

Sparsification for Graph Matching and Matroid Intersection

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Sparsification refers to the process of reducing the size of the input instance while keeping the quality (as much as possible) of the original optimal solution. Such technique is very useful in the era of “big data”. In fact, for many sub-linear-time computational models, such as streaming, parallel computing, distributed computing, communication complexity, sparsification is an extremely powerful tool for designing algorithms.

To see a concrete example, think about finding a maximum matching in a graph. A graph has $O(m)$ edges. Is it possible that we keep only $o(m)$ edges while guaranteeing that in the remaining subgraph, there is still a matching whose size is at least α -approximation of the original graph?

Such a thing is known in the literature.

Definition 1 (from [1]). Let $G = (V, E)$ be a graph, and H a subgraph of G . Given any integer parameters $\beta \geq 2$ and $\beta^- \leq \beta - 1$, we say that a subgraph $H = (V, E_H)$ is a (β, β^-) -EDCS of G if H satisfies the following properties (for $v \in V$, $\deg_H(v)$ denotes the degree of v in H):

- (i) For any edge $(u, v) \in H$, $\deg_H(u) + \deg_H(v) \leq \beta$;
- (ii) For any edge $(u, v) \in G \setminus H$, $\deg_H(u) + \deg_H(v) \geq \beta^-$.

It is known that one can always build such an EDCS of size $O(\frac{n}{\epsilon^2})$ and it always contains a $(2/3 - \epsilon)$ -approximation matching.

Could we do something similar beyond graph matching? It is known that the problem of *matroid intersection* is a generalization of graph matching in a bipartite graph. A recent paper [2] shows that a similar sparsifier is possible. This project is focused on finding further applications of such sparsifiers (for matroid intersection).

References

1. Sepehr Assadi and Aaron Bernstein. Towards a unified theory of sparsification for matching problems. In Jeremy T. Fineman and Michael Mitzenmacher, editors, *2nd Symposium on Simplicity in Algorithms, SOSA 2019, January 8-9, 2019, San Diego, CA, USA*, volume 69 of *OASICS*, pages 11:1–11:20. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2019.
2. Chien-Chung Huang and François Sellier. Robust sparsification for matroid intersection with applications. In David P. Woodruff, editor, *Proceedings of the 2024 ACM-SIAM Symposium on Discrete Algorithms, SODA 2024, Alexandria, VA, USA, January 7-10, 2024*, pages 2916–2940. SIAM, 2024.