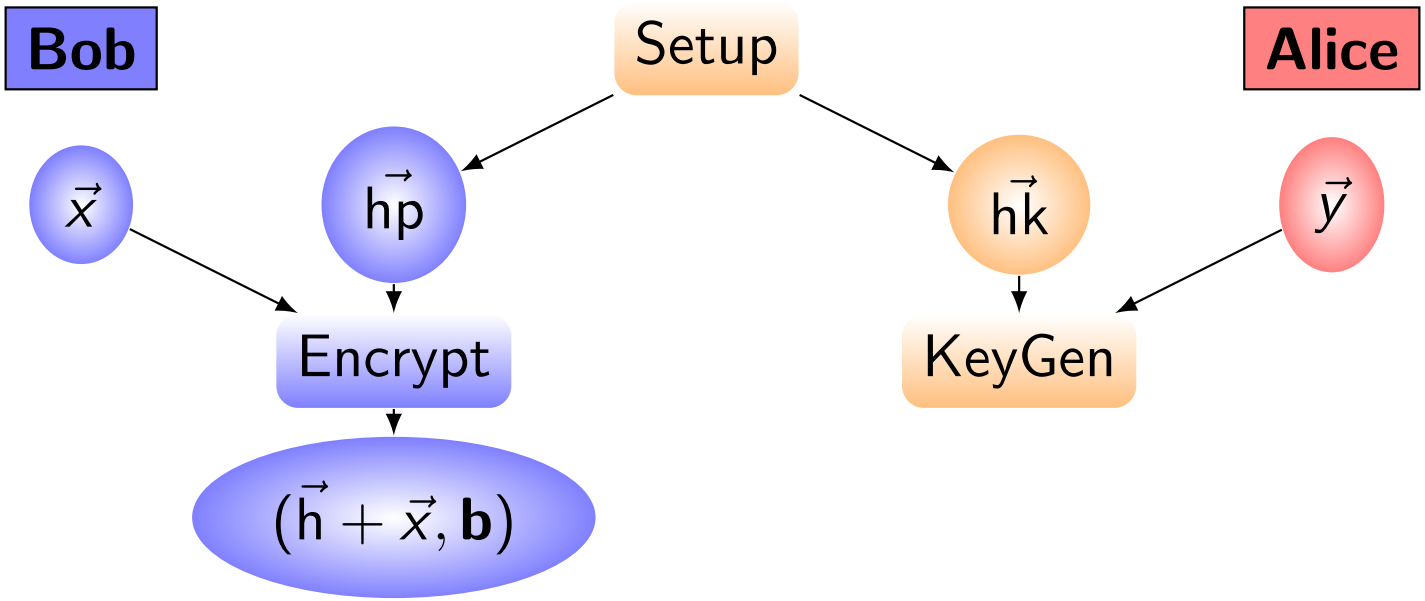

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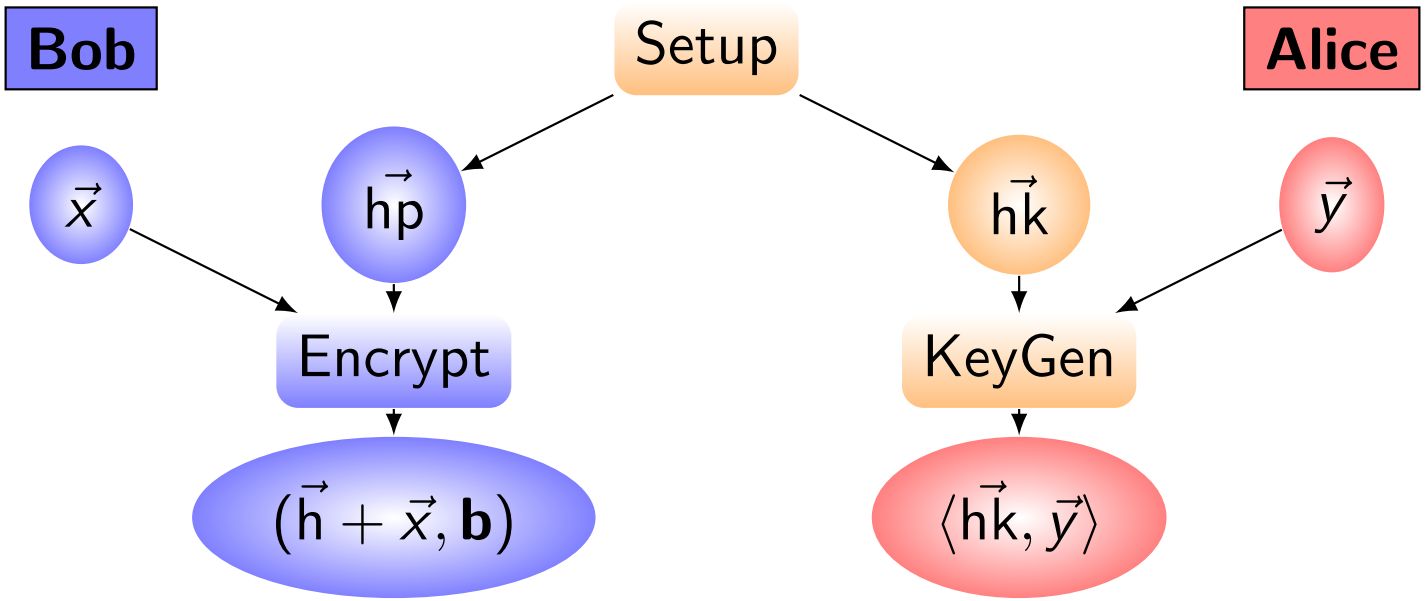

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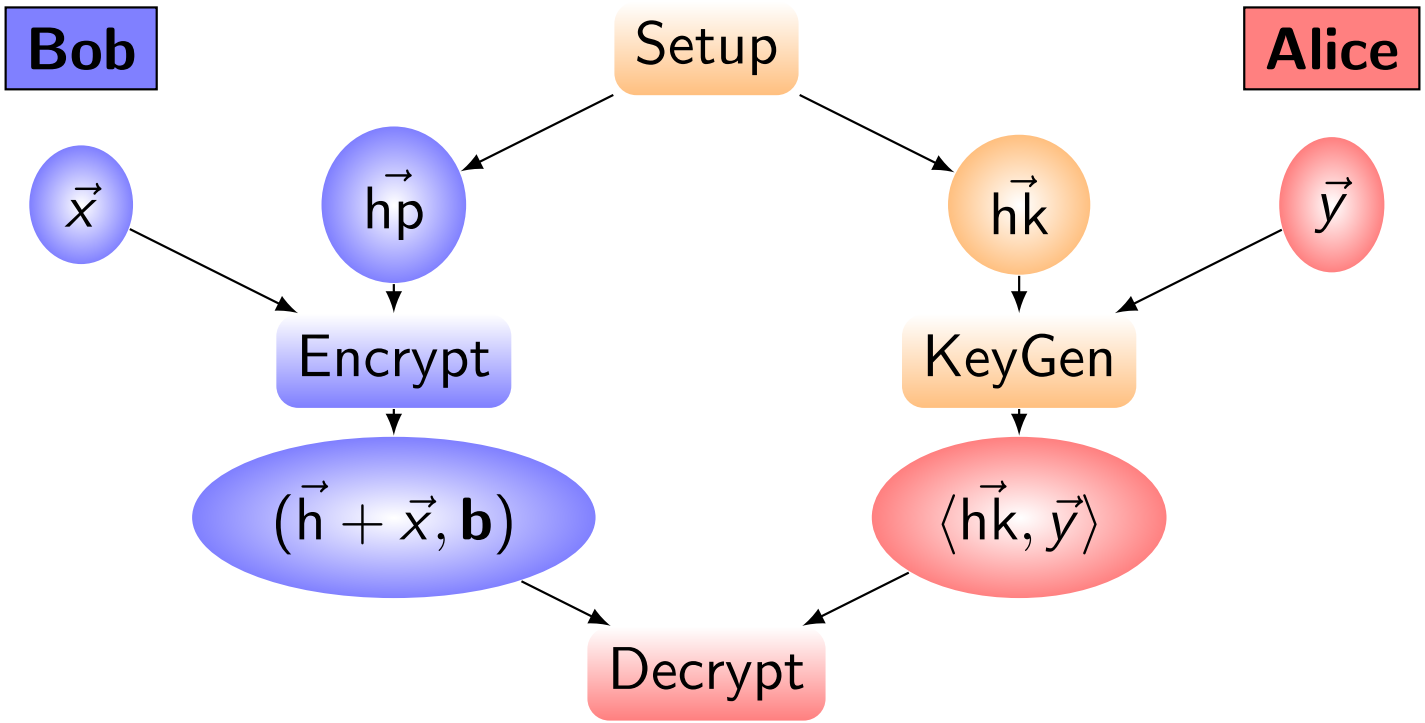

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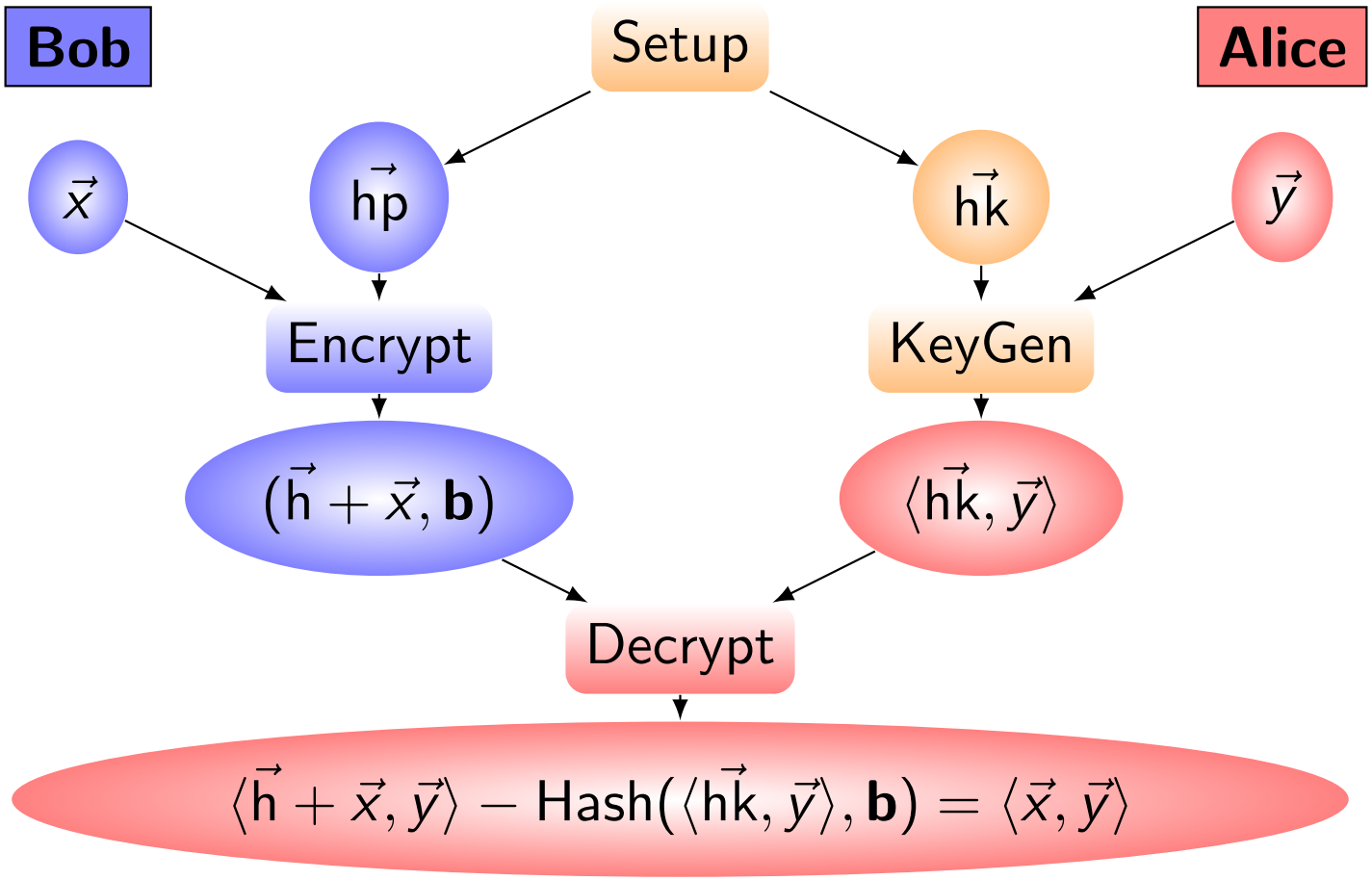
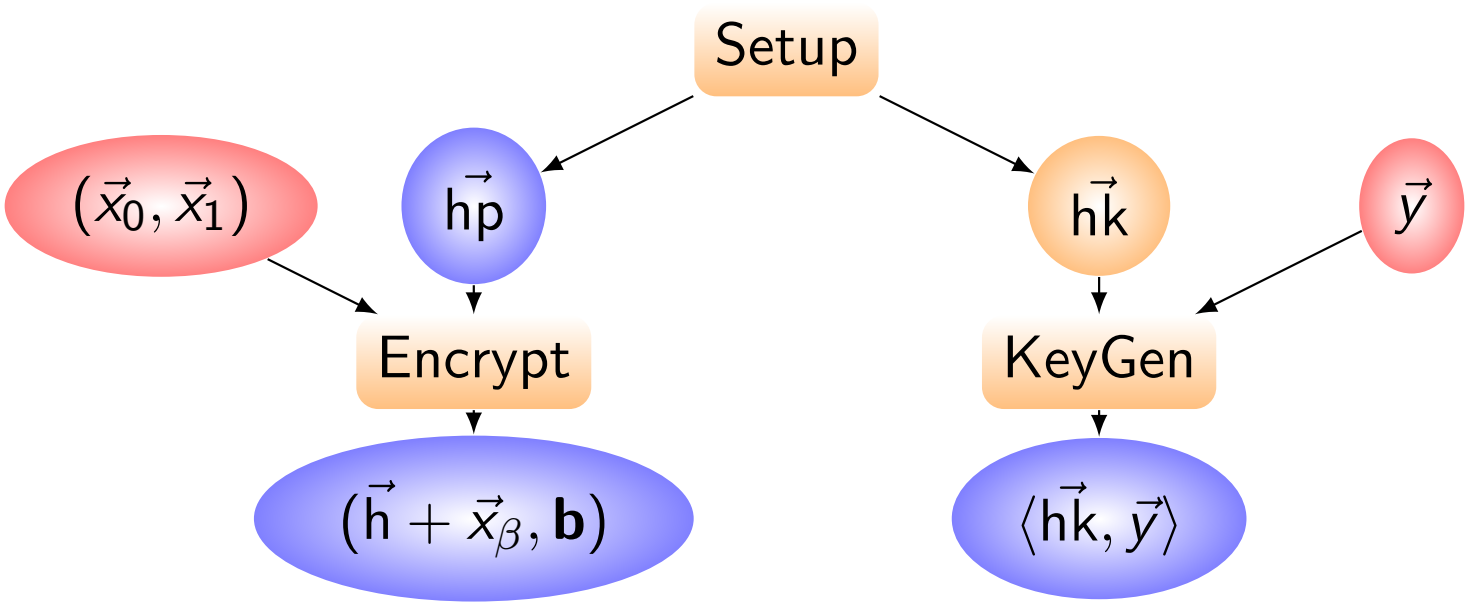


CPA-Security[ALS16]



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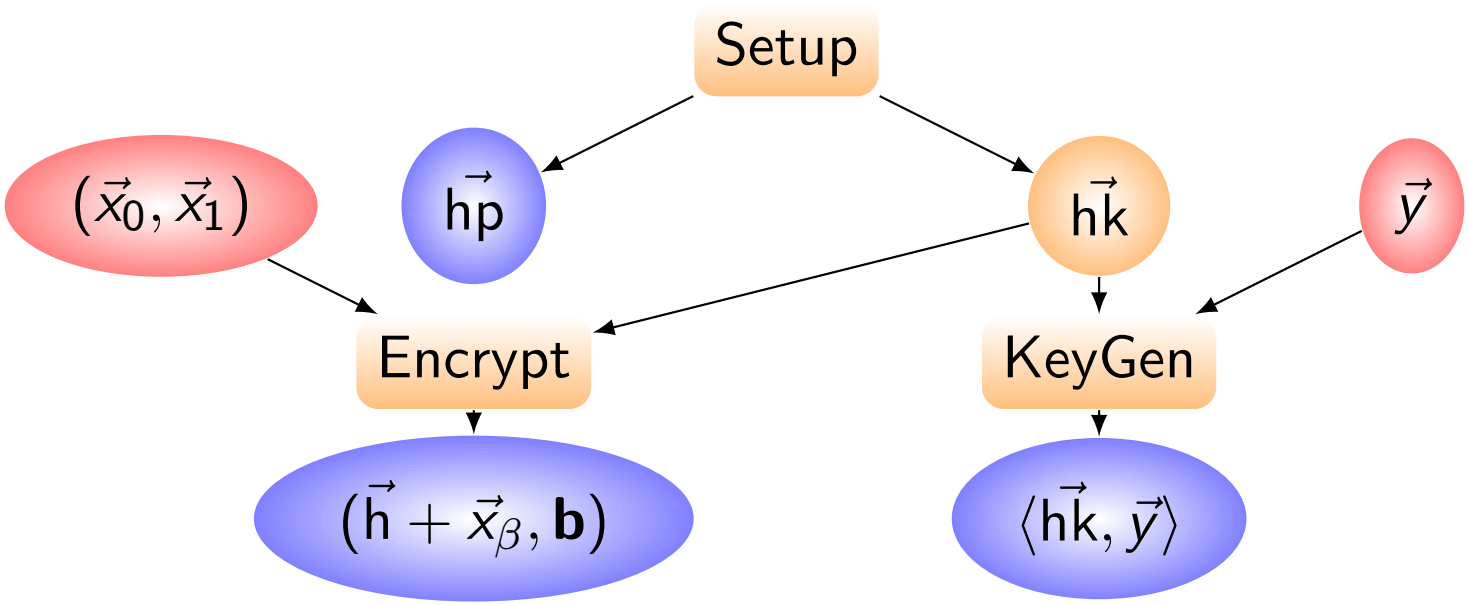

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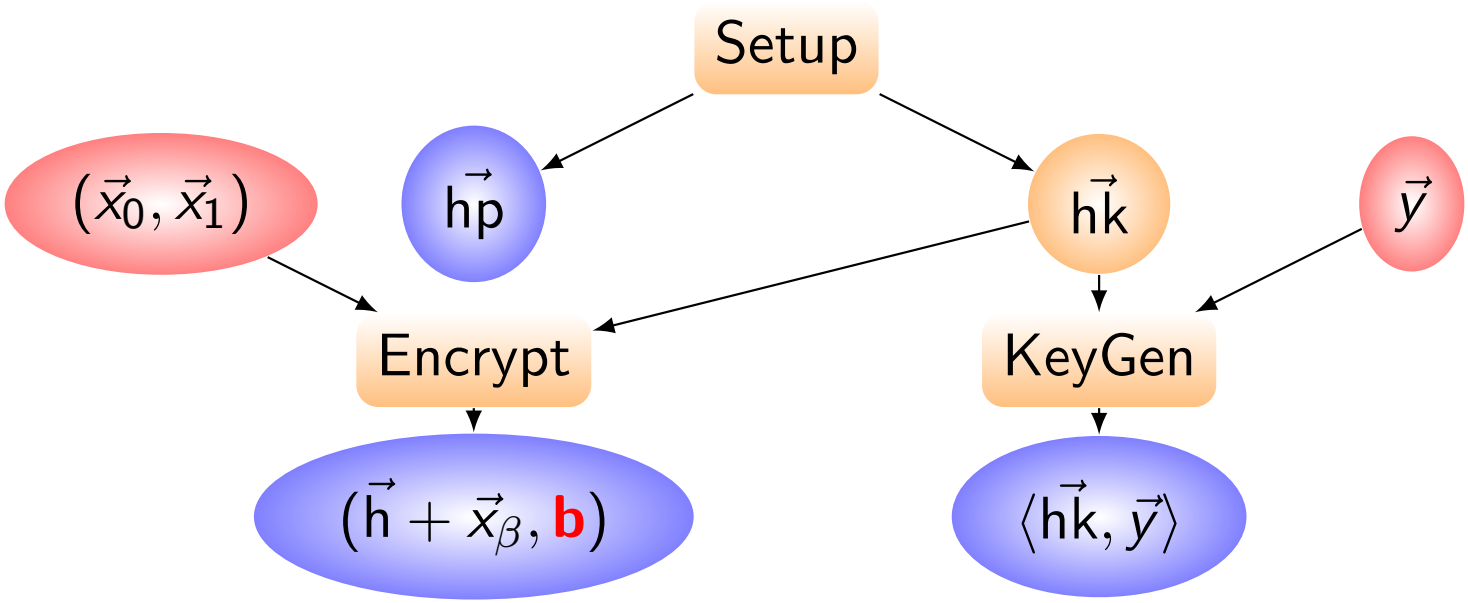

CCA-Security: Issues to fix

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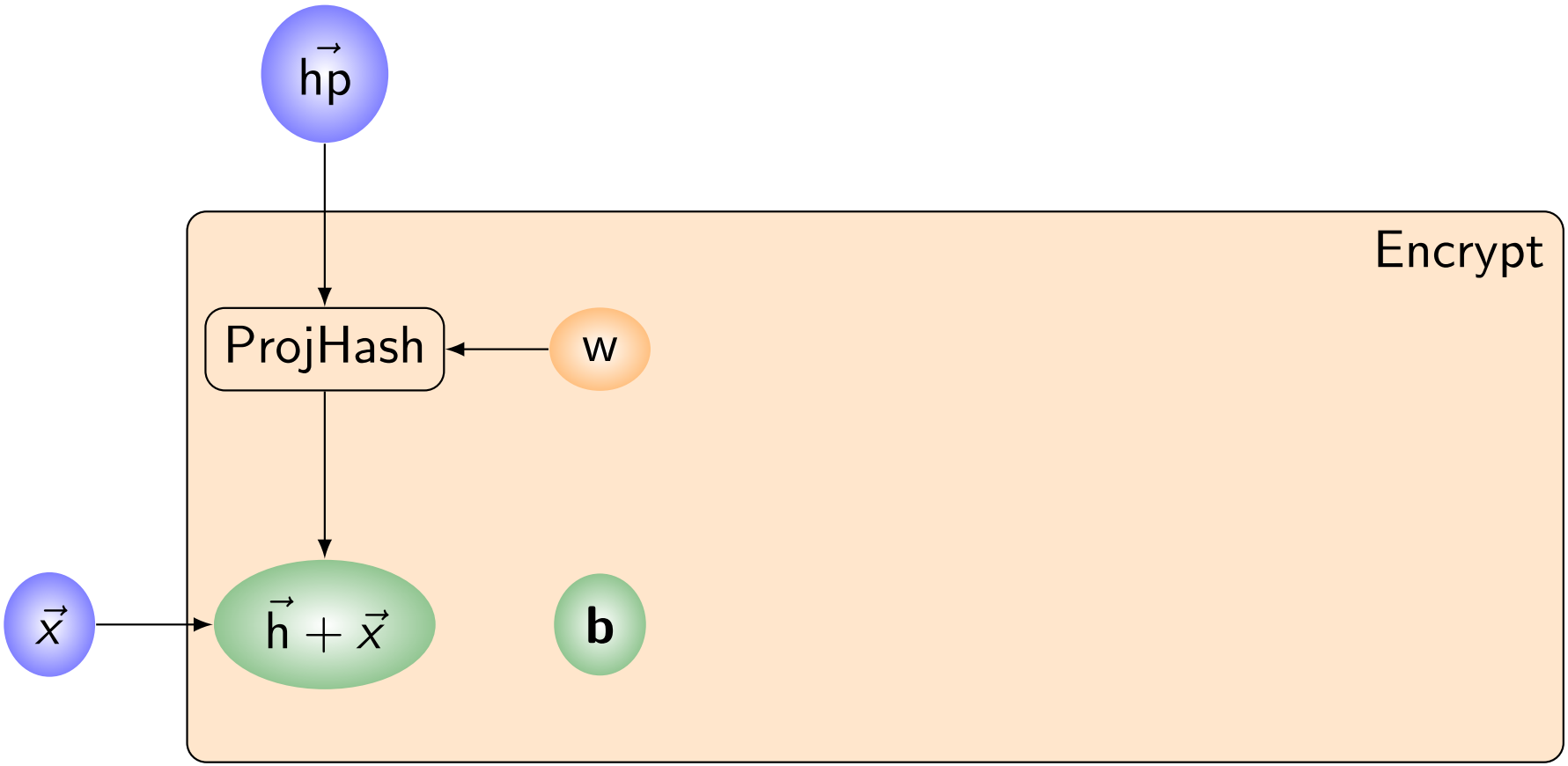
why: Same tag $\Longrightarrow$ cannot sign
Signature $\Longrightarrow$ no parts from other ciphertexts

CCA-Security: Proving $\mathbf{b} \in \mathcal{L}$[CS98]



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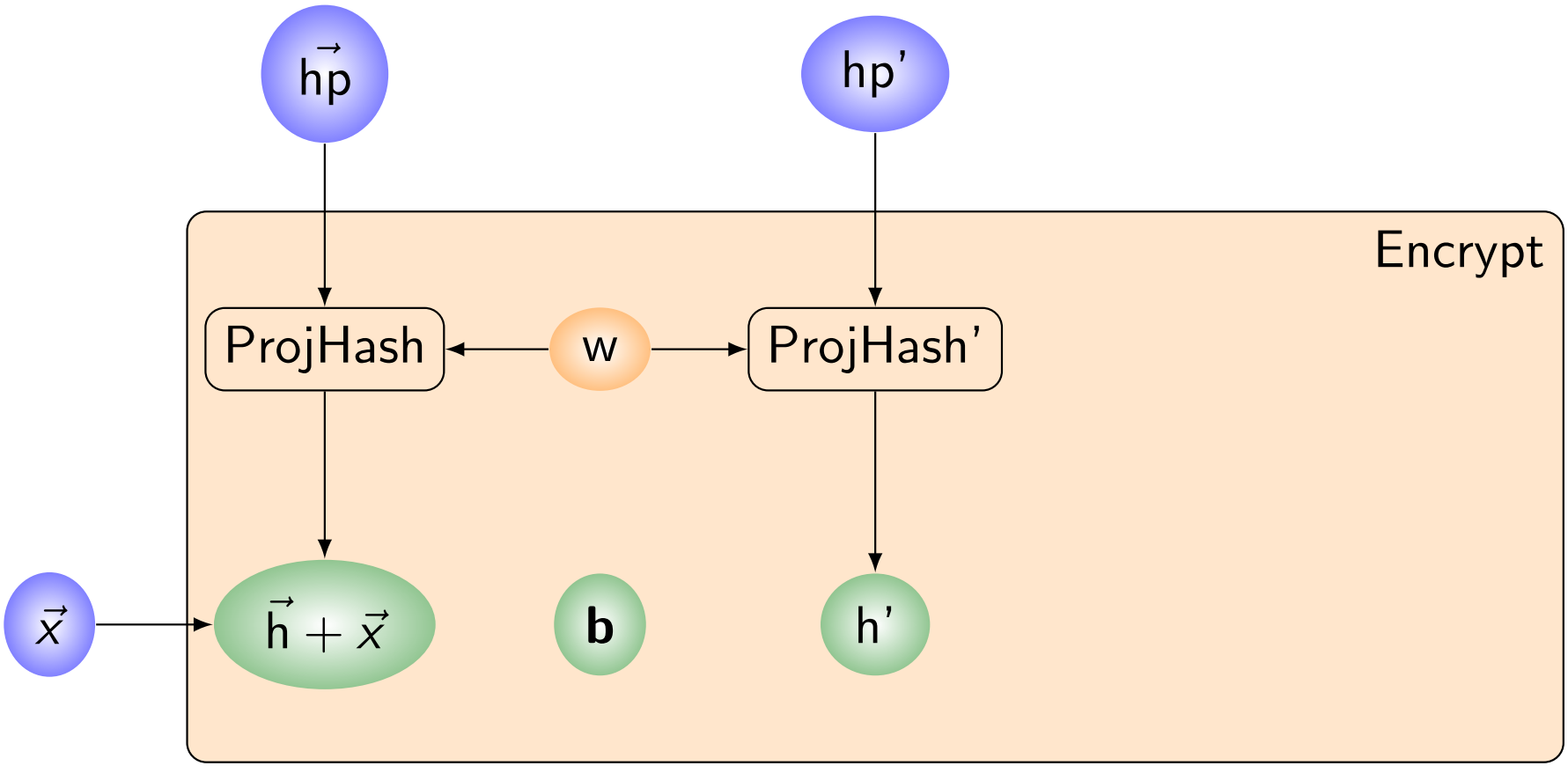
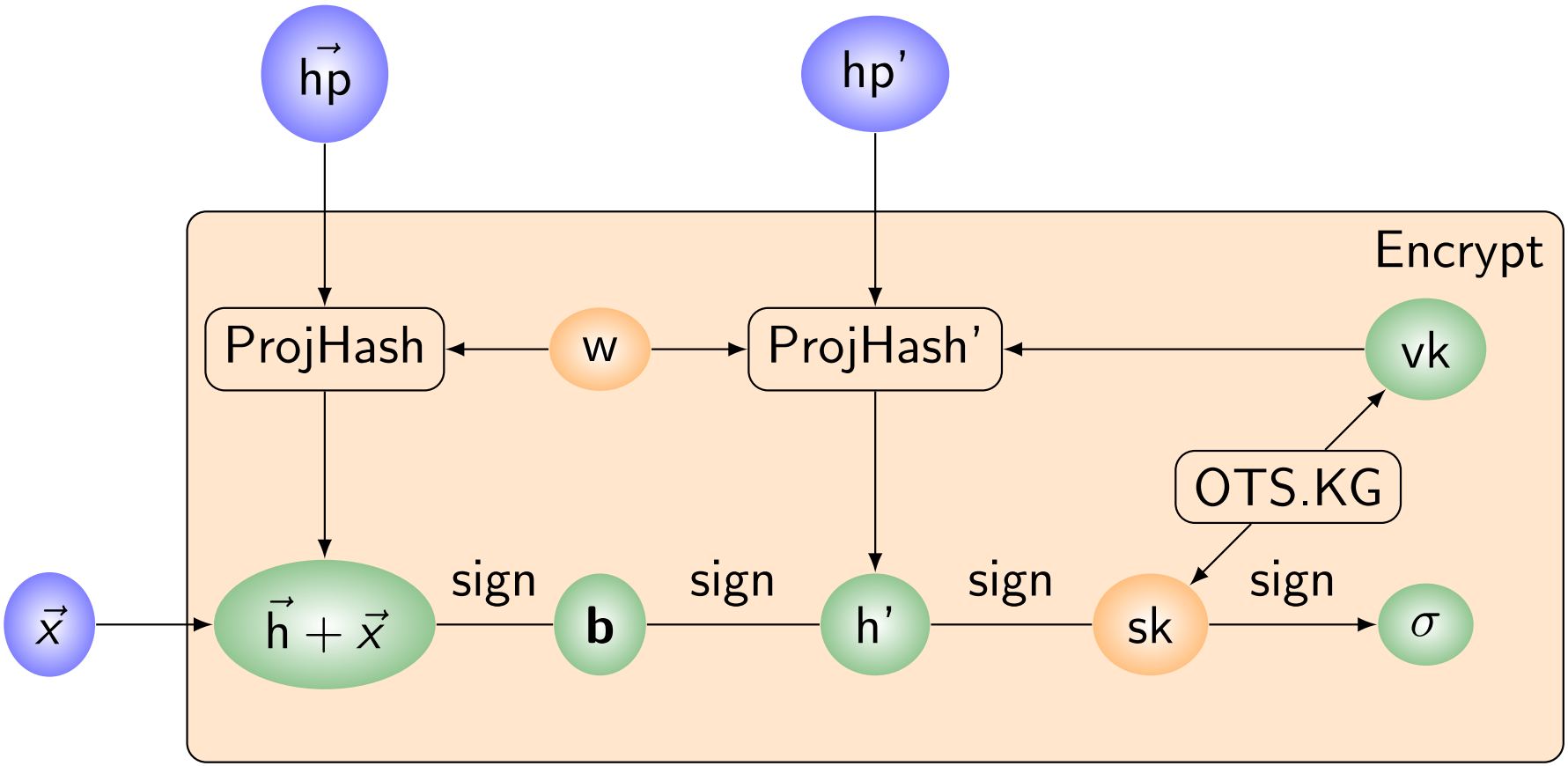


CCA-Security: Removing Malleability



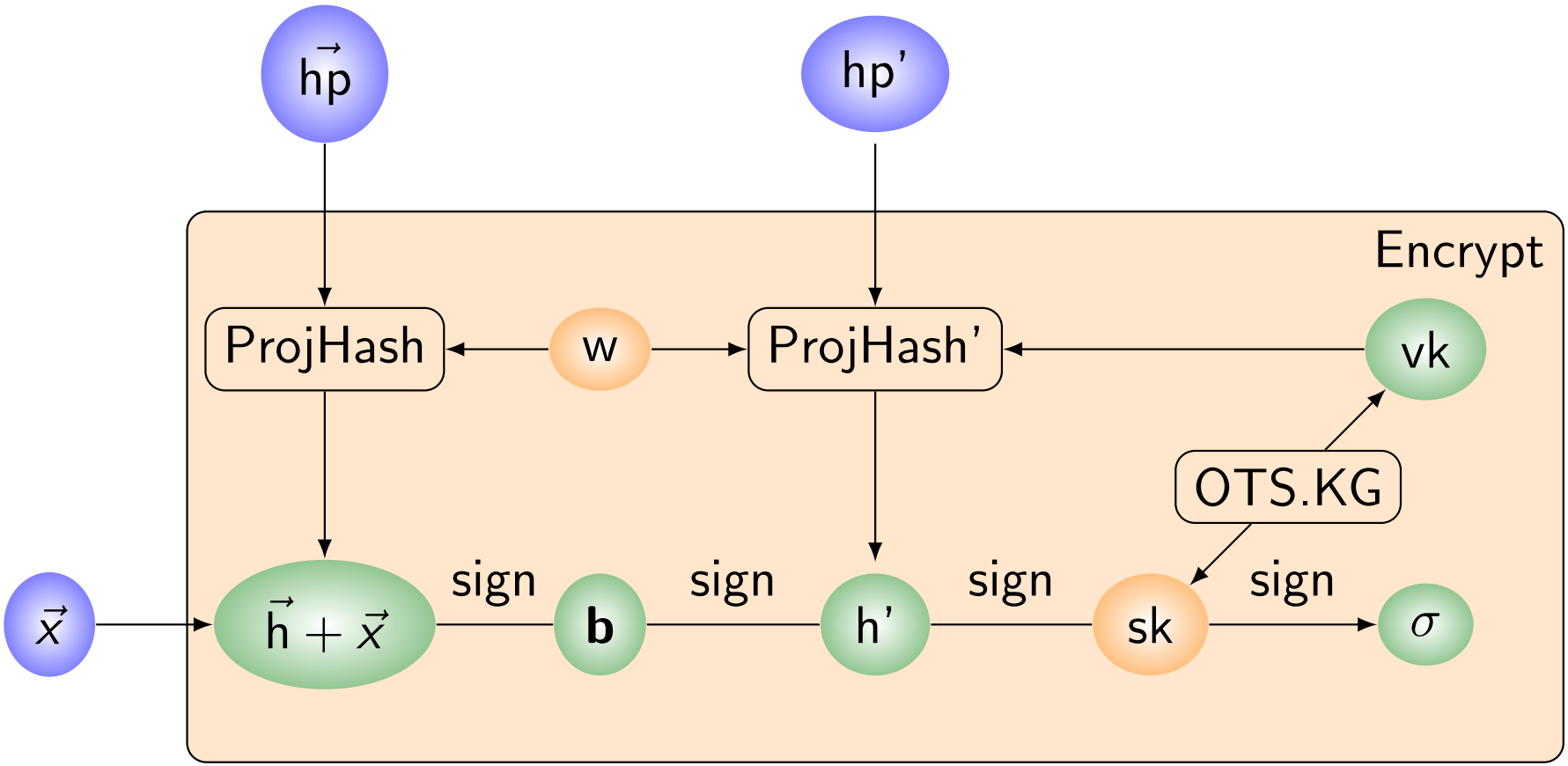
Nope!

This is not enough!

Nope!

This is not enough!
He who can check the proof, can fake it!

CCA-Security



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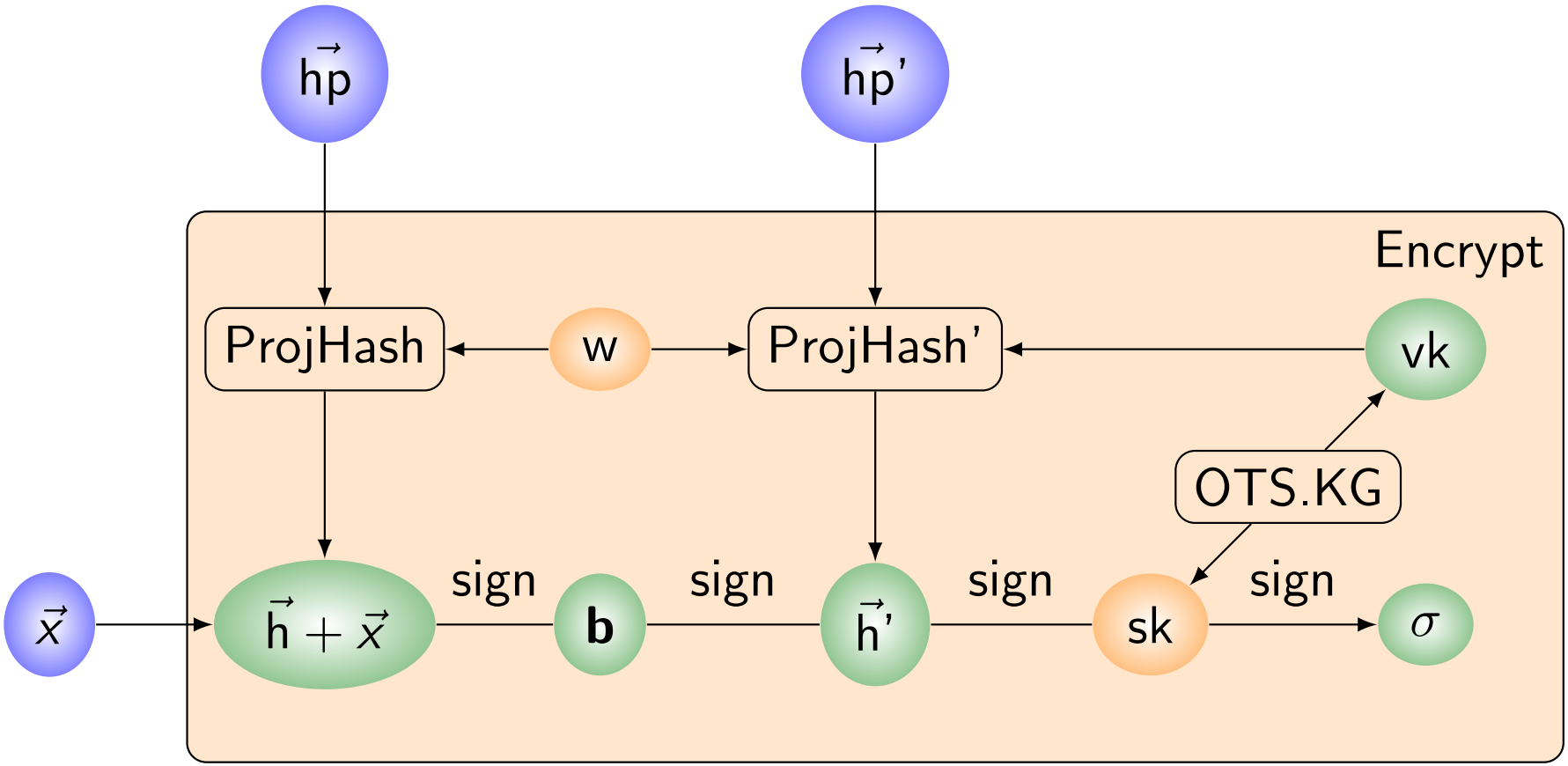

Why?

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Both assumptions can be broken by quantum computers.

Learning With Errors

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$\mathbf{a} \leftarrow \mathbb{Z}_q^n$;

$e \in \mathbb{Z}$ small.

Regev's Encryption Scheme

Bob **Alice**

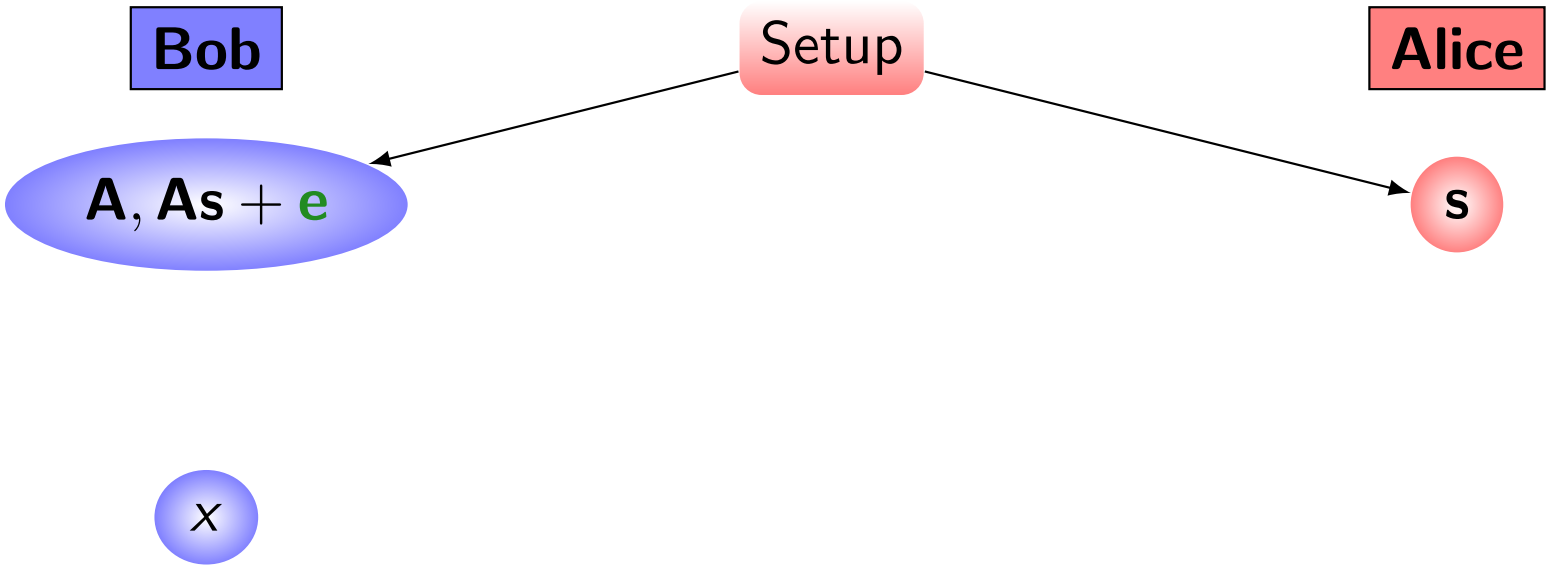
x

Regev's Encryption Scheme

Bob Setup **Alice**

x

Regev's Encryption Scheme



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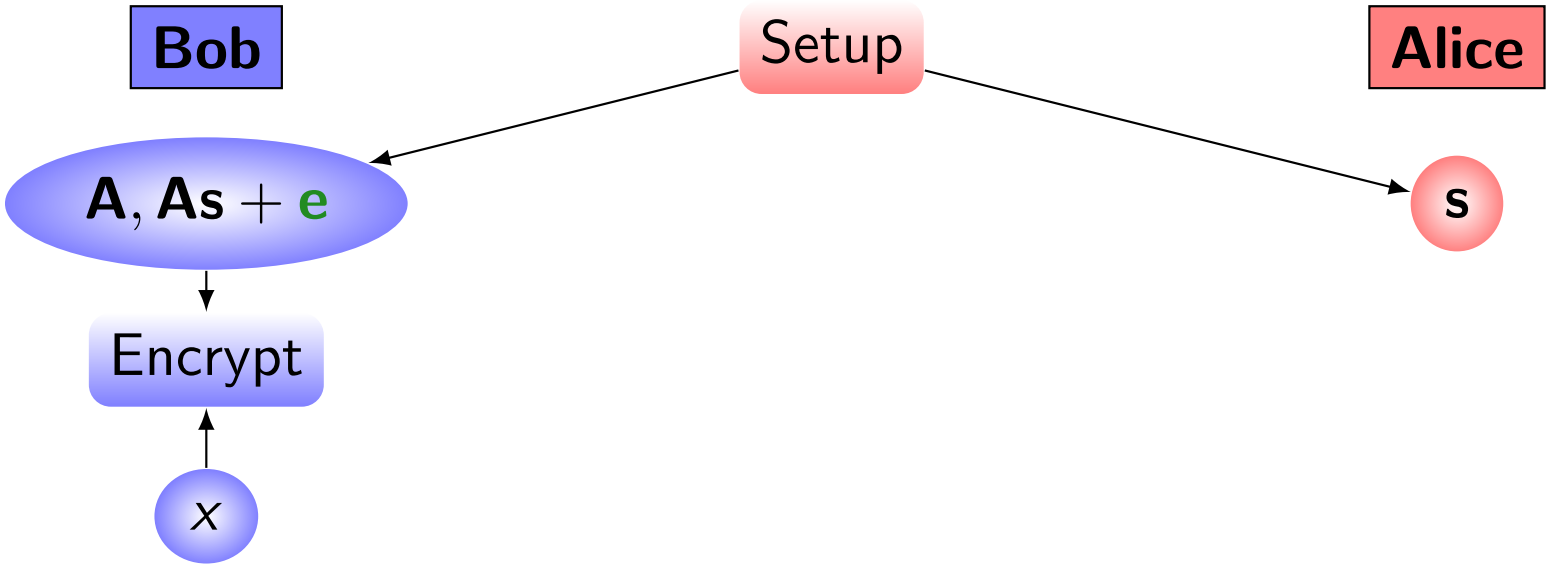

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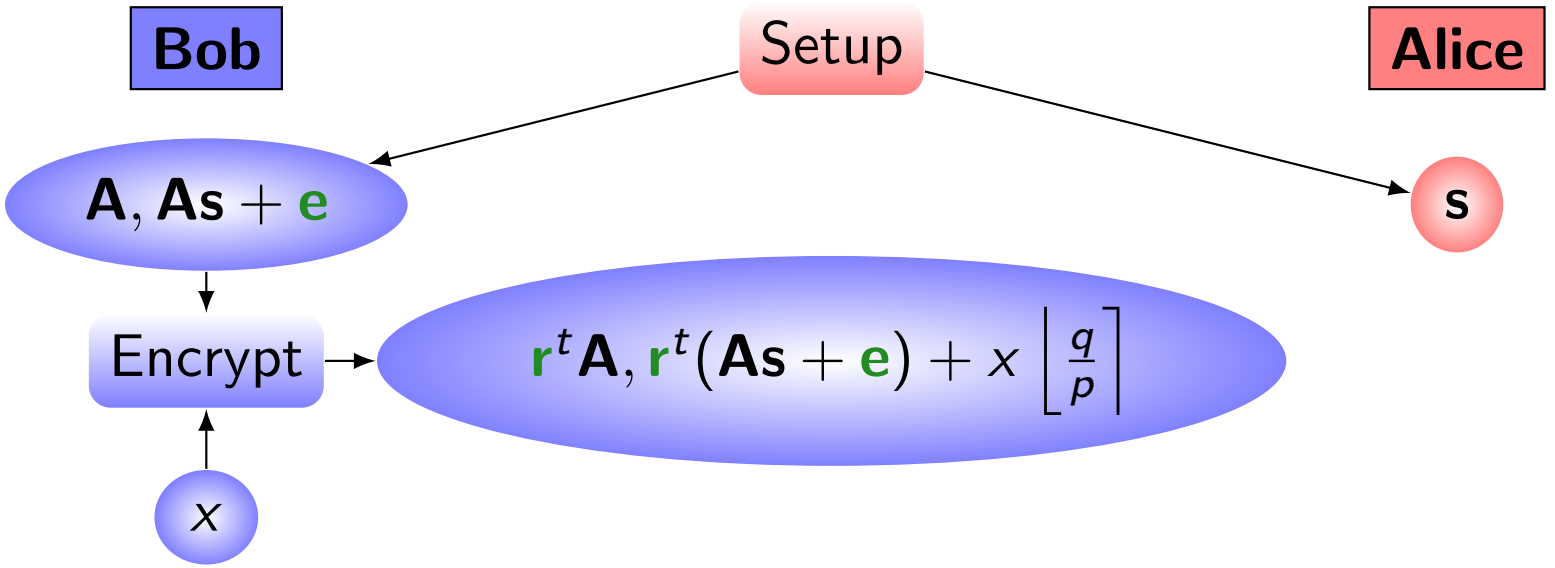

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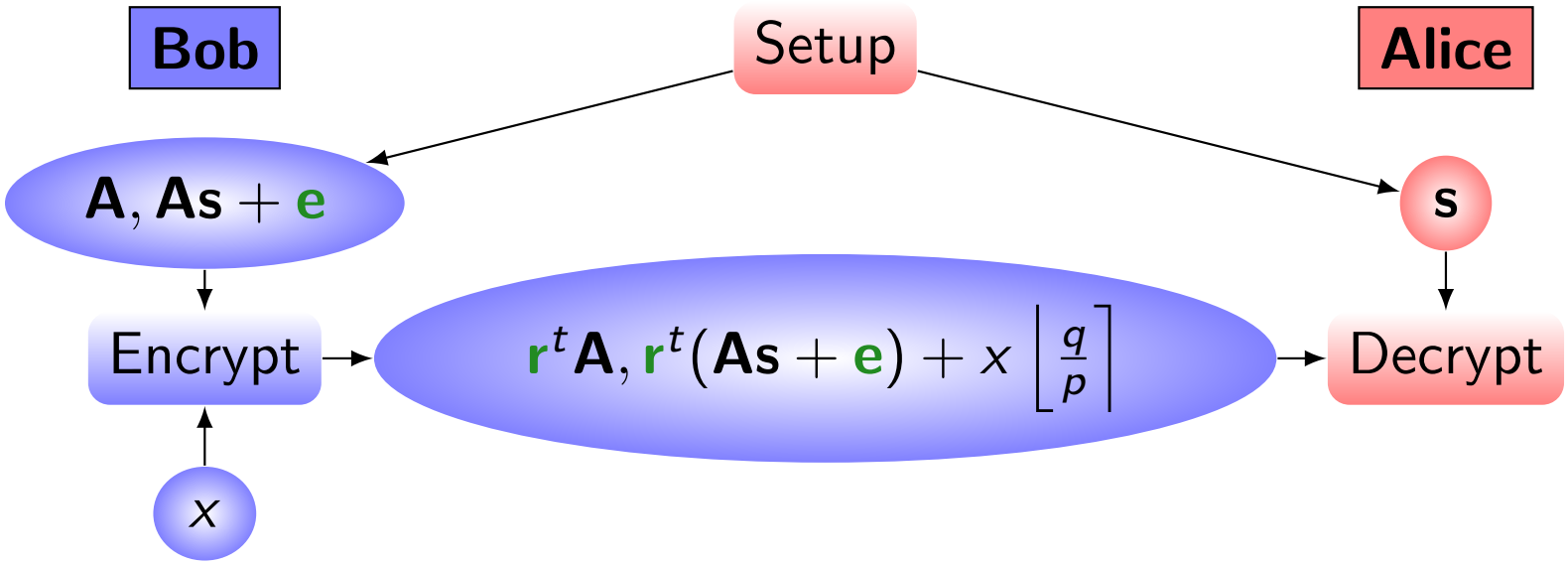

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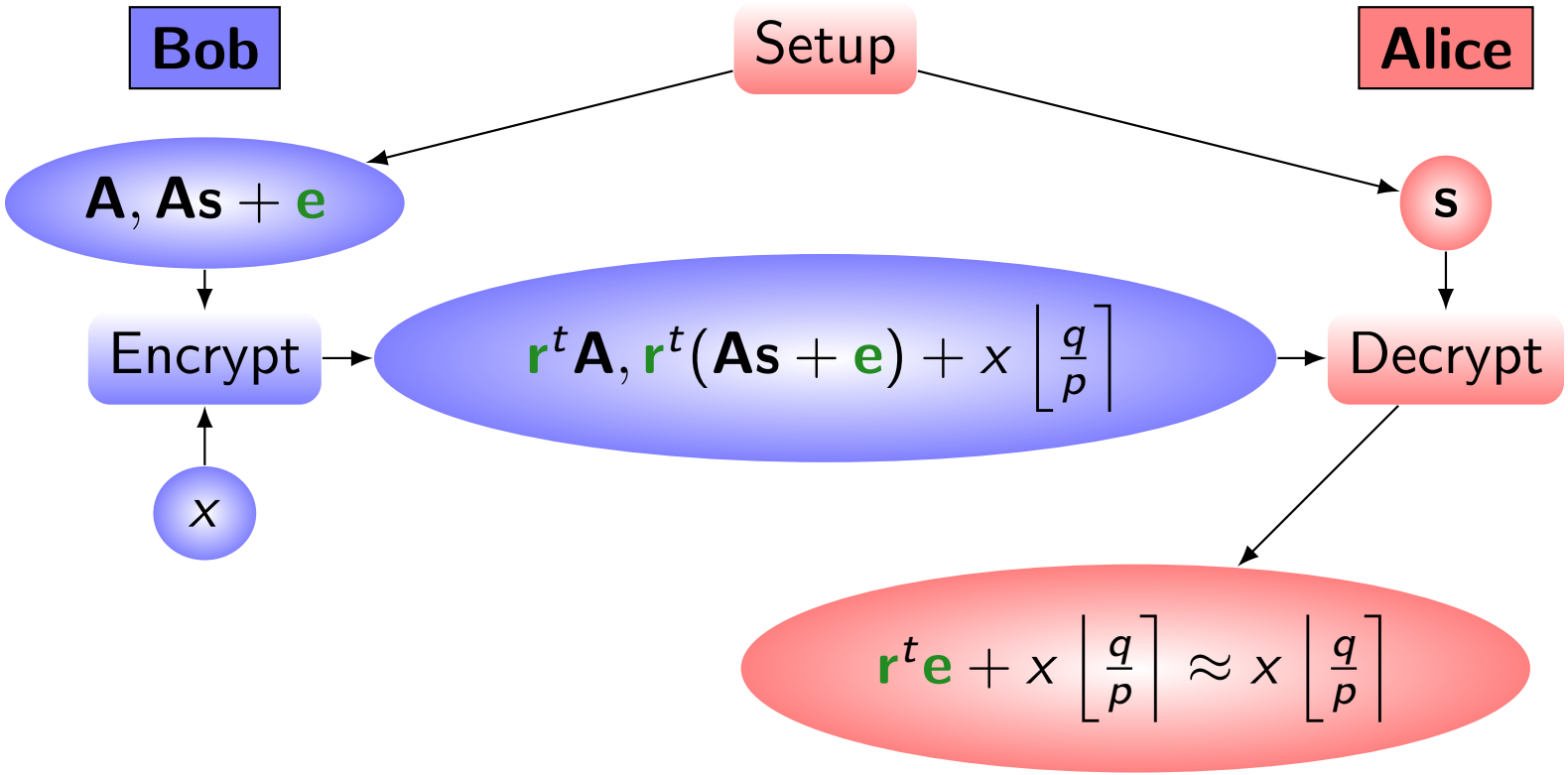

Selectively Secure Scheme

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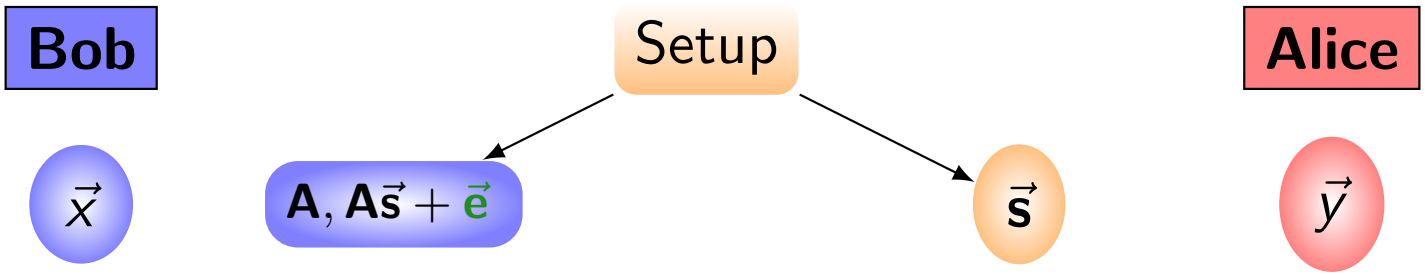

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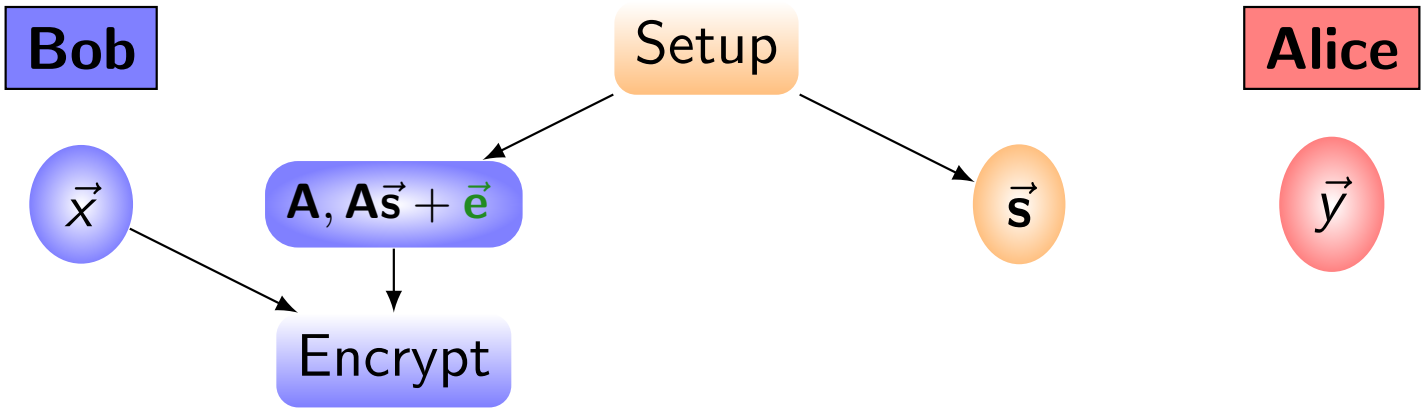

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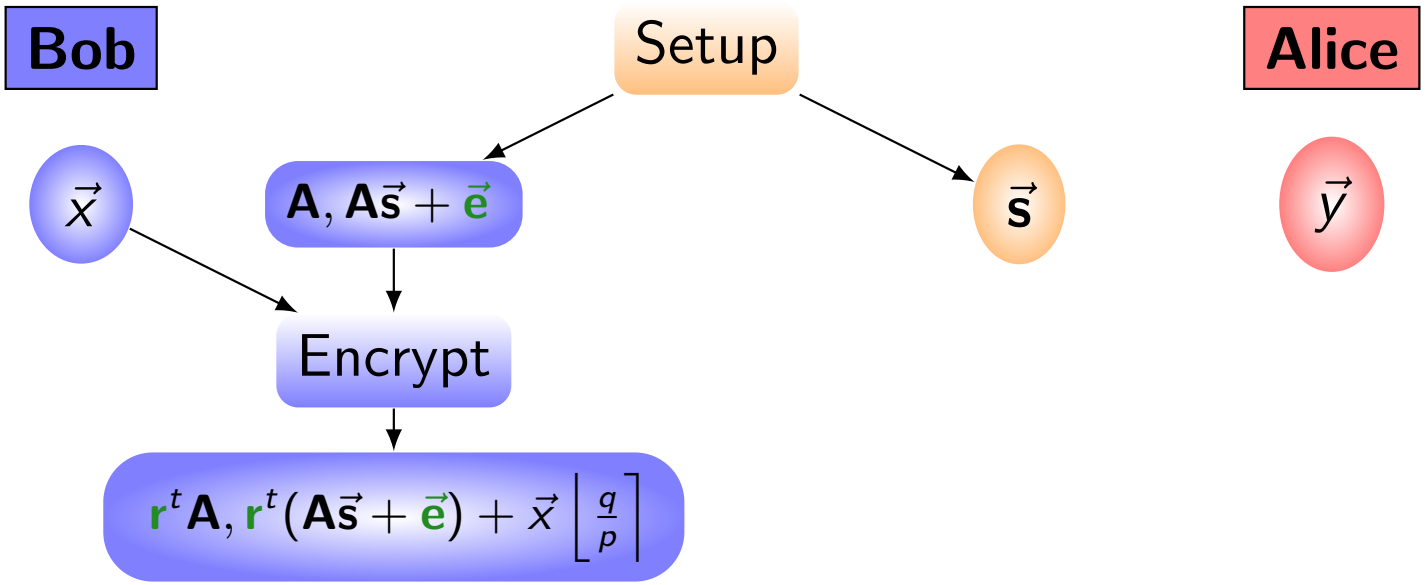
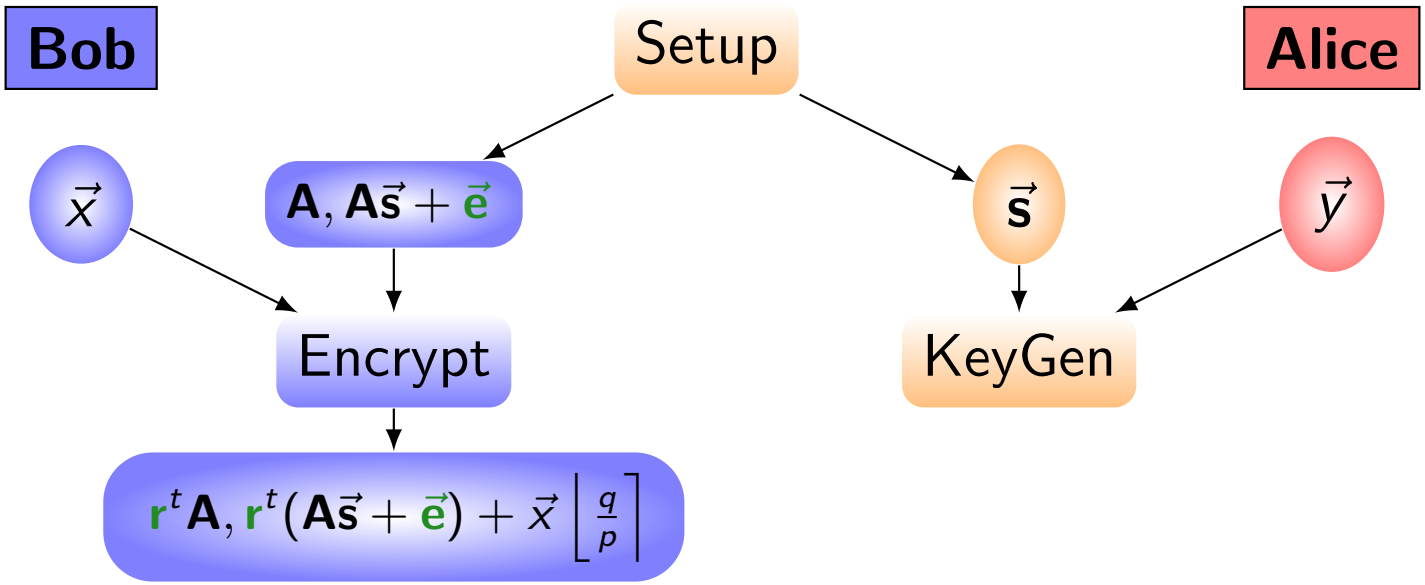
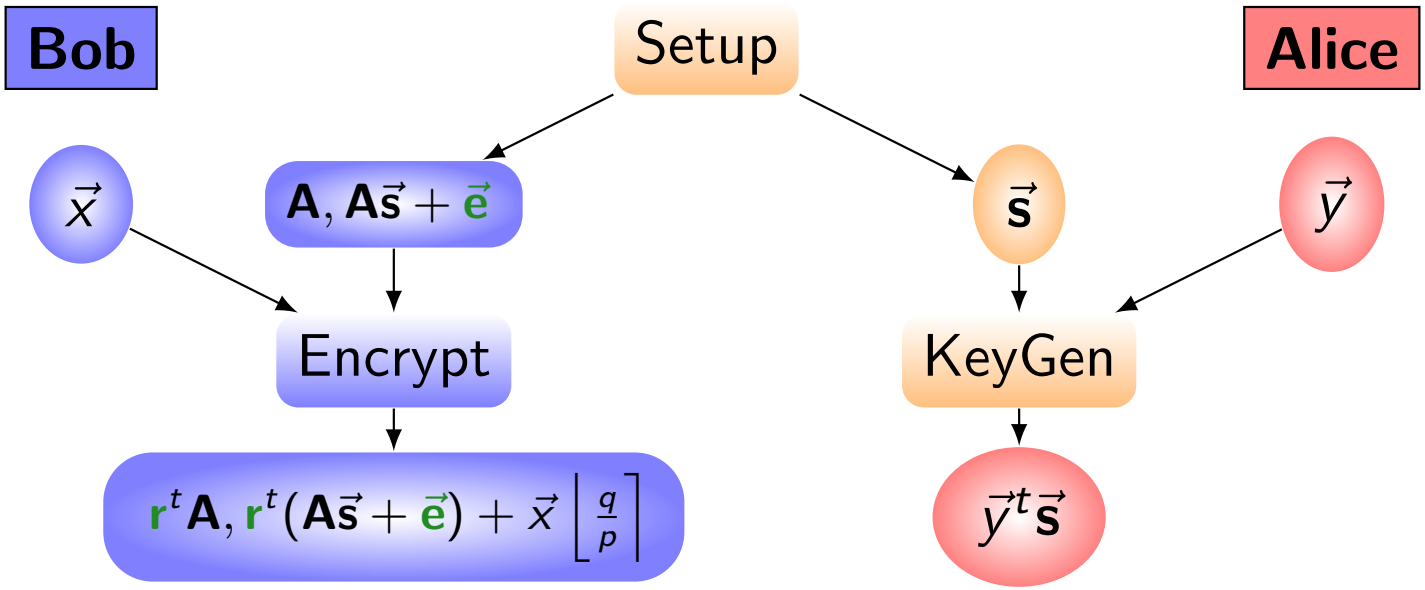


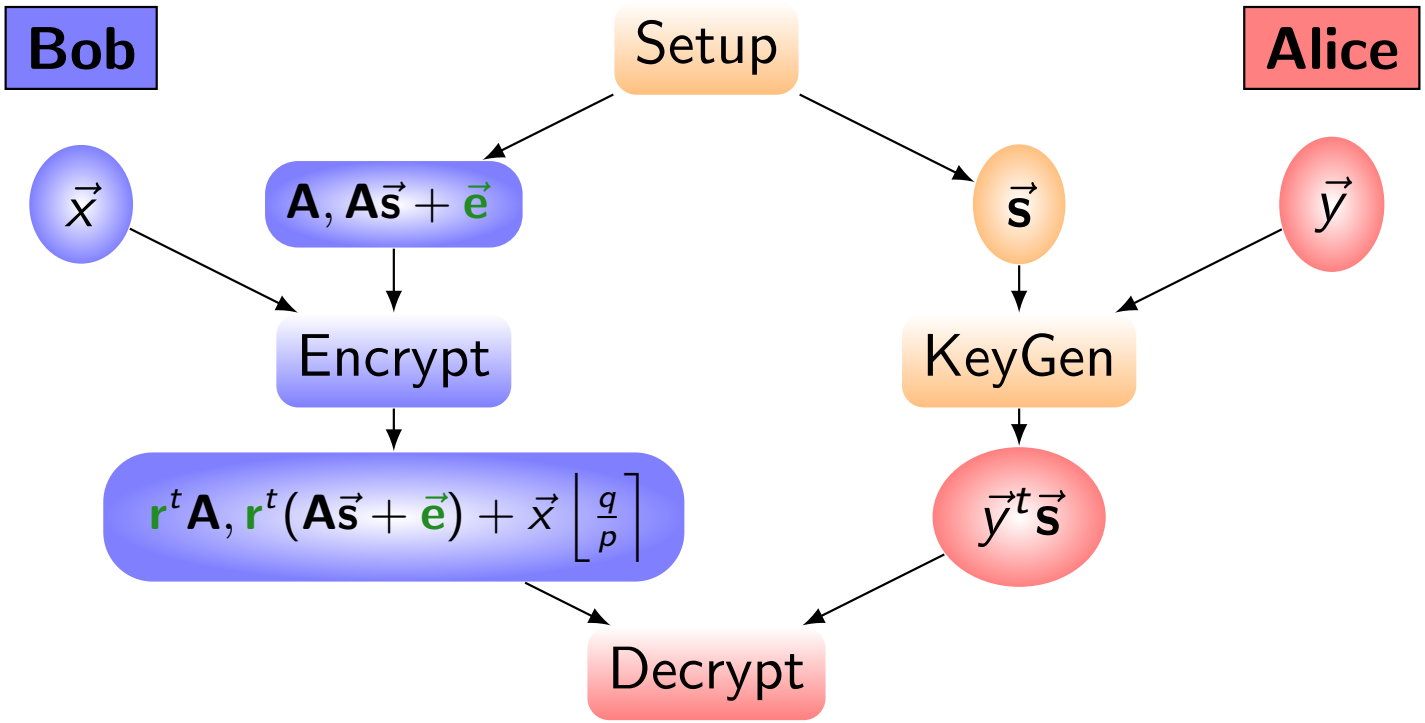
Selectively Secure Scheme



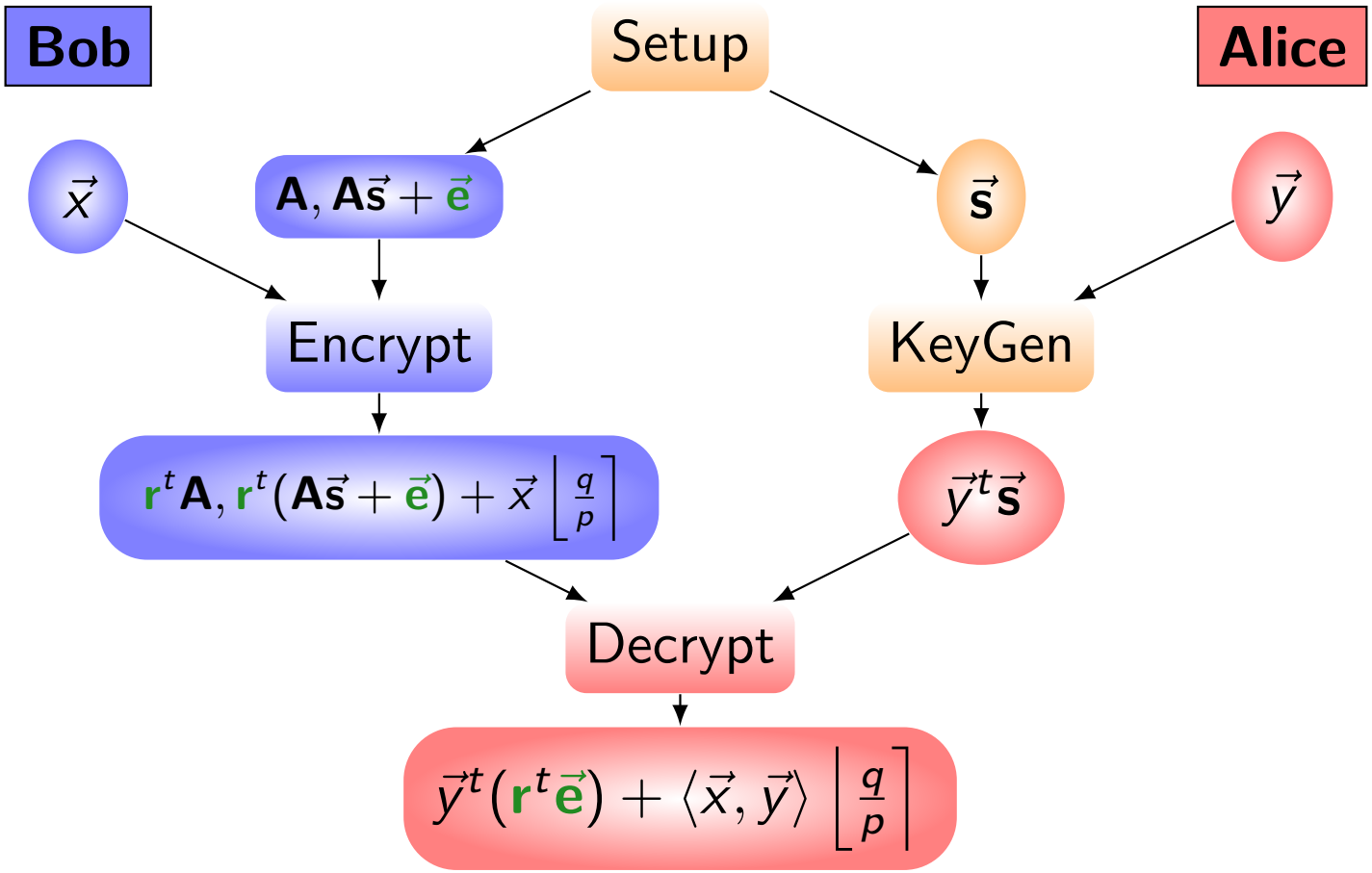
Selectively Secure Scheme



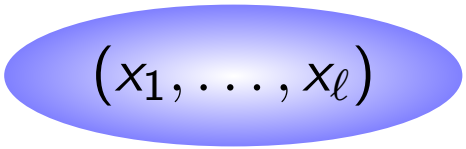
Selectively Secure Scheme

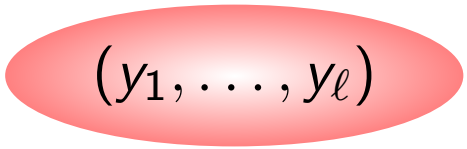


Selectively Secure Scheme



Adaptive Security





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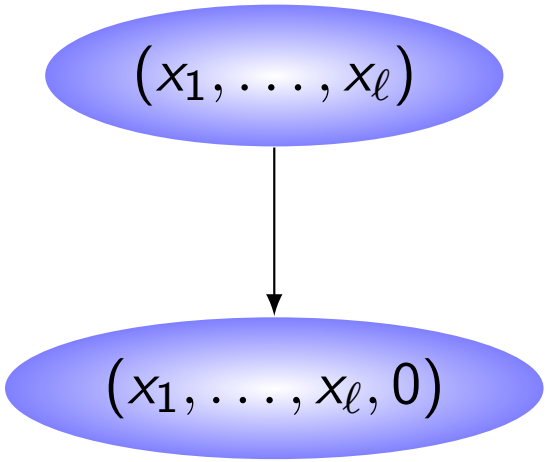

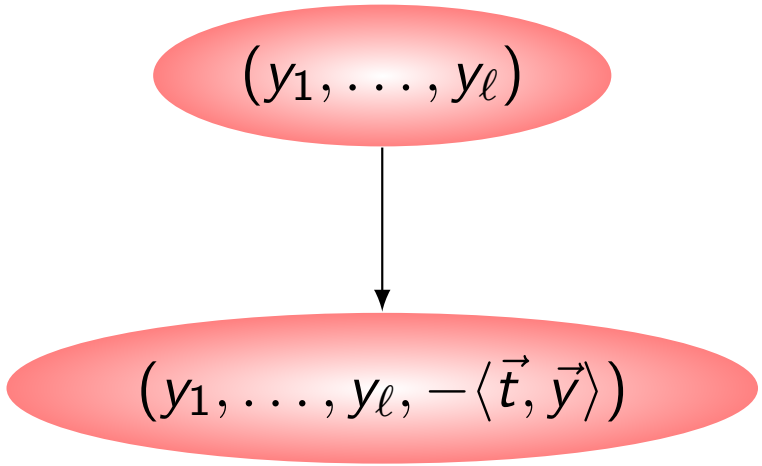

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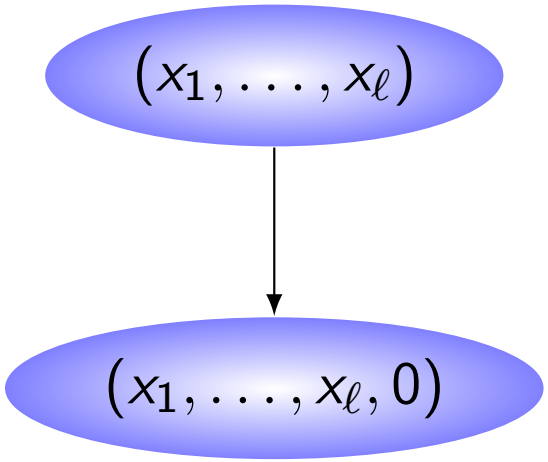
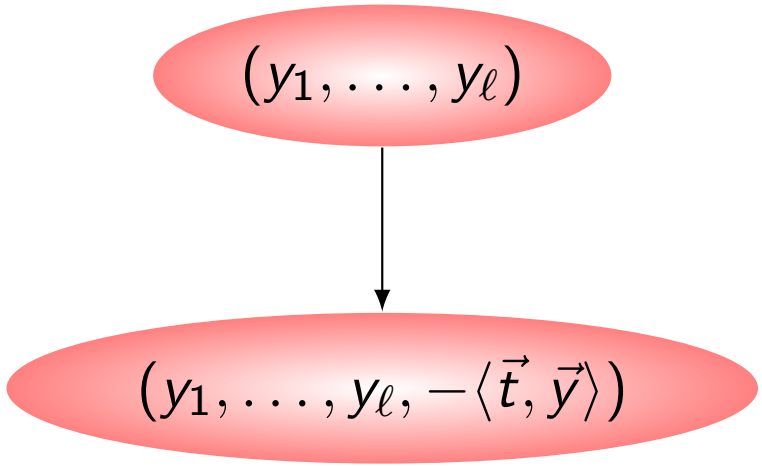




$$\mathsf{Enc}(\vec{0}, 0) \approx_c \mathsf{Enc}(\vec{t}, 1)$$

The Proof

$(\vec{x}_0, 0)$

$(\vec{x}_1, 0)$

The Proof

$(\vec{x}_0, 0)$

$\approx_c$

$(\vec{x}_0 + \vec{t}, 1)$

$(\vec{x}_1, 0)$

The Proof

$(\vec{x}_0, 0)$ $\approx_c$ $(\vec{x}_1, 0)$

$(\vec{x}_0 + \vec{t}, 1)$ $\approx_c$ $(\vec{x}_1 + \vec{t}, 1)$

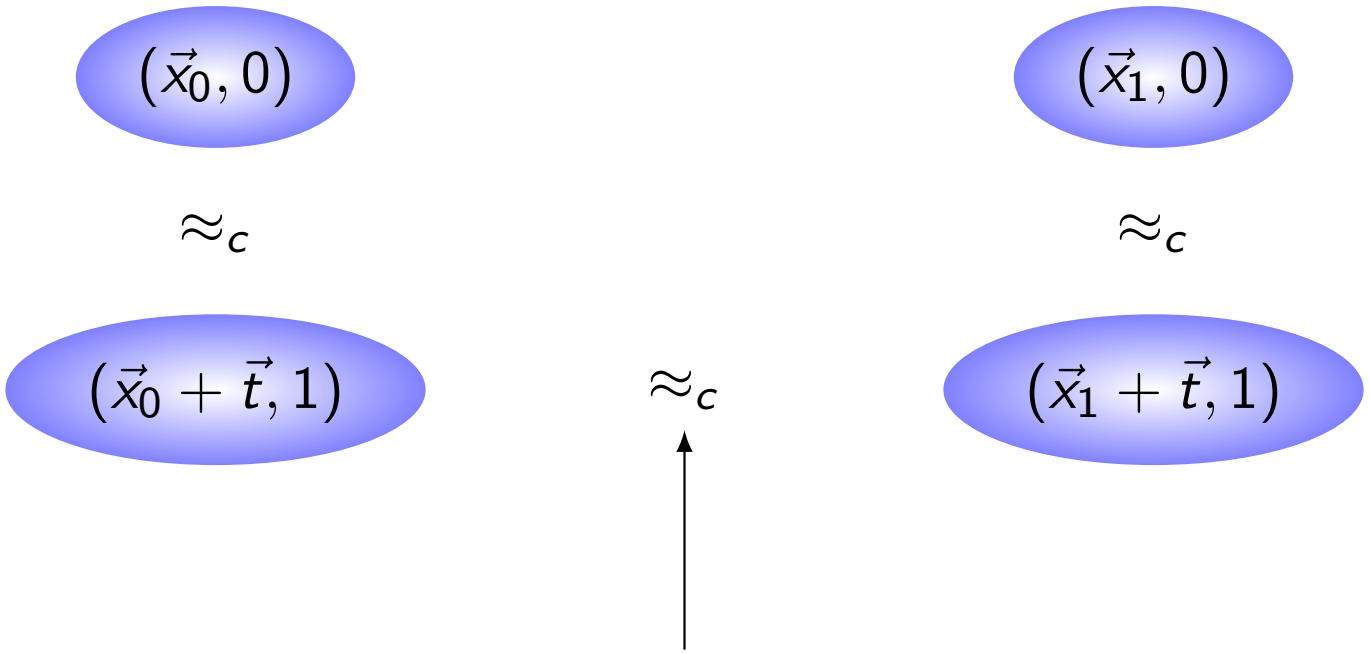
The Proof

$(\vec{x}_0, 0)$ $\approx_c$ $(\vec{x}_1, 0)$

$(\vec{x}_0 + \vec{t}, 1)$ $\approx_c$ $(\vec{x}_1 + \vec{t}, 1)$

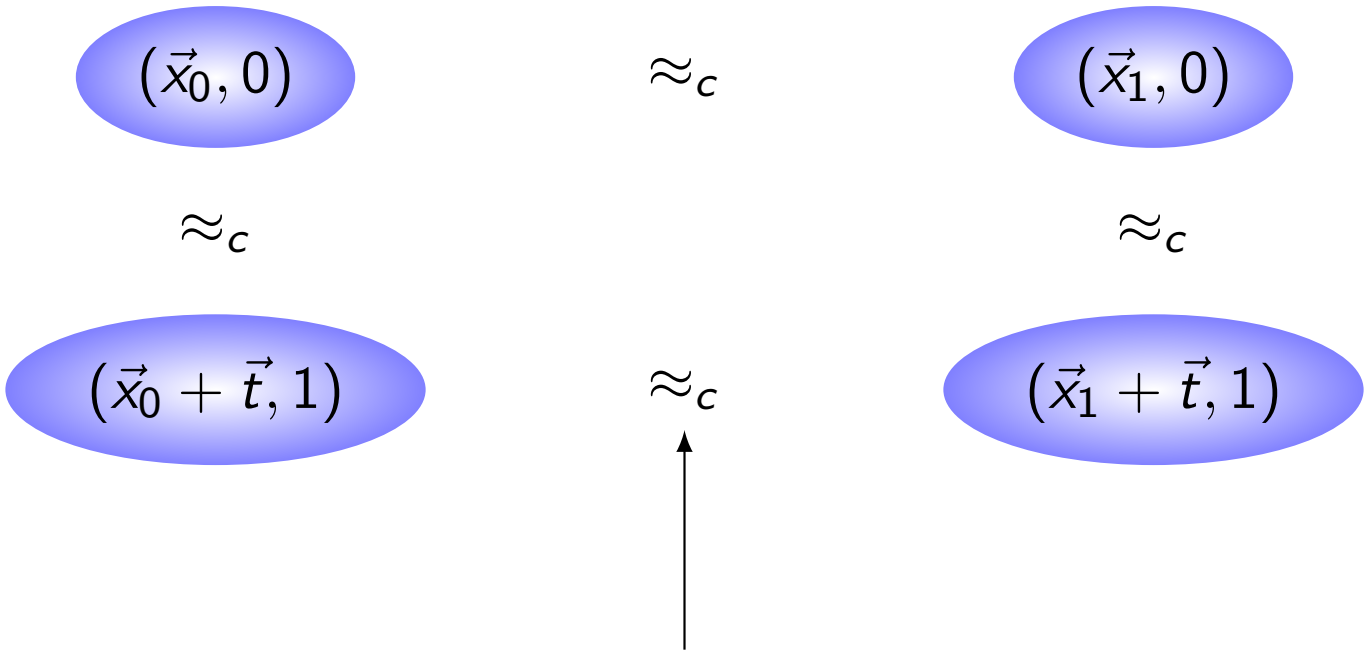
flooding, requires $\vec{t}$ bigger than $\vec{x}$ by a super-polynomial factor

The Proof



flooding, requires $\vec{t}$ bigger than $\vec{x}$ by a super-polynomial factor

The Proof



flooding, requires $\vec{t}$ bigger than $\vec{x}$ by a super-polynomial factor

Table of Comparison

Assumption	CCA-Security	Efficient Decryption	Modular	Post-Quantum
DDH	✓		✓	
DCR	✓	✓	✓ *	
LWE		✓	✓ *	✓

*Stateful KeyGen [ALS16]