HW #1

Exercises 1-4  (Convexity - duality) Do exercises 4.11, 4.26, 5.11, 5.17 in the textbook by Boyd & Vandenberghe.

Exercise 5  (Newton’s method) Write a MATLAB function that takes a matrix $A \in \mathbb{R}^{m \times n}$ and a vector $b \in \mathbb{R}^m$ as inputs and returns the solution to:

$$\min_{x \in \mathbb{R}^n} f(x) := -\sum_{i=1}^{m} \log(b_i - a_i^T x)$$

as an output. This is called the analytic center of the linear inequalities $Ax \leq b$. Use Newton’s method with backtracking line search to compute the minimum with a target precision around $10^{-10}$. Demonstrate your code on random examples (e.g. a random section of the unit box in $\mathbb{R}^2$) and plot the convergence to show that it exhibits quadratic convergence close to the optimum (you can use the minimum value of $f$ you computed as a proxy for $f^*$).

Exercise 6  (Linear programming solver) Write a MATLAB function to solve the following problem:

$$\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad Ax \leq b.
\end{align*}$$

Use the barrier method together with the analytic center code you wrote above to produce a matlab function that takes a feasible point as input and returns both primal and dual solutions. Write another function that computes a strictly feasible solution to $Ax \leq b$ if there is one, or returns an error message if not. Check your results against those produced by CVX.