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Neil Walton, University of Amsterdam.

Ads ⓘ

Joint work
with

Frank Kelly
university of cambridge

Peter Key
microsoft research

This Talk

We give a simple allocation and pricing mechanism

whose Nash equilibrium solves a very large
optimization problem

This Talk

We give a simple allocation and pricing mechanism

whose Nash equilibrium solves a very large
optimization problem

Very Large = over the infinite results of a search engine.

Outline

- A Introduction to Sponsored Search
 - Bids, Impressions, Click-Through Rate, Advertizers, Platform
- Auction or Optimize?
 - Our Mechanism, Generalized 2nd Price, VCG Mechanism, Decomposition.
- Our Results
 - Main Theorem, Implementations
- Further Results and Extensions
 - Dynamics, Multivariate Utilities, General Page Layouts, Budgets.

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Bid

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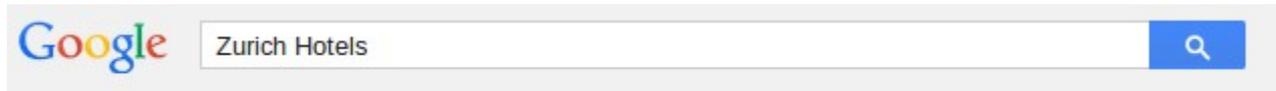

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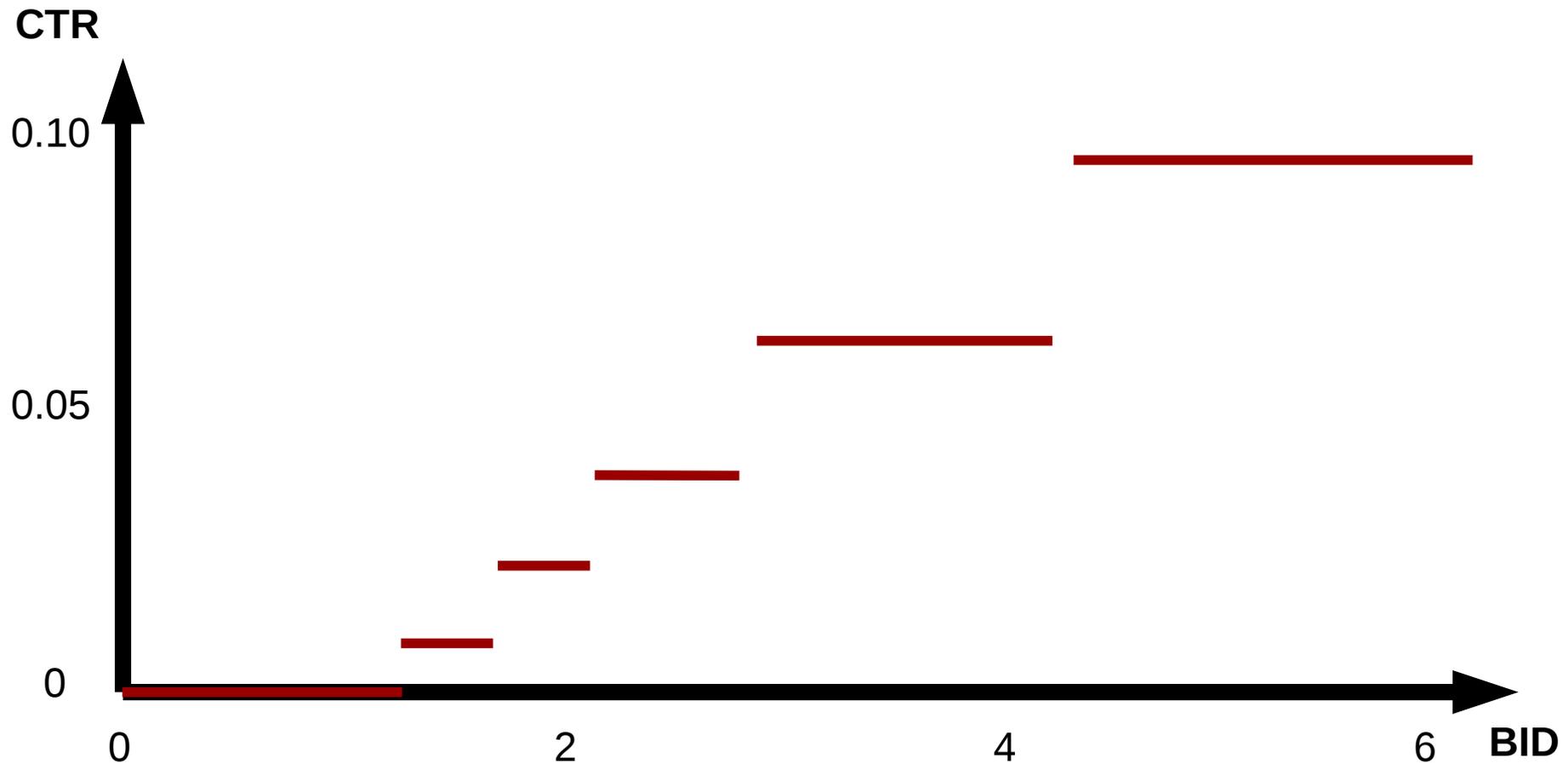


Variability in Sponsored Search

A **one-shot** or repeated auction:

eg. “Dominos Pizza”

(exact match)

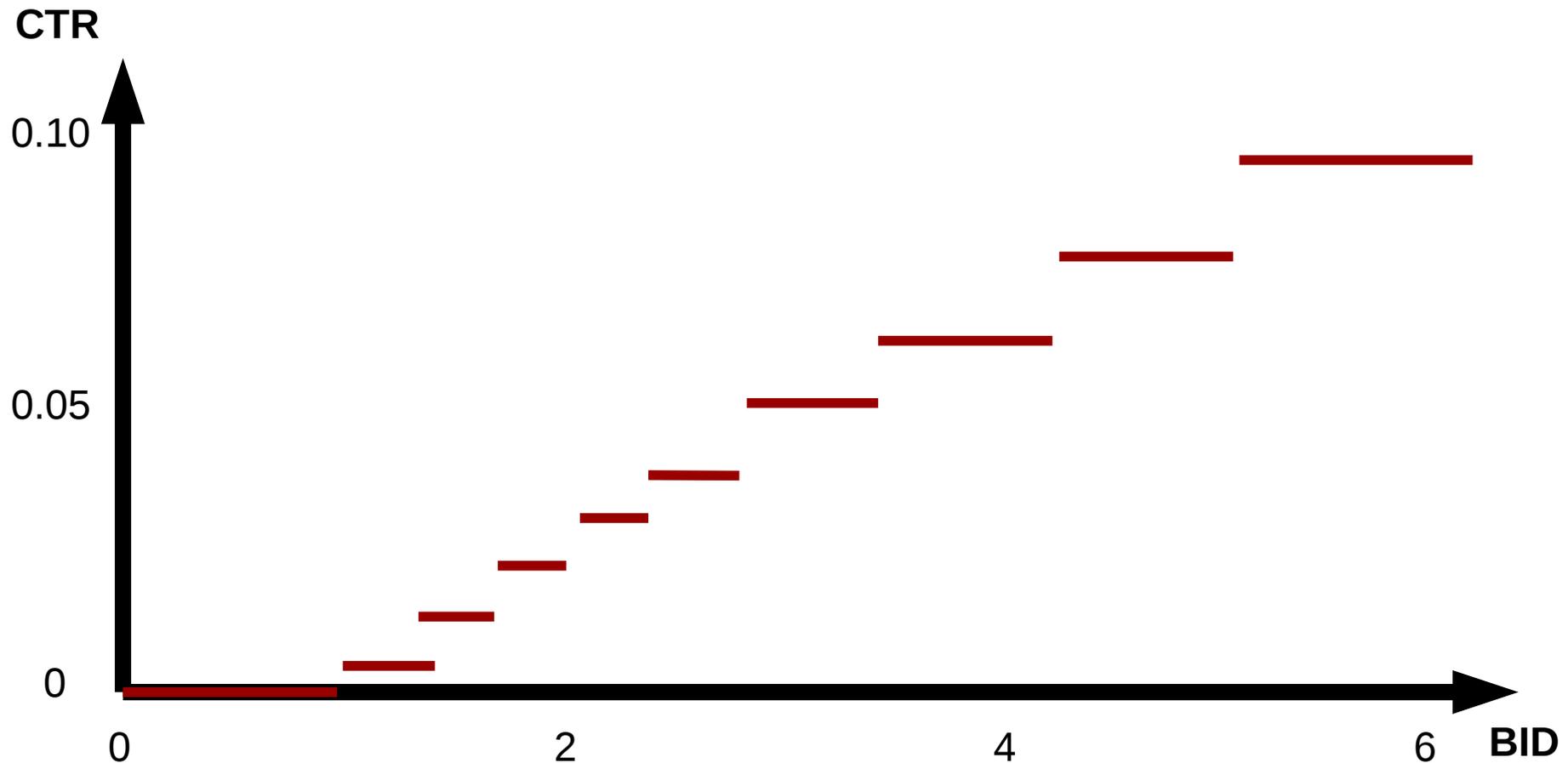


Variability in Sponsored Search

A mixture of auctions:

eg. "... Dominos ... Pizza ..."

(phrase match)

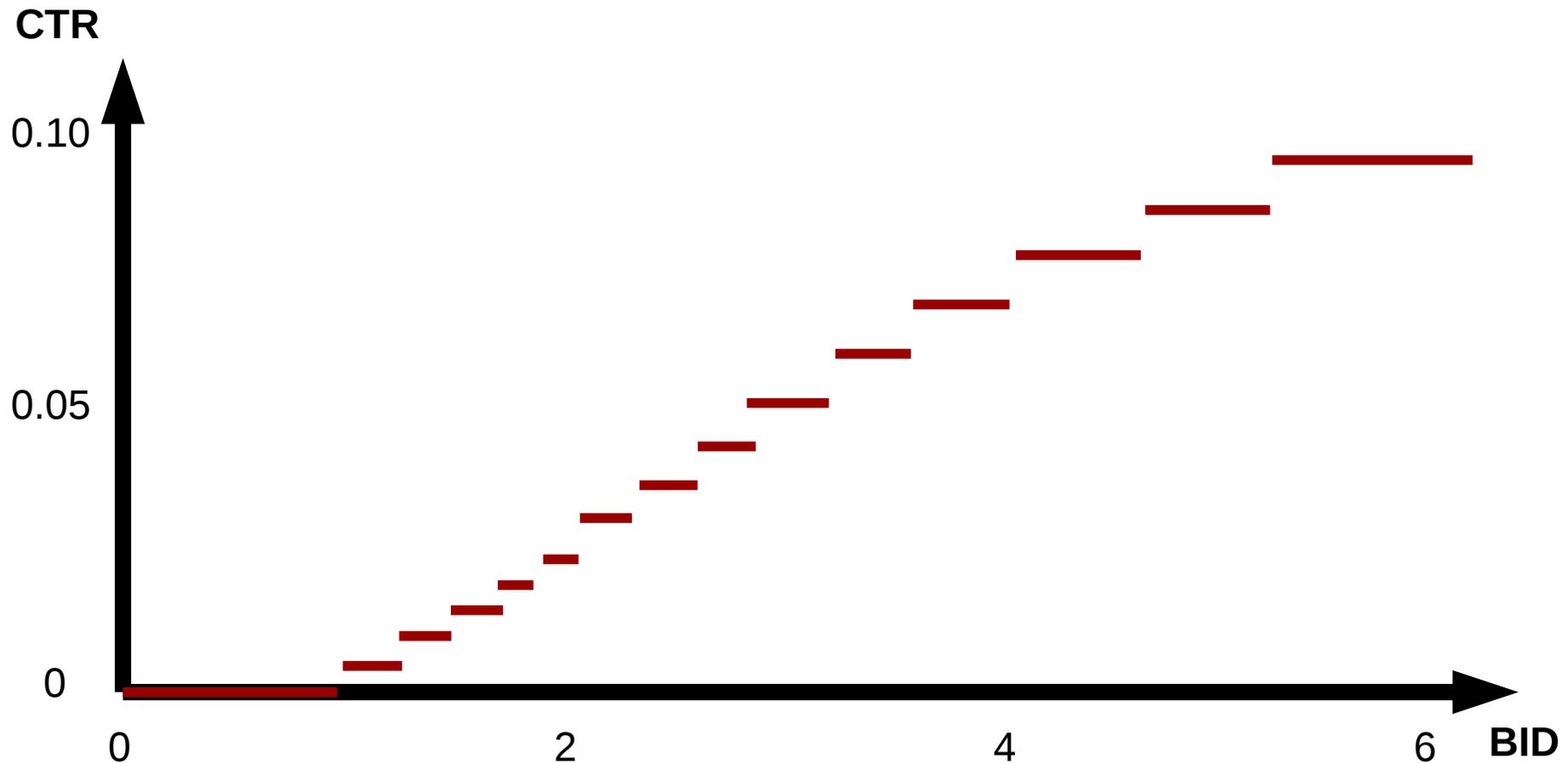


Variability in Sponsored Search

A bigger mixture of auctions:

eg. "... Delivery ... Pizza ..."

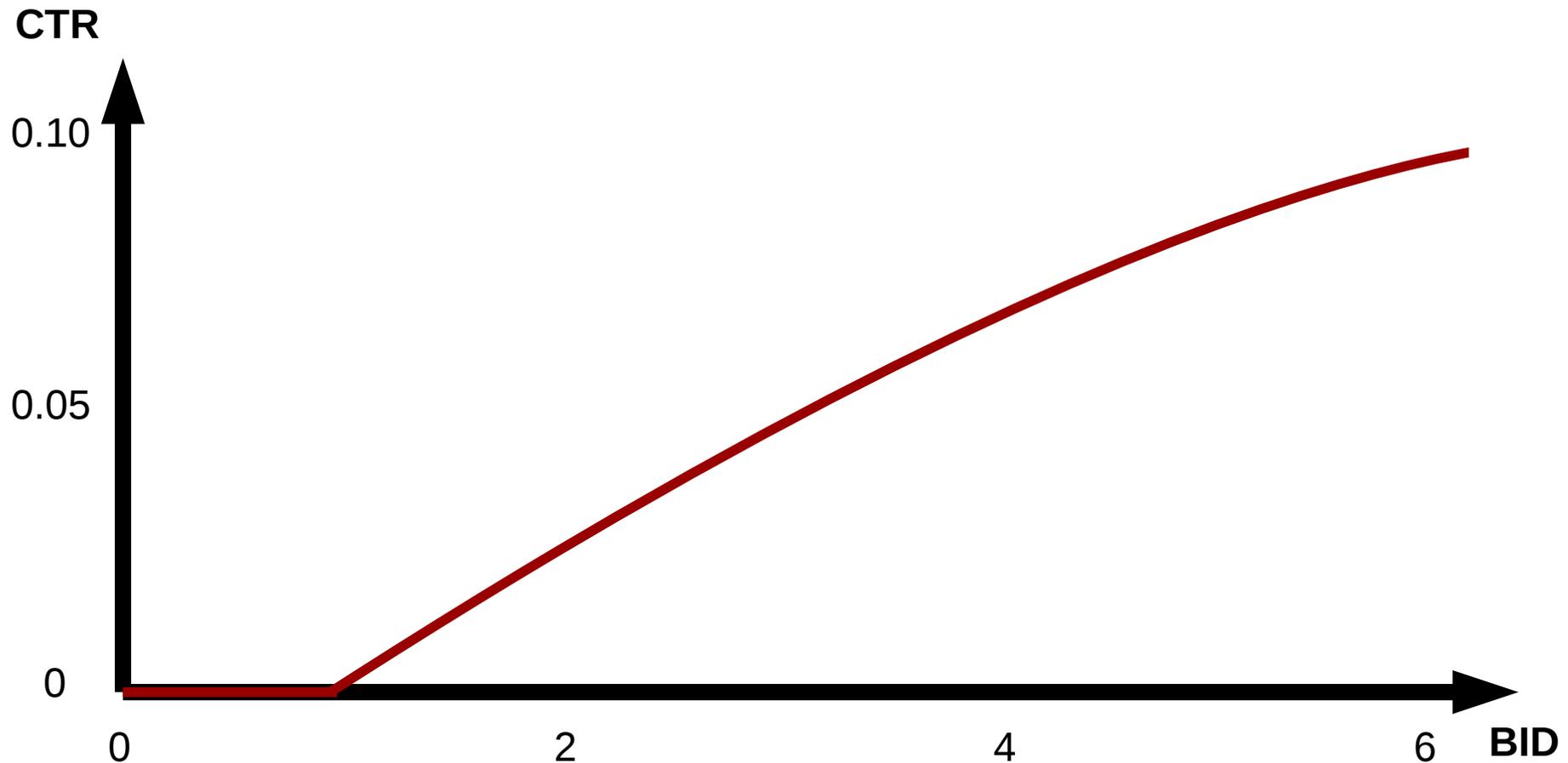
(broad match)



Variability in Sponsored Search

A **continuum** of auctions:

eg. “... Delivery ... Pizza ...” + **location + time**
(broad match) + **Searcher**



Information and Temporal Asymmetry

Information and Temporal Asymmetry

The Searcher



Information and Temporal Asymmetry

The Searcher



The Platform



Information and Temporal Asymmetry

The Searcher



The Platform



The Advertiser



Information and Temporal Asymmetry

The Searcher



Search Distribution

$$P_{\tau}$$

The Platform



The Advertiser



Information and Temporal Asymmetry

The Searcher



Search Distribution

$$P_{\tau}$$

The Platform



When a search occurs

Click-Through p_{il}^{τ}

Assignment x_{il}^{τ}

The Advertiser



Information and Temporal Asymmetry

The Searcher



Search Distribution

$$\mathbb{P}_\tau$$

The Platform



When a search occurs

Click-Through p_{il}^τ
Assignment x_{il}^τ

The Advertiser



Receives average information

Click-Through $\mathbb{E}_\tau \sum_l p_{il}^\tau x_{il}^\tau$
Assignment $\mathbb{E}_\tau x_{il}^\tau$

Information and Temporal Asymmetry

The Searcher



Search Distribution

$$\mathbb{P}_\tau$$

Unknown
To everyone

The Platform



When a search occurs

Click-Through p_{il}^τ

Assignment x_{il}^τ

The Advertiser



Receives average information

Click-Through $\mathbb{E}_\tau \sum_l p_{il}^\tau x_{il}^\tau$

Assignment $\mathbb{E}_\tau x_{il}^\tau$

Information and Temporal Asymmetry

The Searcher



Search Distribution

$$\mathbb{P}_\tau$$

Unknown
To everyone

The Platform



When a search occurs

Click-Through p_{il}^τ

Assignment x_{il}^τ

Platform knows
Advertiser doesn't

The Advertiser



Receives average information

Click-Through $\mathbb{E}_\tau \sum_l p_{il}^\tau x_{il}^\tau$

Assignment $\mathbb{E}_\tau x_{il}^\tau$

Information and Temporal Asymmetry

The Searcher



Search Distribution

$$\mathbb{P}_\tau$$

Unknown
To everyone

The Platform



When a search occurs

Click-Through p_{il}^τ

Assignment x_{il}^τ

Platform knows
Advertiser doesn't

The Advertiser



Receives average information

Click-Through $\mathbb{E}_\tau \sum_l p_{il}^\tau x_{il}^\tau$

Assignment $\mathbb{E}_\tau x_{il}^\tau$

Platform knows
Advertiser knows

Auction or Optimize?

Two Auctions

τ – Search Type

λ_i – Bid of ad i

p_{il}^τ – Click-Through ad i slot l

$y_i^\tau(\lambda)$ – CTR given bids

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Auction 1:

Assign

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Pay, per-click

$$\pi_i^\tau = \frac{\lambda_{(i+1)i+1} p_{(i+1)i+1}^\tau}{p_{(i)i}^\tau}$$

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Auction 2:

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Auction 2:

Assign max matching

$$\max \sum_i \sum_l \lambda_i p_{il}^\tau x_{il}$$

$$\text{s.t. } \sum_i x_{il}^\tau \leq 1, \quad \sum_l x_{il}^\tau \leq 1$$

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Pay, per-click $\mu_i \sim U[0, \lambda_i]$

$$\lambda_i \left(1 - \frac{y_i^\tau(\mu_i)}{y_i^\tau(\lambda_i)} \right)$$

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Generalized 2nd Price

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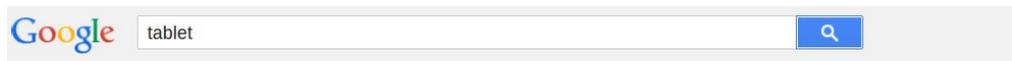
Pay, per-click $\mu_i \sim U[0, \lambda_i]$

$$\lambda_i \left(1 - \frac{y_i^\tau(\mu_i)}{y_i^\tau(\lambda_i)} \right)$$

A VCG Auction

Immediate Advantages

Immediate Advantages



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There is not ordering of Ads!
Generalized 2nd Price Breaks down. Our mechanism and results hold true.

Is it GSP or VCG?

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Google (2006) said:

“Google’s unique auction model uses Nobel Prize winning economic theory ... the AdWords™ Discounter makes sure that they only pay what they need in order to stay ahead of their nearest competitor.”

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Q. Is this really true?
A. Not really.

Q. What did they really mean?
A. The VCG Mechanism...



Vickrey



Clark



Groves

The VCG Mechanism

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- Advertiser's utilities $U_i(\cdot)$

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- Assignment constraints A

The VCG Mechanism

- Advertiser's utilities $U_i(\cdot)$
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- Assignment constraints \mathcal{A}

Platform Assigns:

$$y_i^* \in \operatorname{argmax}_{y \in \mathcal{A}} \sum_i V_i(y_i)$$

The VCG Mechanism

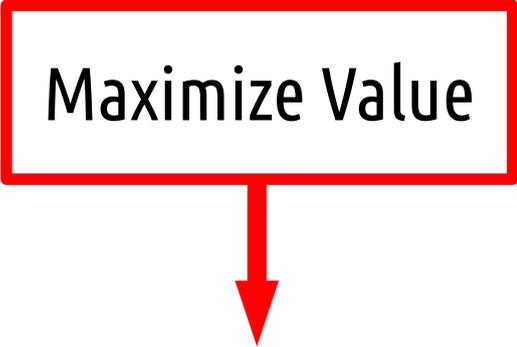
- Advertiser's utilities
- bid utilities
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$$U_i(\cdot)$$

$$V_i(\cdot)$$

$$\mathcal{A}$$

Maximize Value



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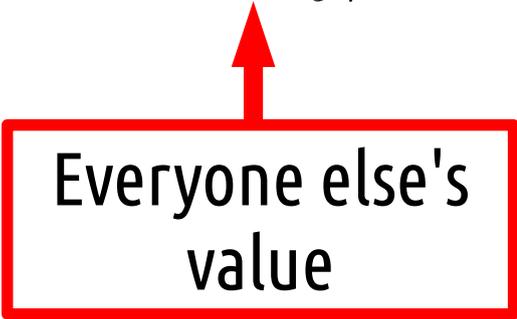
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Everyone else's
value

The VCG Mechanism

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Everyone else's
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Value without
you there

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Equilibrium Advertiser:

$$\max_{V_i} \{U_i(y_i^*) - \pi_i\}$$

The VCG Mechanism

Theorem

The VCG mechanism has a dominate strategies equilibrium that is:

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The VCG mechanism has a dominant strategies equilibrium that is:

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bids are truthful: $V_i(\cdot) = U_i(\cdot)$

The VCG Mechanism

Theorem

The VCG mechanism has a dominant strategies equilibrium that is:

- Incentive compatible

bids are truthful: $V_i(\cdot) = U_i(\cdot)$

- Efficient

allocation is optimal: $y^* \in \operatorname{argmax}_{y \in \mathcal{A}} \sum_i U_i(y_i)$

Directly Applying VCG

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Pros

1. Result applies in very **general** settings
2. Allocation of Adverts is provably **optimal**

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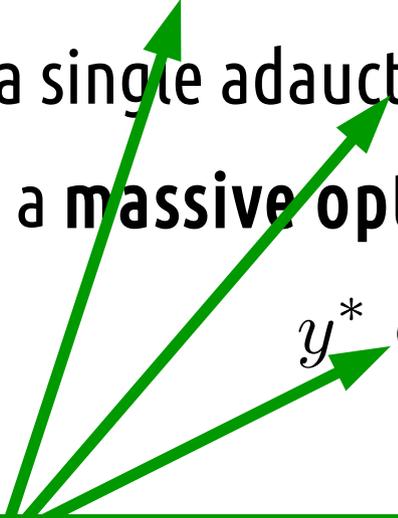
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This talk: We deal with these issue by appropriately decomposing this massive optimization.

A Massive Optimization

$$\begin{aligned} \text{Maximize} \quad & \sum_i U_i(y_i) \\ \text{subject to} \quad & y_i = \mathbb{E}_\tau \left[\sum_l p_{il}^\tau x_{il}^\tau \right], \quad i, \\ & \sum_i x_{il}^\tau \leq 1, \quad l, \tau, \\ & \sum_l x_{il}^\tau \leq 1, \quad i, \tau, \\ \text{over} \quad & x_{il}^\tau \geq 0, y_i \geq 0 \quad l, \quad i, \quad \tau. \end{aligned}$$

A Massive Optimization

Maximize $\sum_i U_i(y_i)$

← **Max Utility**

subject to $y_i = \mathbb{E}_\tau \left[\sum_l p_{il}^\tau x_{il}^\tau \right], \quad i,$

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← **Mean click-rate**

$$\sum_i x_{il}^\tau \leq 1, \quad l, \tau,$$

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Per impression
Assignment
Constraints

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small

A Massive Optimization

Maximize $\sum_i U_i(y_i)$ ← **Max Utility**

subject to $y_i = \mathbb{E}_\tau \left[\sum_l p_{il}^\tau x_{il}^\tau \right], \quad i,$ ← **Mean click-rate**

$\sum_i x_{il}^\tau \leq 1, \quad l, \tau,$ ← **Per impression Assignment Constraints**

$\sum_l x_{il}^\tau \leq 1, \quad i, \tau,$ ← **Per impression Assignment Constraints**

over $x_{il}^\tau \geq 0, y_i \geq 0$ $l, i, \tau.$

small ↑ $l,$ **Large!** ↑ $i,$

A Massive Optimization

Maximize $\sum_i U_i(y_i)$ ← **Max Utility**

subject to $y_i = \mathbb{E}_\tau \left[\sum_l p_{il}^\tau x_{il}^\tau \right], \quad i,$ ← **Mean click-rate**

$\sum_i x_{il}^\tau \leq 1, \quad l, \tau,$ ← **Per impression Assignment Constraints**

$\sum_l x_{il}^\tau \leq 1, \quad i, \tau,$ ← **Per impression Assignment Constraints**

over $x_{il}^\tau \geq 0, y_i \geq 0$

small **Large!** **Uncountably infinite!!**

A Massive Optimization

$$\begin{aligned} &\text{Maximize} && \sum_i U_i(y_i) \\ &\text{subject to} && y_i = \mathbb{E}_\tau \left[\sum_l p_{il}^\tau x_{il}^\tau \right], \quad i, \\ &&& \sum_i x_{il}^\tau \leq 1, \quad l, \tau, \\ &&& \sum_l x_{il}^\tau \leq 1, \quad i, \tau, \\ &\text{over} && x_{il}^\tau \geq 0, y_i \geq 0 \quad l, \quad i, \quad \tau. \end{aligned}$$

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- even if we knew all the parameters, it's impossible to solve this optimization off-line
- Still ... maybe we can solve a lot of small optimizations...

A Small Optimization

When a search τ occurs, solve:

Maximize
$$\sum_i \sum_l \lambda_i p_{il}^\tau x_{il}$$

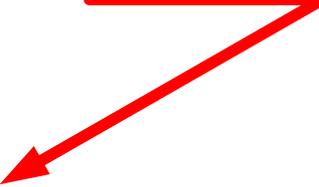
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A Small Optimization

Assignment Problem



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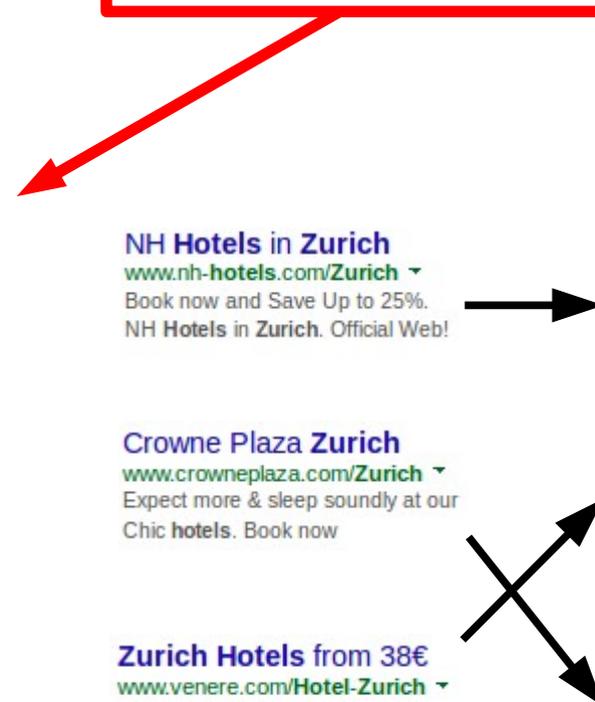
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A Small Optimization

Assignment Problem

When a search τ occurs, solve:

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Lots of polynomial time algorithms:

A Small Optimization

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Lots of polynomial time algorithms:
Hungarian ; Hopcroft-Karp ; Bertsekas' Auction ...

Optimization Decomposition

Solve the big optimization with many little optimizations

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Solve the big optimization with many little optimizations

1. Substitution:

$$x^* \in \arg \max_{x \in \mathcal{X}} U(x)$$

$$\iff x^* \in \arg \max_{x \in \mathcal{X}} V(x; \lambda^*),$$

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Optimization Decomposition

Solve the big optimization with many little optimizations

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$$\max_{x \in \mathcal{X}, y \in \mathcal{Y}} \left\{ f(x) + g(y) \right\}$$

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Solve the big optimization with many little optimizations

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THE RESULT: A massively distributed VCG Mechanism

Our Results

A Preliminary Calculation

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$$\max_{x,y} L(x, y; \lambda) = \max_{x,y} \left[\sum_i U_i(y_i) + \sum_i \lambda_i \left(y_i - \mathbb{E}_\tau \left[\sum_l p_{il}^\tau x_{il}^\tau \right] \right) \right]$$

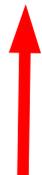
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LF-Transform



Assignment Problem



A Preliminary Optimization Result

PROPOSITION 2 (**Decomposition**). Variables \tilde{y} , $\tilde{x}^\tau, \tau \in \mathcal{T}$, satisfying the feasibility conditions (7b-7e) are optimal for $SYSTEM(U, \mathcal{I}, \mathbb{P}_\tau)$ if and only if there exist $\tilde{\lambda}_i, i \in \mathcal{I}$, such that

A. $\tilde{\lambda}_i$ minimizes $U_i^*(\lambda_i) + \lambda_i \tilde{y}_i$ over $\lambda_i \geq 0$, for each $i \in \mathcal{I}$,

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Main Theorem and Mechanism Design

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solve the Massive Optimization

Proof of Main Theorem

Optimality condition for the dual:

$$\begin{aligned}
 \min_{\lambda \geq 0} \sum_i [U_i^*(\lambda_i) + \lambda_i y_i(\lambda)] &\stackrel{\text{envelope thm}}{\iff} \frac{d}{d\lambda_i} U_i^*(\lambda_i) + y_i(\lambda) = 0, \quad \forall i \\
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The correct price!!

How to Implement the Prices

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A massively distributed VCG mechanism

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A very simple pay-per click mechanism:

A massively distributed VCG mechanism

A very simple pay-per click mechanism:

Assignment

$$\max \sum_i \sum_l \lambda_i p_{il}^\tau x_{il}^\tau$$

Pricing

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at Nash equilibrium solves the Massive Optimization

$$\begin{aligned} & \text{Maximize} && \sum_i U_i(y_i) \\ & \text{subject to} && y_i = \mathbb{E}_\tau \left[\sum_l p_{il}^\tau x_{il}^\tau \right], \quad i, \\ & && \sum_i x_{il}^\tau \leq 1, \quad l, \tau, \\ & && \sum_l x_{il}^\tau \leq 1, \quad i, \tau, \\ & \text{over} && x_{il}^\tau \geq 0, y_i \geq 0 \quad l, i, \tau. \end{aligned}$$

Further Results and Extensions

Dynamics and Convergence

Dynamics and Convergence

A natural dynamic: $\frac{d}{dt} \lambda_i(t) \geq 0$ according as $\lambda_i(t) \leq U'_i(y_i(\lambda(t)))$.

Dynamics and Convergence

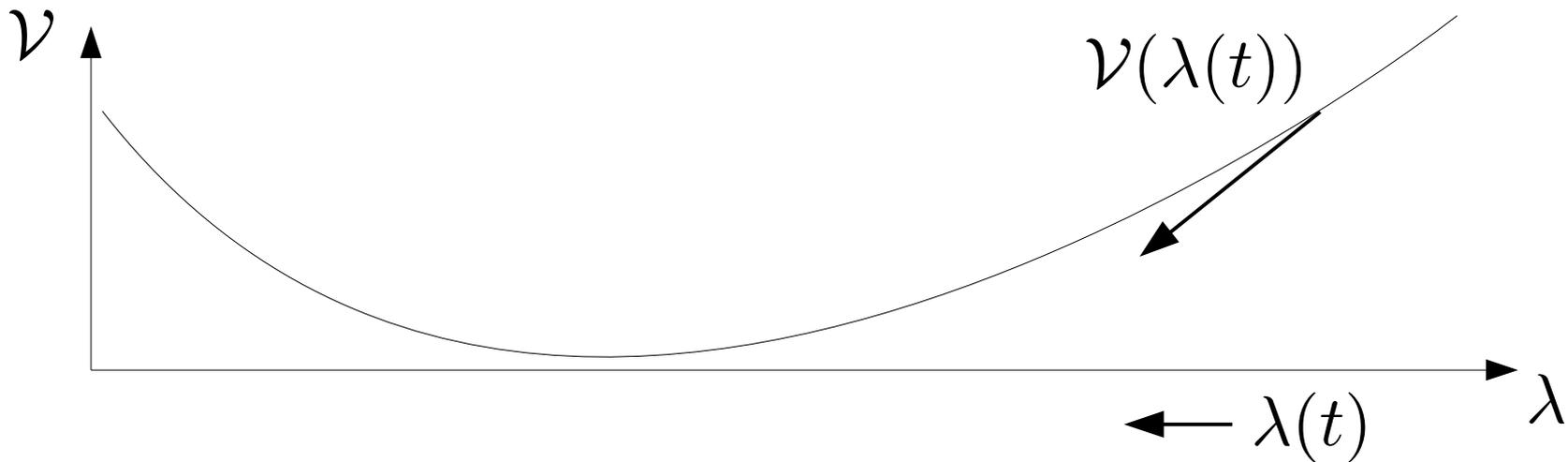
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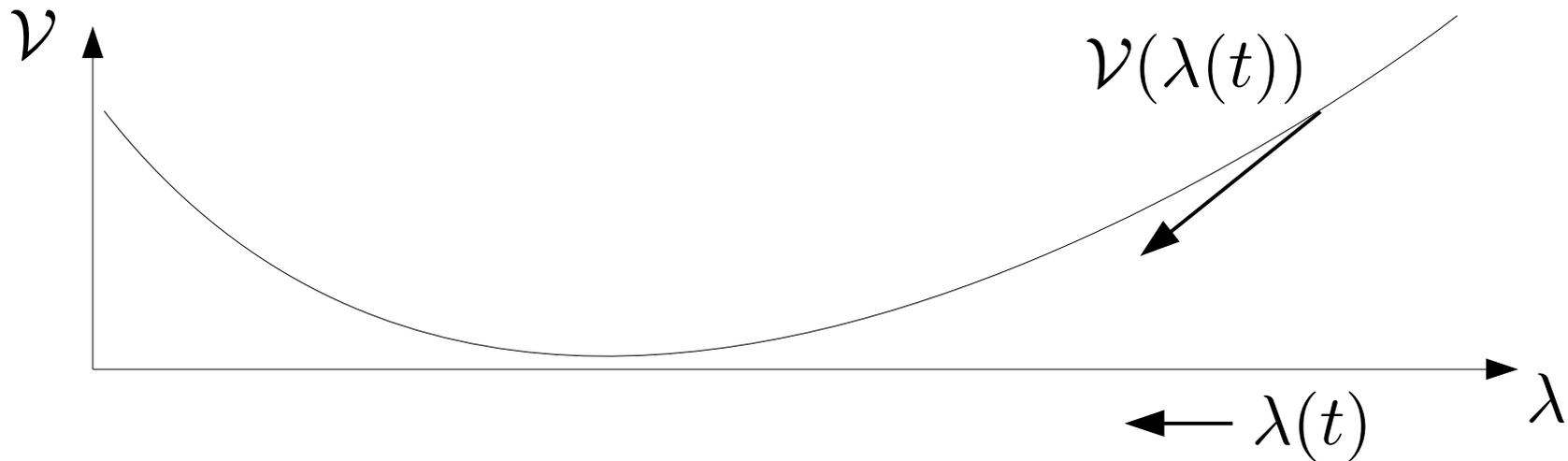
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Further Extensions

Controlling number of slots:

$$\max_{x^\tau} \sum_{i,l} (\lambda_i p_{il}^\tau - q_{il}^\tau) x_{il}^\tau$$

The image shows a Google search interface for 'eth zurich library'. The search results page displays 'About 600,000 results (0.60 seconds)'. The top result is 'ETH-Bibliothek - ETH Zürich' with the URL 'www.library.ethz.ch/en/'. Below the main title, there are several links: 'Wissensportal ETH-Bibliothek', 'Locations, addresses, opening hours...', 'ETH E-Collection', 'Archival holdings ...', 'Physics Library', and 'Using electronic resources'. On the right side, there is a map showing the location of the library at Rämistrasse 101, 8092 Zürich, near the Universitätsspital Zürich. A photo of the library interior is also visible.

ETH-Bibliothek - ETH Zürich
www.library.ethz.ch/en/
Main library of the Swiss Federal Institute of Technology Zurich.

Wissensportal ETH-Bibliothek
Homepage der ETH-Bibliothek Zürich mit online Katalog.

Locations, addresses, opening hours...
Locations, addresses, opening hours. Home · Contact ...

ETH E-Collection
The ETH E-Collection is the Institutional Repository of the ...

Archival holdings ...
Home · Resources · Archival holdings, documentations. The ...

Physics Library
ETH Zurich, Department of Physics. ... Extend your loans. For books ...

Using electronic resources
When using the electronic resources, please note the ...

[More results from ethz.ch »](#)

ETH-Bibliothek
Directions Be the first to review

Address: Rämistrasse 101, 8092 Zürich
Phone: 044 632 21 35

Further Extensions

Multivariate utilities:

$$U_i(y_{ik} : k \in \mathcal{K}_i)$$

<input type="checkbox"/>	<input type="checkbox"/>	Keyword	Status [?]	Max. CPC [?] ↓	Clicks [?]	Impr. [?]	CTR [?]	Avg. CPC [?]	Cost [?]	Avg. Pos. [?]	Labels [?]
<input type="checkbox"/>	⏸	ifor eth	🗨 Paused	€5.25 ☑	0	0	0.00%	€0.00	€0.00	0.0	--
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<input type="checkbox"/>	●	Advert Zurich	🗨 Eligible	€5.00 ☑	0	20	0.00%	€0.00	€0.00	1.2	--
<input type="checkbox"/>	●	optimization zurich	🗨 Eligible	€5.00 ☑	0	1	0.00%	€0.00	€0.00	1.0	--
<input type="checkbox"/>	●	hotel zurich	🗨 Eligible	€3.50 ☑	0	564	0.00%	€0.00	€0.00	3.0	--
<input type="checkbox"/>	●	optimization eth	🗨 Eligible	€2.25 ☑	0	0	0.00%	€0.00	€0.00	0.0	--
<input type="checkbox"/>	⏸	eth zentrum	🗨 Paused	€0.10 ☑	0	3	0.00%	€0.00	€0.00	1.0	--

Further Extensions

Budget constraints: $U_i(y_{ik} : k \in \mathcal{K}_i) = \frac{b_i}{q} \log \sum_{k \in \mathcal{K}_i} (w_{ik} y_{ik})^q$

Bid strategy Basic options | [Advanced options](#)

I'll manually set my bids for clicks

AdWords will set my bids to help maximize clicks within my target budget

Budget CA\$ per day

Actual daily spend may vary.

Summary of the talk

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 - Simple
 - Flexible

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 - Applies to different page layouts
- Provably solves an Infinitely Large Optimization.

Thank you for listening!

NEIL WALTON'S WEBSITE

RESEARCH

NEIL'S NOTES

LINKS

[Efficient Advert Assignment](#)

F.P. Kelly, P. Key, N.S. Walton. (2014). (Preprint)

(an earlier version was presented at EC'14 see below)

[\[pdf\]](#) [\[arxiv\]](#)

[Incentivized Optimal Advert Assignment via Utility Decomposition](#)

F.P. Kelly, P. Key, N.S. Walton (2014). *ACM Conference on Economics and Computation*.

[\[pdf\]](#) [\[arxiv\]](#) [\[proceedings\]](#) [\[bibtex\]](#)