DEMAND RESPONSE USING SERVICE CURVES

[Le Boudec, Tomozei; Innovative Smart Grids Techn, 2011]

Demand response: find algorithm to get a "good service" under some constraints.

a) direct control by the supplier → intrusive, does not scale well
b) consumers adapt their load by reacting to pricing signals → price volatility

Service curve: → control the rate at which a consumer may draw power (real time)
→ guarantee on the minimum amount of power for the consumer + max rate + contract with fixed price.

→ how to control
- consumer side
- provider side.

I Service curve approach: contract.

assumption: "the rate at which a consumer may draw power from the grid may be controlled through a load control signal".

$u(t)$ control signal received by one specific consumer
$z(t)$ the power drawn by this consumer from the grid at time t (Watt).

contract:
- $z(t) \leq u(t) \leq z_{max} \quad \forall t$
- $\int_{t'}^{t} u(z)\,dz \geq \beta(t-t') \quad \forall t' \leq t \qquad \beta: \begin{pmatrix}\mathbb{R}_+ \\ \mathbb{N}\end{pmatrix} \to \mathbb{R}_+$ service curve

Example:

(a) load switching "every 24h, 30 min switch off"

(b) Two level load control "allow a constant minim. power at all time"

Calculus of service curves

- β can be assumed to be super-additive: $\underbrace{\int_{t''}^{t'} u(z)\,dz}_{\geq \beta(t''-t')} + \underbrace{\int_{t'}^{t} u(z)\,dz}_{\geq \beta(t-t'')} = \int_{t''}^{t} u(z)\,dz \geq \beta(t-t'')$
[β_1 & β_2 are super-additive]

Th: $u(t)$ control sequence up to T, $u(t) \leq z_{max} \forall t \leq T$. Equivalence

(i) $\int_{t'}^{t} u(z)\,dz \geq \beta_2(t-t') \quad \forall t' < t \leq T$

(ii) $\int_{t}^{t+t_1} u(z)\,dz \geq \beta_2(t_1) \quad \forall t + t_1 \leq T \qquad \cancel{u(z) \geq z_{min} \forall z \leq T}$

→ the control sequence can be checked to satisfy β_i by keeping in memory the t_1 last units of time.

II Consumer side problem

A consumer wants to charge his battery [PEV: plug in electric vehicule] at time horizon T. The current charge is $B(0)$.

Optimizat° pb: minimize $K(B_{max} - B(T)) + \int_0^T z(\tau)\,d\tau$ $[K > T z_{max}]$

obj 1: full battery
obj 2: min. energy consump.

Battery level at time t:
$$B(t) - B(0) = \int_0^t \eta z(\tau) - \rho z(\tau)^2 - \gamma \mathbb{1}_{B(t)>0} \, d\tau$$

η: efficiency ; γ: battery leakage ; ρ: thermal loss _ discrete time now.

We assume that the PEV is the only elastic appliance. The other appliances use z_{min}.
$z(t+1) + m(t+1) \leq u(t+1) \Rightarrow z(t+1) \leq u(t+1) - z_{min}$.
$u(t+1)$ not known when the decision is made.

Let $C(\pi, u)$ be the total cost when policy π is used and the signal was u.
We want $\max_u (C(\pi, u))$ to be minimized$_\pi$: the policy that minimizes the worst-case

Note that full charge cannot be garanteed due to the battery leakage.
We assume that $z_{max} - z_{min} < \eta/2\rho$ and $\eta^2 > 4\gamma\rho$ and $T < t_1$.

Th: let V^* be the optimal value of min $V = \sum_{\tau=1}^{T} (\eta x(\tau) - \rho x(\tau)^2)$
subject to $0 \leq x(\tau) \leq z_{max} - z_{min}$ $\forall \tau$
$\sum_{\tau=t+1}^{t+t_1} x(\tau) \geq (z_{max} - z_{min})(t_1 - t_0)$ $\forall t \leq T - t_1$, $1 - t_1 \leq$

If $V^* \geq B_{max} - B(0) + T\gamma$, then
$\exists \pi$ that achieves $B(T) \geq B_{max} - \gamma T$
This policy achieves full battery at some time $t \in [0, T]$.

Dém: Let $u(t)$ be the actual control value. $x(t) = u(t) - z_{min}$
B': battery in the ideal case [no leakage]. $B'(0) = 0$ and $B'(t+1) - B'(t) = \eta x(t) - \rho x(t)^2$
$\Rightarrow B'(T) - B'(0) = \sum_\tau (\eta x(\tau) - \rho (x(\tau))^2) \geq V^* \geq B_{max} - B(0) + T\gamma$.
$\Rightarrow B'(T) \geq B_{max} + T\gamma$

Define policy π: $z(t+1) = u(t+1) - z_{min}$ if $B(t) < B_{max}$
$z(t+1) = \min(u(t+1) - z_{min}, \gamma)$ if $B(t) = B_{max}$

$B'(t) - \gamma t \leq B(t) \leq B'(t) \rightarrow B(T) \geq B_{max} - \gamma T$
If $B(t)$ never reaches B_{max}, then $B'(T) \leq B(T) + \gamma T < B_{max} + \gamma T$
and $B'(t) - \gamma T < B_{max}$ ξ.

III Provider side pb

a) Minimum variance policy [highly idealized scenario]
infinite populat° of identical customers with service curve β_c.
customers arrive according to PP(λ) $\{S(n), n \in \mathbb{Z}\}$ and consume an amount $\beta_c(T)$ of energy with $t_0 < T < t_1$.
$w = \beta_c(T) = z_{max}(T) - (z_{max} - z_{min}) t_0$ [known by the provider].

The provider chooses $u(s)$ with $\int_{t'}^{t} u(s)\,ds \geq \beta_c(t - t')$ $\forall t' \leq t$.
The customer chooses $z(t) = u(t) \mathbb{1}_{0 \leq t \leq T}$.

The provider wants to have $\int_0^T u(s)\,ds = \beta_c(T)$ → satisfies contract → minimize power.

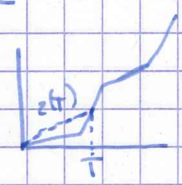
Aggregate instant energy: $Z(t) = \sum_{n=-\infty}^{+\infty} z(t - S(n))$
↑ shift the start point

We know that
$E[Z] = \lambda \int_{-\infty}^{+\infty} z(s)\,ds = \lambda\omega \qquad Var(Z) = \lambda \int_{-\infty}^{+\infty} z^2(s)\,ds.$ Goal: minimize the variance.

minimize $\int_{-\infty}^{+\infty} z^2(s)\,ds$ under $\begin{cases} u(t) \leq z_{max} \\ \int_{t'}^{t} u(s)\,ds \geq \beta_2 (t-t') \quad \forall t' \leq T \leq T. \end{cases}$

Try a binary policy: $u_b(t) = \begin{cases} z_{min} & \forall 0 \leq t \leq t_0 \\ z_{max} & \text{otherwise.} \end{cases}$

$$\int_0^T u_b^2(s)\,ds = t_0 z_{min}^2 + (T-t_0) z_{max}^2 \underset{\text{convexity of } x^2}{\geq} T \Big(\underbrace{\frac{t_0}{T} z_{min} + \frac{T-t_0}{T} z_{max}}_{z(T)} \Big)^2$$



New policy: $u_T^*(t) = \begin{cases} z(T) & \text{if } 0 \leq t \leq T \\ z_{max} & \text{otherwise} \end{cases}$ [Minimum constant allocation policy].

Th: u_T^* satisfies β_2 and minimizes the variance of $z(t)$.

$\forall$ control u: $\int_0^T u^2(s)\,ds \underbrace{\int_0^T 1\,ds}_{T} \geq \left(\int_0^T u(s)\,ds\right)^2 = [\beta_2(T)]^2 = T \int_0^T u_T^*(s)\,ds.$

b) Bursty arrivals: Quota binary allocation policy
N users with contract β_2. Let user i receives $u_i(t)$ and consumes $z_i(t) \leq u_i(t)$
PEV = unique non elastic appliance; no th. loss, no leakage.
$z_{max} = z_{min} + z_b$ ← max charging power of PEV.
↑ non elastic

→ detect when PEV charging → replace z_{min} by $z'_{min} = z_{min} + \varepsilon$.
We look at binary policies: z'_{min} or z_{max}

Quota N_a: # max of customers that can receive z_{max} at each time.

Idea: provide z_{max} to customers with contracts "the less respected"

$P_i(t)$: # z'_{min} that can be proposed in a row from t without breaking the contract.
It can be computed from $u_i(t-t_1) \dots u_i(t)$ and updated.

The control algorithm is the following:

$P_i = t_0$; $t \leftarrow 1$
from $t=1$ to T_{max} do:
. Retrieve consumption for each user at time $t-1$
. $A \leftarrow \{ i \mid z_i(t-1) > z_{min} \}$
. Sort A according to P_i : $P_i \leq P_j$ "$\Leftrightarrow$" $\pi(i) \leq \pi(j)$ $\forall i,j \in A$
for each user i
if $i \in A$ and $\pi(i) > N_a$ and $P_i \geq 1$ then $u_i \leftarrow z_{min} + \varepsilon$
else $u_i \leftarrow z_{max}$
update P_i

Experimentally, this reduces the peak rate when the arrivals are normally distributed.