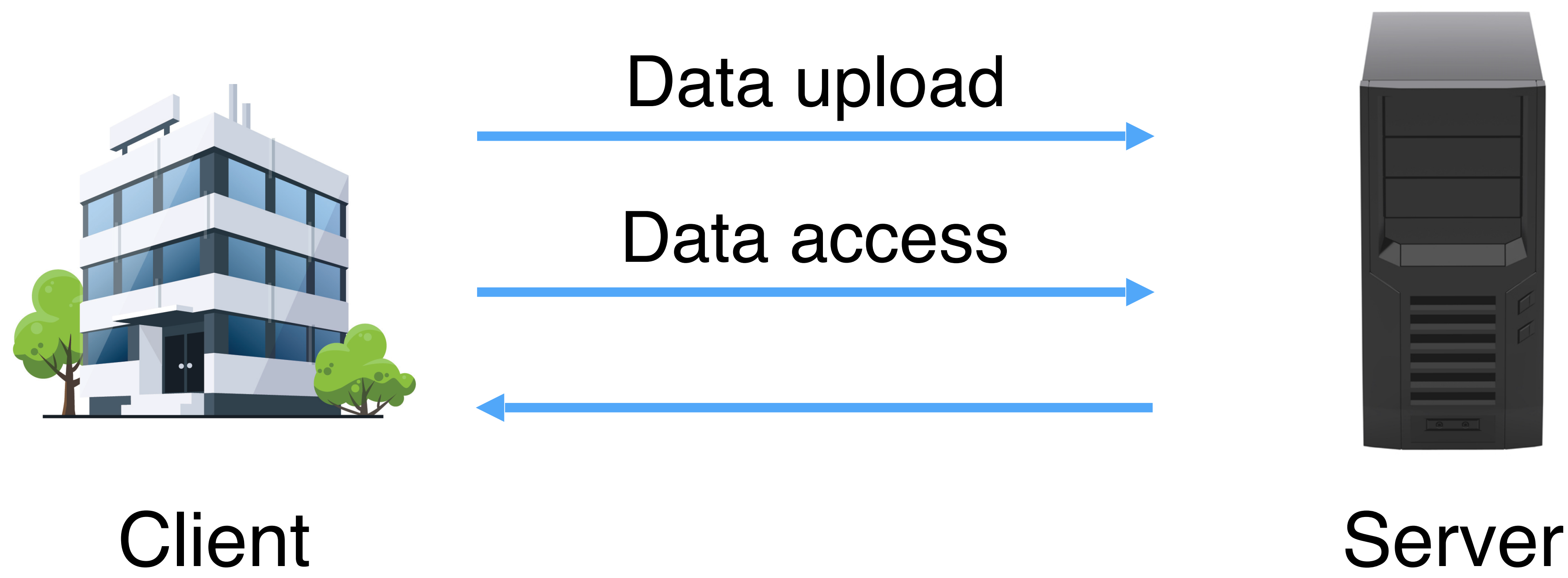
Searchable Encryption, Leakage-Abuse Attacks, and Statistical Learning Theory

Paul Grubbs, Marie-Sarah Lacharité, Brice Minaud, Kenny Paterson

eprint 2019/011 and IEEE S&P 2019.
(also eprint 2018/965, CCS 2018.)

AriC crypto seminar, ENS Lyon, 2019

Outsourcing Data



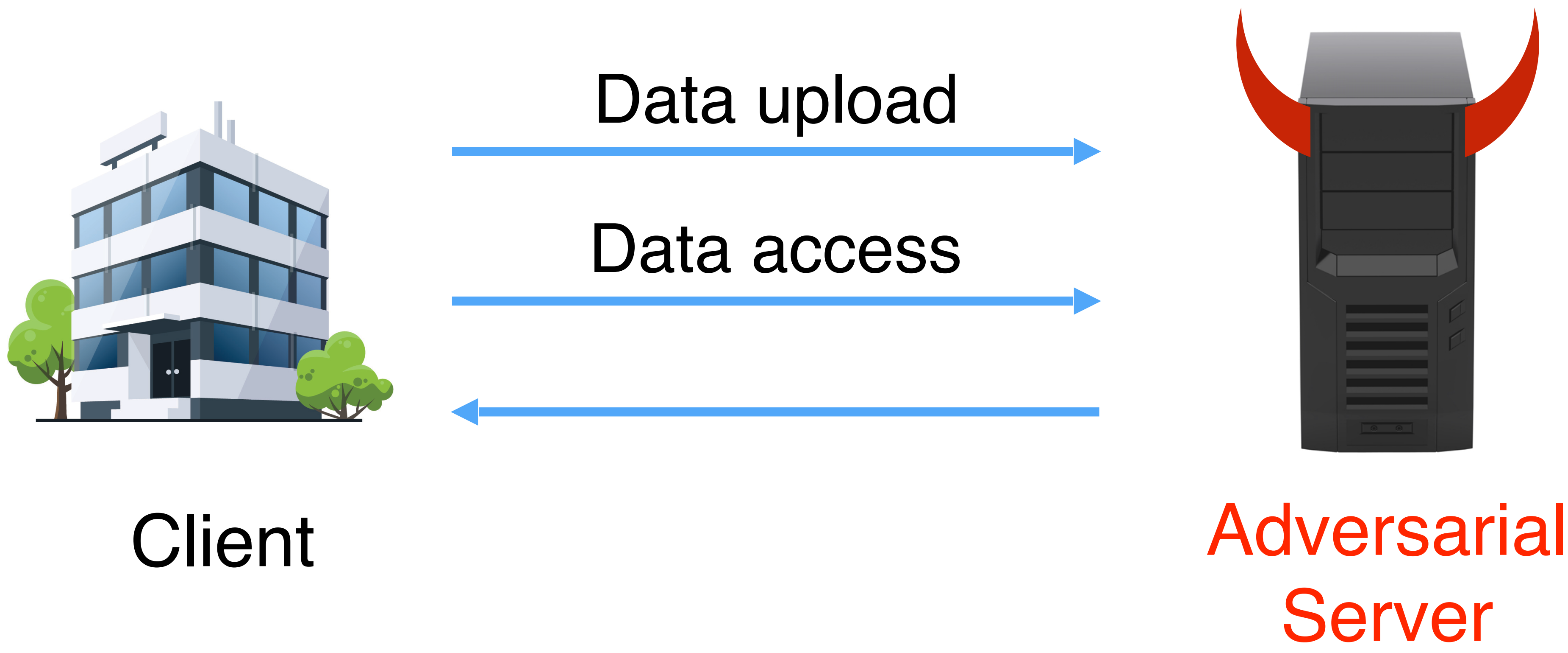
Sensitive data → encryption needed.

An **encrypted database** is of little use if it cannot be searched.

→ **Searchable Encryption**.

Examples: Private message server. Company/hospital outsourcing client/patient info.

Searchable Encryption



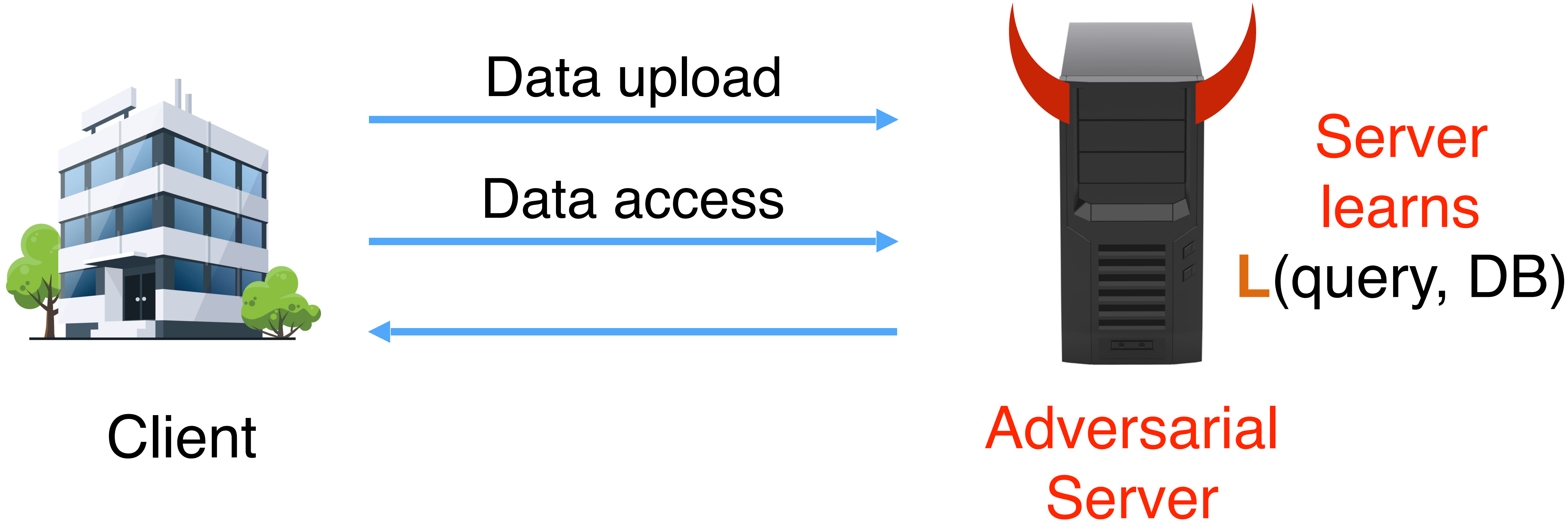
Adversary: **honest-but-curious** host server.

Security goal: **confidentiality** of **data** and **queries**.

Very active topic in research and industry.

[AKSX04], [BCLO09], [PKV+14], [BLR+15], [NKW15], [KKNO16], [LW16], [FVY+17], [SDY+17], [DP17], [HLK18], [PVC18], [MPC+18]…

Security Model



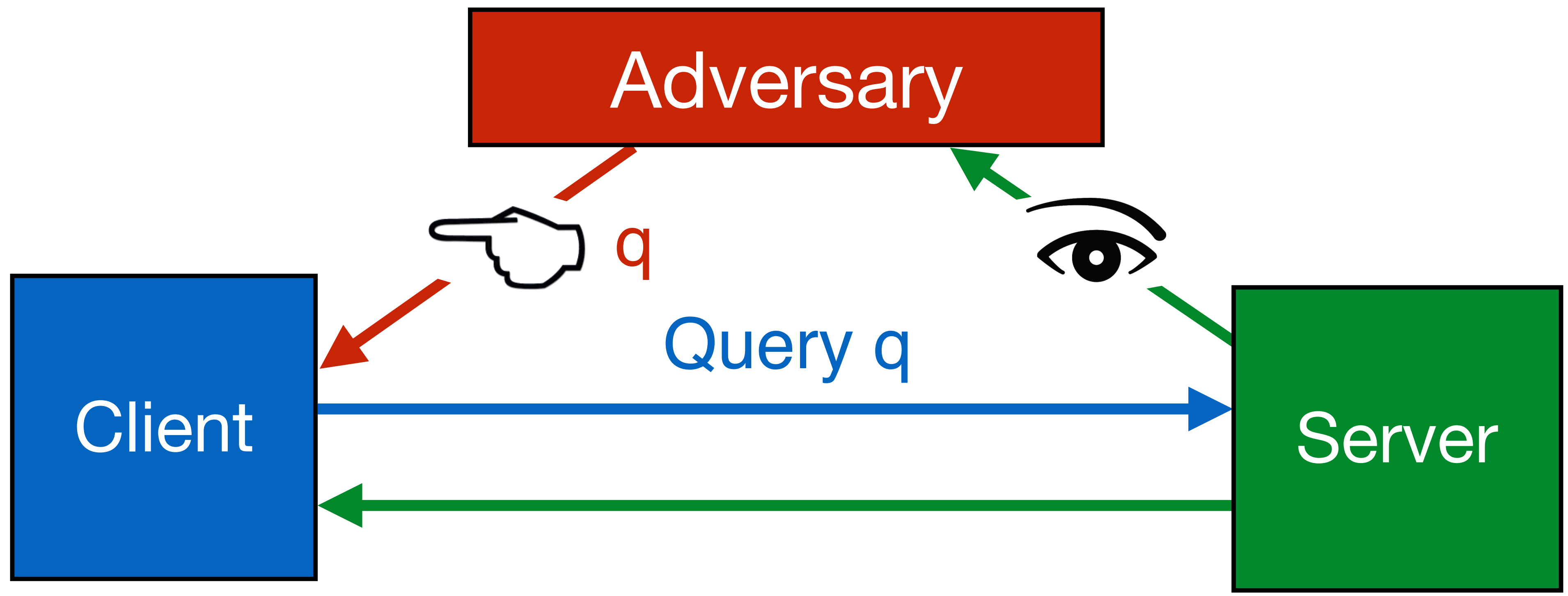
Generic solutions (FHE) are infeasible at scale → for efficiency reasons, some **leakage** is allowed.

Security model: parametrized by a **leakage function L**.

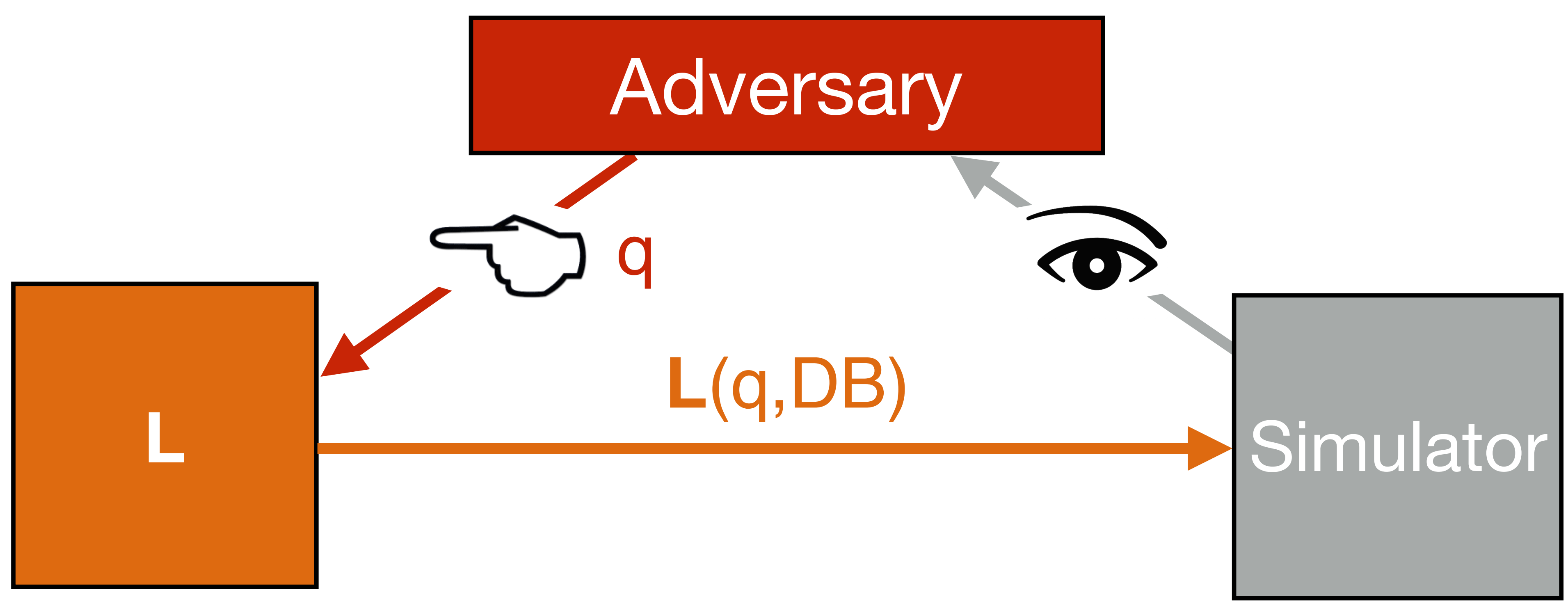
Server learns **nothing** except for the output of the leakage function.

Security Model

Real world



Ideal world



Keyword Search

Symmetric Searchable Encryption (SSE) = keyword search:

- Data = collection of documents. *e.g. messages.*
- Serch query = find documents containing given keyword(s).

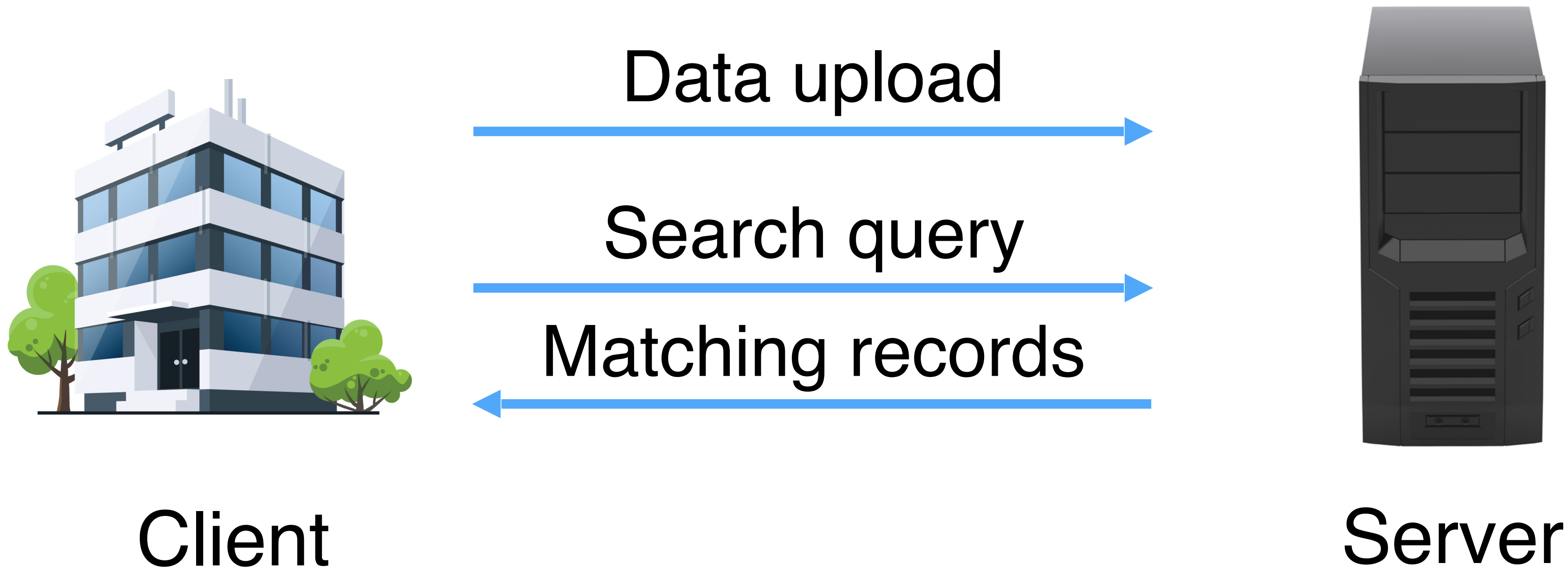
Efficient solutions for leakage = **search pattern** + **access pattern**.

Some active topics:

- **Forward** and **backward privacy** [B16][BMO17][CPPJ18][SYL+18]...

- **Locality** [CT14][ANSS16][DPP18]...

Beyond Keyword Search



For an **encrypted database management system**:

- Data = collection of records. *e.g. health records.*
- Basic query examples:
  - find records with given value. *e.g. patients aged 57.*
  - find records within a given range. *e.g. patients aged 55-65.*

Range Queries

In this talk: **range queries**.

- Fundamental for any encrypted DB system.
- Many constructions out there.
- Simplest type of query that can't “just” be handled by an index.

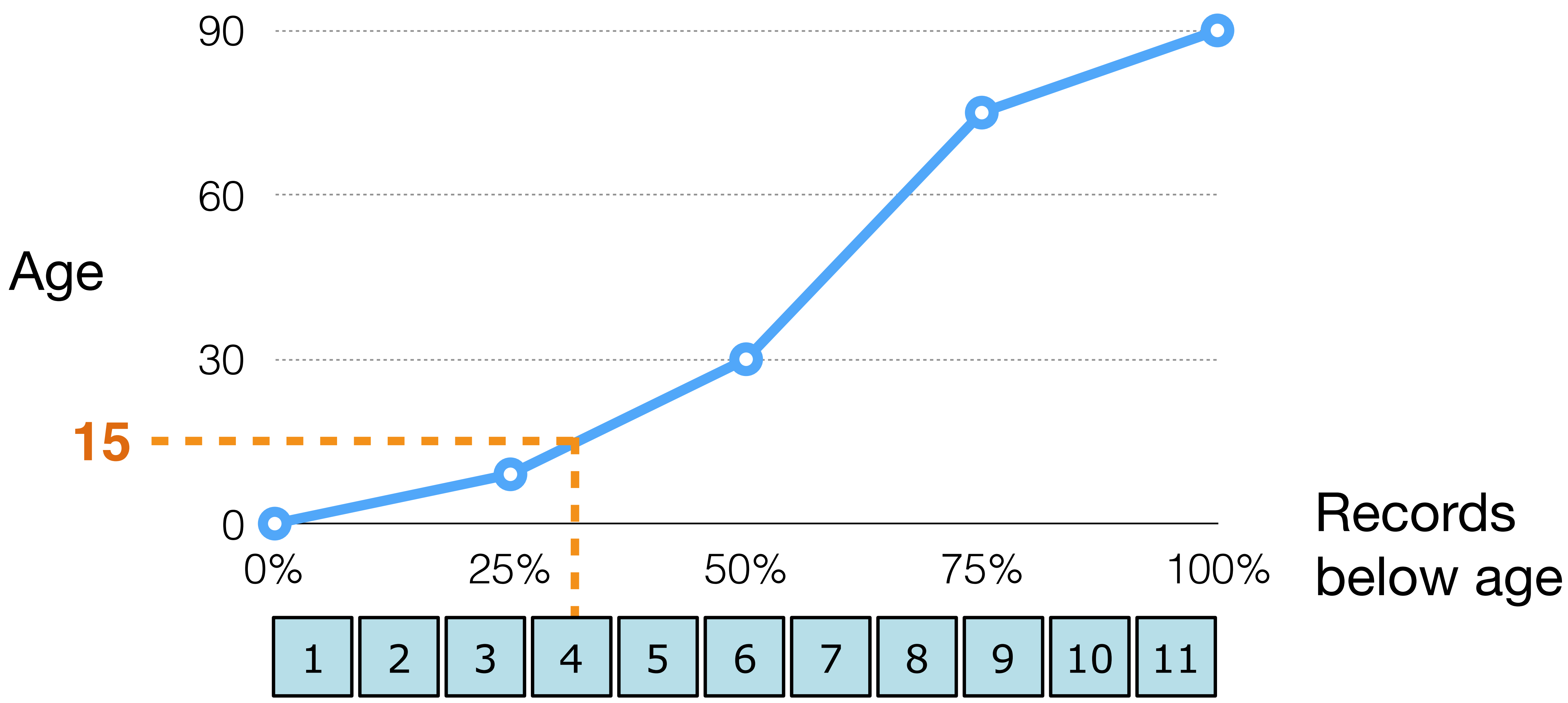
Initial solutions:

Order-Preserving, Order-Revealing Encryption.

- Plaintexts are **ordered**, ciphertexts are **ordered**.
- The encryption map **preserves order**.

Attacks Exploiting ORE

- **"Sorting" attack**: if every possible value appears in the DB... Just sort the ciphertexts and you learn their value!
- **"CDF-matching" attack**: say the attacker has an approximation of the **Cumulative Distribution Function** of DB values...



Leakage-Abuse Attacks

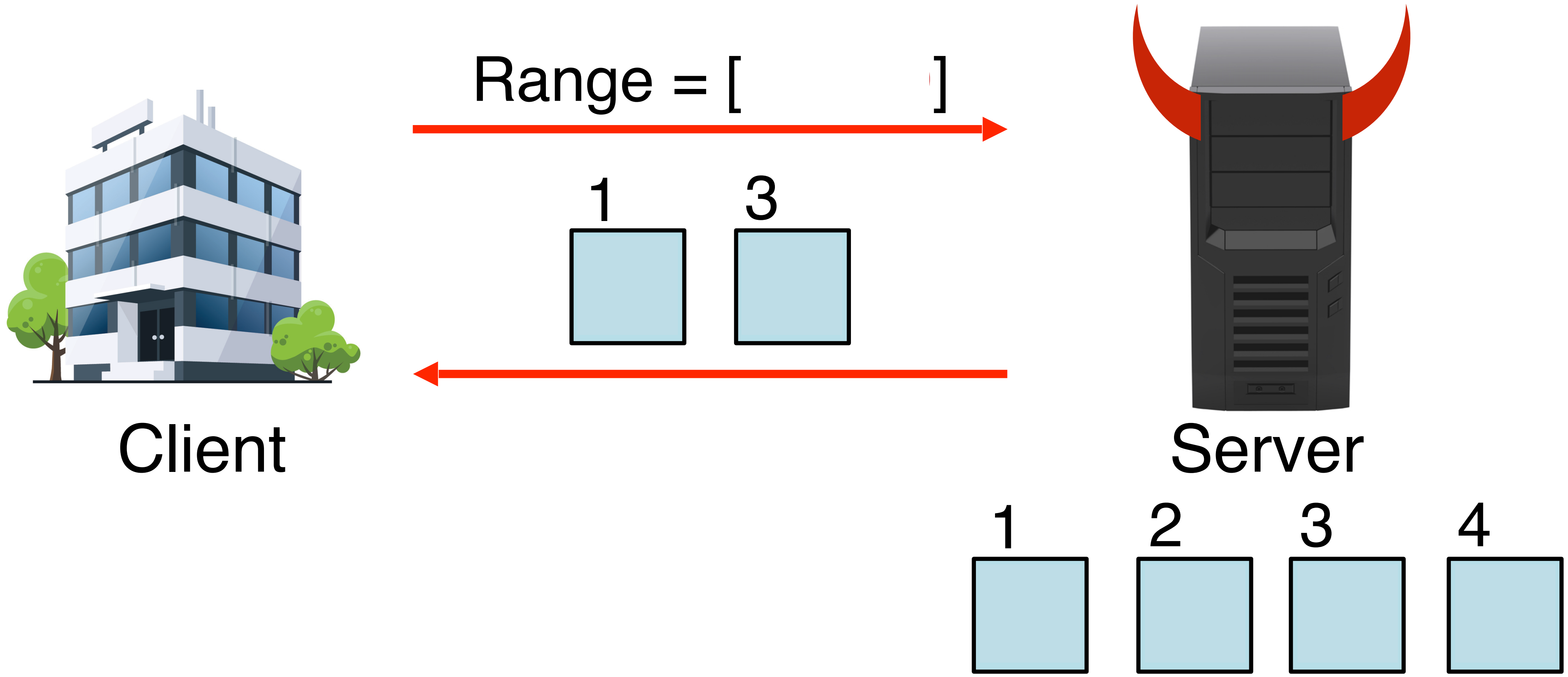
"Leakage-abuse attacks" (coined by Cash et al. CCS'15):

- Do not contradict security proofs.
- Can be devastating in practice.

ORE: order information can be used to infer (approximate) values.
Leaking order is too revealing.

→ **"Second-generation" schemes** enable range queries *without* relying on OPE/ORE.

Range Queries



SE schemes supporting range queries are proven secure w.r.t. a leakage function including **access pattern leakage**.

What can the server learn from the above leakage?

Database Reconstruction

Let ***N*** = number of possible values for the target attribute.

Strongest goal: **full** database reconstruction = recovering the exact value of every record.

More general: **approximate** database reconstruction = recovering all values within εN.

$\varepsilon = 0.05$ is recovery within 5%. $\varepsilon = 1/N$ is full recovery.

(“Sacrificial” recovery: values very close to 1 and *N* are excluded.)

[KKNO16]: full reconstruction in $O(N^4 \log N)$ queries, assuming i.i.d. uniform queries!

Database Reconstruction

[KKNO16]: full reconstruction in $O(N^4 \log N)$ queries!

recovers

This talk ([GLMP19], [LMP18]):

	Full. Rec.	**Lower Bound**
‣ $O(\varepsilon^{-4} \log \varepsilon^{-1})$ for approx. reconstruction.	$O(N^4 \log N)$	$\Omega(\varepsilon^{-4})$
‣ $O(\varepsilon^{-2} \log \varepsilon^{-1})$ with very mild hypothesis.	$O(N^2 \log N)$	$\Omega(\varepsilon^{-2})$
‣ $O(\varepsilon^{-1} \log \varepsilon^{-1})$ for approx. *order* rec.	$O(N \log N)$	$\Omega(\varepsilon^{-1} \log \varepsilon^{-1})$

implies

Full reconstruction in $O(N \log N)$ for *dense* DBs.

Scale-free: does not depend on size of DB or number of possible values.

→ Recovering all values in DB within 5% costs O(1) queries!

Database Reconstruction

[KKNO16]: full reconstruction in $O(N^4 \log N)$ queries!

This talk ([GLMP19], subsuming [LMP18]):

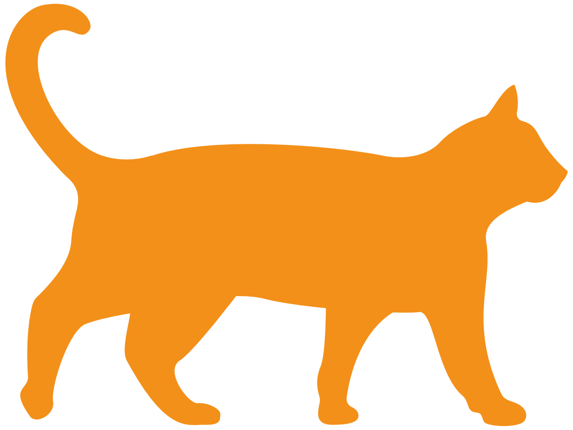
	Full. Rec.	**Lower Bound**
‣ $O(\varepsilon^{-4} \log \varepsilon^{-1})$ for approx. reconstruction.	$O(N^4 \log N)$	$\Omega(\varepsilon^{-4})$
‣ $O(\varepsilon^{-2} \log \varepsilon^{-1})$ with very mild hypothesis.	$O(N^2 \log N)$	$\Omega(\varepsilon^{-2})$
‣ $O(\varepsilon^{-1} \log \varepsilon^{-1})$ for approx. *order* rec.	$O(N \log N)$	$\Omega(\varepsilon^{-1} \log \varepsilon^{-1})$

→ This talk.

Main tool:

- connection with **statistical learning theory**;
- especially, **VC theory**.

VC Theory



C

VC Theory

Foundational paper: **Vapnik and Chervonenkis, 1971**.

Uniform convergence result.

Now a foundation of learning theory, especially PAC (*probably approximately correct*) learning.

Wide applicability.

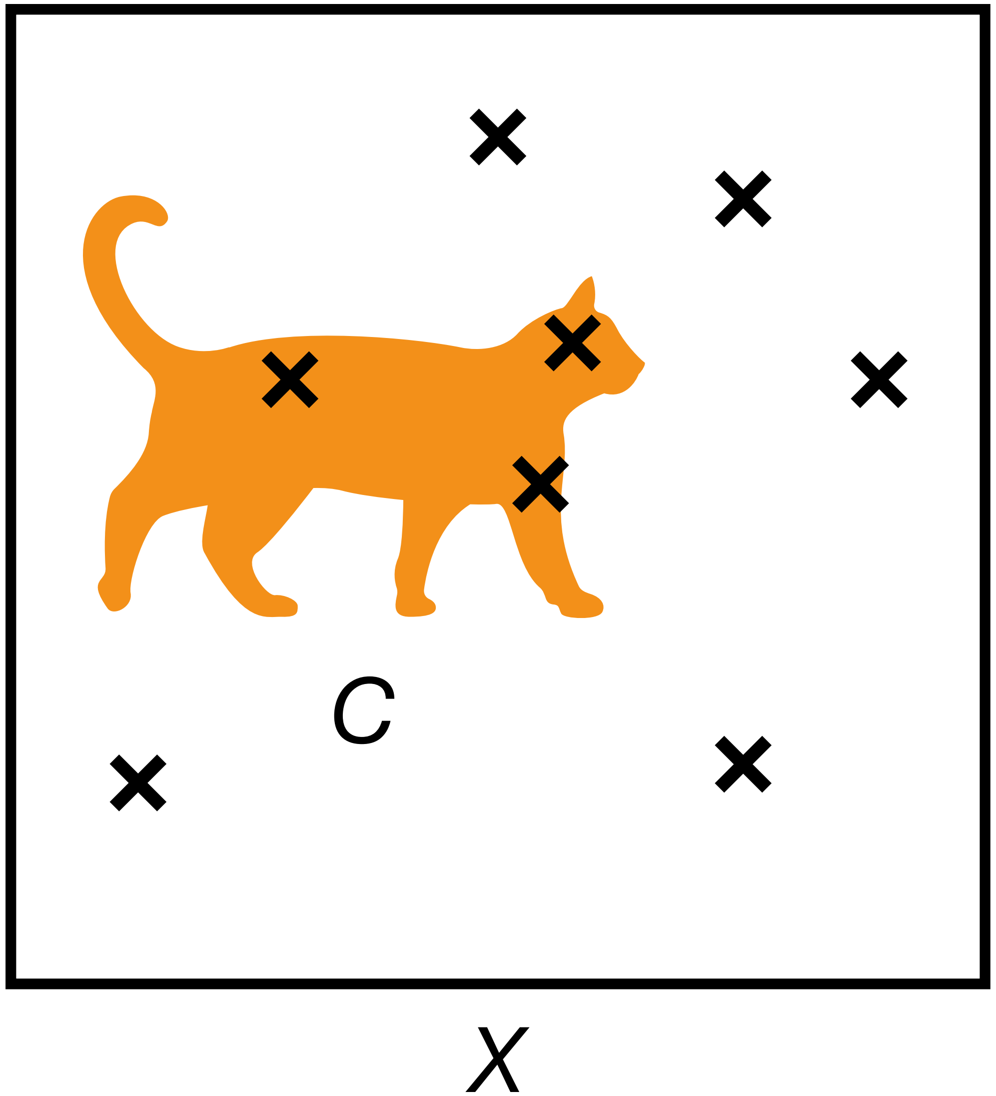
Fairly easy to state/use.

(You don't have to read the original article in Russian.)

Warm-up

Set X with probability distribution D.

Let $C \subseteq X$. Call it a *concept*.



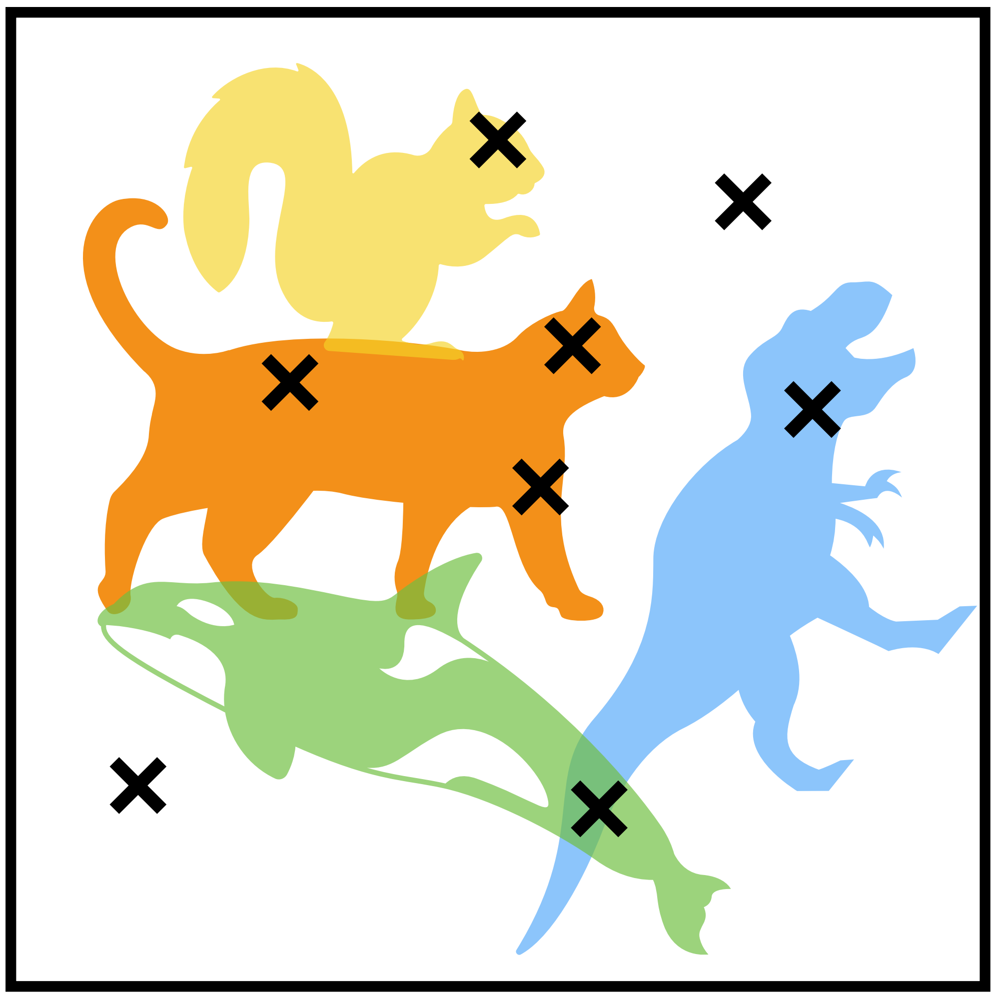
$$\Pr(C) \approx \frac{\#\text{points in } C}{\#\text{points total}}$$

Sample complexity:
to measure Pr(C) within ε,
you need $O(1/\varepsilon^2)$ samples.

Approximating a Concept Set

Now: set $\mathscr{C}$ of concepts.

Goal: approximate their probabilities *simultaneously*.

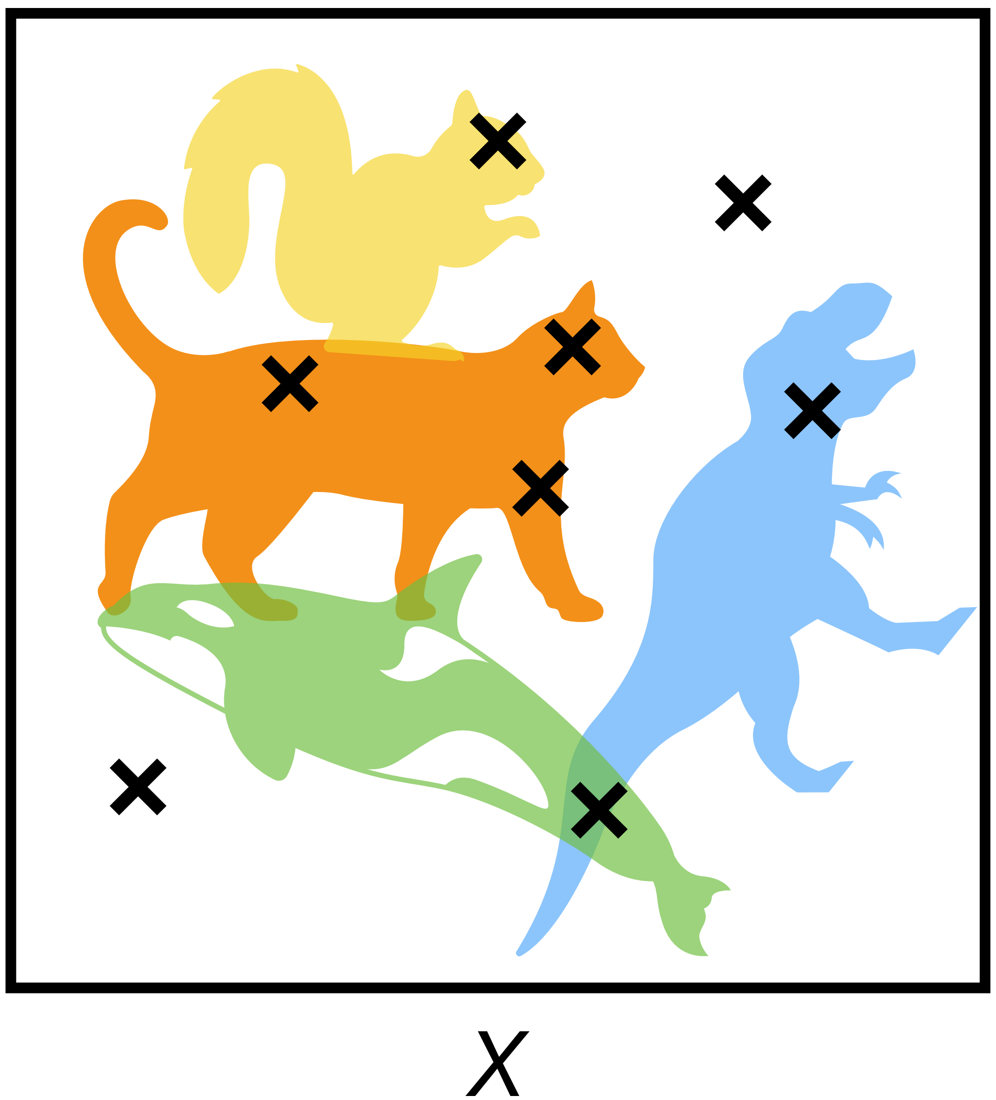


The set of samples drawn from X is an **ε-sample** iff for **all** C in $\mathscr{C}$:

$$\left| \Pr(C) - \frac{\#\text{points in } C}{\#\text{points total}} \right| \leq \epsilon$$

ε-sample Theorem

How many samples do we need to get an ε-sample whp?



Union bound: yields a sample complexity that depends on $|\mathscr{C}|$.

V & C 1971:

If $\mathscr{C}$ has **VC dimension** d, then the number of points to get an ε-sample whp is

$$O\left(\frac{d}{\epsilon^2} \log \frac{d}{\epsilon}\right).$$

Does not depend on $|\mathscr{C}|$*!*

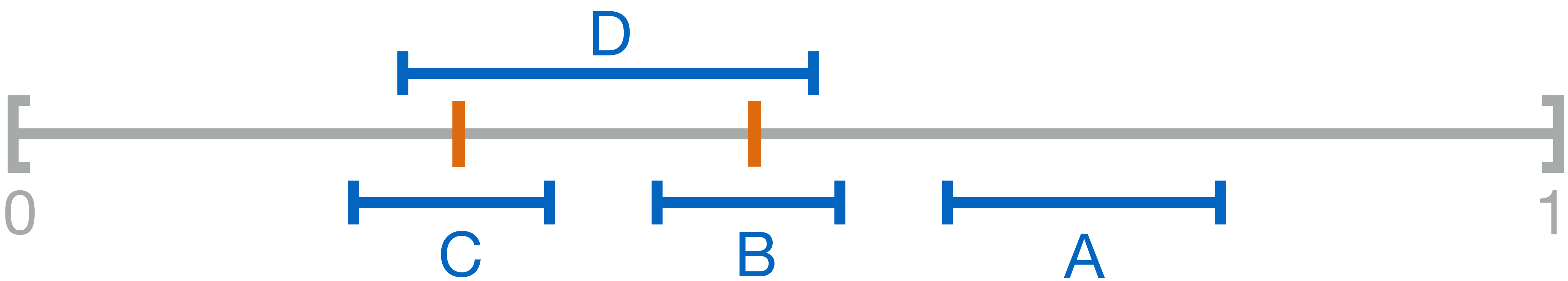
VC Dimension

Remaining Q: *what is the VC dimension?*

A set of points is **shattered** by $\mathscr{C}$ iff:

every subset of S is equal to $C \cap S$ for some C in $\mathscr{C}$.

Example. Take **2 points** in X=[0,1]. Concepts $\mathscr{C}$ = all ranges.



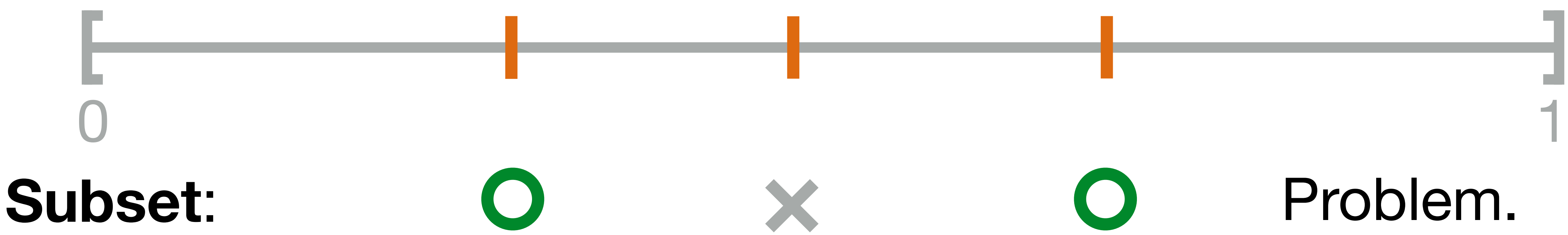
Subsets:

2 points = SHATTERED

✗	✗	**OK**. Range A.
✗	O	**OK**. Range B.
O	✗	**OK**. Range C.
O	O	**OK**. Range D.

VC Dimension

Example. Take **3 points** in X=[0,1]. Concepts $\mathscr{C}$ = all ranges.



3 points = NOT SHATTERED

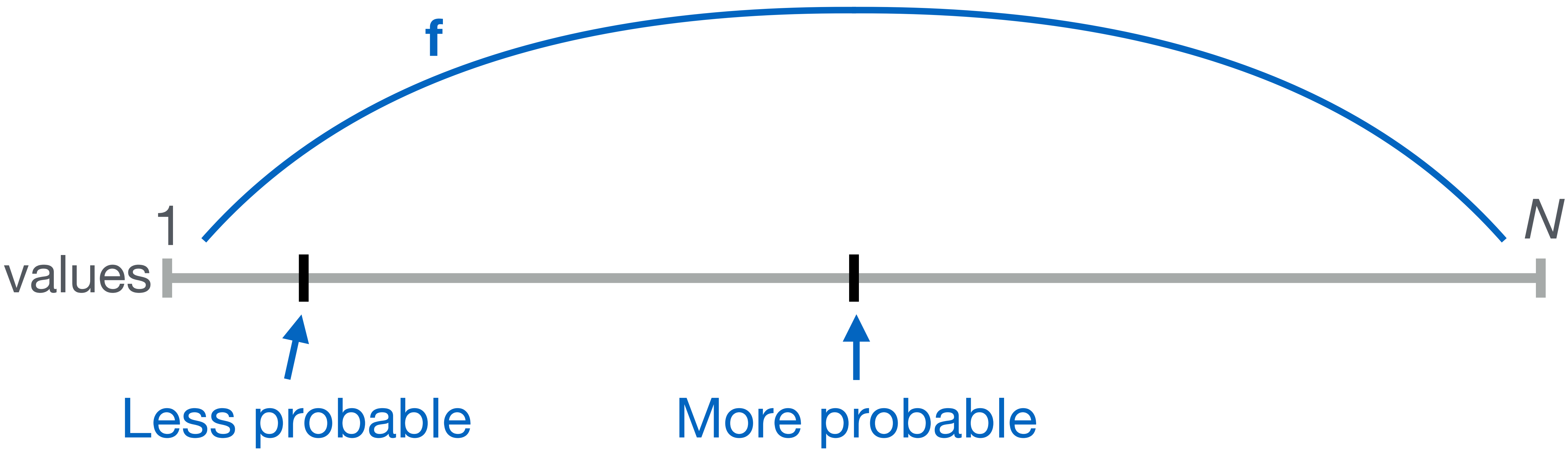
> **VC dimension** of $\mathscr{C}$ = largest cardinality of a set of points in X that is shattered by $\mathscr{C}$.

E.g. VC dimension of ranges is 2.

What typically matters is just that VC dim is finite.

Database Reconstruction

KKNO16-like Attack



Assume a **uniform distribution** on range queries.

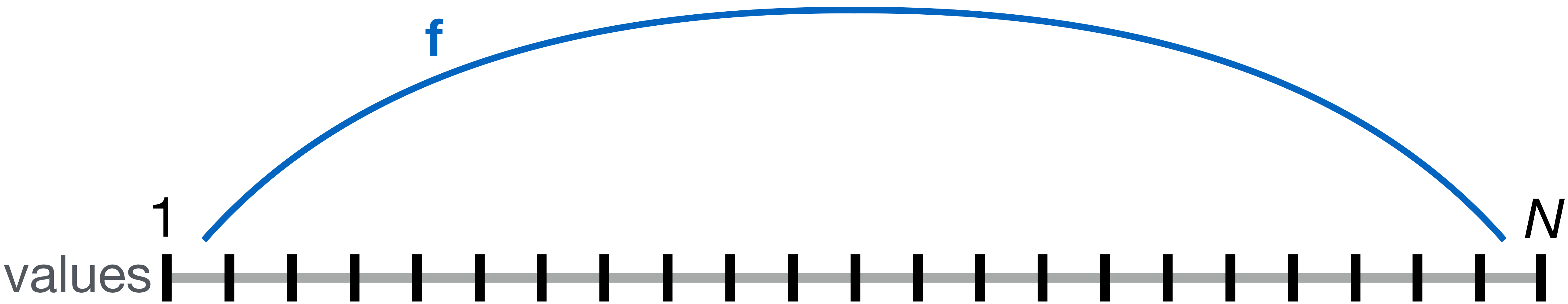
Induces a distribution **f** on the prob. that a given value is hit.

Idea: for each record...

1. Count frequency at which the record is hit.
→ gives estimate of probability it's hit by uniform query.

2. deduce estimate of its value by "inverting" **f**.

KKNO16-like Attack



Step 1: for **all** records, estimate prob of the record being hit.

This is an ε-sample!

X = ranges $\mathscr{C}$ = {{ranges ∋ x}: x ∈ [1,N]}

so we need $O(\varepsilon^{-2} \log \varepsilon^{-1})$ queries.

Step 2: because **f** is quadratic, "inverting" **f** adds a square.

After $O(\varepsilon^{-4} \log \varepsilon^{-1})$ queries, the value of all records is recovered within εN.

On the i.i.d. Assumption

We are assuming **uniformly distributed** queries.

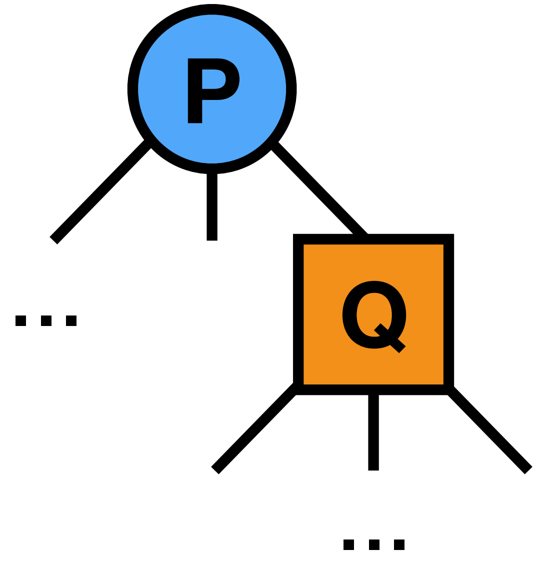
In reality we are assuming:

- The advesary knows the query distribution.
- Queries are uniform.
- More fundamentally, queries are **independent and identically distributed** (i.i.d.).

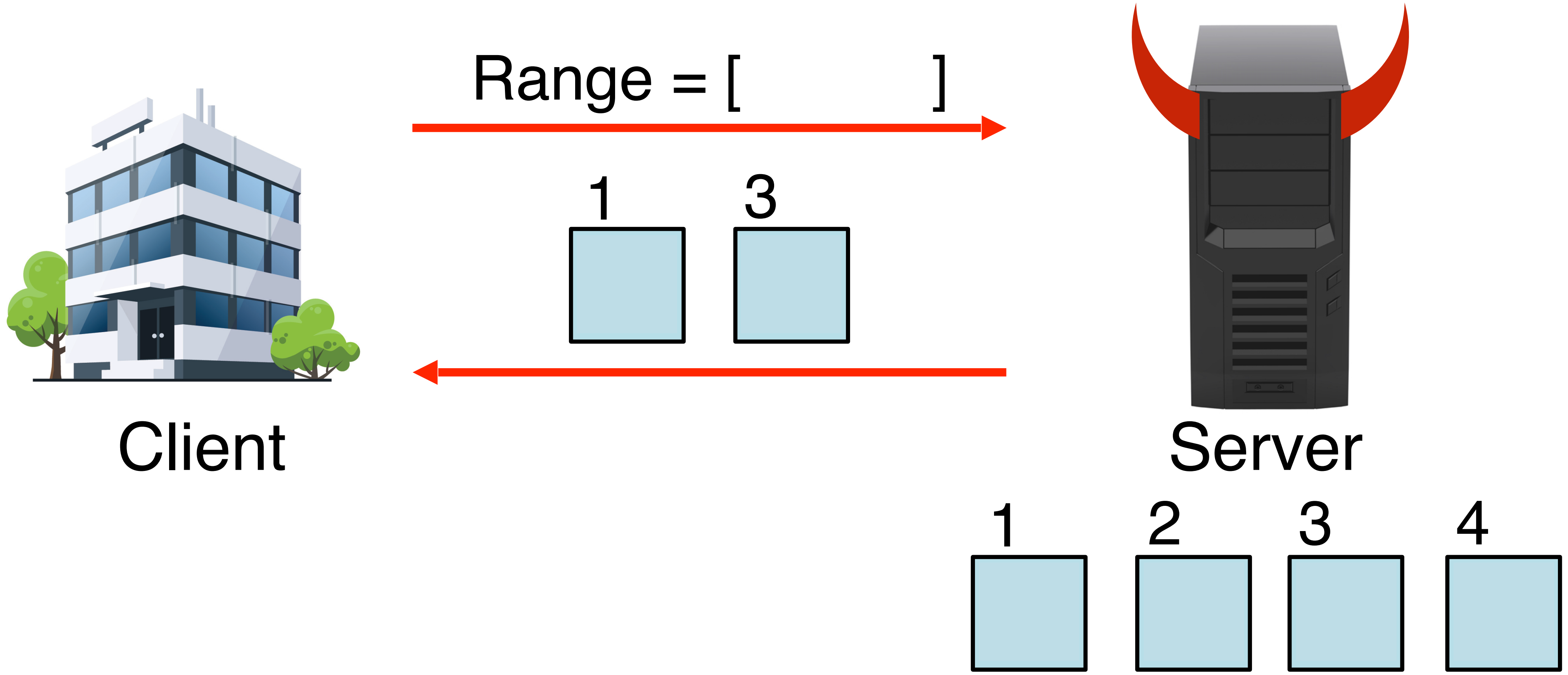
This is not realistic.

What can we learn without that hypothesis?

Order Reconstruction



Problem Statement



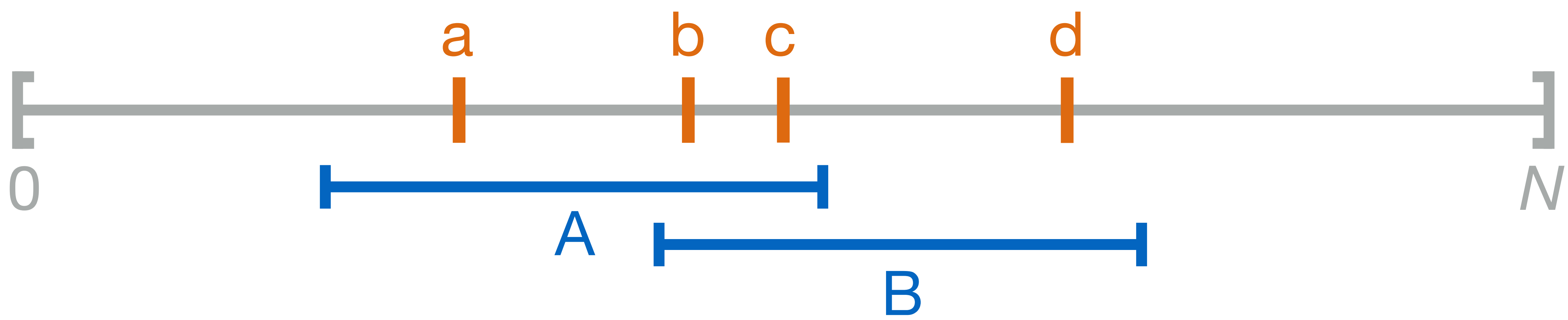
What can the server learn from the above leakage?

This time we **don't assume** i.i.d. queries, or knowledge of their distribution.

Range Query Leakage

Query A matches records a, b, c.

Query B matches records b, c, d.



Then this is the only configuration (up to symmetry)!

→ we learn that records b, c are *between* a and d.

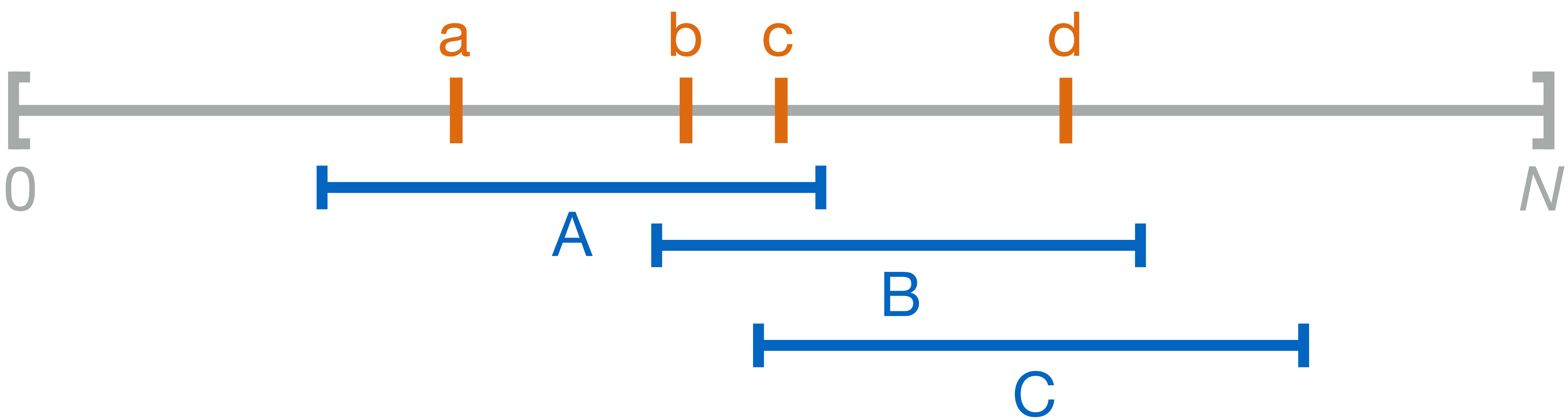
We learn something about the **order** of records.

Range Query Leakage

Query A matches records a, b, c.

Query B matches records b, c, d.

Query C matches records c, d.



Then the only possible order is a, b, c, d (or d, c, b, a)!

Challenges:

- How do we extract order information? (What **algorithm**?)
- How do we **quantify** and **analyze** how fast order is learned as more queries are observed?

Challenge 1: the Algorithm

Short answer: there is already an algorithm!

Long answer: **PQ-trees**.

X: linearly ordered set. Order is unknown.

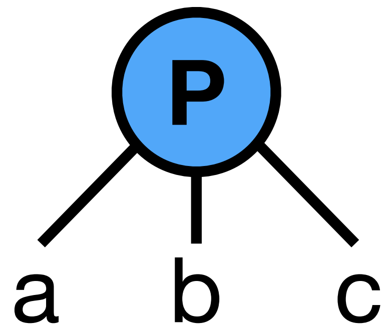
You are given a set *S* containing some intervals in *X*.

A **PQ tree** is a compact (linear in $|X|$) representation of the set of all permutations of *X* that are compatible with *S*.

Can be updated in linear time.

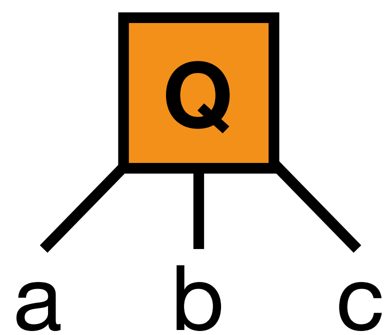
Note: was used in [DR13], didn't target reconstruction.

PQ Trees



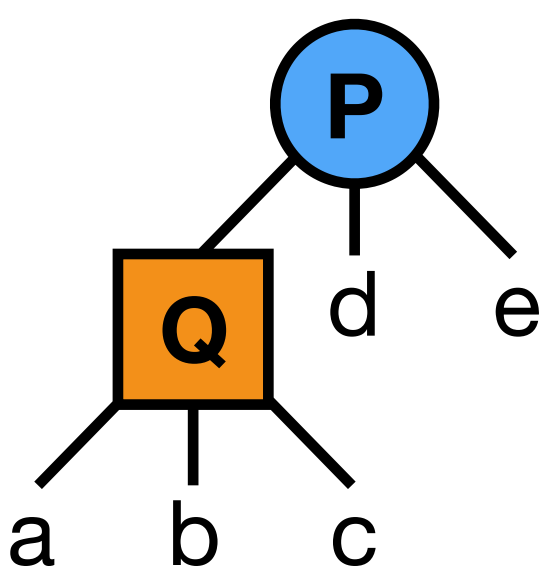
Order is completely **unknown**.

- any permutation of abc.



Order is completely **known** (up to reflection).

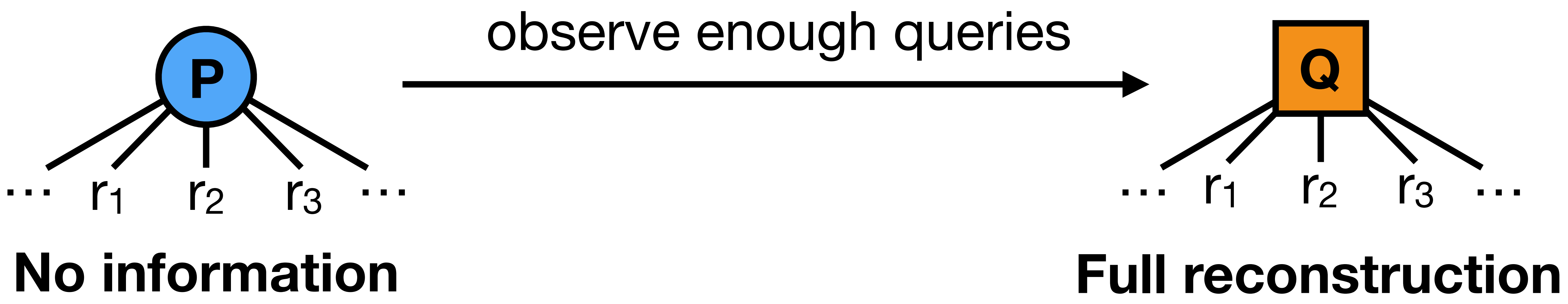
- abc’or ‘cba’.



Combines in the natural way.

- ‘abcde’, ‘abced’, ‘dabce’, ‘eabcd’, ‘deabc’, ‘edabc’, ‘cbade’ etc.

Full Order Reconstruction



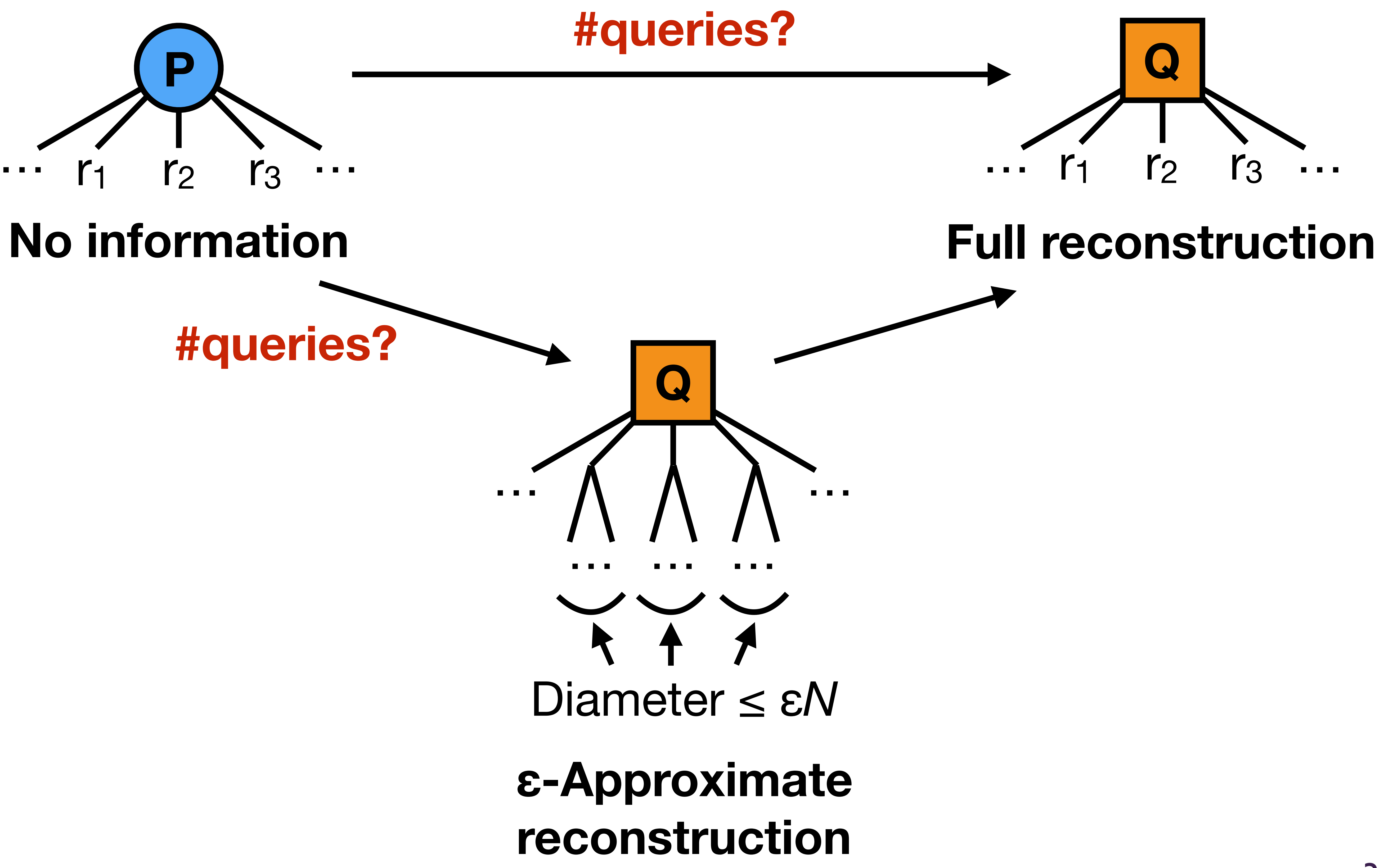
We want to **quantify** order learning...

Challenge 2a: Quantify Order Learning

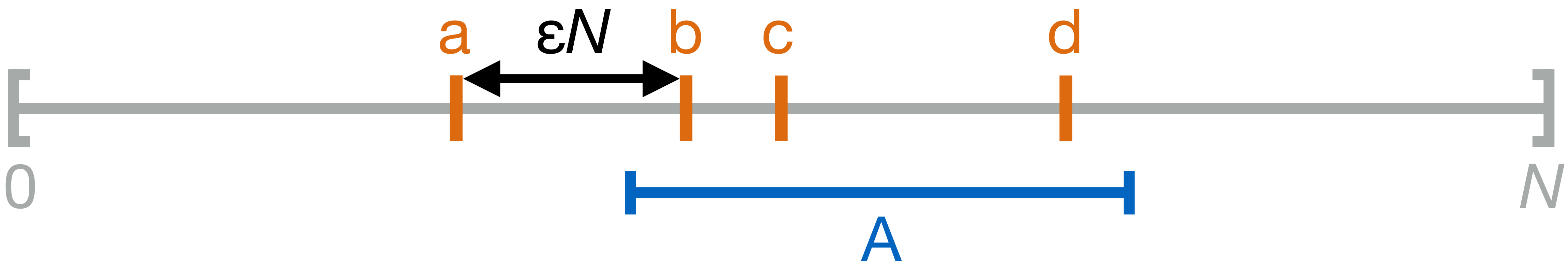
ε-Approximate order reconstruction.

Roughly: we learn the order between two records as soon as their values are $\geq \varepsilon N$ apart. ($\varepsilon = 1/N$ is full reconstruction)

Approximate Order Reconstruction



Challenge 2b: Analyze Query Complexity



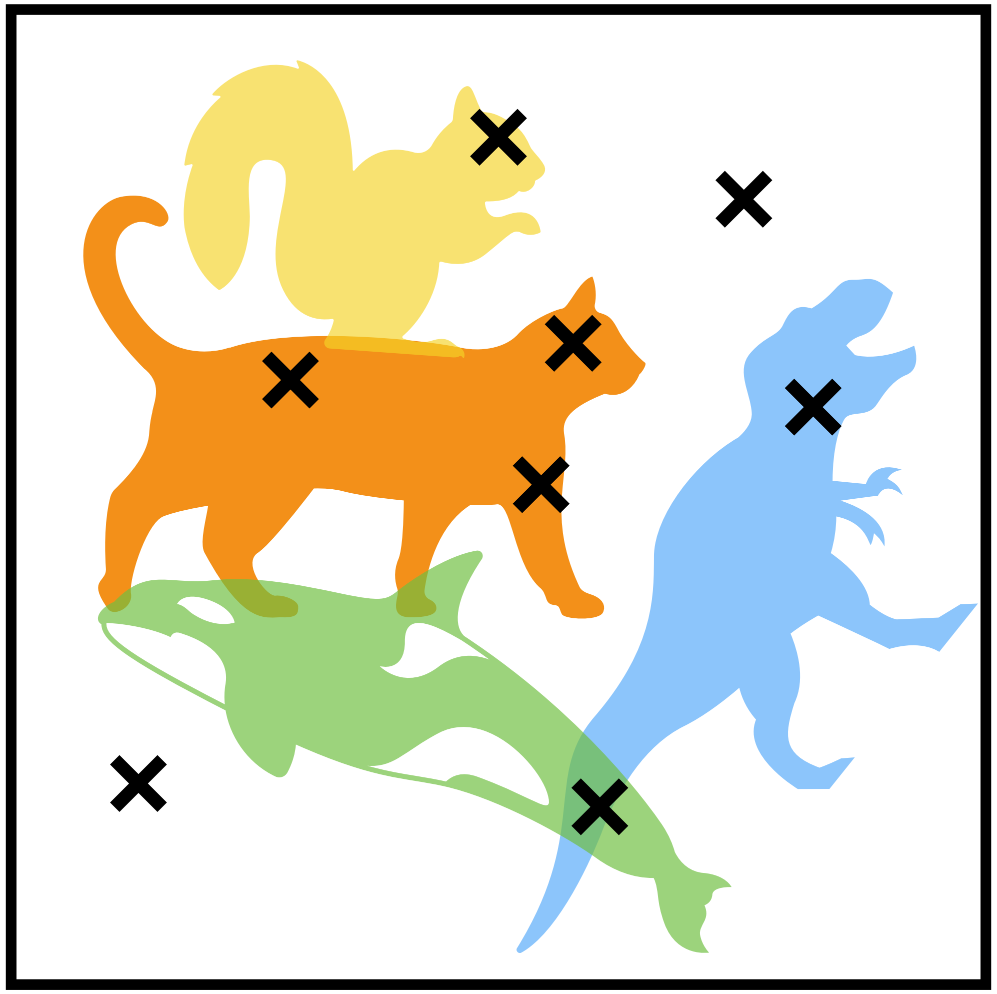
Intuition: if no query has an endpoint between a and b, then a and b can't be separated.

→ ε-approximate reconstruction is impossible.

You want a query endpoint to hit every interval ≥ ε*N*.
Conversely with some other conditions it's enough.

Heavy sweeping of details under rug.

VC Theory Saves the Day (again)



ε-samples: the ratio of points hitting each concept is close to its probability.

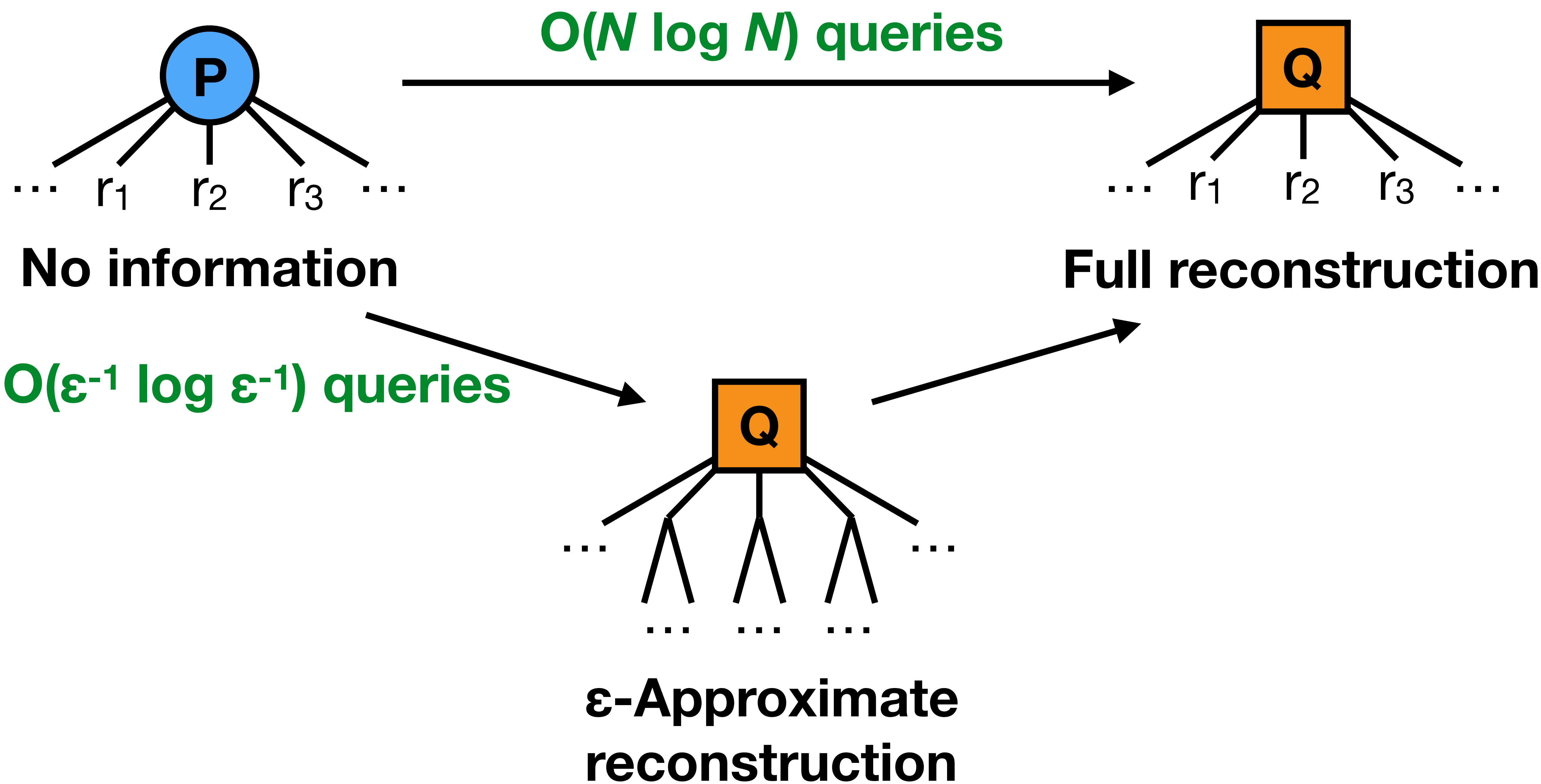
What we want now: if a concept has high enough probability, it is hit by at least one point.

The set of samples drawn from X is an **ε-net** iff for all C in $\mathscr{C}$:

$$\Pr(C) \geq \epsilon \Rightarrow C \text{ contains a sample}$$

→ Number of points to get an ε-net whp: $O\left(\frac{d}{\epsilon}\log\frac{d}{\epsilon}\right)$

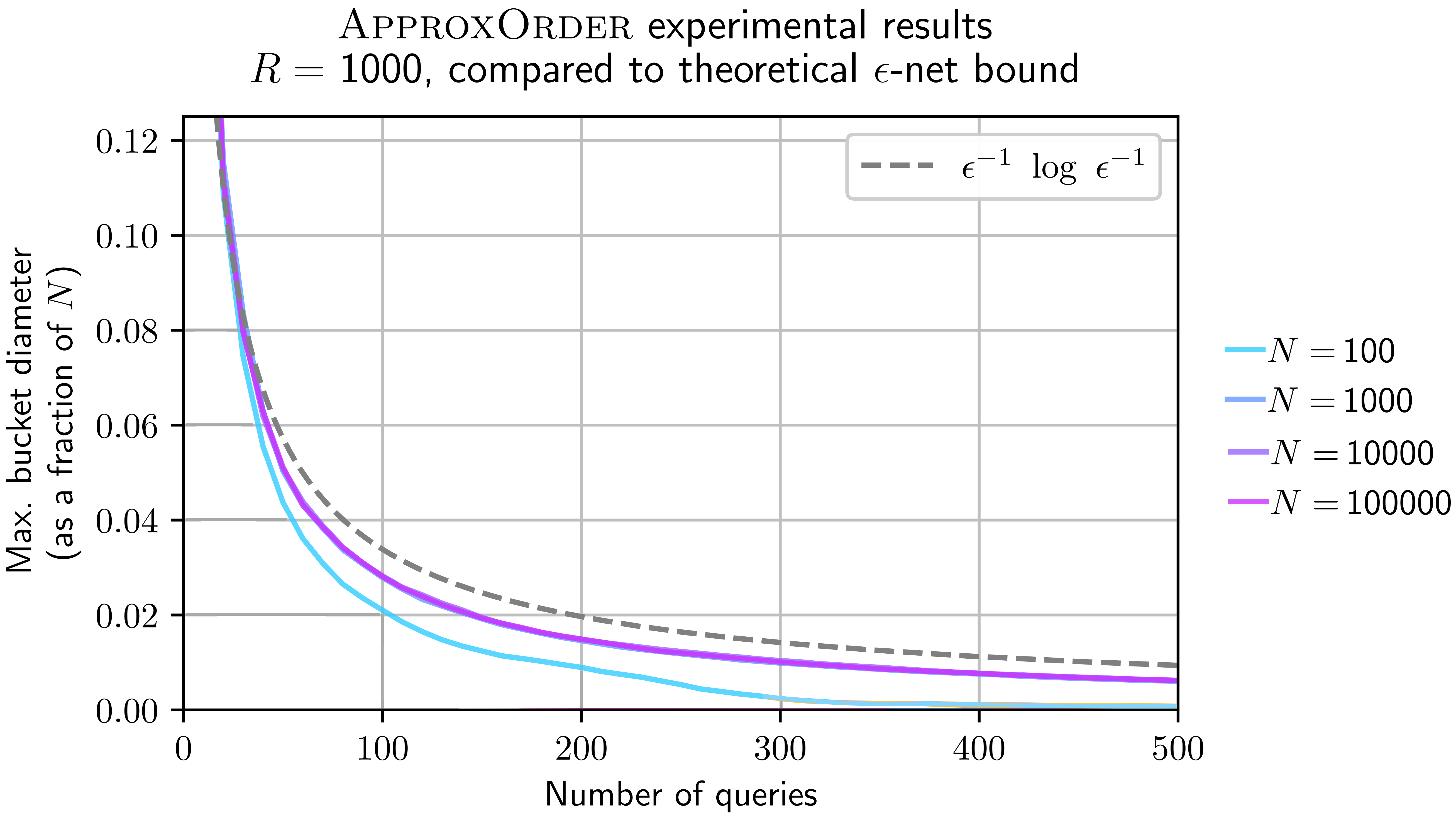
Approximate Order Reconstruction



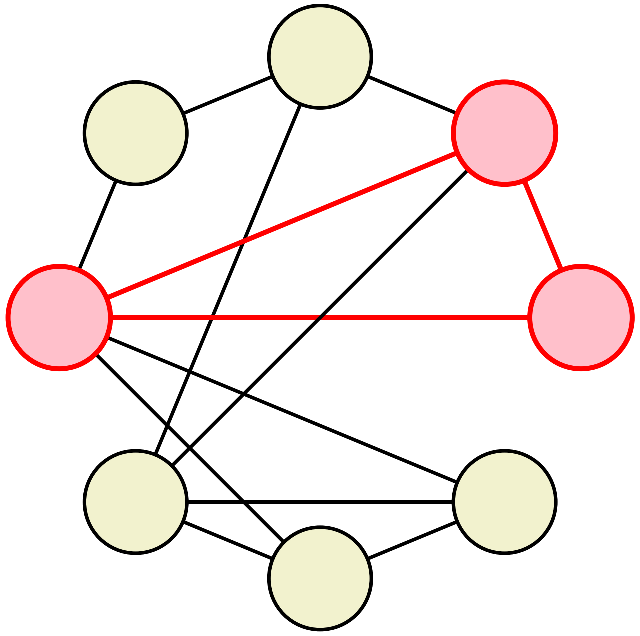
Conclusion: learn order very quickly. Almost back to ORE...

Note: some (weak) assumptions are swept under the rug.

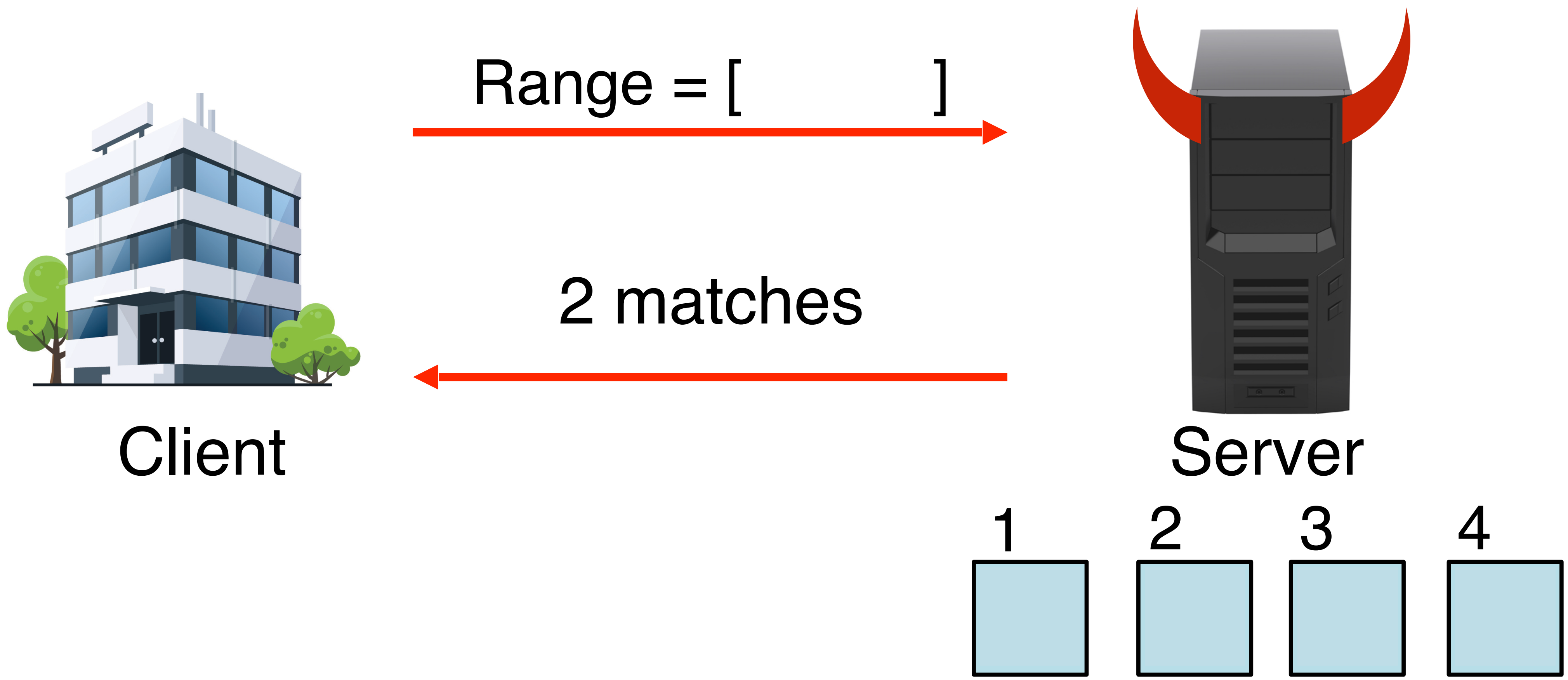
Experiments



Volume Leakage



Problem Statement

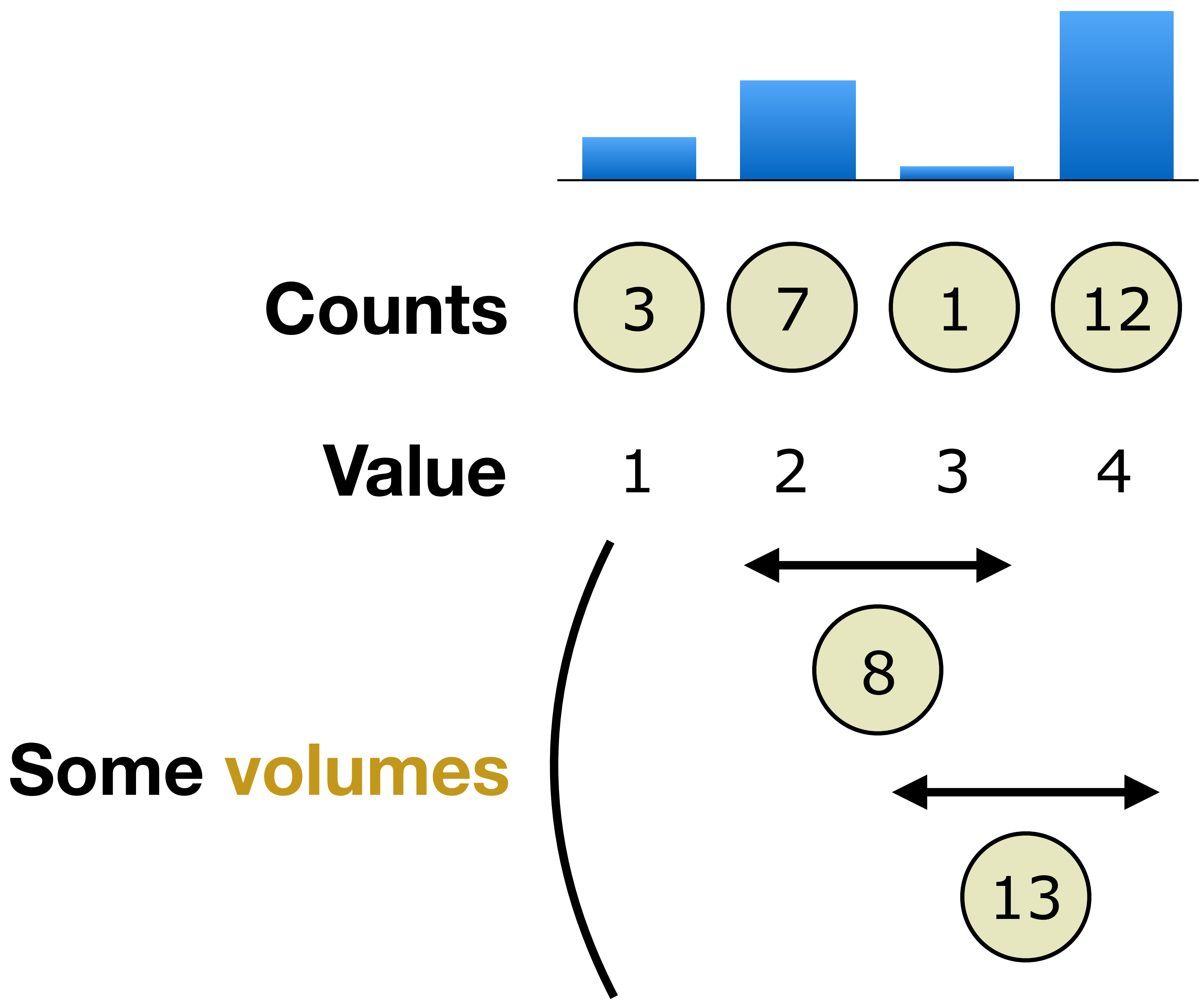


Attacker *only* sees **volumes** = **number of records** matching each query.

What can the server learn from the above leakage?

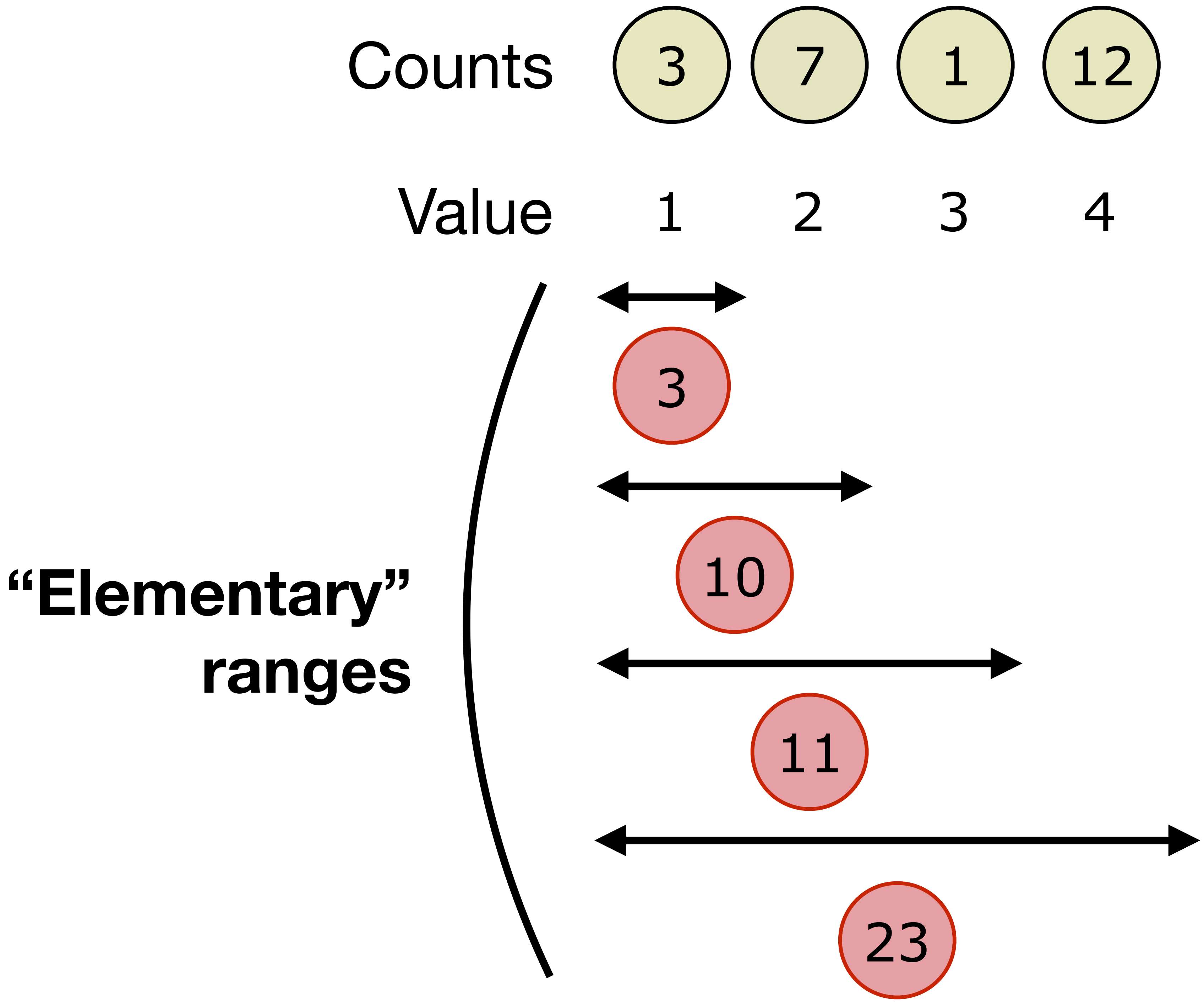
Volumes

The attacker wants to learn exact **counts.**



A **volume** = number of records matching some range.

Elementary Volumes



Elementary volumes = volumes of ranges [1,1], [1,2], [1,3]...

Elementary Volumes

Counts	3	7	1	12
Value	1	2	3	4

Fact:

vol([a,b]) = vol([1,b]) - vol([1,a])

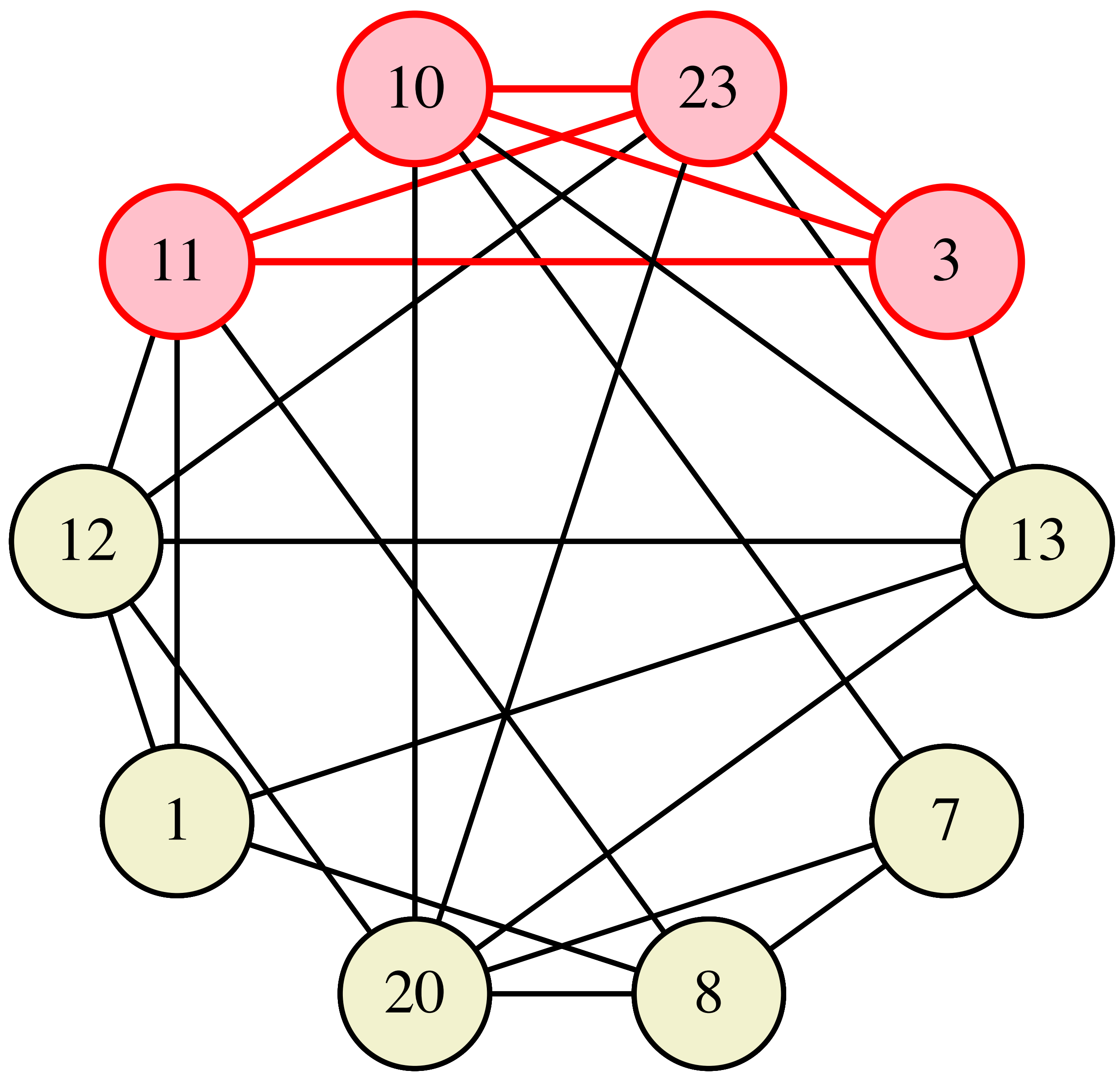
so...

- Every volume is = difference of two elementary volumes.
- Knowing set of elementary volumes ⇔ knowing counts.

Our goal: finding elementary volumes.

The Attack

Assumption: the volumes of all queries are observed.



Draw an **edge** between volumes **a** and **b** iff **|b-a|** is a volume.

Summary

Attack: **elementary volumes** form a clique in the volume graph → clique-finding algorithm reveals them.

For structured queries, even just volume leakage can be quite damaging. Attack requires strong assumption.

In the article:

- *Pre-processing to avoid clique finding.*
- *Analysis of parameters + experiments.*
- *Other attacks.*

Closing Remarks

On Range Queries

Access pattern: severe attacks under minimal assumptions.

Please don't use OPE/ORE.

Also avoid current encrypted DBs if you don't trust the server and care about privacy.

New solutions needed. E.g. efficient specialized ORAMs.

Even then, need to hide volumes.

Many open problems...

Connection to Machine Learning

- In this talk: VC theory.
- In the article: known query setting = PAC learning.
- Some results for general query classes.

Machine learning in crypto: also used for **side channel attacks**. Same general setting!

> Natural connection between **reconstructing secret information from leakage** and **machine learning**.

Seems to be a powerful tool to understand the security implications of leakage. **In side channels** - use learning algorithms; **here** - use learning theory.