Project Proposal: Matroid-Constrained Clustering

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Introduction

Given a finite set V, a matroid is a pair $\mathcal{M} = (V, \mathcal{I})$ where $\mathcal{I} \subseteq \mathcal{P}(V)$ is a family of subsets in V that satisfies the following three conditions:

(1) $\emptyset \in \mathcal{I}$,

(2) if $X \subseteq Y \in \mathcal{I}$, then $X \in \mathcal{I}$,

(3) if $X, Y \in \mathcal{I}, |Y| > |X|$, then there exists an element $e \in Y \backslash X$ so that $X \cup \{e\} \in \mathcal{I}$.

The set V is called the *ground set* of the matroid and the elements of $\mathcal{I}$ are called the *independent sets*. The notion of matroid generalizes that of linear independence in vector spaces; moreover, it also generalizes that of graph: in a *graphic matroid*, the edges of the graph form the ground set and the independent sets are the acyclic sets of edges, namely, the forests. As a result, matroids are of fundamental importance in combinatorial optimization, as they generalize a large variety of constraints.

In this project will be focused on *clustering*, such as the famous k-center, k-means, and facility location problems. These problems are of fundamental importance in theoretical computer science and have myriad applications (especially in the area of machine learning). We would be interested in studying the variants of these problems, where the matroids are imposed as constraints.

To see why matroids constraints are interesting, think about this simple example. For some applications, for the sake of diversity or fairness, it is desirable that in each cluster, *some* members of each group are present. One can thus imagine that we are given a bunch of points of various colours, and we want to make sure that each cluster has all the colors presents—this is the same as imposing a partition matroid constraint on the clusters.

Some background knowledge about matroids in general would be helpful for this project.

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