# MODÉLISATION STOCHASTIQUE DES PROTOCOLES

Bartłomiej Błaszczyszyn

(INRIA/ENS & Univ. of Wrocław)

Rescom 2009

La Palmyre 7–12 juin 2009

# SPATIAL ALOHA MAC PROTOCOL for wireless mobile ad-hoc networks

based on

*“Stochastic Geometry and Wireless Networks”*

*Chapter 16. Spatial Aloha*

by François Baccelli and Bartłomiej Błaszczyszyn

to appear in *Frontiers and Trends in Networking*

Now Publishers.

# OUTLINE

- PRELIMINARIES
- ALOHA IN BASIC POISSON BIPOLAR NETWORK MODEL
- OPPORTUNISTIC ALOHA
- BEYOND THE POISSON BIPOLAR NETWORK MODEL
- LOCAL DELAYS IN ALOHA
- CONCLUDING REMARKS

# PRELIMINARIES

# Medium Access Control (MAC) Protocol

In packet switching networks logically addressed packets are sent from their source toward their ultimate destination, possibly through intermediate nodes.

A set of standard rules in charge of this process (called the communication protocol) is typically structured in a few layers.

The Medium Access Control (MAC) layer is a part of the data communication protocol organizing simultaneous packet transmissions in the network.

# Aloha MAC

In our talk we will consider the, perhaps most simple, algorithm used in the MAC layer, called Aloha:

at each time slot (we will consider only slotted; i.e., discrete, time case), each potential transmitter independently tosses a coin with some bias $p$; it accesses the medium (transmits) if the outcome is heads and it delays its transmission otherwise.

# Tuning Aloha Parameter $p$

In Aloha algorithm it is important to tune the value of the Medium Access Probability (MAP) $p$, so as to realize a compromise between two contradicting types of wishes:

- a "social one" to have as many concurrent transmissions as possible in the network and
- an "individual one" to have high chances that authorized transmissions be successful and/or efficient.

The contradiction between these two wishes stems from the fact that the very nature of the "medium" in which the transmissions take place (Ethernet cable or electromagnetic field in the case of wireless communications) imposes some constraints on the maximal number and configuration of successful concurrent transmissions.

# Aloha in Wireless Ad-hoc Networks

In this talk we will focus on Aloha in wireless ad-hoc networks; i.e.:

- networks made of nodes arbitrarily repartitioned in some region,
- nodes exchange packets either transmitting or receiving them on a common frequency,
- (in contrast to cellular networks) do not relay on any fixed infrastructure to carry the packets on long distances,
- but use intermediary retransmissions by nodes lying on the path between the packet source node and its destination node.

# Analysis of Stochastic Models

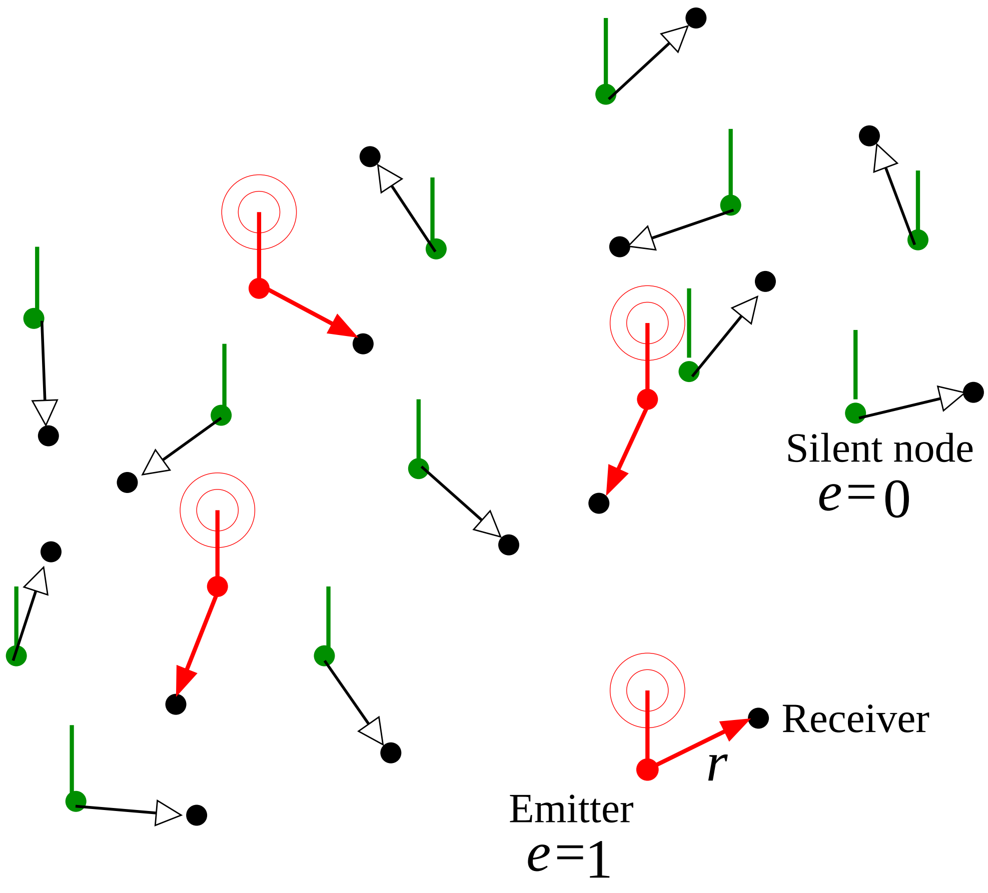
In this context, the variability of radio channel conditions (so called fading) and arbitrary geometry of the network make spatial, stochastic modeling particularly pertinent as it allows to capture all these uncertainties in a statistical manner.

Mathematical analysis of simple yet not-simplistic stochastic models is an important alternative for (crude) simulations of these networks.

In our talk we will show a few such models and results difficult (or impossible) to obtain via crude simulations.

# ALOHA
# IN BASIC POISSON BIPOLAR NETWORK MODEL

# Bipolar Ad-hoc Network — Snapshot



# Bipolar Ad-hoc Network Model with Aloha

Independently marked Poisson point process (p.p.) $\widetilde{\Phi} = \{(X_i, e_i, y_i, \mathbf{F}_i)\}$, where

1. $\Phi = \{X_i\}$ denotes the locations of the nodes (the potential transmitters); $\Phi$ is always assumed Poisson with positive and finite intensity $\lambda$;

# Bipolar Ad-hoc Network Model with Aloha

Independently marked Poisson point process (p.p.) $\widetilde{\Phi} = \{(X_i, e_i, y_i, \mathbf{F}_i)\}$, where

1. $\Phi = \{X_i\}$ denotes the locations of the nodes (the potential transmitters); $\Phi$ is always assumed Poisson with positive and finite intensity $\lambda$;
2. $\{e_i\}$ is the medium access indicator of node $i$; ($e_i = 1$ if node $i$ is allowed to transmit and $0$ otherwise).
   Aloha principle:The random variables $e_i$ are i.i.d. and independent of everything else, with $\mathsf{P}(e_i = 1) = p$ ($p$ is the MAP).

# Bipolar Ad-hoc Network Model with Aloha

Independently marked Poisson point process (p.p.) $\widetilde{\Phi} = \{(X_i, e_i, y_i, \mathbf{F}_i)\}$, where

1. $\Phi = \{X_i\}$ denotes the locations of the nodes (the potential transmitters); $\Phi$ is always assumed Poisson with positive and finite intensity $\lambda$;
2. $\{e_i\}$ is the medium access indicator of node $i$; ($e_i = 1$ if node $i$ is allowed to transmit and $0$ otherwise). Aloha principle:The random variables $e_i$ are i.i.d. and independent of everything else, with $\mathsf{P}(e_i = 1) = p$ ($p$ is the MAP). Consequence of Aloha: the set of nodes that transmit $\Phi^1 = \{X_i : e_i = 1\}$ is a Poisson p.p. with intensity $\lambda_1 = \lambda p$ (as an independent thinning of $\Phi$).

# Receivers in Bipolar Model

3. $\{y_i\}$ denotes the location of the receiver for node $X_i$ (we assume here that no two transmitters have the same receiver). We assume that $\{X_i - y_i\}$ are i.i.d random vectors with $|X_i - y_i| = r$; i.e. each receiver is at distance $r$ from its transmitter.

# Wireless Channel Conditions — Fading

4. $\{\mathbf{F}_i = (F_i^j : j)\}$ where $F_i^j$ denotes the virtual power emitted by node $i$ (provided $e_i = 1$) towards receiver $y_j$; by this we understand the product of the (effective) power of transmitter $i$ and of the random fading from this node to receiver $y_j$.

# Wireless Channel Conditions — Fading

4. $\{\mathbf{F}_i = (F_i^j : j)\}$ where $F_i^j$ denotes the virtual power emitted by node $i$ (provided $e_i = 1$) towards receiver $y_j$; by this we understand the product of the (effective) power of transmitter $i$ and of the random fading from this node to receiver $y_j$.

The random vectors $\{\mathbf{F}_i\}$ are assumed to be i.i.d. and the components $(F_i^j, j)$ are assumed to be i.i.d. as a generic r.v. denoted by $F$ with mean $1/\mu$ assumed finite.

# Wireless Channel Conditions — Fading

4. $\{\mathbf{F}_i = (F_i^j : j)\}$ where $F_i^j$ denotes the virtual power emitted by node $i$ (provided $e_i = 1$) towards receiver $y_j$; by this we understand the product of the (effective) power of transmitter $i$ and of the random fading from this node to receiver $y_j$.

The random vectors $\{\mathbf{F}_i\}$ are assumed to be i.i.d. and the components $(F_i^j, j)$ are assumed to be i.i.d. as a generic r.v. denoted by $F$ with mean $1/\mu$ assumed finite.

A spacial important case consists in assuming constant emitted power and Rayleigh fading which implies exponential $F$.

# Omnidirectional Path-loss

Select some omnidirectional path-loss (OPL) model $l(\cdot)$. The receiver of node $i$ receives the transmitter located at node $j$ with a power equal to $F_i^j / l(|X_j - y_i|)$, where $|\cdot|$ denotes the Euclidean distance on the plane.

# Omnidirectional Path-loss

Select some omnidirectional path-loss (OPL) model $l(\cdot)$.
The receiver of node $i$ receives the transmitter located at node $j$ with a power equal to $F_i^j/l(|X_j - y_i|)$, where $|\cdot|$ denotes the Euclidean distance on the plane.

An important special case consists in taking

$$l(u) = (Au)^\beta \quad \text{for } A > 0 \text{ and } \beta > 2, \tag{1}$$

which we call in what follows OPL 3.
Note that $1/l(u)$ has a pole at $u = 0$, and thus in particular *is not* correct for small distances. Despite it, the OPL 3 path-loss model (1), we will use it as our default model, because it is precise enough for large enough values of $u$, it simplifies analysis and reveals important scaling laws.

# Coverage (Successful Transmission)

We will say that transmitter $\{X_i\}$ covers its receiver $y_i$ in the reference time slot if

(2) $$\mathrm{SINR}_i = \frac{F_i^i / l(|X_i - y_i|)}{W + I_i^1} \geq T\,,$$

where

- $I_i^1 = \sum_{X_j \in \widetilde{\Phi}^1,\, j \neq i} F_j^i / l(|X_j - y_i|)$ is the shot-noise of $\widetilde{\Phi}^1$, namely, and models the interference,
- $W > 0$ is the external (thermal) noise — a r. v. independent of everything else; default assumption in this talk $W = \text{const}$.
- and where $T$ is some SINR threshold.

We say equivalently that $x_i$ is successfully received by $y_i$.

# Coverage Indicator as a New Mark

Denote by $\boldsymbol{\delta_i}$ the indicator that location $\boldsymbol{y_i}$ is covered by transmitter $\boldsymbol{X_i}$; i.e., that the SINR condition (2) holds. We will consider $\boldsymbol{\delta_i}$ as a new mark of $\boldsymbol{X_i}$.

# Coverage Indicator as a New Mark

Denote by $\delta_i$ the indicator that location $y_i$ is covered by transmitter $X_i$; i.e., that the SINR condition (2) holds. We will consider $\delta_i$ as a new mark of $X_i$.

The marked point process $\widetilde{\Phi}$ enriched by $\delta_i$ is stationary; i.e., its distribution is invariant with respect to any transition. However, in contrast to the original marks $e_i, y_i, \mathbf{F}_i$, given the points of $\Phi$, the random variables $\{\delta_i\}$ are neither independent nor identically distributed.
Indeed, the points of $\Phi$ lying in dense clusters have a smaller probability of coverage than more isolated points due to interference; in addition, the shot noise variables $I_i^1$ make that $\delta_i$'s dependent.

# Coverage Indicator as a New Mark

Denote by $\delta_i$ the indicator that location $y_i$ is covered by transmitter $X_i$; i.e., that the SINR condition (2) holds. We will consider $\delta_i$ as a new mark of $X_i$.

The marked point process $\widetilde{\Phi}$ enriched by $\delta_i$ is stationary; i.e., its distribution is invariant with respect to any transition. However, in contrast to the original marks $e_i, y_i, \mathbf{F}_i$, given the points of $\Phi$, the random variables $\{\delta_i\}$ are neither independent nor identically distributed.
Indeed, the points of $\Phi$ lying in dense clusters have a smaller probability of coverage than more isolated points due to interference; in addition, the shot noise variables $I_i^1$ make that $\delta_i$'s dependent.

Do we have some typical node?

# Coverage Probability for the Typical Node

By probability of coverage of the typical node, given it is a transmitter, we understand

$$\mathsf{P}^{\mathbf{0}}\{\,\boldsymbol{\delta_0 = 1 \,|\, e_0 = 1}\} = \mathsf{E}^{\mathbf{0}}[\boldsymbol{\delta_0 \,|\, e_0 = 1}],$$

where $\mathsf{P}^{\mathbf{0}}$ is the Palm probability associated to the (marked) stationary point process $\widetilde{\boldsymbol{\Phi}}$ and where $\boldsymbol{\delta_0}$ is the mark of the point $\boldsymbol{X_0 = 0}$ a.s. located at the origin $\mathbf{0}$ under $\mathsf{P}^{\mathbf{0}}$.

# Palm Probability — a Reminder

This Palm probability $\mathsf{P}^0$ is derived from the original (stationary) probability $\mathsf{P}$ by the following relation

$$\mathsf{P}^0\{\,\delta_0 = 1 \,|\, e_0 = 1\} = \frac{1}{\lambda_1 |B|} \mathsf{E}\Big[\sum_i \delta_i 1(X_i \in B)\Big]\,;$$

$B$ is an arbitrary subset of the plane and $|B|$ is its surface.

# Palm Probability — a Reminder

This Palm probability $\mathsf{P}^{\mathbf{0}}$ is derived from the original (stationary) probability $\mathsf{P}$ by the following relation

$$\mathsf{P}^{\mathbf{0}}\{\, \boldsymbol{\delta_0 = 1 \,|\, e_0 = 1}\} = \frac{\mathbf{1}}{\boldsymbol{\lambda_1 |B|}} \mathsf{E}\Big[\sum_{\boldsymbol{i}} \boldsymbol{\delta_i 1(X_i \in B)}\Big]\,;$$

$\boldsymbol{B}$ is an arbitrary subset of the plane and $|\boldsymbol{B}|$ is its surface.

Knowing that $\boldsymbol{\lambda_1 |B|}$ is the expected number of transmitters in $\boldsymbol{B}$, the typical node coverage probability is the mean number of transmitters which cover their receivers in any given window $\boldsymbol{B}$ in which we observe our network. Note that this mean is based on a double averaging: a mathematical expectation – over all possible realizations of the network and, for each realization, a spatial averaging – over all nodes in $\boldsymbol{B}$.

# Palm Probability — Ergodic Interpretation

If the underlying point process is ergodic (as it is the cased for our i.m. Poisson p.p. $\tilde{\mathbf{\Phi}}$) the typical node coverage probability can also be interpreted as a spatial average of the number of transmitters which cover their receiver in almost every given realization of the network and large $\boldsymbol{B}$ (tending to the whole plane).

# Palm Probability — Poisson p.p. Case

For a stationary i.m. Poisson p.p. the probability $\mathbf{P}^{\mathbf{0}}$ can easily be constructed due to Slivnyak's theorem: under $\mathbf{P}^{\mathbf{0}}$, the nodes of our Poisson network and their marks follow the distribution

$$\widetilde{\boldsymbol{\Phi}} \cup \{(\boldsymbol{X_0} = \mathbf{0}, \boldsymbol{e_0}, \boldsymbol{y_0}, \mathbf{F_0})\},$$

where $\widetilde{\boldsymbol{\Phi}}$ is the original stationary i.m. Poisson p.p. (i.e. that seen under the original probability $\mathbf{P}$) and $(\boldsymbol{e_0}, \boldsymbol{y_0}, \mathbf{F_0})$ is a new copy of the mark independent of everything else and distributed like all other i.i.d. marks $(\boldsymbol{e_i}, \boldsymbol{y_i}, \mathbf{F}_i)$ of $\widetilde{\boldsymbol{\Phi}}$ under $\mathbf{P}$.

# Palm Probability — Poisson p.p. Case

For a stationary i.m. Poisson p.p. the probability $\mathsf{P}^{\mathbf{0}}$ can easily be constructed due to Slivnyak's theorem:
under $\mathsf{P}^{\mathbf{0}}$, the nodes of our Poisson network and their marks follow the distribution

$$\widetilde{\boldsymbol{\Phi}} \cup \{(\boldsymbol{X_0 = 0, e_0, y_0, \mathrm{F}_0})\},$$

where $\widetilde{\boldsymbol{\Phi}}$ is the original stationary i.m. Poisson p.p. (i.e. that seen under the original probability $\mathsf{P}$) and $(\boldsymbol{e_0, y_0, \mathrm{F}_0})$ is a new copy of the mark independent of everything else and distributed like all other i.i.d. marks $(\boldsymbol{e_i, y_i, \mathrm{F}_i})$ of $\widetilde{\boldsymbol{\Phi}}$ under $\mathsf{P}$.

Under $\mathsf{P}^{\mathbf{0}}$, the node at the origin $\boldsymbol{X_0 = 0}$ is called the the typical node. Note that the typical node, is not necessarily a transmitter; $\boldsymbol{e_0}$ is equal to 1 or 0 with probability $\boldsymbol{p}$ and $\boldsymbol{1-p}$ respectively.

# Back to the Coverage Probability

Denote by $p_c(r, \lambda_1, T) = \mathsf{E}^0[\delta_0 \,|\, e_0 = 1]$ the probability of coverage of the typical node given it is a transmitter.
It follows from the above construction (Slivnyak's theorem) that this probability only depends on the density of effective transmitters $\lambda_1 = \lambda p$, on the distance $r$ and on the SINR threshold $T$; it can be expressed using three independent generic random variables $F, I^1, W$ by the following formula:

$$
\begin{aligned}
p_c(r, \lambda_1, T) &= \mathsf{P}^0\{\, F_0^0 > l(r)T(W + I_0^1) \mid e_0 = 1 \,\} \\
&= \mathsf{P}\{\, F \geq Tl(r)(I^1 + W)\,\}\,. \qquad (3)
\end{aligned}
$$

# Back to the Coverage Probability

Denote by $p_c(r, \lambda_1, T) = \mathsf{E}^0[\delta_0 \,|\, e_0 = 1]$ the probability of coverage of the typical node given it is a transmitter.
It follows from the above construction (Slivnyak's theorem) that this probability only depends on the density of effective transmitters $\lambda_1 = \lambda p$, on the distance $r$ and on the SINR threshold $T$; it can be expressed using three independent generic random variables $F, I^1, W$ by the following formula:

$$\begin{aligned} p_c(r, \lambda_1, T) &= \mathsf{P}^0\{\, F_0^0 > l(r)T(W + I_0^1) \mid e_0 = 1 \,\} \\ &= \mathsf{P}\{\, F \geq Tl(r)(I^1 + W) \,\}\,. \end{aligned} \tag{3}$$

First goal: evaluate $p_c(r, \lambda_1, T)$.

# Coverage Probability with Rayleigh Fading

**Proposition 1** *In Poisson bipolar network model with exponential* $F$

$$p_c(r,\lambda_1,T) = \exp\left\{-\mu WTl(r) - 2\pi\lambda_1 \int_0^\infty \frac{u}{1 + l(u)/(Tl(r))}\,du\right\}.$$

(4)

*In particular if* $W \equiv 0$ *and that the path-loss model (1) is used then*

(5) $$p_c(r,\lambda_1,T) = \exp(-\lambda_1 r^2 T^{2/\beta} K(\beta)),$$

*where*

(6) $$K(\beta) = \frac{2\pi\Gamma(2/\beta)\Gamma(1-2/\beta)}{\beta} = \frac{2\pi^2}{\beta\sin(2\pi/\beta)}.$$

# Proof of Proposition 1

From (3) with exponential $F$ (of parameter $\mu$) by independence we obtain

$$\begin{aligned} p_c(r, \lambda_1, T) &= \exp\Big[-\mu T l(r)(I^1 + W)\Big] \\ &= e^{-\mu W T l(r)} \mathsf{E}[e^{-\mu T l(r) I^1}]\,. \end{aligned}$$

The second factor in the above expression is just the Laplace transform of the Poisson Shot-noise $\mathcal{L}_{I^1}(s)$ evaluated at $s = \mu T l(r)$. It admits the following closed form expression

$$\mathcal{L}_{I^1}(s) = \mathsf{E}[e^{-I^1 s}] = \exp\Big\{-\lambda_1 2\pi \int_0^\infty t\Big(1 - \mathcal{L}_F(s/l(t))\Big)\,\mathrm{d}t\Big\}\,, \tag{7}$$

where $\mathcal{L}_F$ is the Laplace transform of $F$ (here exponential).

# Coverage Probability with General Fading

The results of Proposition 1 can be extended to a general case of $F$ using Plancherel-Parseval theorem. We skip the details for simplicity.

# Application

**Example 1** Assume one wants to operate a network with Aloha MAC where each transmitter-receiver distance is $r$ and a successful transmission is guaranteed with a probability at least $1 - \varepsilon$, where $\varepsilon$ is a predefined QoS. Then, the MAP $p$ parameter of Aloha should be such that $p_c(r, \lambda p, T) = 1 - \varepsilon$. In particular, assuming the path-loss setting (1), one should take

$$p = \min\left(1, \frac{-\ln(1-\varepsilon)}{\lambda r^2 T^{2/\beta} K(\beta)}\right) \approx \min\left(1, \frac{\varepsilon}{\lambda r^2 T^{2/\beta} K(\beta)}\right).$$

For example, for $T = 10$dB [a] and OPL 3 model with $\beta = 4$, $r = 1$, one should take $p \approx \min(1, 0.064\,\varepsilon/\lambda)$.

[a] A positive real number $x$ is $10 \log_{10}(x)$ dB.

# Mean Packet Progress

When trying to maximize the coverage probability $p_c(r, \lambda_1, T)$ one obtains degenerate maximum at $r = 0$.

Assuming that our network relays packets, which have to reach some distant destination nodes, a more meaningful optimization consists in maximizing some transmission distance-based characteristics. For example the mean progress made in a typical transmission:

$$prog(r, \lambda_1, T) = r\mathsf{E}^0[\delta_0] = rp_c(r, \lambda_1, T)\,. \tag{8}$$

# Optimizing Mean Packet Progress

Assume now some given map intensity $\lambda_1 = p\lambda$ of transmitters.

We look for the distance $r$ which maximizes the mean packet progress $prog(r, \lambda_1, T)$.

Obviously small $r$ makes the transmissions more sure but involves more relaying nodes to communicate on some given (large) distance. On the other hand large $r$ reduces the number of hops but might increase the number faults and retransmissions on a given hop.

We denote by

$$\boldsymbol{r}_{\text{max}} = \boldsymbol{r}_{\text{max}}(\boldsymbol{\lambda_1}) = \arg\max_{\boldsymbol{r} \geq \boldsymbol{0}} \boldsymbol{prog}(\boldsymbol{r}, \boldsymbol{\lambda_1}, \boldsymbol{T})$$

the best transmission distance for the density of transmitters $\boldsymbol{\lambda_1}$ whenever such a value exists and is unique.

Let

$$\boldsymbol{\rho} = \boldsymbol{\rho}(\boldsymbol{\lambda_1}) = \boldsymbol{prog}(\boldsymbol{r}_{\text{max}}, \boldsymbol{\lambda_1}, \boldsymbol{T})$$

be the optimal mean packet progress.

**Proposition 2** *In the Poisson bipolar network model (with a general fading), OPL 3 function and* $W = 0$

$$r_{\max}(\lambda) = \frac{\mathit{const}_3}{T^{1/\beta}\sqrt{\lambda_1}},$$

$$\rho(\lambda) = \frac{\mathit{const}_4}{T^{1/\beta}\sqrt{\lambda_1}},$$

*where the constants* $\mathit{const}_3$ *and* $\mathit{const}_4$ *do not depend on* $R, T, \mu$, *provided* $r_{\max}$ *is well defined. If* $F$ *is exponential (i.e. for Rayleigh fading) and* $l(r)$ *given by (1) then* $\mathit{const}_3 = 1/\sqrt{2K(\beta)}$ *and* $\mathit{const}_4 = 1/\sqrt{2eK(\beta)}$.

We can conclude that the optimal distance $r_{\max}(\lambda_1)$ from transmitter to receiver should be of the order of the distance to the nearest neighbor of the transmitter, namely $\sim 1/\sqrt{\lambda}$.

Mean packet progress still does not lead to pertinent optimizations of the network model, as it regards one (typical) transmission.

In particular, $\boldsymbol{prog(r, \lambda p, T)}$ is trivially maximized when $\boldsymbol{p \longrightarrow 0}$, i.e.; when transmissions are very efficient but very rare in the network.

# Density of Successful Transmissions

In fact, we will need some network (social) performance metrics; as e.g., (spatial) density of successful transmissions

$$d_{suc}(r, \lambda_1.T) = \frac{1}{|B|}\mathsf{E}\Big[\sum_i e_i\delta_i \mathbf{1}(X_i \in B)\Big]$$

that, by stationarity, does not depend on the particular choice of set $B$ and by Campbell's formula is equal to

$$d_{suc}(r, \lambda_1.T) = \lambda_1 p_c(r, \lambda_1, T) = \lambda p\, p_c(r, \lambda p, T). \tag{9}$$

# Other Spatial/Social Performance Metrics

The following characteristics can also be expressed in terms of the coverage probability $p_c(r, \lambda_1, T)$.

- spatial density of progress, $d_{prog}$, the mean number of meters progressed by all transmissions taking place per unit surface unit;
- spatial density of Shannon throughput, $d_{throu}$, the mean throughput per unit surface unit;
- spatial density of transport, $d_{trans}$, the mean number of bit-meters transported per second and per unit of surface.

We skip the details.

# Optimal Density of Successful Trans.

In what follows we will be interested in optimizing $d_{suc}(r, \lambda p, T)$ in MAP parameter $p$ of Aloha in order to find a compromise between the average number of concurrent transmissions per unit area and the probability that a given authorized transmission will be successful.

Define

$$\lambda_{\max} = \arg\max_{0 \le \lambda < \infty} d_{suc}(r, \lambda, T)$$

whenever such a value of $\lambda$ exists and is unique.

**Proposition 3** *Under the assumptions of Proposition 1 with* $p = 1$ *the unique maximum of* $d_{suc}(r, \lambda, T)$ *is attained at*

$$\lambda_{\max} = \left(2\pi \int_0^{\infty} \frac{u}{1 + l(u)/(Tl(r))}\, du\right)^{-1},$$

*and the maximal value is equal to*

$$d_{suc}(r, \lambda_{\max}, T) = e^{-1}\lambda_{\max} e^{-\mu W T l(r)}.$$

*In particular, assuming* $W \equiv 0$ *and OPL 3 model (1)*

$$\begin{aligned} \lambda_{\max} &= \frac{1}{K(\beta) r^2 T^{2/\beta}}, \\ d_{suc}(r, \lambda_{\max}, T) &= \frac{1}{eK(\beta) r^2 T^{2/\beta}}. \end{aligned}$$

# Optimal MAP $p$

**Corollary 1** *Under assumptions of Proposition 1 with some given $r$ the value of the MAP $p$ that maximizes the density of successful transmissions is*

$$p_{\max} = \min(1, \lambda_{\mathsf{max}}/\lambda)\,.$$

# Degeneracy of Joint Optimization in $\boldsymbol{p}$ and $\boldsymbol{r}$

Assume for simplicity a $\boldsymbol{W} = \mathbf{0}$ and OPL 3 path-loss (1).

# Degeneracy of Joint Optimization in $p$ and $r$

Assume for simplicity a $W = 0$ and OPL 3 path-loss (1). In Proposition 3 we found that for fixed $r$, the optimal density of successful transmissions $d_{suc}$ is attained when the density of transmitters is equal to $\lambda_1 = \lambda_{\text{max}}$.It is now natural to look for the distance $r$ maximizing the mean progress for the network with this optimal density of transmitters. We obtain

$$\begin{aligned}
\sup_{r\geq 0} prog(r, \lambda_{\text{max}}, T) &= \sup_{r\geq 0} r\, p_c(r, \lambda_{\text{max}}, T) \\
&= \sup_{r\geq 0} r \frac{d_{suc}(r, \lambda_{\text{max}}, T)}{\lambda_{\text{max}}} \\
&= \sup_{r\geq 0} r \frac{\text{const}_2}{\text{const}_1} = \infty
\end{aligned}$$

and thus the optimal choice of $r$ consists in taking $r = \infty$, and consequently $\lambda_{\text{max}} = 0$.

# Bipolar Model — Conclusions

Simple yet not simplistic model. Allows for

- closed form expression for the successful transmission probability.
- pertinent optimization of many network performance metrics in $\boldsymbol{r}$ or in $\boldsymbol{p}$.

A better receiver model is needed to study the joint optimization in the transmission distance and in Aloha MAP $\boldsymbol{p}$.
We will propose such models in
BEYOND THE POISSON BIPOLAR NETWORK MODEL.

# OPPORTUNISTIC ALOHA

# Idea

In the basic Spatial Aloha scheme, each node tosses a coin to access the medium independently of the fading variables. It is clear that something more clever can be done by combining the random selection of transmitters with the occurrence of good channel conditions.

The general idea of Opportunistic Aloha is to select the nodes with the channel fading larger than a certain threshold as transmitters in the reference time slot.

# Model Modification

Opportunistic Aloha can be described by $\widetilde{\Phi} = \{(X_i, \theta_i, y_i, \mathbf{F}_i)\}$, where $\{(X_i, y_i, \mathbf{F}_i)\}$ is as in the basic Poisson Bipolar Model (1)–(4), with item (2) replaced by:

(2') Opportunistic Aloha principle: The MAC indicator $e_i$ of node $i$ ($e_i = 1$ if node $i$ is allowed to transmit and 0 otherwise) is the following function of the channel condition to its receiver $F_i^i$: $e_i = \mathbf{1}(F_i^i > \theta_i)$, where $\{\theta_i\}$ are new random i.i.d. marks, with a generic mark denoted by $\theta$.

# Model Modification

Opportunistic Aloha can be described by $\widetilde{\Phi} = \{(X_i, \theta_i, y_i, \mathbf{F}_i)\}$, where $\{(X_i, y_i, \mathbf{F}_i)\}$ is as in the basic Poisson Bipolar Model (1)–(4), with item (2) replaced by:

(2') Opportunistic Aloha principle: The MAC indicator $e_i$ of node $i$ ($e_i = 1$ if node $i$ is allowed to transmit and 0 otherwise) is the following function of the channel condition to its receiver $F_i^i$: $e_i = 1(F_i^i > \theta_i)$, where $\{\theta_i\}$ are new random i.i.d. marks, with a generic mark denoted by $\theta$.

Special cases of interest are that where $\theta$ is constant, and that where $\theta$ is exponential with parameter $\nu$. (allows for close-form expression for the coverage probability).

# Model Modification

Opportunistic Aloha can be described by $\widetilde{\Phi} = \{(X_i, \theta_i, y_i, \mathrm{F}_i)\}$, where $\{(X_i, y_i, \mathrm{F}_i)\}$ is as in the basic Poisson Bipolar Model (1)–(4), with item (2) replaced by:

(2') Opportunistic Aloha principle: The MAC indicator $e_i$ of node $i$ ($e_i = 1$ if node $i$ is allowed to transmit and 0 otherwise) is the following function of the channel condition to its receiver $F_i^i$: $e_i = 1(F_i^i > \theta_i)$, where $\{\theta_i\}$ are new random i.i.d. marks, with a generic mark denoted by $\theta$.

Special cases of interest are that where $\theta$ is constant, and that where $\theta$ is exponential with parameter $\nu$. (allows for close-form expression for the coverage probability).
As in Aloha $\{e_i\}$ are again i.i.d. marks of the point process $\widetilde{\Phi}$, which now depend on $\{\theta_i, F_i^i\}$.

In Opportunistic Aloha the set of transmitters is a Poisson p.p. $\Phi^1$ (different from that in plain Aloha) with intensity $\lambda$P$(F > \theta)$ (where $F$ is a generic $F_i^i$ and $\theta$ a generic $\theta_i$, with $(F, \theta)$ independent).

Thus in order to compare Opportunistic Aloha to the plain Aloha one can take $p =$ P$\{ F > \theta \}$, where $p$ is the MAP of plain Aloha, which guarantees the same density of (selected) transmitters at a given time slot.

Note that the virtual power emitted by any node to its receiver, given it is selected by Opportunistic Aloha has for law the distribution of $F$ conditional on $F > \theta$. Below, we will denote by $F_\theta$ a random variable with this law. However, by

independence of $(F_i^j, j)$, the virtual powers $F_i^j$, $j \neq i$, toward other receivers are still distributed as $F$. Consequently, the interference $I_i^1$ experienced at any receiver has exactly the same distribution as in plain Aloha.

# Coverage Probability

Hence, the coverage probability for the typical transmitter can be expressed by the following three independent generic random variables

$$\widehat{p}_c(r, \lambda_1, T) = \mathsf{P}\{\, F_\theta > Tl(r)(I^1 + W)\,\}\,,$$

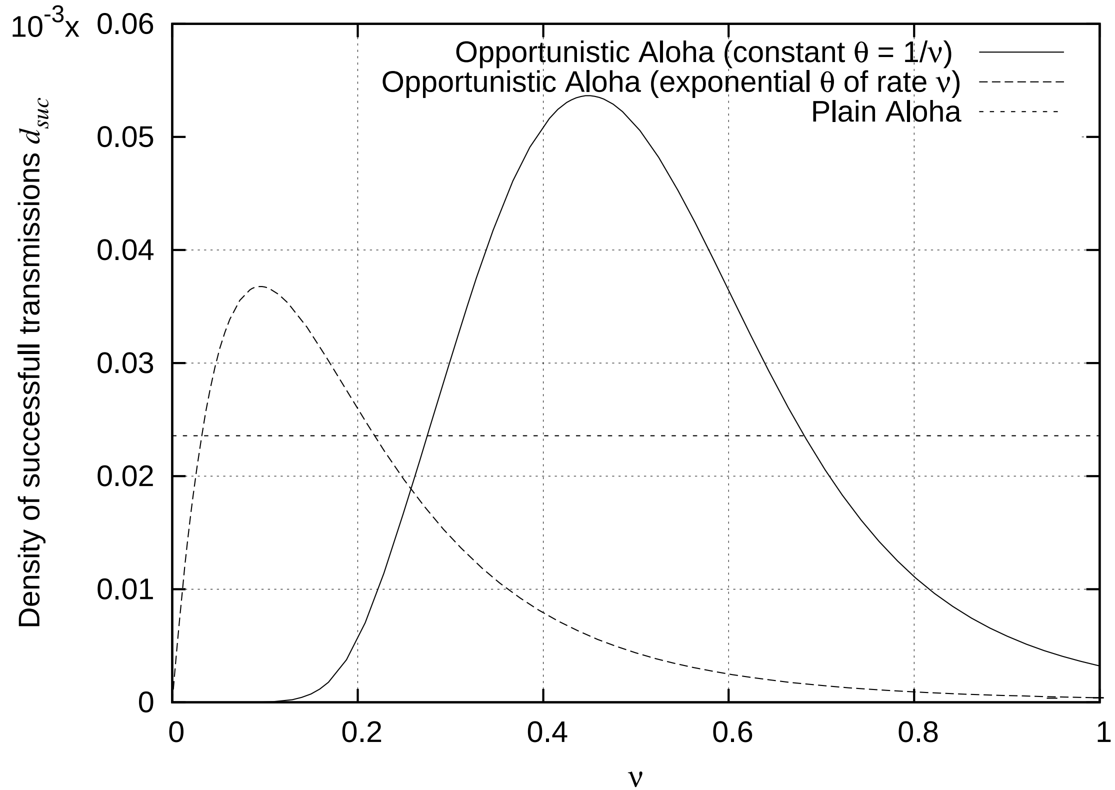
where $I^1$ is the generic shot-noise generated by Poisson p.p. with intensity $\lambda_1 = \mathsf{P}\{F > \theta\}\lambda$ and (non-conditioned) fading variables $F_j$ (as in (3)).

**Proposition 4** *Assume Rayleigh fading (exponential* $\boldsymbol{F}$ *with parameter* $\boldsymbol{\mu}$*), exponential distribution of the threshold* $\boldsymbol{\theta}$ *with parameter* $\boldsymbol{\nu}$*, and (for simplicity)* $\boldsymbol{W \equiv 0}$ *and the OPL 3 model (1). Then*

$$
\begin{aligned}
&\widehat{p}_c(r, \lambda_1, \nu) \\
&= \frac{\mu+\nu}{\nu} \exp\{-\lambda_1 T^{2/\beta} r^2 K(\beta)\} \\
&\quad - \frac{\mu}{\nu} \exp\Big\{-\lambda_1 \left(\frac{(\mu+\nu)T}{\mu}\right)^{2/\beta} r^2 K(\beta)\Big\},
\end{aligned}
$$

*with* $\boldsymbol{\lambda_1 = \lambda\nu/(\mu+\nu)}$*.*

# Opportunistic vs plain Aloha




The density of successful transmissions $d_{suc}$ of Opportunistic Aloha for various choices of $\theta$. The propagation model is (1). We assume Rayleigh fading with mean $1$ and $W = 0$, $\lambda = 0.001$, $T = 10$dB, $r = \sqrt{1/\lambda}$ and $\beta = 4$. For comparison the constant value $\lambda_{\text{max}} p_c(r, \lambda_{\text{max}})$ of plain Aloha is plotted.

# BEYOND THE POISSON BIPOLAR NETWORK MODEL

# Locations of Receivers — Network Split

We will consider now a few possible scenarios where the receiver of a given transmitter is not necessarily at distance $r$, as in the Poisson bipolar model considered so far.

# Locations of Receivers — Network Split

We will consider now a few possible scenarios where the receiver of a given transmitter is not necessarily at distance $r$, as in the Poisson bipolar model considered so far.

In what follows we will always assume that the transmitters choose their (actual) receivers in the original set $\Phi$ of nodes of the network. More precisely, nodes $\Phi^0 = \Phi \setminus \Phi^1$ not allowed to access the medium (those with $e_i = 0$) form the set of potential receivers.

# Locations of Receivers — Network Split

- We will consider now a few possible scenarios where the receiver of a given transmitter is not necessarily at distance $r$, as in the Poisson bipolar model considered so far.
- In what follows we will always assume that the transmitters choose their (actual) receivers in the original set $\Phi$ of nodes of the network. More precisely, nodes $\Phi^0 = \Phi \setminus \Phi^1$ not allowed to access the medium (those with $e_i = 0$) form the set of potential receivers.
- The virtual powers $F_i^j$ have now the following modified interpretation:

(4') $F_i^j$ denotes the virtual power (modified by the channel condition) emitted by node $i$ (provided $e_i = 1$) towards node $j$ in $\Phi^0$.

# Aloha and Network Split

In case of Aloha MAC, at a given time slot, the transmitters $\Phi^1$ and the potential receivers $\Phi^0$ form two independent Poisson p.p's. with intensities, respectively, $\lambda_1 = \lambda p$ and $\lambda_0 = \lambda(1-p)$.

# MAC and Routing Interplay

In practice, some routing algorithmspecifies the receivers(s) (relay node(s)) of each given transmitter. The joint design and analysis of MAC and routing is a difficult task even if we assume the simplest MAC (Aloha).

# MAC and Routing Interplay

In practice, some routing algorithmspecifies the receivers(s) (relay node(s)) of each given transmitter. The joint design and analysis of MAC and routing is a difficult task even if we assume the simplest MAC (Aloha).

We propose models based on simplifying assumptions on the routing layer. Specifically, we will assume that each transmitter selects its receiver according to one of the following two routing principles:

- as close by as possible,
- the most distant successful receiver (in a given direction).

# Nearest Receiver (NR) Model

Assume that each transmitter selects the nearest point in $\mathbf{\Phi^0}$ of nodes which do not emit a the considered time slot. Formally, this consists in replacing the assumption concerning the distribution of $\{\boldsymbol{y_i}\}$ in (3) of the definition of the Poisson bipolar model by the following one

(3') The receiver $\boldsymbol{y_i}$ of the transmitter $\boldsymbol{X_i} \in \mathbf{\Phi}$ is the point $\boldsymbol{y_i} = \boldsymbol{Y_i^*} = \arg\min_{\boldsymbol{Y_i} \in \mathbf{\Phi^0}}\{|\boldsymbol{Y_i} - \boldsymbol{X_i}|\}$.

# Nearest Receiver (NR) Model

Assume that each transmitter selects the nearest point in $\Phi^0$ of nodes which do not emit a the considered time slot. Formally, this consists in replacing the assumption concerning the distribution of $\{y_i\}$ in (3) of the definition of the Poisson bipolar model by the following one

(3') The receiver $y_i$ of the transmitter $X_i \in \Phi$ is the point $y_i = Y_i^* = \arg\min_{Y_i \in \Phi^0} \{|Y_i - X_i|\}$.

The nearest receiver $y_i$ is almost surely well defined for all $i$ (due to homogeneous Poisson assumption) however some additional specifications is required on what happens if two or more transmitters pick the same receiver. In what follows that either the receivers are capable of multi-receptions or the SINR threshold $T > 1$, which (by a simple algebraic argument) excludes such multi-receptions.

# Coverage Probability in NR Model

Since $\Phi^1$ and $\Phi^0$ are independent Poisson p.p.s it is easy to calculate the probability $p_c(\text{NR}, \lambda_1, T)$ of successful reception for the typical emitter in the NR model conditioning on of the distance from the origin to the nearest point of $\Phi^0$.

# Coverage Probability in NR Model

Since $\Phi^1$ and $\Phi^0$ are independent Poisson p.p.s it is easy to calculate the probability $p_c(\mathrm{NR}, \lambda_1, T)$ of successful reception for the typical emitter in the NR model conditioning on of the distance from the origin to the nearest point of $\Phi^0$.

**Proposition 5** *The coverage probability in the NR model is equal to*

$$p_c(NR, \lambda_1, T) = 2\pi\lambda_0 \int_0^\infty r \exp(-\lambda_0 \pi r^2) p_c(r, \lambda_1, T)\, dr \,,$$

*where $p_c(r, \lambda_1, T)$ is the probability of coverage at distance $r$ evaluated for the Poisson bipolar model under the same assumptions except for the receiver location.*

# Coverage Probability in NR Model

Since $\Phi^1$ and $\Phi^0$ are independent Poisson p.p.s it is easy to calculate the probability $p_c(\mathrm{NR}, \lambda_1, T)$ of successful reception for the typical emitter in the NR model conditioning on of the distance from the origin to the nearest point of $\Phi^0$.

**Proposition 5** *The coverage probability in the NR model is equal to*

$$p_c(NR, \lambda_1, T) = 2\pi\lambda_0 \int_0^\infty r \exp(-\lambda_0 \pi r^2) p_c(r, \lambda_1, T)\, dr\,,$$

*where $p_c(r, \lambda_1, T)$ is the probability of coverage at distance $r$ evaluated for the Poisson bipolar model under the same assumptions except for the receiver location.*

Other characteristics (mean progress, density of successful transmission) can also be evaluated in a similar way.

# Density of Successful Trans. in NR Model

We are now interested in optimizing $d_{suc}(NR, \lambda p, T) = \lambda p \, p_c(NR, \lambda p, T)$ in MAP parameter $p$ of Aloha.

Assume for simplicity $W \equiv 0$, OPL 3 model (1) and exponential $F$. Then by Propositions and 1 we have

$$d_{suc}(NR, \lambda p, T) = \frac{\lambda p(1-p)}{(1-p) + pT^{2/\beta} K(\beta)/\pi} \, .$$

# Density of Successful Trans. in NR Model

We are now interested in optimizing $d_{suc}(NR, \lambda p, T) = \lambda p\, p_c(NR, \lambda p, T)$ in MAP parameter $p$ of Aloha.
Assume for simplicity $W \equiv 0$, OPL 3 model (1) and exponential $F$. Then by Propositions  and 1 we have

$$d_{suc}(NR, \lambda p, T) = \frac{\lambda p(1-p)}{(1-p) + pT^{2/\beta}K(\beta)/\pi}.$$

The above function (of $p$) attains the unique non-degenerate maximum for some $0 < p^* < 1$, which gives a compromise between the average number of concurrent transmissions per unit area and the probability that a given authorized transmission will be successful in the NR model.
Moreover, the optimal tuning of $p^*$ does not depend on the node density $\lambda$.

# Most Distant Successful Receiver (MDSR)

We need to extend the i.m. p.p. $\tilde{\Phi}$ by introducing marks $\mathrm{d}_i$; the latter are i.i.d. unit vectors in $\mathbb{R}^2$ representing directions in which the nodes aim to send packets. We assume following assumption concerning the choice of receivers:

(3”) The receiver $y_i$ of the transmitter $X_i \in \Phi$ is its SINR neighbor that maximizes the effective progress of the transmitted packet in the direction $\mathrm{d}_i$

$$y_i = \operatorname*{arg\,max}_{X_j \in V(X_i)} \left\{ \langle X_j - X_i, \mathrm{d}_i \rangle \right\},$$

By the set SINR neighbors $V(X_i)$ of $X_i$ we understand subset of potential receivers $\Phi^0$, which successfully capture the packet transmitted by $X_i$ when $X_i$ transmits at the given time slot, plus $X_i$ itself.

A major difference between this opportunistic mechanism and all previously considered cases is that it has much more chance to lead to a successful transmission. Indeed, it suffices that at least one receiver with a positive abscissa in a given direction successfully captures the packet. For this reason we can call this scheme an opportunistic selection of the receiver.

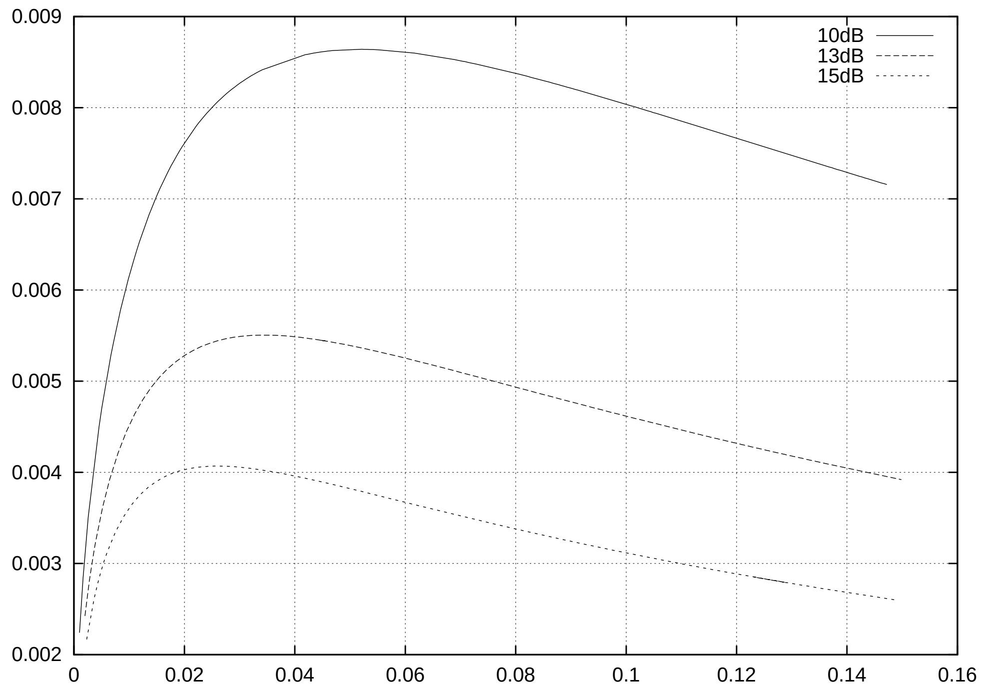
# Analysis of the MDSR Model

Unfortunately, calculating the probability of successful transmission in MDSR model is more tricky. We have only bounds.

We can prove that, as for NR model, the density of progress $d_{prog}(\lambda, p)$ attains a non-degenerate maximum for some MAP $0 < p^* < 1$. In case of $W \equiv 0$, OPL 3 model (1) and exponential $F$, the optimal tuning of $p^*$ does not depend on the node density $\lambda$.

# Optimal tuning of MAP $p$ in MDSR Model

$d_{\widetilde{prog}}(\lambda, p)$




$p$

Density of modified progress $d_{\widetilde{prog}}(\lambda, p)$ (lower bound of the "true" one) in the MDSR model. Here, $\beta = 3$, $\lambda = 1$ and with $T = \{10, 13, 15\}$dB (curves from top to bottom).

# LOCAL DELAYS IN ALOHA — TOWARD SPACE–TIME ANALYSIS

Our aim is to discuss the mean time to transmit a packet under Aloha, which will be referred to as the local delay in what follows.

This will require the introduction of the underlying time-space structure.

# Time-Space Scenario in Poisson Network

We add time-dimension to the basic Poisson Bipolar model and its extensions.

We assume a sequence of time slots $n = 0, 1, \ldots$ to which all nodes are synchronized.

# Time-Space Scenario in Poisson Network

We add time-dimension to the basic Poisson Bipolar model and its extensions.

We assume a sequence of time slots $n = 0, 1, \ldots$ to which all nodes are synchronized.

Poisson p.p. $\Phi = \{X_i\}$ modeling locations of nodes remains unchanged in time.

# Time-Space Scenario in Poisson Network

We add time-dimension to the basic Poisson Bipolar model and its extensions.
We assume a sequence of time slots $n = 0, 1, \ldots$ to which all nodes are synchronized.
Poisson p.p. $\Phi = \{X_i\}$ modeling locations of nodes remains unchanged in time.

Marks $e_i = e_i(n)$, $\mathbf{F}_i = \mathbf{F}_i(n)$ representing, respectively, MAC status and virtual power (channel quality) are re-sampled independently, identically, in each time slot $n$.

# Time-Space Scenario in Poisson Network

We add time-dimension to the basic Poisson Bipolar model and its extensions.
We assume a sequence of time slots $n = 0, 1, \ldots$ to which all nodes are synchronized.
Poisson p.p. $\Phi = \{X_i\}$ modeling locations of nodes remains unchanged in time.

Marks $e_i = e_i(n)$, $\mathbf{F}_i = \mathbf{F}_i(n)$ representing, respectively, MAC status and virtual power (channel quality) are re-sampled independently, identically, in each time slot $n$.

Noise $W$ is random but static in time; i.e., $W = W(n)$.

# Time-Space Scenario in Poisson Network

We add time-dimension to the basic Poisson Bipolar model and its extensions.
We assume a sequence of time slots $n = 0, 1, \ldots$ to which all nodes are synchronized.
Poisson p.p. $\Phi = \{X_i\}$ modeling locations of nodes remains unchanged in time.

Marks $e_i = e_i(n)$, $\mathbf{F}_i = \mathbf{F}_i(n)$ representing, respectively, MAC status and virtual power (channel quality) are re-sampled independently, identically, in each time slot $n$.

Noise $W$ is random but static in time; i.e., $W = W(n)$.

The above scenario of fast fading, static noise is an example; other scenarios can be considered.

# Time-Space Scenario, cont.

Locations of receivers $y_i$, depend on the model:

- in the basic Poisson bipolar model remain constant in time at a distance $r$,
- in the NR and MDSR model are found in each tome slot according to the corresponding role. Note that the set of potential receivers depend on $e_i(n)$ and thus vary in time.

# Mean Local Delay

By the mean local delay $\ell = \mathsf{E}^{\mathbf{0}}[L]$ we understand the expected number of time slots $L$ required for the typical node to successfully transmit one packet.

It turns out that this mean time very much depends on the receiver model which is chosen. In several “reasonable” cases surprisingly(?) $\ell = \infty$. In fact this is not a surprise (to be explained).

# Wireless Contention Phase Transition

For some models, we observe the following wireless contention phase transition: $\ell < \infty$ or $\ell = \infty$ depending on the choice of model parameters, as the transmission distance $r$ or MAP $p$ or mean noise $\mathsf{E}[W]$ or mean signal power $\mathsf{E}[F] = 1/\mu$.
We explain it on the simplest example of the basic Poisson Bipolar model.

# Mean Local Delay in Poisson Bipolar Model

Given repartition of nodes $\Phi$ and noise $W$ (and all other random model components that do not change in time, if any) under Palm probability $\mathsf{P}^0$, the events $E_n = \{$node $X_0 = 0$ successfully transmits at time $n\}$ $n \geq 0$ are independent (Bernoulli) trials with probability of success

$$p(\Phi, W) = p\,\mathsf{P}\{\, F > l(r)T(W + I^1(\Phi)\}\,,$$

where $I^1(\Phi)$ is the "conditional realization" of the shot noise given node positions $\Phi$ and noise $W$.

# Mean Local Delay in Poisson Bipolar Model

Given repartition of nodes $\Phi$ and noise $W$ (and all other random model components that do not change in time, if any) under Palm probability $\mathsf{P}^0$, the events $E_n = \{\text{node } X_0 = 0 \text{ successfully transmits at time } n\}$ $n \geq 0$ are independent (Bernoulli) trials with probability of success

$$p(\Phi, W) = p\,\mathsf{P}\{\, F > l(r)T(W + I^1(\Phi)\}\,,$$

where $I^1(\Phi)$ is the “conditional realization” of the shot noise given node positions $\Phi$ and noise $W$.
The number of trials before the first success is thus geometric r.v. with parameter $p(\Phi, W)$. Its mean is know to be $1/p(\Phi, W)$. De-conditioning w.r.t. $\Phi$ we obtain

$$\mathsf{E}^0[\ell] = \mathsf{E}^0\Big[\frac{1}{p(\Phi, W)}\Big]\,.$$

# Local Delay in Poisson Bipolar Model, cont.

**Proposition 6** *In the Poisson Bipolar model with fast Rayleigh fading and static noise we have*

$$\ell = \frac{\mathsf{E}[e^{\mu W T l(r)}]}{p} \exp\Big\{ 2\pi p \lambda \int_0^\infty \frac{v T l(r)}{l(v) + (1-p) T l(r)} \, dv \Big\}.$$

# Local Delay in Poisson Bipolar Model, cont.

**Proposition 6** *In the Poisson Bipolar model with fast Rayleigh fading and static noise we have*

$$\ell = \frac{\mathsf{E}[e^{\mu W T l(r)}]}{p} \exp\Big\{2\pi p\lambda \int_0^\infty \frac{vTl(r)}{l(v)+(1-p)Tl(r)}\, dv\Big\}.$$

The $\exp\{\ldots\}$ in the above formula is finite for any $p, T, r$ for path loss model OPL 3. However this is typically not the case for $\mathsf{E}[e^{\mu W T l(r)}]$, which is exponential moment of $W$ of order $Tl(r)\mu$. Often this moment is finite only for some sufficiently small value of $Tl(r)\mu$. This may give rise to the wireless contention phase transition due to noise limitations (see the next slide).

# Example of Phase Transition for $\ell$

Assume exponential noise $W$ of parameter $\nu$. Then $\mathsf{E}[e^{\mu W T l(r)}] = \frac{\nu}{\nu - T l(r)} < \infty$ for $T l(r) \mu < \nu$ and infinite for $T l(r) \mu > \nu$. Thus whe have (in this fast Rayleigh fading, static exponential noise case) the following incarnations of the wireless contention phase transition: all other parameters fixed, there is a threshold on

# Example of Phase Transition for $\ell$

Assume exponential noise $W$ of parameter $\nu$. Then E$[e^{\mu W T l(r)}] = \frac{\nu}{\nu - Tl(r)} < \infty$ for $Tl(r)\mu < \nu$ and infinite for $Tl(r)\mu > \nu$. Thus whe have (in this fast Rayleigh fading, static exponential noise case) the following incarnations of the wireless contention phase transition: all other parameters fixed, there is a threshold on

- distance $r$ to the receiver below which the mean local delay is finite and above which it is infinite;

# Example of Phase Transition for $\ell$

Assume exponential noise $W$ of parameter $\nu$. Then $\mathsf{E}[e^{\mu W T l(r)}] = \frac{\nu}{\nu - T l(r)} < \infty$ for $T l(r) \mu < \nu$ and infinite for $T l(r) \mu > \nu$. Thus whe have (in this fast Rayleigh fading, static exponential noise case) the following incarnations of the wireless contention phase transition: all other parameters fixed, there is a threshold on

- distance $r$ to the receiver below which the mean local delay is finite and above which it is infinite;
- mean transmission power $1/\mu$ above which the mean local delay is finite and below which it is infinite;

# Example of Phase Transition for $\ell$

Assume exponential noise $W$ of parameter $\nu$. Then $\mathsf{E}[e^{\mu W T l(r)}] = \frac{\nu}{\nu - T l(r)} < \infty$ for $T l(r) \mu < \nu$ and infinite for $T l(r) \mu > \nu$. Thus whe have (in this fast Rayleigh fading, static exponential noise case) the following incarnations of the wireless contention phase transition: all other parameters fixed, there is a threshold on

- distance $r$ to the receiver below which the mean local delay is finite and above which it is infinite;
- mean transmission power $1/\mu$ above which the mean local delay is finite and below which it is infinite;
- mean thermal noise power $1/\nu$ below which the mean local delay is finite and above which its is infinite.

# Example of Phase Transition for $\ell$

Assume exponential noise $W$ of parameter $\nu$. Then $\mathsf{E}[e^{\mu W T l(r)}] = \frac{\nu}{\nu - T l(r)} < \infty$ for $T l(r)\mu < \nu$ and infinite for $T l(r)\mu > \nu$. Thus whe have (in this fast Rayleigh fading, static exponential noise case) the following incarnations of the wireless contention phase transition: all other parameters fixed, there is a threshold on

- distance $r$ to the receiver below which the mean local delay is finite and above which it is infinite;
- mean transmission power $1/\mu$ above which the mean local delay is finite and below which it is infinite;
- mean thermal noise power $1/\nu$ below which the mean local delay is finite and above which its is infinite.
- SINR threshold $T$, below which the mean local delays is finite and above which it is infinite.

# Phase Transition for $\ell$ — Interpretation

Under the above assumptions for the Poisson bipolar model

- individually each node has (some) positive transmission probability (depending on its position in the network), finite local delay and thus positive temporal throughput.
- However, for an important fraction of nodes this delay is so large that the spatial average of the local delay is infinite.

# Phase Transition for $\ell$ — Interpretation

Under the above assumptions for the Poisson bipolar model

- individually each node has (some) positive transmission probability (depending on its position in the network), finite local delay and thus positive temporal throughput.
- However, for an important fraction of nodes this delay is so large that the spatial average of the local delay is infinite.

One observes similar phenomena for NR and MDSR models, (even for constant $W$ — thus having all exponential moments). The wireless contention phase transition in these models are due to the randomness of the distance to the receiver.

# Local Delay and Restart Algorithm

The observed problem of $\ell$ possibly being infinite is a consequences of the fact that the local delays typically are heavy-tailed r.v.s, and this even if all variables generating our model are light-tailed.

# Local Delay and Restart Algorithm

The observed problem of $\ell$ possibly being infinite is a consequences of the fact that the local delays typically are heavy-tailed r.v.s, and this even if all variables generating our model are light-tailed.

It is primarily because of the SINR coverage logic, where one transmits full packets at time slots when the receiver is covered at the required SINR and where one wastes all the other time slots. For a “classical” example of such behavior see the so called “Restart algorithm” [Jelenkovic, Asmussen], where it is explained why we have heavy tails and possibly infinite means.

# Beyond the SINR Coverage / Restart Algo.

Adaptive coding offers the possibility of breaking the coverage/Restart logic: it gives up with minimal requirements on SINR and it provides some non-null throughput at each time slot, where this throughput depends on the current value of the SINR e.g. via Shannon's formula.

# Beyond the SINR Coverage / Restart Algo.

Adaptive coding offers the possibility of breaking the coverage/Restart logic: it gives up with minimal requirements on SINR and it provides some non-null throughput at each time slot, where this throughput depends on the current value of the SINR e.g. via Shannon's formula.

In this latter case, one can show that the mean local delay (given the throughput prescribed by the Shannon's formula) is finite under very mild assumptions.

# Beyond the SINR Coverage / Restart Algo.

Adaptive coding offers the possibility of breaking the coverage/Restart logic: it gives up with minimal requirements on SINR and it provides some non-null throughput at each time slot, where this throughput depends on the current value of the SINR e.g. via Shannon's formula.

In this latter case, one can show that the mean local delay (given the throughput prescribed by the Shannon's formula) is finite under very mild assumptions.

See more details in the monograph *"Stochastic Geometry and Wireless Networks" by F. Baccelli and B.B. to appear in* Frontiers and Trends in Networking *Now Publishers.*

# STOCHASTIC ANALYSIS — CONCLUDING REMARKS

In stochastic modeling we are not interested in one particular configuration of nodes but in some “ensemble” of possible configurations, which are observed with some “chances”. (example: Poisson repartition of nodes).
Problem: when/what statistical assumptions are valid?

In stochastic modeling we are not interested in one particular configuration of nodes but in some "ensemble" of possible configurations, which are observed with some "chances". (example: Poisson repartition of nodes). Problem: when/what statistical assumptions are valid?

Stochastic analysis gives universal answers in the form of mathematical expectations which can and have to interpreted in terms of space/time averages.

In stochastic modeling we are not interested in one particular configuration of nodes but in some “ensemble” of possible configurations, which are observed with some “chances”. (example: Poisson repartition of nodes). Problem: when/what statistical assumptions are valid?

Stochastic analysis gives universal answers in the form of mathematical expectations which can and have to interpreted in terms of space/time averages.

For the stochastic analysis to be feasible one needs to construct simple yet not simplistic models. In particular: we have considered simplified version of MAC (slotted Aloha), without nodes mobility, and routing “caricatures”.