Efficient Advert Assignment

Search

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Ad **Efficient Advert Talk**

@google.com

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Ads ⓘ

Joint work with

Frank Kelly
university of cambridge

Peter Key
microsoft research

This Talk

We give a simple allocation and pricing mechanism

whose Nash equilibrium solves a very large optimization problem

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whose Nash equilibrium solves a very large optimization problem

Very Large = over the infinite results of a search engine.

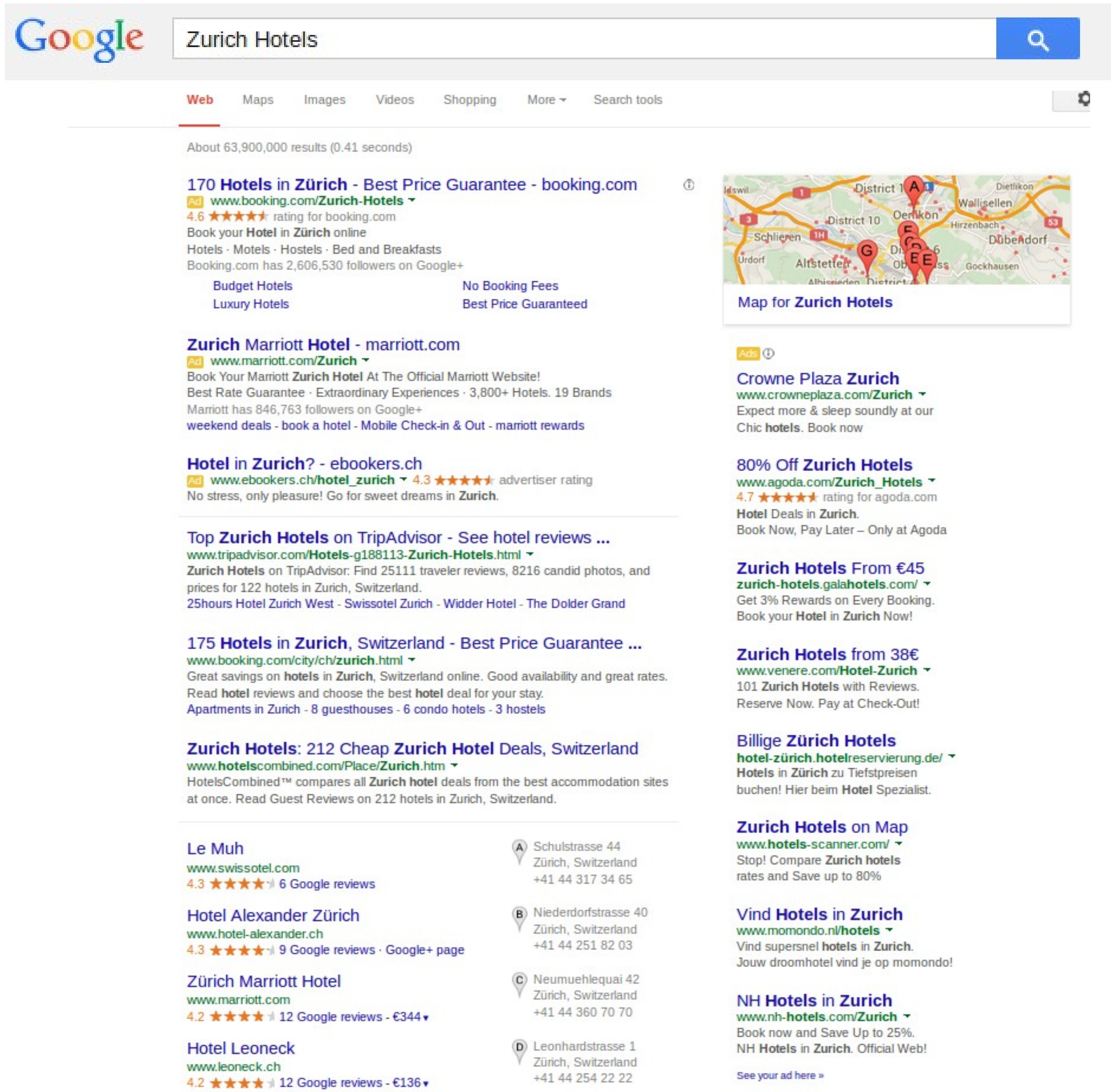
Outline

- A Introduction to Sponsored Search
  - Bids, Impressions, Click-Through Rate, Advertizers, Platform
- Auction or Optimize?
  - Our Mechanism, Generalized 2nd Price, VCG Mechanism, Decomposition.
- Our Results
  - Main Theorem, Implementations
- Further Results and Extensions
  - Dynamics, Multivariate Utilities, General Page Layouts, Budgets.

Introduction to Sponsored Search

Sponsored Search

Sponsored Search



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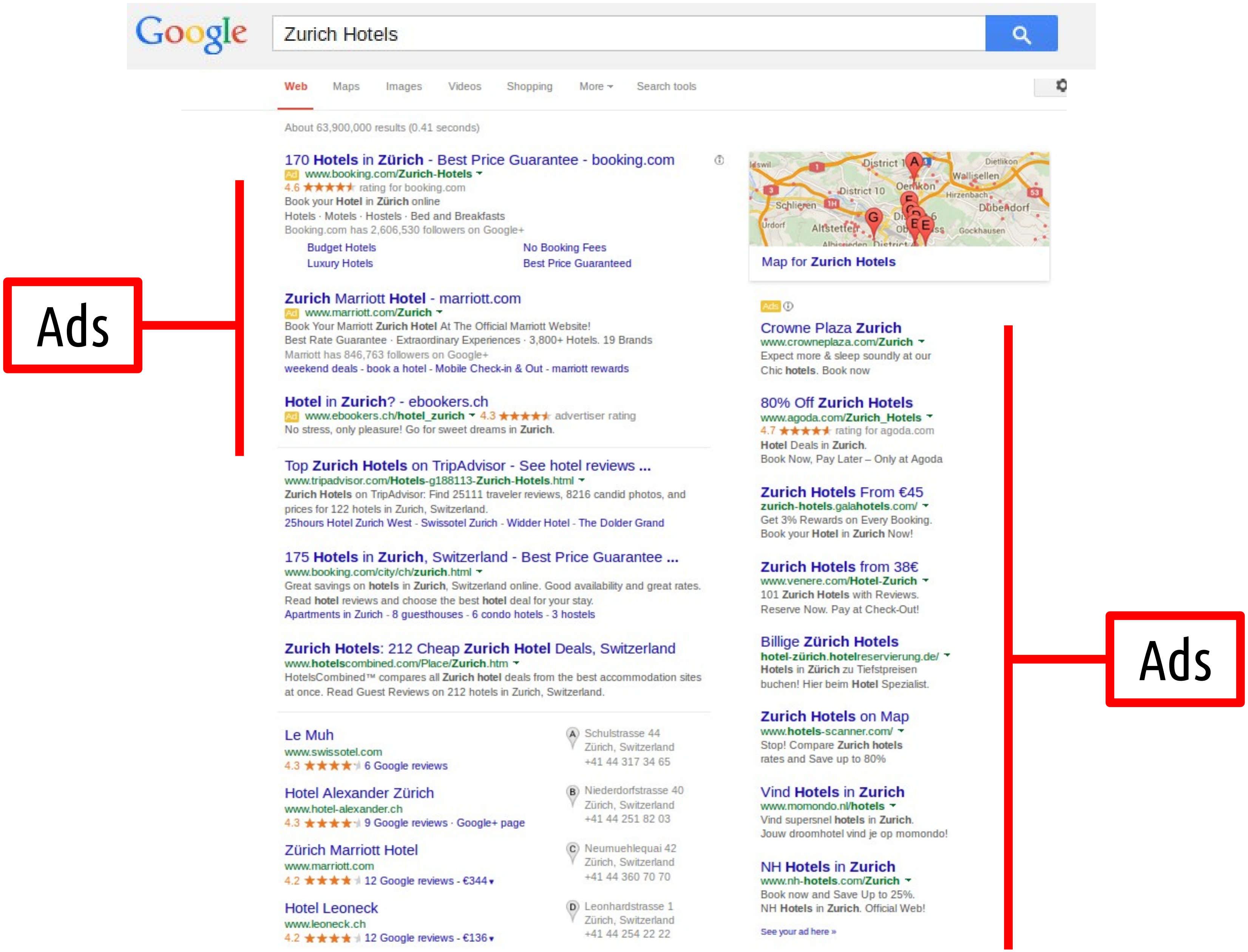

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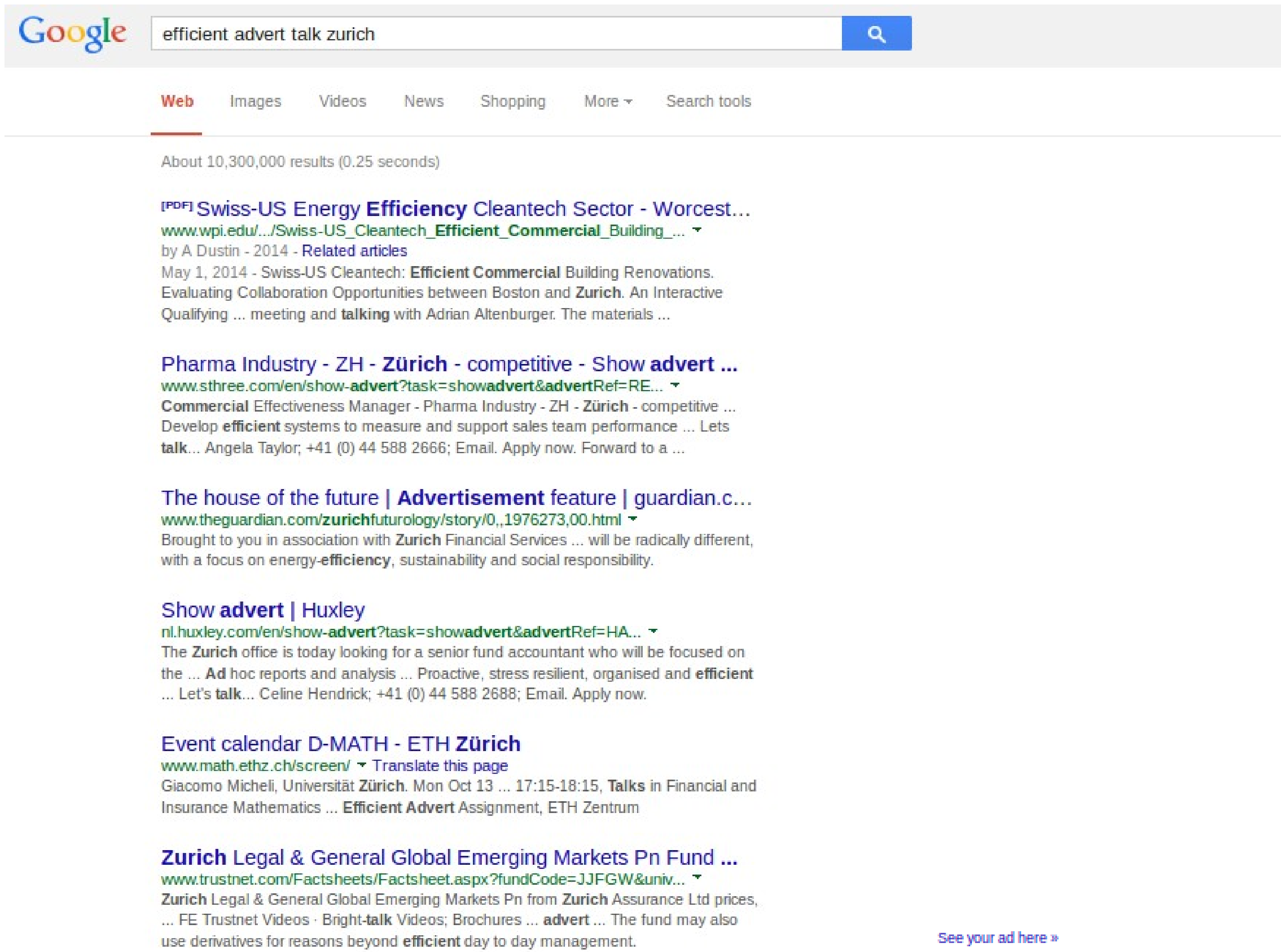

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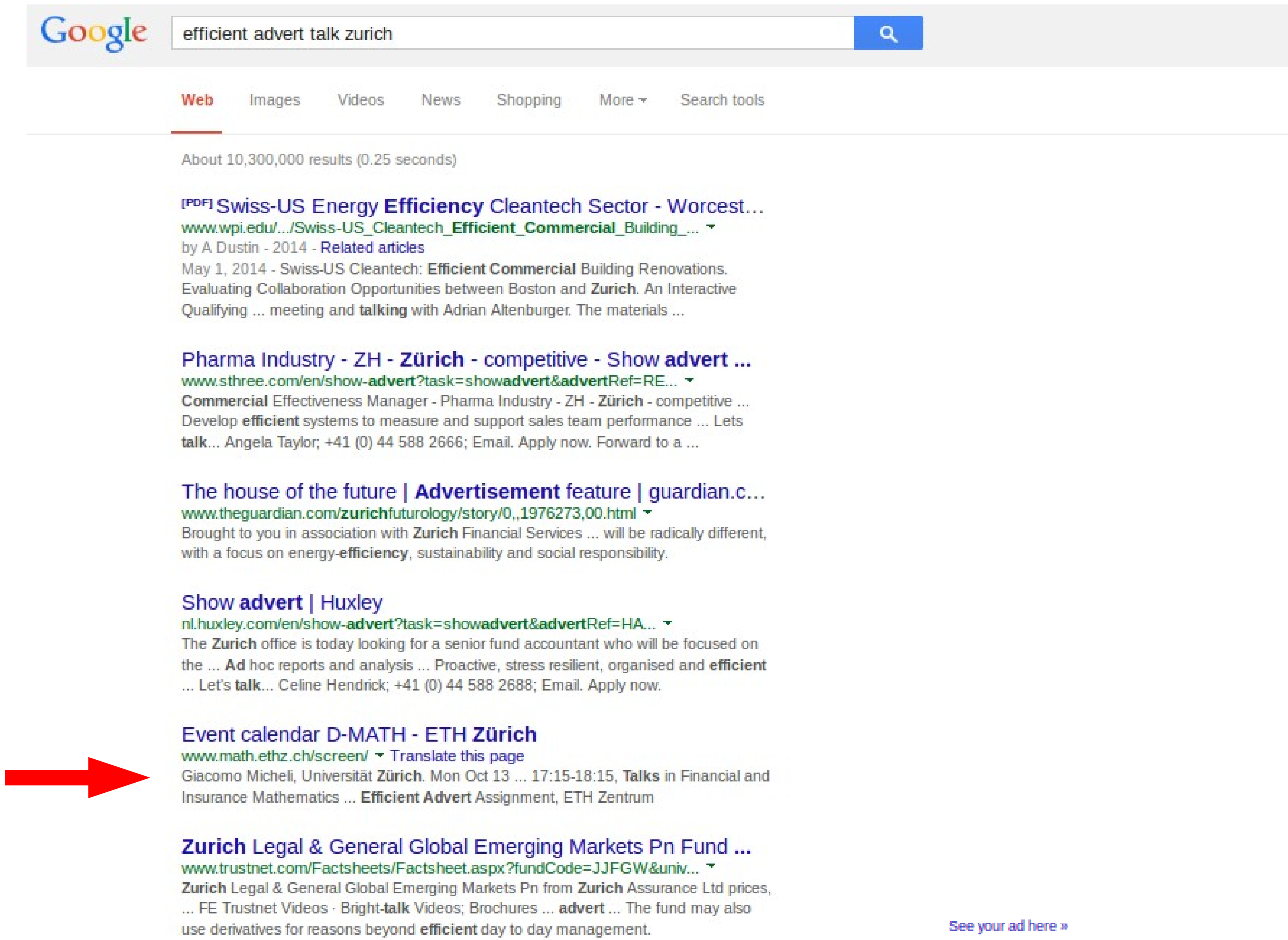

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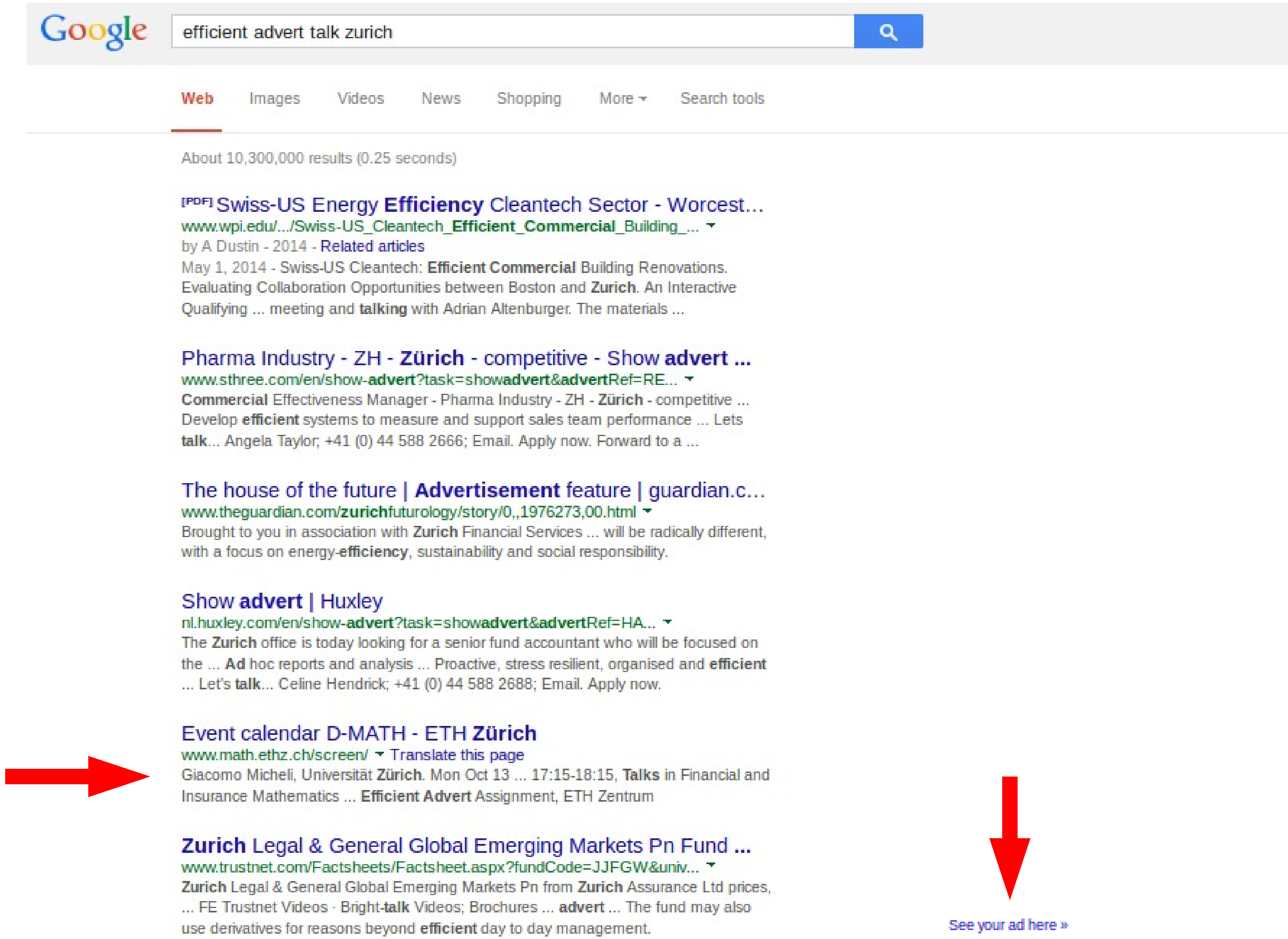

Sponsored Search

		Ad group	Status	Default Max. CPC	Clicks	Impr.	CTR	Avg. CPC	Cost	Avg. Pos.
		Ad Group #1	Eligible	€0.15	0	0	0.00%	€0.00	€0.00	0.0

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Bid

Sponsored Search

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Sponsored Search

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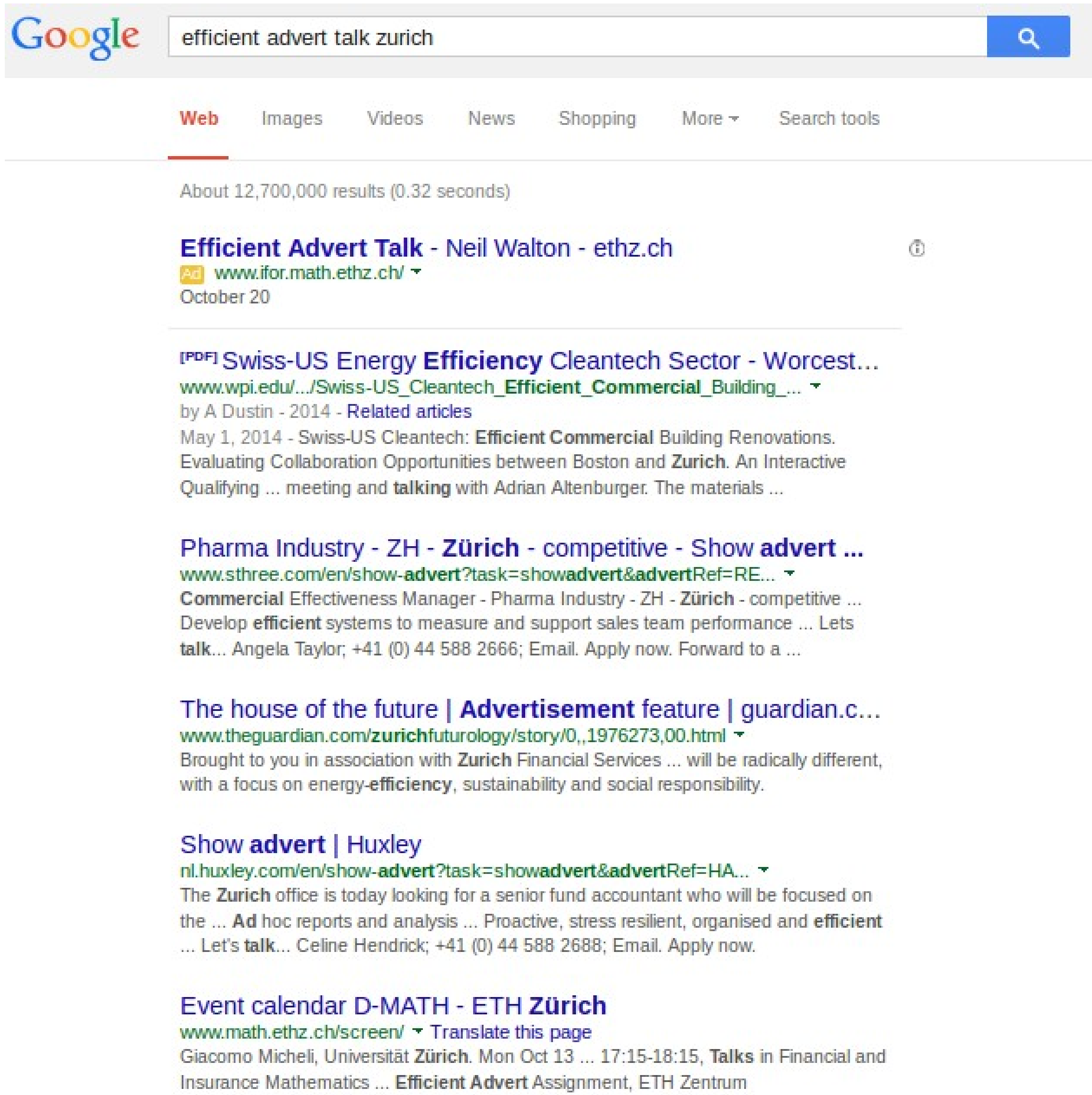
Bid

Price per click

Sponsored Search



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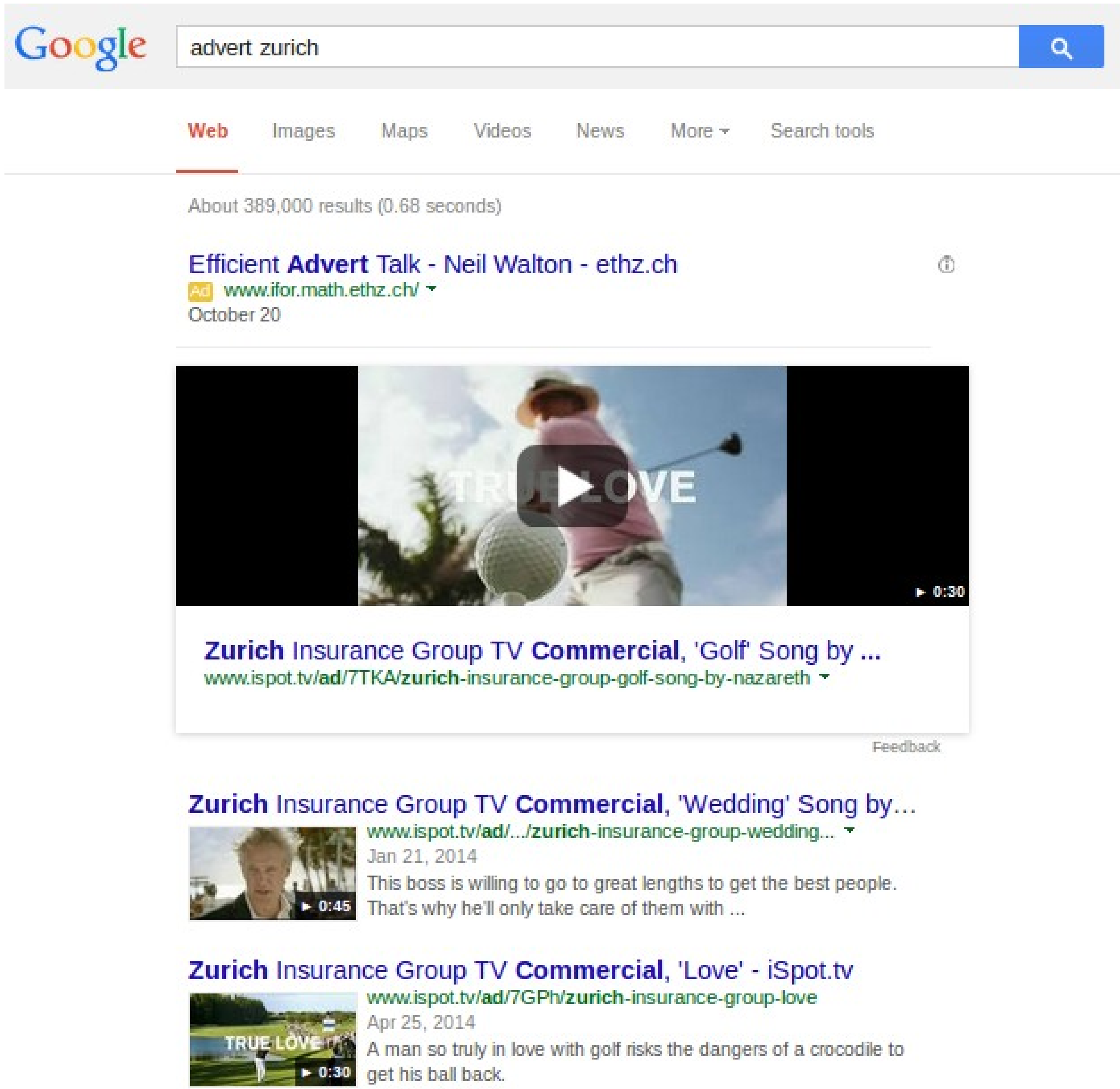
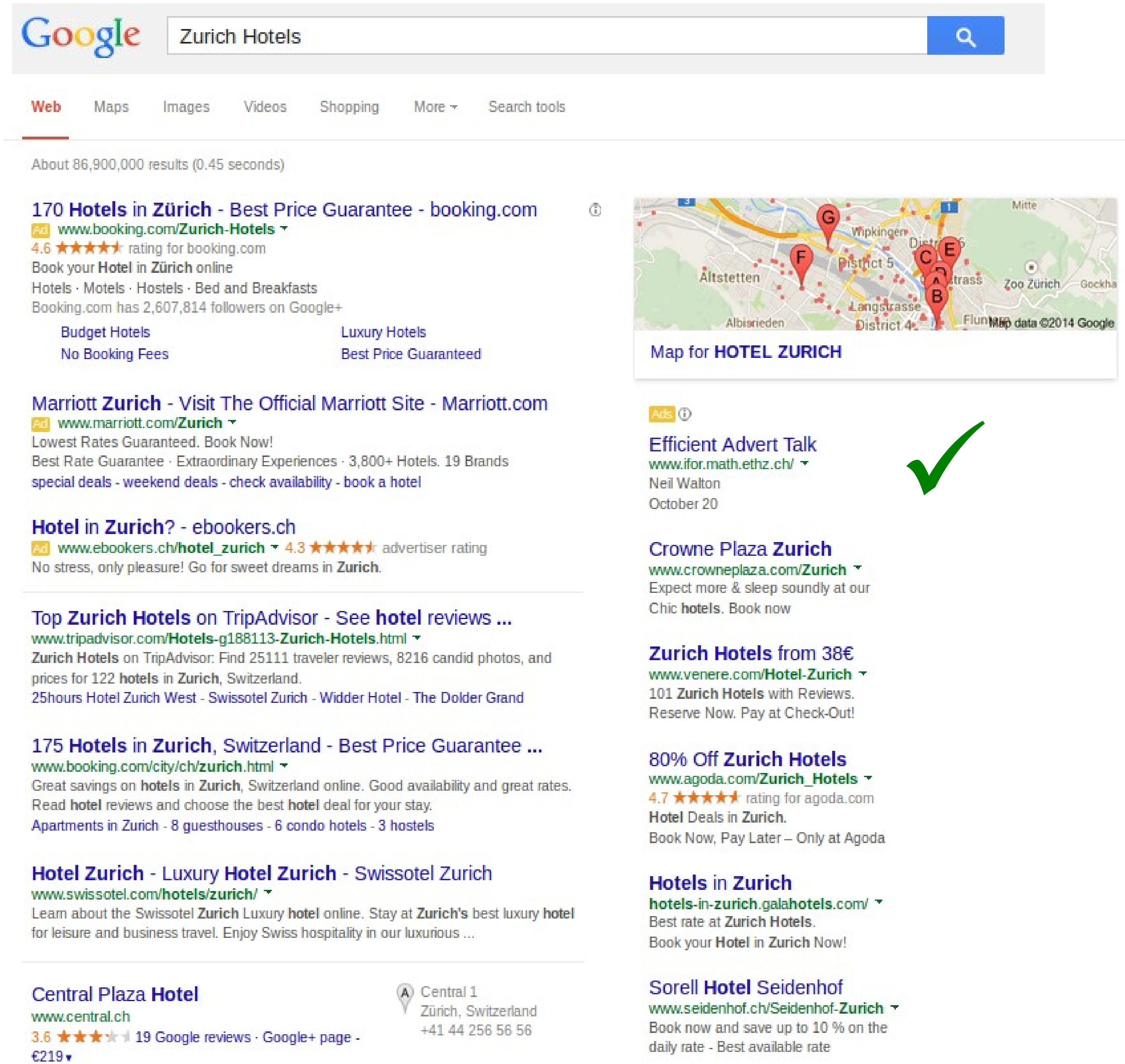


Click

Sponsored Search

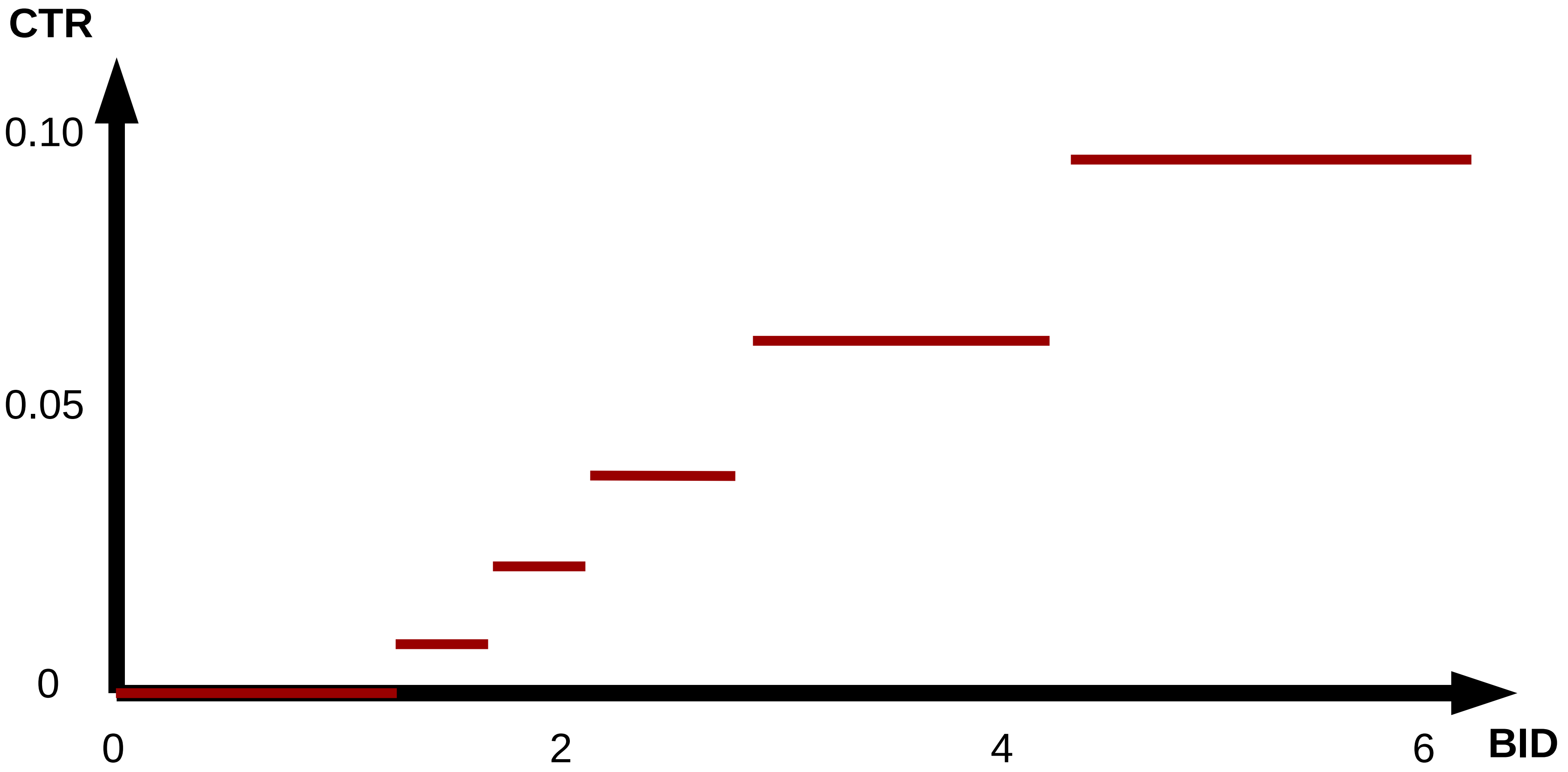


Variability in Sponsored Search

A **one-shot** or repeated auction:

eg. “Dominos Pizza”

(exact match)

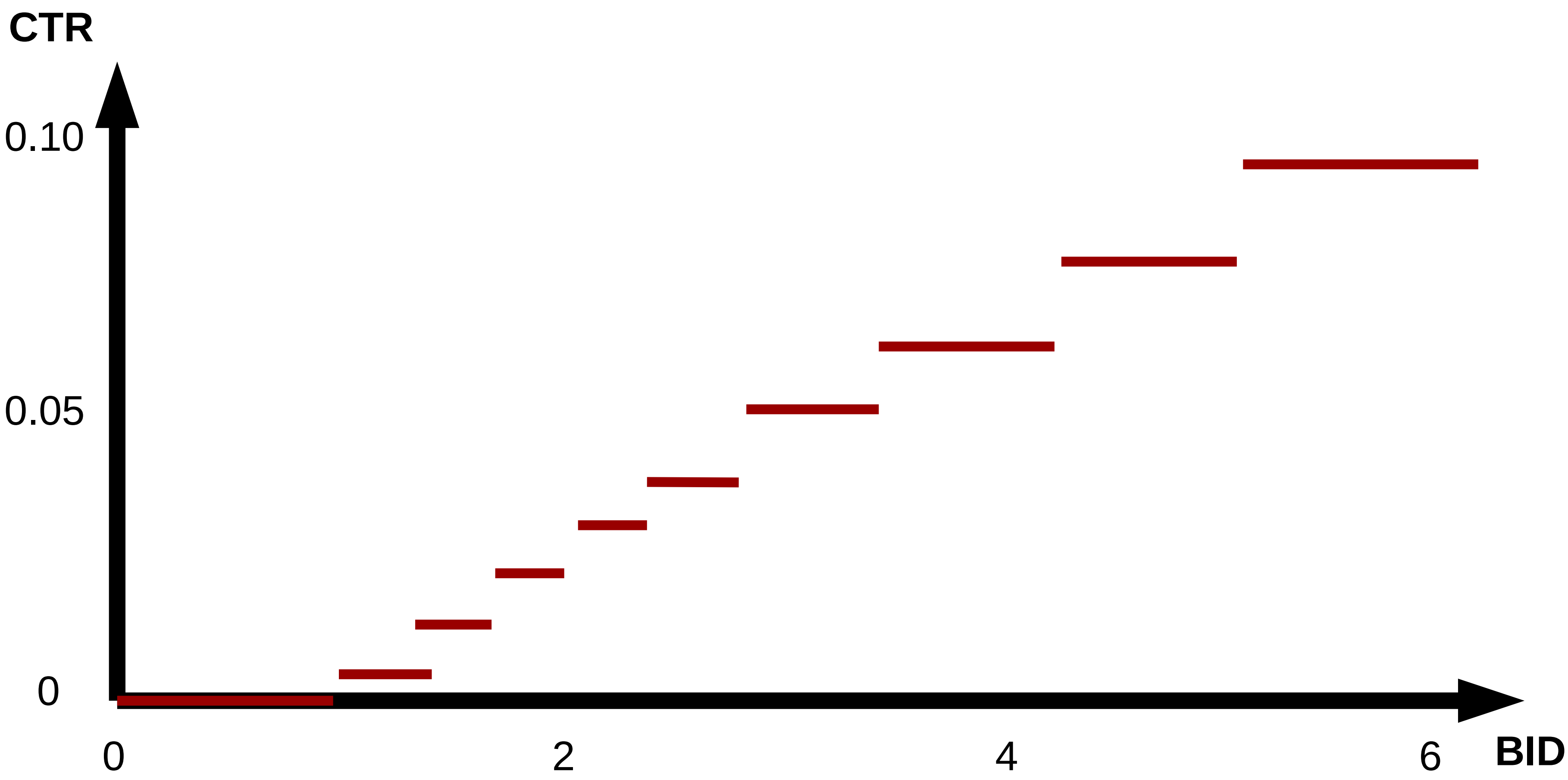


Variability in Sponsored Search

A **mixture** of auctions:

eg. “ … Dominos … Pizza …”

(phrase match)

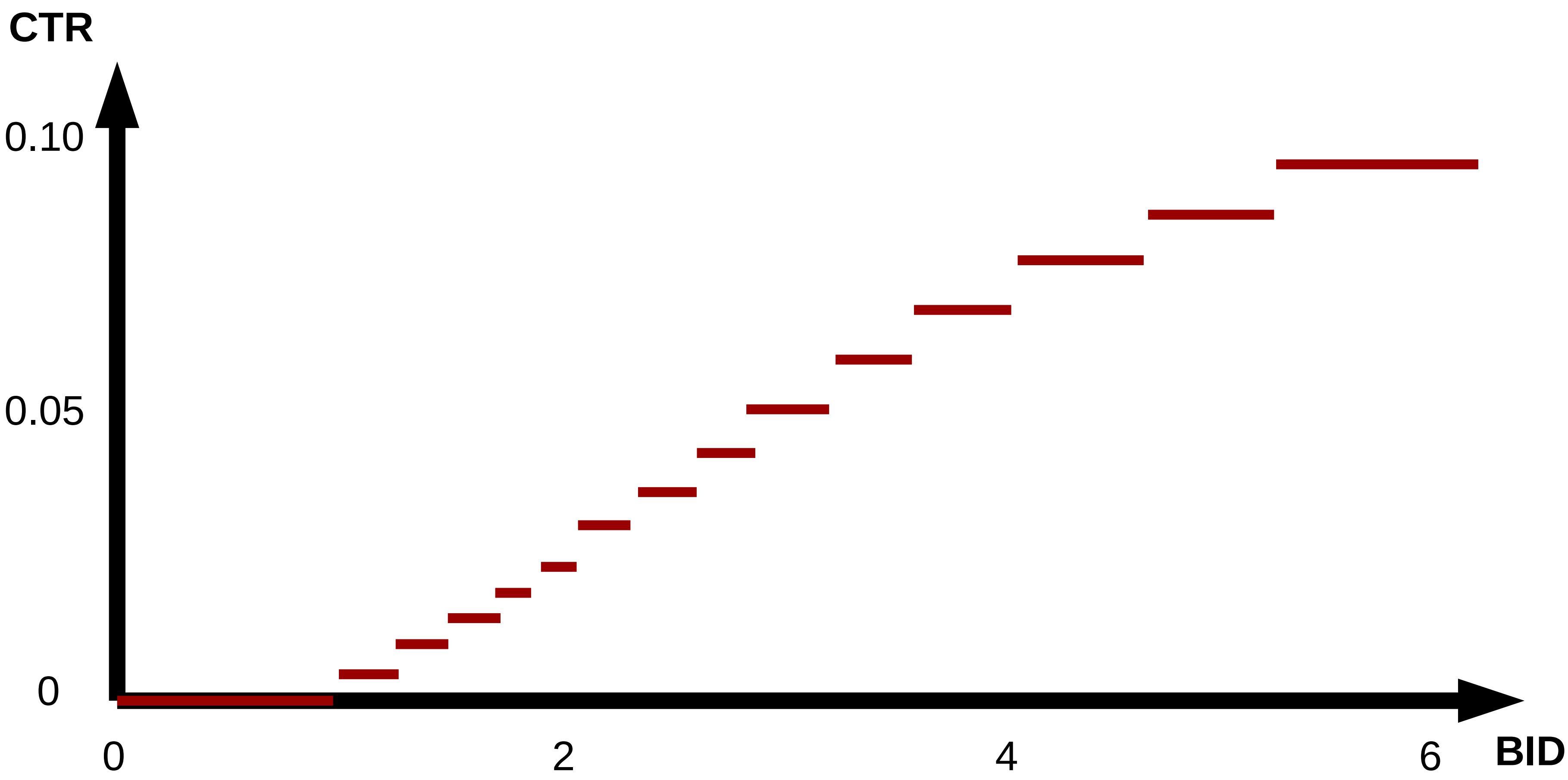


Variability in Sponsored Search

A **bigger mixture** of auctions:

eg. " ... Delivery ... Pizza ..."

(broad match)

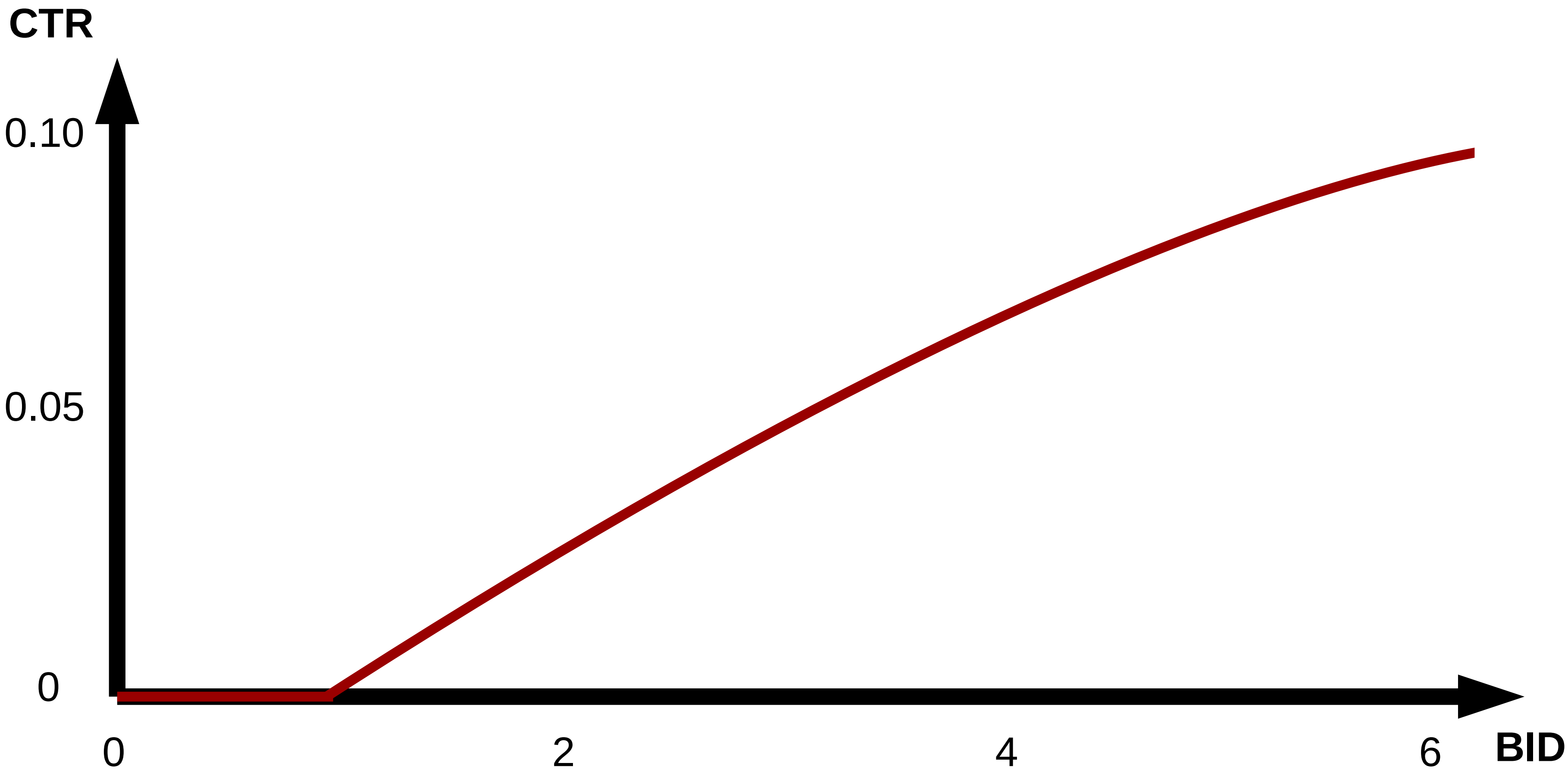


Variability in Sponsored Search

A **continuum** of auctions:

eg. " … Delivery … Pizza …" + **location** + **time**

(broad match) + **Searcher**



Information and Temporal Asymmetry

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The Searcher

Information and Temporal Asymmetry

The Searcher

The Platform

Google

bing

YAHOO!

Information and Temporal Asymmetry

The Searcher

The Platform

Google

bing

YAHOO!

The Advertiser

Information and Temporal Asymmetry

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The Advertiser

Search Distribution

$$\mathbb{P}_\tau$$

Information and Temporal Asymmetry

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The Platform

When a search occurs

Click-Through p_{il}^{τ}

Assignment x_{il}^{τ}

The Advertiser

Information and Temporal Asymmetry

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When a search occurs

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The Advertiser

Receives average information

Click-Through $\mathbb{E}_\tau \sum_l p_{il}^{\tau} x_{il}^{\tau}$

Assignment $\mathbb{E}_\tau x_{il}^{\tau}$

Information and Temporal Asymmetry

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Search Distribution

$$\mathbb{P}_\tau$$

Unknown To everyone

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Advertiser doesn't

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Auction or Optimize?

Two Auctions

τ – Search Type

λ_i – Bid of ad i

p_{il}^{τ} – Click-Through ad i slot l

$y_i^{\tau}(\lambda)$ – CTR given bids

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Auction 2:

Two Auctions

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Auction 2:

Assign max matching

$$\max \sum_i \sum_l \lambda_i p^{\tau}_{il} x_{il}$$

$$\text{s.t.} \sum_i x^{\tau}_{il} \leq 1, \sum_l x^{\tau}_{il} \leq 1$$

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$$\lambda_i \left(1 - \frac{y_i^{\tau}(\mu_i)}{y_i^{\tau}(\lambda_i)}\right)$$

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Generalized 2nd Price

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A VCG Auction

Immediate Advantages

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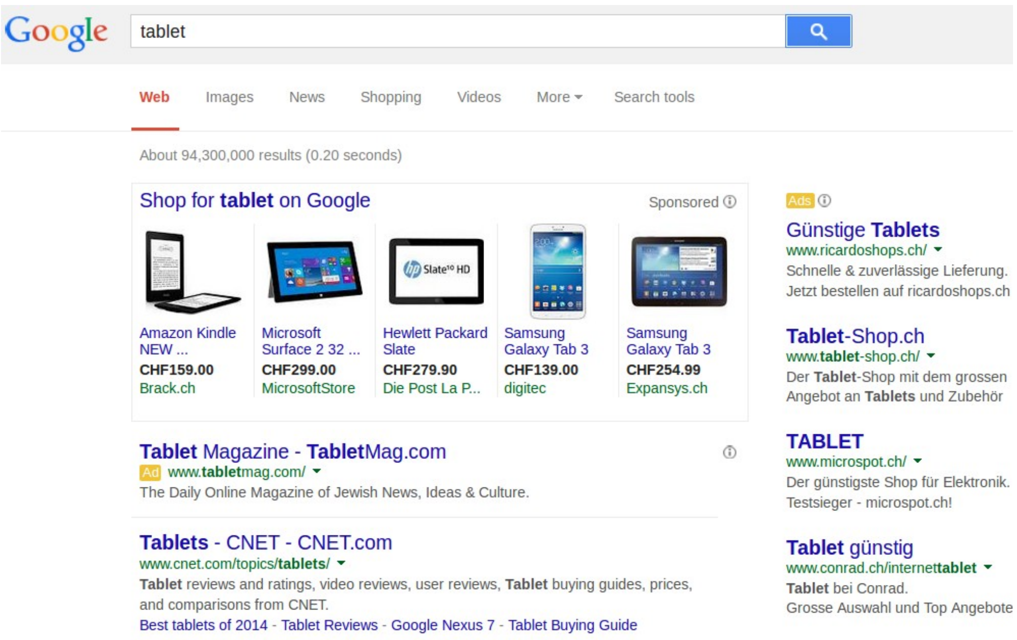
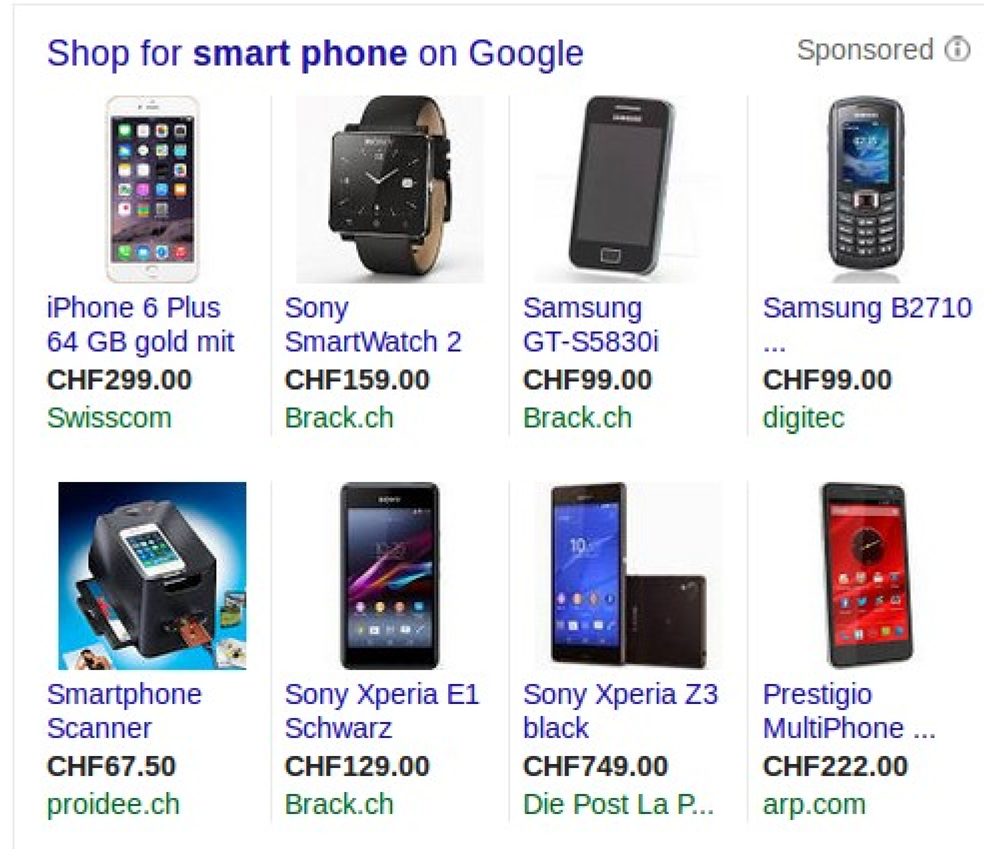
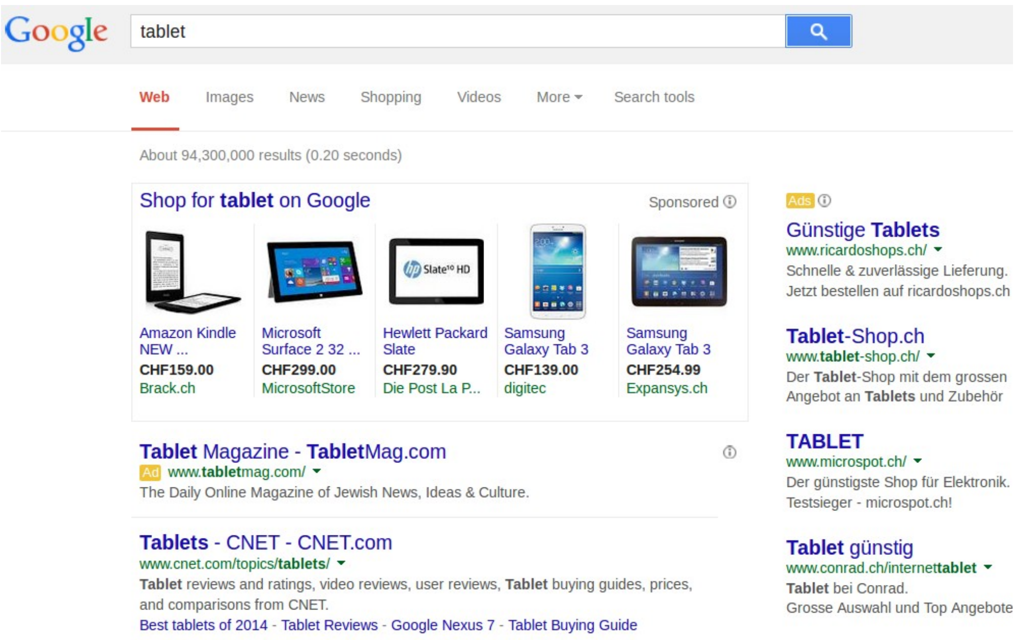
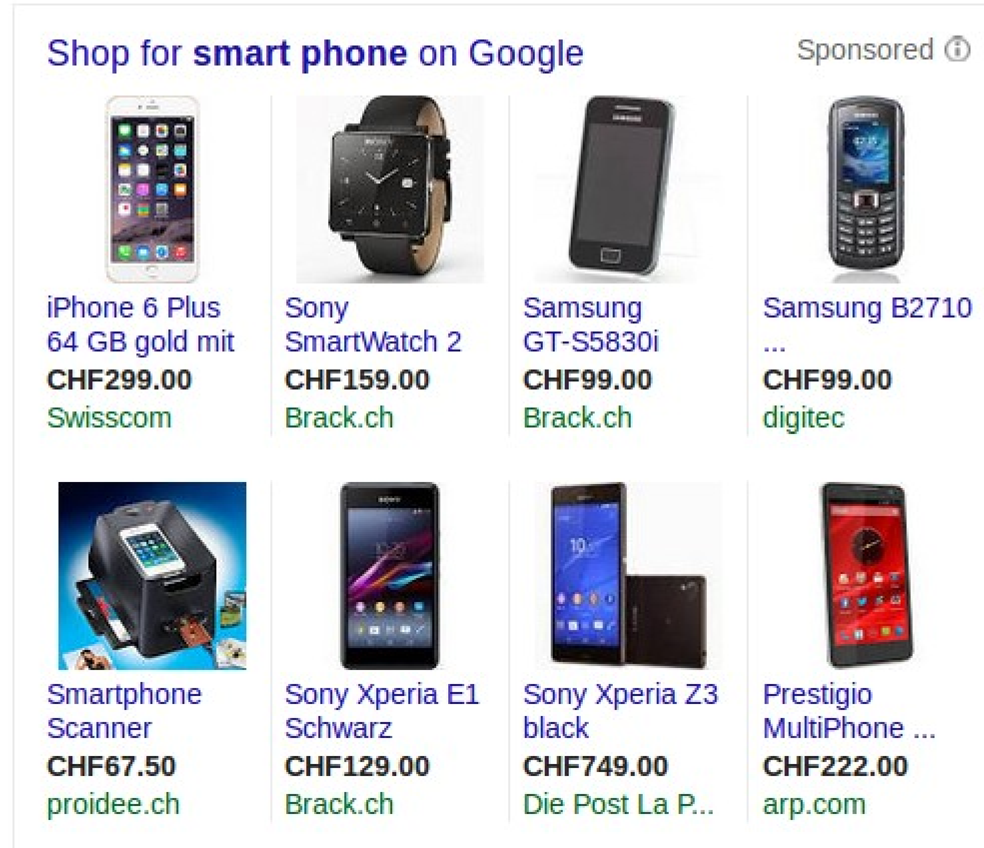




Immediate Advantages





There is not ordering of Ads!

Generalized 2nd Price Breaks down. Our mechanism and results hold true.

Is it GSP or VCG?

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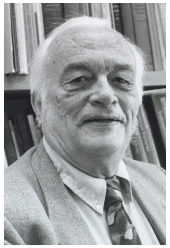
A. The VCG Mechanism…



Vickrey

Clark

Groves

The VCG Mechanism

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- Advertiser's utilities $U_i(\cdot)$

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The VCG Mechanism

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Maximize Value

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Everyone else's value

Value without you there

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Equilibrium Advertizer: $$\max_{V_i} \{U_i(y_i^*) - \pi_i\}$$

The VCG Mechanism

Theorem

The VCG mechanism has a dominate strategies equilibrium that is:

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The VCG Mechanism

Theorem

The VCG mechanism has a dominate strategies equilibrium that is:

- Incentive compatible

$$\text{bids are truthful:} \quad V_i(\cdot) = U_i(\cdot)$$

- Efficient

$$\text{allocation is optimal:} \quad y^* \in \text{argmax}_{y \in \mathcal{A}} \sum_i U_i(y_i)$$

Directly Applying VCG

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Pros

1. Result applies in very **general** settings
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This talk: We deal with these issue by appropriately decomposing this massive optimization.

A Massive Optimization

$$
\begin{aligned}
\text{Maximize} \quad & \sum_i U_i(y_i) \\
\text{subject to} \quad & y_i = \mathbb{E}_\tau\Big[\sum_l p_{il}^\tau x_{il}^\tau\Big], \quad i, \\
& \sum_i x_{il}^\tau \leq 1, \qquad l, \tau, \\
& \sum_l x_{il}^\tau \leq 1, \qquad i, \tau, \\
\text{over} \quad & x_{il}^\tau \geq 0, y_i \geq 0 \qquad l, \qquad i, \qquad \tau.
\end{aligned}
$$

A Massive Optimization

$$\text{Maximize} \quad \sum_i U_i(y_i)$$

Max Utility

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Max Utility

Mean click-rate

A Massive Optimization

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Max Utility

Mean click-rate

Per impression Assignment Constraints

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Per impression Assignment Constraints

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small

A Massive Optimization

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Max Utility

Mean click-rate

Per impression Assignment Constraints

small (l)

Large! (i)

A Massive Optimization

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\text{subject to} \quad & y_i = \mathbb{E}_\tau \left[\sum_l p_{il}^\tau x_{il}^\tau \right], \quad i, \\
& \sum_i x_{il}^\tau \leq 1, \quad l, \tau, \\
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\text{over} \quad & x_{il}^\tau \geq 0, y_i \geq 0 \quad l, \quad i, \quad \tau.
\end{aligned}$$

Max Utility

Mean click-rate

Per impression Assignment Constraints

small

Large!

Uncountably infinite!!

A Massive Optimization

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$$

- even if we knew all the parameters, it's impossible to solve this optimization off-line
- Still … maybe we can solve a lot of small optimizations...

A Small Optimization

When a search τ occurs, solve:

$$\begin{aligned}
&\text{Maximize} && \sum_i \sum_l \lambda_i p_{il}^\tau x_{il} \\
&\text{subject to} && \sum_i x_{il}^\tau \leq 1, \\
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Assignment Problem

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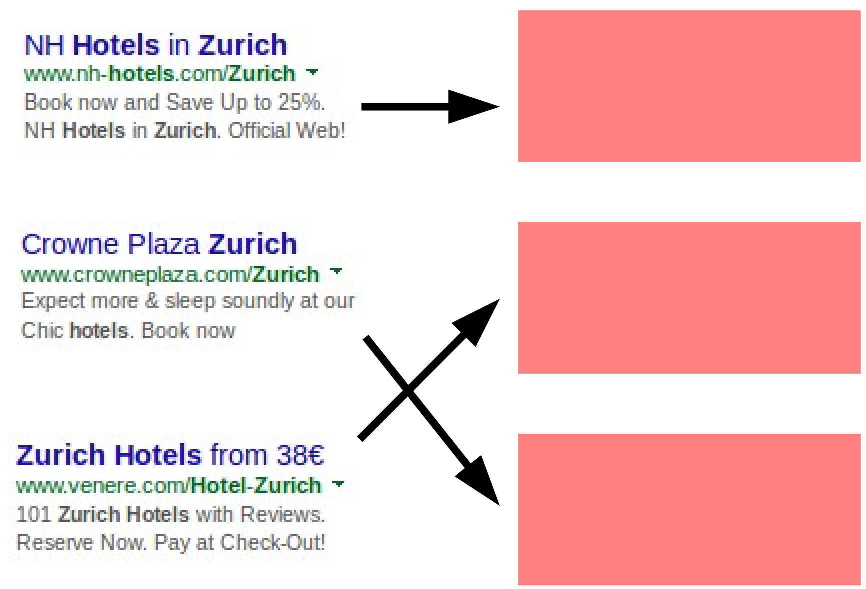

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Lots of polynomial time algorithms:

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Lots of polynomial time algorithms:

Hungarian ; Hopcroft-Karp ; Bertsekas' Auction ...

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Solve the big optimization with many little optimizations

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1. Substitution:

$$x^* \in \arg\max_{x \in \mathcal{X}} U(x)$$

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2. Separability:

$$\max_{x \in \mathcal{X}, y \in \mathcal{Y}} \left\{ f(x) + g(y) \right\}$$

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THE RESULT: A massively distributed VCG Mechanism

Our Results

A Preliminary Calculation

A Preliminary Calculation

$$\max_{x,y} L(x, y; \lambda) = \max_{x,y} \left[\sum_i U_i(y_i) + \sum_i \lambda_i \left(y_i - \mathbb{E}_\tau \Big[\sum_l p_{il}^\tau x_{il}^\tau \Big] \right) \right]$$

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LF-Transform

Assignment Problem

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Proposition 2 **(Decomposition)**. *Variables* $\tilde{y}$, $\tilde{x}^{\tau}, \tau \in \mathcal{T}$, *satisfying the feasibility conditions (7b-7e) are optimal for SYSTEM(*$U, \mathcal{I}, \mathbb{P}_{\tau}$*) if and only if there exist* $\tilde{\lambda}_i$, $i \in \mathcal{I}$, *such that*

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- Decomposed on the timescales of Platform and Advertisers.
- Search distribution is not required.
- But it's an optimization result, we must incentivize this behaviour.

Main Theorem and Mechanism Design

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Advertizers maximizes rewards:

$$r_i(\lambda) = U_i(y_i(\lambda)) - \pi_i(\lambda) y_i(\lambda).$$

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solve the Massive Optimization

Proof of Main Theorem

Optimality condition for the dual:

$$\min_{\lambda \geq 0} \quad \sum_i \left[U_i^*(\lambda_i) + \lambda_i y_i(\lambda)\right] \quad \underset{\text{envelope thrm}}{\iff} \quad \frac{d}{d\lambda_i} U_i^*(\lambda_i) + y_i(\lambda) = 0, \quad \forall i$$

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The correct price!!

How to Implement the Prices

How to Implement the Prices

Two Price Implementations:

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A massively distributed VCG mechanism

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A very simple pay-per click mechanism:

A massively distributed VCG mechanism

A very simple pay-per click mechanism:

Assignment

$$\max \sum_i \sum_l \lambda_i p_{il}^{\tau} x_{il}^{\tau}$$

Pricing

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A massively distributed VCG mechanism

A very simple pay-per click mechanism:

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Further Results and Extensions

Dynamics and Convergence

Dynamics and Convergence

A natural dynamic: $\frac{d}{dt}\lambda_i(t) \gtrless 0$ according as $\lambda_i(t) \lessgtr U_i'(y_i(\lambda(t)))$.

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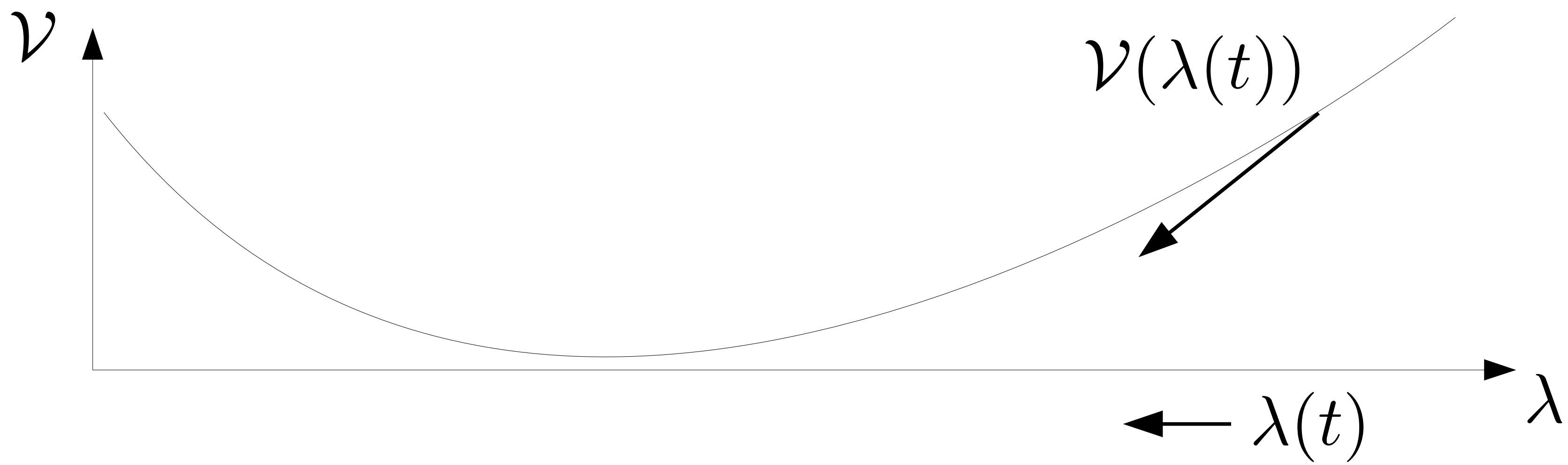

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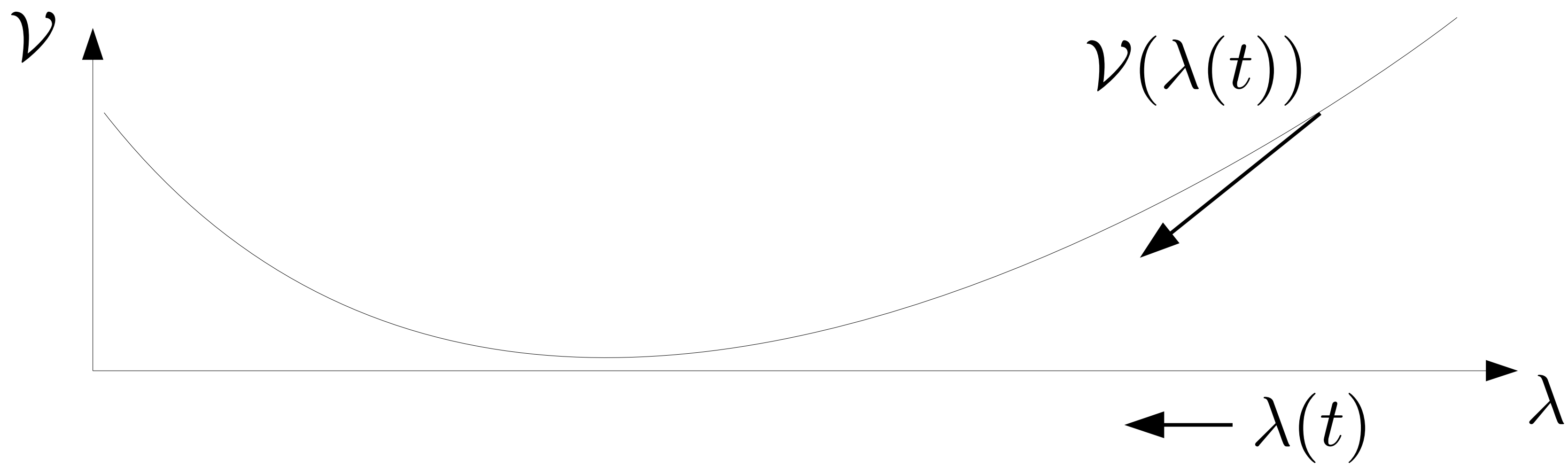
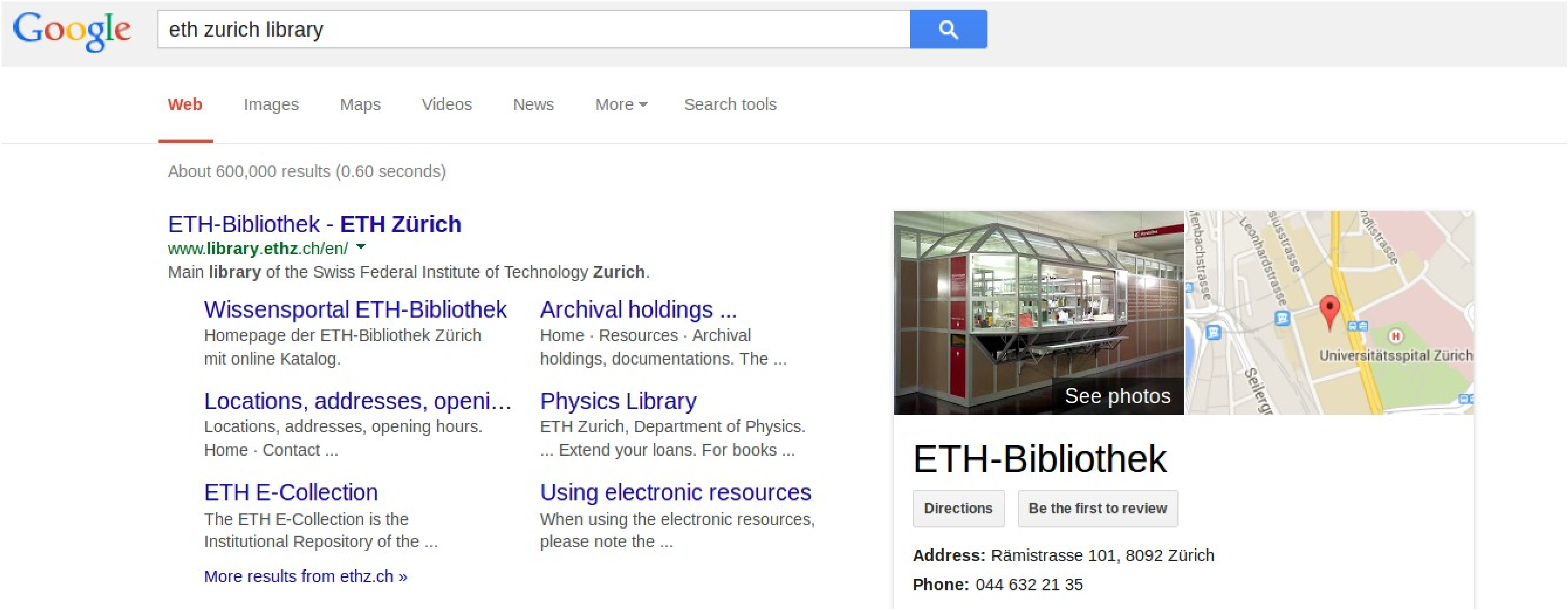


Further Extensions

Controlling number of slots: $$\max_{x^T} \sum_{i,l} (\lambda_i p_{il}^T - q_{il}^T) x_{il}^T$$



Further Extensions

Multivariate utilities:

$$U_i(y_{ik} : k \in \mathcal{K}_i)$$

+ KEYWORDS | Edit | Details | Bid strategy | Automate | Labels

Keyword	Status	Max. CPC	Clicks	Impr.	CTR	Avg. CPC	Cost	Avg. Pos.	Labels
ifor eth	Paused	€5.25	0	0	0.00%	€0.00	€0.00	0.0	--
eth ifor	Paused	€5.00	0	0	0.00%	€0.00	€0.00	0.0	--
Advert Zurich	Eligible	€5.00	0	20	0.00%	€0.00	€0.00	1.2	--
optimization zurich	Eligible	€5.00	0	1	0.00%	€0.00	€0.00	1.0	--
hotel zurich	Eligible	€3.50	0	564	0.00%	€0.00	€0.00	3.0	--
optimization eth	Eligible	€2.25	0	0	0.00%	€0.00	€0.00	0.0	--
eth zentrum	Paused	€0.10	0	3	0.00%	€0.00	€0.00	1.0	--

Further Extensions

Budget constraints: $$U_i(y_{ik} : k \in \mathcal{K}_i) = \frac{b_i}{q} \log \sum_{k \in \mathcal{K}_i} (w_{ik} y_{ik})^q$$

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- Provably solves an Infinitely Large Optimization.

Thank you for listening!

NEIL WALTON'S WEBSITE RESEARCH NEIL'S NOTES LINKS

Efficient Advert Assignment

F.P. Kelly, P. Key, N.S. Walton. (2014). (Preprint)

(an earlier version was presented at EC'14 see below)

[pdf] [arxiv]

Incentivized Optimal Advert Assignment via Utility Decomposition

F.P. Kelly, P. Key, N.S. Walton (2014). *ACM Conference on Economics and Computation*.

[pdf] [arxiv] [proceedings] [bibtex]