Assignment 3

Please type your answers using the Latex. Please hand in your assignment before October 9th, 2019.

\( \alpha \)-min-cut

Let us assume that a undirected \( G = (V, E) \) is given. We can assume that all edges are unweighted (but multi-edges are allowed). The \( \alpha \)-min-cut is a natural generalisation of min-cut. Let \( c_1 \) be size of the min-cut in \( G \). A node set \( U \subseteq V \) is an \( \alpha \)-min-cut if \( |\delta(U)| \leq \alpha c_1 \).

Prove that the number of different \( \alpha \)-min-cuts is at most \( 2^{2\alpha - 1} n^{2\alpha} \). You can assume that \( \alpha \) is an integer.

multiple-criteria-min-cut

Let \( G = (V, E) \) be a graph, where each edge \( e \in E \) is associated with two weights, \( c_1(e) \) and \( c_2(e) \). Given any cut \( U \), two cut values are naturally induced, namely \( \sum_{e \in \delta(U)} c_1(e) \) and \( \sum_{e \in \delta(U)} c_2(e) \). What we want is a cut \( U \) so that

\[ \max_{i=1}^2 c_i(U), \]

is minimised.

Design a randomized polynomial algorithm to find such a cut with high probability. Hint: the previous exercise can be useful here.

Hyper-Graph Coloring

Let \( G = (V, E) \) be a hypergraph—here each hyperedge \( e \in E \) spans a subset of vertices \( U \subseteq V \). \( G \) is said to be \( k \)-uniform if each edge \( e \in E \) has size \( |e| = k \).

The notion of “coloring” generalizes naturally to hypergraphs. A coloring of vertices is feasible if not all vertices spanned by the same hyperedge \( e \) have the same color.

Prove that if \( |E| < 2^{k-1} \), the graph is always 2-colorable. Hint: assign a random coloring to \( V \).