Hawks and Doves in Segmented Markets: Profit Maximization with Varying Competitive Aggressiveness

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Abstract

We consider a duopoly, each firm supplying a captive and a contested segment. Starting from two extreme formulations of the profit maximization objective, we introduce a parameterized class of managerial objective functions, involving price-quantity pairs as strategic variables but corresponding to different managerial attitudes. By specifying the parameters of competitive aggressiveness, we recover classical competition regimes. The model is extended to analyze the strategic choice of managerial aggressiveness, and to examine the implications of changes in the intensity of competition. In order to endogenize the competitive aggressiveness parameters, a more primitive price-quantity model introduces the possibility of consumer price discounting.

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I. Introduction

Although the postulate of rationality as applied to the theory of the firm seems simple to state, namely that a firm should maximize profit, this objective becomes ambiguous as soon as there are several competing firms. Firms can use different kinds of strategies (*e.g.* prices or quantities) leading to different outcomes, and these strategies are chosen by managers who may adopt different attitudes towards their rivals. Managers may be motivated by animosity or respect, or be more or less confident and optimistic, so that in the end their firm has a more or less aggressive managerial style.[1] This has started to be documented in the branch of behavioral economics applied to firms, both through experiments and from empirical investigations, and a common conclusion is that

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[1] As John Gapper puts it in a paper written for the *Financial Times* ("Opening the windows on Microsoft", 27.02.08), the software industries are moving from an age ruled by the Silicon Valley motto "Only the paranoid survive" to the internet age of companies that "no longer try to eradicate competition at all costs", an age for which a better slogan would be "Only the sociable thrive". And he adds that "it is no coincidence that Mr Gates, the company's co-founder and guiding spirit for more than 30 years [...] has handed over his title of chief software architect to Ray Ozzie, an industry veteran who is instinctively more open to collaboration."

managerial behavior departs from strict profit maximizing behavior (see the survey by Armstrong and Huck (2010)).

Our purpose here is to argue that such a variety of managerial attitudes is compatible with profit maximization, because profit maximization in a competitive world can be formulated in various ways corresponding to different competitive regimes. Our argument will be developed from two different perspectives. First, adopting a practical point of view, we shall introduce a class of managerial objective functions, corresponding to different formulations of the profit maximizing objective, but allowing for a large variety of managerial styles, from the most to the least aggressive (from hawks to doves). A basic ingredient in this approach, already used in Shubik (1959), is to suppose that the sellers do not privilege one strategic variable but that they compete simultaneously in price-quantity pairs. This is illustrated in a duopoly model *à la* Hotelling (1929), where the possibility of market segmentation is explicitly introduced and where (as in Narasimhan (1988)) firms cannot price discriminate between loyal customers in their "hinterland" (their captive segment) and possibly brand switching customers in the contested segment of the market. The duopoly game, where firms compete in both prices and quantities, is parameterized by the pair $\theta = (\theta_1, \theta_2)$ of parameters indicating the degree of "competitive aggressiveness" of firm 1 and firm 2. By varying these two parameters we change the competition regime. We will show that several well identified regimes are nested in this general oligopoly model: the (Hotelling) price-competition regime, the quantity-competition regime, as well as a Cournot-like regime where firms adjust their prices in order to accommodate their competitors' sales targets. The corresponding solutions can hence be recovered as Nash equilibria of these θ-games for some θ. Moreover, for asymmetric values of the parameters, various kinds of Stackelberg equilibria (in prices, in quantities, or in price-quantity pairs) can also be obtained. The practical advantage of this approach is to provide a single modelling tool for a whole spectrum of oligopolistic regimes that simplifies computations and facilitates comparisons.[2]

As a further step in this analysis, we will consider the possibility for each firm to choose, in a first stage, the managerial style it finds best when anticipating the managerial style of its competitor and the second stage equilibrium that would result. We shall adopt this strategic approach for the selection of θ in our model and then analyze how the associated subgame perfect equilibrium (and hence the equilibrium value of θ) varies with the intensity of competition in the market.

The second perspective in our argument to defend the compatibility of profit maximization with various managerial attitudes is more fundamental. It consists in deriving our class of managerial objective functions from more primitive assumptions and thus to explain the interpretation of the parameters θ. In this more primitive model, still with market segmentation and no price discrimination, and with each firm playing both in price (advertising a list price) and in quantity (announcing a sales target), such an explanation will be based on the possibility for each manager to offer price discounts to the customers[3]. This is used as a threat both to reach the firm sales target and to keep its loyal customers. We prove that in such a game the existence of an equilibrium implies that each firm selects a price-quantity pair so as to maximize profit under two feasibility constraints, one for price competitiveness, the other for target compatibility, a result

[2]From an empirical point of view this means that, as done in the *New Empirical Industrial Organization* approach (e.g. Bresnahan (1989)), one could use simultaneous econometric equations to identify all these competition regimes. In that respect our approach is related to the conjectural variations approach but with better theoretical foundations.

[3]Similar use of discounting is to be found in Narasimhan (1988). See also Holt and Scheffman (1987).

similar to the ones established in different models by d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (2007), and d'Aspremont and Dos Santos Ferreira (2009). The set of equilibria thus generated can be parameterized, for each firm and each equilibrium, by a coefficient equal to the relative weight of the Lagrange multiplier associated with its competitiveness constraint. These coefficients correspond precisely to (θ_1, θ_2), the indices of competitive aggressiveness we have started with.

Related Literature. The present work is related to several lines of research in the literature. The first is the marketing management literature dealing with aggressiveness. This work, empirically based, argues that competitive aggressiveness has to be viewed, not as a mere individual psychological characteristic, but rather as a constructed feature of the organization itself (*e.g.* J. G. Covin and T. J. Covin (1990), Venkatraman (1989)). This is also related to behavioral economics as applied to firms (recent references can be found in the above mentioned survey by Armstrong and Huck (2010)). Evidence about aggressive or vengeful behavior (Huck, W. Muller, and Normann (2001)) or the role of trust in sustaining collusion (Leslie (2004)) are important observations in our context.

Still another line of research, well reviewed in Miller and Pazgal (2002), is pursued in the Industrial Organization literature on managerial incentives, under separation of ownership and control. Starting with Vickers (1985), this literature interprets the managerial objective of the firm as a monetary compensation given to the CEO who seeks to maximize his revenue. As justly remarked by Miller and Pazgal, there is only a semantic difference between the delegation approach and the behavioral approach. By contrast with these approaches, we insist that different managerial attitudes are compatible with profit maximization, whereas they use various parameterized classes of managerial objective functions, departing from pure profit maximization.[4] Moreover, the (second-stage) games derived from these classes of objective functions resort to single strategic variables (either quantities or prices) and thus generate two parameterized set of equilibria, price- and quantity-equilibria. The choice of the strategic variable remains exogenous (even at the first stage). In our approach the strategies are price-quantity pairs allowing to recover both types of equilibria in the same parameterized class, so that introducing a first stage endogenizes the choice of the strategic variable. Last, the classes of objective functions analyzed by these alternative approaches are not derived from a more primitive model, as we will do for our parameterization.

Finally, another line of research is related to our work, in particular to our analysis of the influence of the intensity of competition on the firm equilibrium degree of competitive aggressiveness. This is the IO literature dealing with consumer brand loyalty. Typically, the market is there divided into several segments, a captive segment of loyal consumers for each firm and a contested segment of possibly switching consumers (Narasimhan (1988), Fershtman and E. Muller (1993), Deneckere, Kovenock, and Lee (1992)).

Outline of the paper. The paper is organized as follows. In Section II we start from two extreme formulations of the profit maximization objective, and introduce a parameterized class of managerial objective functions as convex combinations of these two formulations, all involving price-quantity pairs as strategic variables. We then propose a spatial duopoly model with market segmentation where, by specifying the parameters of competitive aggressiveness we can recover various classical competition regimes as nested regimes. Finally, we extend this duopoly model to analyze the strategic choice of managerial aggressiveness in a delegation game. In Section III, in order to endogenize the competitive

[4]These functions are linear combinations of either the firm profit and sales (*e.g.* Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Fumas (1992)) or of both firms profits (*e.g.* Miller and Pazgal (2002)).

aggressiveness parameters, we propose a more primitive price-quantity model introducing the possibility of consumer price discounting. We conclude in Section IV.

II. Price-Quantity Competition

We study a market where two firms compete to sell two different brands of the same product. We denote by $D_i(p_i, p_j)$ the demand for brand i at price p_i, given the price p_j posted for brand j $(i, j = 1, 2,\ i \neq j)$. The aggregate demand $D_i(p_i, p_j) + D_j(p_j, p_i)$ will be denoted $\overline{D}(p_i, p_j)$. Demand functions D_i and $\overline{D}$ are assumed decreasing in p_i. The quantity q_i of brand i is produced at constant unit cost $c_i \geq 0$.

A basic ingredient in our approach is to suppose that the sellers do not privilege one strategic variable but, following the pioneering analysis of Shubik (1959), that they compete in both prices and quantities: they simultaneously fix the prices p_i and sales targets q_i, $i = 1, 2$. The sales targets represent some desired output levels which are *a priori* announced by the firms. We shall first introduce a general formulation of the firms' managerial objectives on this basis, and then analyze the impact of this assumption in the Hotelling (1929) framework.

II.1. A General Formulation of Managerial Objectives

A first managerial objective to consider is the standard profit maximization under pure price competition. For a firm i, it is equivalent with constrained profit maximization in both price and quantity as follows:

$$\max_{(p_i, q_i) \in \mathbb{R}^2_+} (p_i - c_i) q_i, \text{ s.t. } q_i \leq D_i(p_i, p_j). \tag{1}$$

Since it is always advantageous for firm i to have q_i equal to $D_i(p_i, p_j)$, profit can be simply written as $(p_i - c_i)D_i(p_i, p_j)$, so that the game reduces to pure price (Bertrand-like) competition. We may characterize this managerial objective as oriented towards *price competitiveness*. No attention is paid to the rival's sales target. When both firms behave in such a way, we obtain the toughest type of competition.

A second way to express profit maximization is for firm i to take the rival's quantity effectively into account. Firm i considers that it will sell the quantity $\left(\overline{D}(p_i, p_j) - q_j\right)^+ \equiv \max\{\overline{D}(p_i, p_j) - q_j, 0\}$ left by firm j according to its sales target q_j. Then constrained profit maximization in both price and quantity becomes

$$\max_{(p_i, q_i) \in \mathbb{R}^2_+} (p_i - c_i) q_i, \text{ s.t. } q_i \leq \left(\overline{D}(p_i, p_j) - q_j\right)^+. \tag{2}$$

The rival's quantity q_j must now be taken into account (along with price p_j) in a Cournot-like way.[5] This managerial objective may be motivated by the desire not to upset the opponent; it thus represents a conciliatory attitude, oriented towards *target accomodativeness*.

[5]Contrary to what is commonly believed, price strategies play also a role in Cournot's equilibrium concept, along with quantity strategies: "Proprietor (1) can have no direct influence on the determination of D_2: all that he can do, when D_2 has been determined by proprietor (2), is to choose for D_1 the value which is best for him. This he will be able to accomplish *by properly adjusting his price*, except as proprietor (2), who, seeing himself *forced to accept this price and this value of* D_1, may adopt a new value for D_2, more favourable to his interests than the preceding one" (Cournot (1838, 1897), p. 80; our emphasis).

This suggests to formulate a general managerial objective of profit maximization covering intermediate regimes, as it is done by d'Aspremont and Dos Santos Ferreira (2009), when the oligopoly is homogeneous and the extreme objectives are those associated with Bertrand and Cournot. Any intermediate regime is characterized for each firm by a specific feature of its manager's behavior which is referred to as "competitive aggressiveness" in the management literature (J. G. Covin and T. J. Covin (1990), Venkatraman (1989)).

For any *degrees of competitive aggressiveness* $(\theta_1, \theta_2) \in [0,1]^2$, we shall define a (generalized)[6] game, called the *θ-game*, where each player's strategy set $S_i(p_j)$ and payoff function $\pi_i^\theta(p_i, q_i, p_j, q_j)$ are defined by

$$S_i(p_j) = \{(p_i, q_i) \in \mathbb{R}_+^2 : q_i \leq D_i(p_i, p_j)\}, \tag{3}$$

$$\pi_i^\theta(p_i, q_i, p_j, q_j) = (p_i - c_i)\Big(\theta_i q_i + (1-\theta_i)\big(\overline{D}(p_i, p_j) - q_j\big)^+\Big). \tag{4}$$

We should emphasize that the main role of the sales target q_i as a strategic variable is to have this quantity taken as given by the competitor, so determining firm j's residual demand. At equilibrium, sales targets and residual demands coincide for both firms, and so do payoffs and profits. We should also notice that it is always advantageous for firm i to have $p_i > c_i$ and, as soon as $\theta_i > 0$, $q_i = D_i(p_i, p_j)$. It follows that the first order condition for maximization of firm i's profit at equilibrium (p^*, q^*) can be computed to be

$$D_i(p_i^*, p_j^*) + (p_i^* - c_i)\left(\frac{\partial D_i(p_i^*, p_j^*)}{\partial p_i} + (1-\theta_i)\frac{\partial D_j(p_j^*, p_i^*)}{\partial p_i}\right) = 0. \tag{5}$$

The two extreme formulations of profit maximization are nested in this general formulation: the most aggressive competition regime corresponds to the parameter values $\theta_1 = \theta_2 = 1$ and the least to $\theta_1 = \theta_2 = 0$. We will show in the following that, in addition to these Bertrand-like and Cournot-like regimes, other standard competition regimes – such as pure quantity competition and Stackelberg regimes – are also nested in this general formulation.

II.2. Consumers' Demand in an Extended Hotelling Model

Consider the Hotelling spatial duopoly where a continuum of consumers, each likely to buy one unit of the product, is uniformly distributed on the interval $[0, 1+2a]$. All consumers have the same valuation v of the product. Brand 1 is positioned at point a and brand 2, symmetrically, at point $1+a$, so that they are separated by a distance normalized to 1.

The market is thus divided into three segments: $[0, a]$ on the side of firm 1 and $[1+a, 1+2a]$ on the side of firm 2, plus the middle segment $[a, 1+a]$. Consumers located on any segment will buy the brand offering the lowest delivered price (including the travel cost) provided it does not exceed their reservation price v. Consumers are in principle indifferent between the two brands when the delivered prices coincide. For indifferent consumers on the extreme segments, we assume that preference is given to the closest brand. The two extreme segments may thus be called *captive*, the middle one *contested*. The travel costs differ according to the market segment considered. The consumer located in firm i's captive segment bears a travel unit cost t_i; the consumer located in the contested segment bears a travel unit cost t.

On the captive segment for brand i, a customer located at distance $d \in [0, a]$ from firm i will purchase that brand if the *delivered price* $p_i + t_i d$ does not exceed the reservation price v (we disregard for the moment opportunities which arise with undercutting,

[6] The game is "generalized" in the sense of Debreu (1952), since the choice of each firm is restricted to a subset of $\mathbb{R}_+^2$ which depends upon the competitor's choice.

i.e. $p_j + t < p_i$). The quantity demanded to firm i on its captive segment is consequently $\min\{(v - p_i)/t_i, a\}$. On the contested segment, the demand is upper bounded by $\min\{(v - p_i)/t, 1\}$, but there is an additional requirement: a customer located at distance d from firm i will only purchase brand i if the delivered price $p_i + td$ does not exceed the delivered price $p_j + t(1 - d)$ for the other brand, that is, if $d \leq (t + p_j - p_i)/2t$. Thus, if again we momentarily disregard undercutting opportunities associated with $|p_i - p_j| > t$, so that $(t + p_j - p_i)/2t \in [0, 1]$, the demand for brand i on the contested segment is $\min\{(v - p_i)/t, (t + p_j - p_i)/2t\}$.

Taking an abstract view of the characteristics space, beyond its strict geographical interpretation, a decrease in the travel cost t on the sole contested segment appears as a convenient way to make the two products more substitutable, and hence to increase, independently of any autonomous change in competitive aggressiveness, the structural intensity of competition (reflected in the slope $-1/2t$ of the demand on the contested segment). Also, an increase in the travel cost t_i on the captive segment of firm i reduces the absolute value of the slope $-1/t_i$ of firm i's demand function on the captive segment, enhancing its market power on this segment. A decrease in the ratio $\tau_i \equiv t/t_i$ consequently describes a redistribution of market power from the contested to the captive segment. However, an increase in t_i also reduces the size v/t_i of the captive segment relative to the fixed unit size of the contested segment, thus increasing the degree of exposure of firm i to competition.

To simplify the analysis by avoiding an excessive number of cases, we introduce the following assumption.

Assumption 1.

$$1 < \frac{v}{t} < a \min\left\{\frac{t_1}{t}, \frac{t_2}{t}\right\}.$$

The left inequality implies that $(t + p_j - p_i)/2t < (v - p_i)/t$ for any pair of prices which do not exceed the monopoly price $v/2$, thus excluding the case of coexisting local monopolies with disconnected market areas. As to the right inequality, it implies that the captive segment of each firm is never covered even for a nil price: $(v - p_i)/t_i \leq v/t_i < a$. Assumption 1 thus ensures that the two firms are active competitors for customers on the contested segment, and that the demand on the captive segments is always responsive to price changes. It also entails that the demand for brand i is given by $F_i(p_i) \equiv (v - p_i)/t_i$ on the corresponding captive segment and by $G(p_i, p_j) \equiv (t + p_j - p_i)/2t$ on the contested segment.

We thus obtain the following specifications:

$$D_i(p_i, p_j) = F_i(p_i) + G(p_i, p_j) = \frac{v - p_i}{t_i} + \frac{t + p_j - p_i}{2t}, \tag{6}$$
$$\overline{D}(p_i, p_j) = 1 + F_i(p_i) + F_j(p_j) = 1 + \frac{v - p_i}{t_i} + \frac{v - p_j}{t_j}.$$

The simplification obtained through Assumption 1, by eliminating boundary effects, opens the way to a straightforward extension of the model to the n firm oligopoly, with the demand for brand i on the captive segment given by $F_i(p_i)$ specified as before, and with the demand for brand i on the contested segment given by

$$G(p_i, \overline{p}_{-i}) = \frac{t + \overline{p}_{-i} - p_i}{nt}, \text{ where } \overline{p}_{-i} = \frac{\sum_{j \neq i} p_j}{n - 1}. \tag{7}$$

If we want to stick to a spatial interpretation of this extended model, we can refer to the spokes model (Chen and Riordan (2007)), where the customers are uniformly distributed on a spokes network and each firm is located at the origin of some spoke.[7]

II.3. The θ-Game and its Equilibrium

We assume for simplicity that unit costs are normalized to zero. First order conditions (5) apply under the restriction $q_i = D_i(p_i, p_j)$ for $i, j = 1, 2,\ i \neq j$ (holding at equilibrium). Under this restriction and under the specifications (6), they can be written as

$$\frac{v - p_i}{t_i} + \frac{t + p_j - p_i}{2t} - \left(\frac{1}{t_i} + \frac{\theta_i}{2t}\right) p_i = 0,\ i, j = 1, 2,\ i \neq j, \tag{8}$$

giving the following "best reply function":

$$p_i = \frac{1 + 2v/t_i + p_j/t}{1 + 4t/t_i + \theta_i} t,\ i, j = 1, 2,\ i \neq j. \tag{9}$$

The best reply in p_i is an increasing function of price p_j (strategic complementarity) and of the captive segment size[8] v/t_i. It is a decreasing function of the ratio $\tau_i \equiv t/t_i$ of the travel unit costs, on the contested segment with respect to the captive one. The two best reply functions lead together to the equilibrium prices

$$p_i^*(\theta_i, \theta_j) = \frac{(1 + 4\tau_j + \theta_j)(1 + 2v/t_i) + (1 + 2v/t_j)}{(1 + 4\tau_i + \theta_i)(1 + 4\tau_j + \theta_j) - 1} t,\ i, j = 1, 2,\ i \neq j. \tag{10}$$

The equilibrium price $p_i^*(\theta_i, \theta_j)$ of firm i is decreasing with respect to its own aggressiveness θ_i and to that, θ_j, of its competitor. Moreover, it is an increasing function of both captive segment sizes v/t_i and v/t_j, and a decreasing function of both ratios τ_i and τ_j, although more responsive to the own values v/t_i and τ_i.[9]

The system (10) does not ensure that $p_1^*(\theta_1, \theta_2)$ and $p_2^*(\theta_2, \theta_1)$ are indeed equilibrium prices for *any* pair (θ_1, θ_2) in $[0, 1]^2$. As well known, the possibility of undercutting threatens existence of a Nash equilibrium of the Hotelling pricing game. In this section, we shall simply avoid the problem by forbidding undercutting. This amounts to introduce a *no-mill price undercutting condition*, which has been associated in the literature with the suspension of the "zero conjectural variation" hypothesis when undercutting leads to the elimination of the competitor (see Eaton (1976), Eaton and Lipsey (1978)).[10] In Section IV, we will introduce a more primitive version of the game where this condition will be replaced by the possibility for each manager to offer price discounts to their customers. Here though, under this condition, we get a simple proof by introducing a further restriction on the parameters. In the standard Hotelling model where $t_1 = t_2 = t$,

[7] For any firm i, existence of a captive segment results from unavailability of the second preferred brand. If the origins of the spokes are all occupied by active firms, captive segments disappear, and the market for brand i actually reduces to its contested segment. By slightly modifying Chen and Riordan's model through the introduction of a *hinterland* for each firm, we would obtain exactly our specification.

[8] Hence, the firm with a larger captive segment (less exposed to competition) sets, *ceteris paribus*, a higher price and is, in this sense, less "aggressive" (see Narasimhan (1988), p. 435).

[9] For simplicity of notations we have normalized to 1 the distance between the two firms. Should it be taken as the variable l, this variable (instead of 1) would be added to $2v/t_i$ and $2v/t_j$ in the numerator of the expression on the right hand side of (10). Hence, equilibrium prices are naturally increasing in the distance between the two firms, which measures the size of the contested segment.

[10] Commenting on Eaton (1976), Novshek (1980) suggests to view the equilibrium outcome resulting from this condition as a Nash equilibrium for an extended game with prices and response price functions as strategies. For another route see d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (1995).

this amounts to assume: $3/2 < v/t < 14$. In the general case, this new assumption becomes:

Assumption 2.

$$1 + \frac{1 + 2\tau_j}{\tau_i + \tau_j + 4\tau_i\tau_j} < \frac{v}{t} < 6 + 8\tau_j.$$

We now prove existence of an equilibrium with active competition between the two firms for any degrees of their competitive aggressiveness.

Proposition 1. Under Assumption 2 and no-mill price undercutting, for any $\theta \in [0,1]^2$ there is a Nash equilibrium

$$((p_1^*(\theta), D_1(p_1^*(\theta), p_2^*(\theta))), (p_2^*(\theta), D_2(p_2^*(\theta), p_1^*(\theta))))$$

of the θ-game (with $p_i^*(\theta)$ given by (10)), such that both firms are active competitors on the contested segment.

Proof. See Appendix. □

Using $q_i^* = D_i(p_i^*, p_j^*)$, for $i = 1, 2$, $i \neq j$, we can then compute the equilibrium profit

$$\begin{aligned} \Pi_i(\theta_i, \theta_j) &= p_i^*(\theta_i, \theta_j) D_i\big(p_i^*(\theta_i, \theta_j), p_j^*(\theta_j, \theta_i)\big) \\ &= \left(\frac{(1 + 4\tau_j + \theta_j)(1 + 2v/t_i) + (1 + 2v/t_j)}{(1 + 4\tau_i + \theta_i)(1 + 4\tau_j + \theta_j) - 1} \right)^2 \frac{2\tau_i + \theta_i}{2} t. \end{aligned} \tag{11}$$

It is easy to check that Π_i is decreasing in θ_j, and unimodal with respect to θ_i, with[11]

$$\arg \max_{\theta_i \in [0,1]} \Pi_i(\theta_i, \theta_j) = \frac{4\tau_j + \theta_j}{1 + 4\tau_j + \theta_j}, \ i = 1, 2, \ i \neq j. \tag{12}$$

II.4. Varying the Competition Regimes

As shown above, the standard Bertrand-Hotelling pure price competition regime and the Cournot-like regime preserving target compatibility are nested in our general formulation as two polar cases, corresponding respectively to maximal ($\theta_1 = \theta_2 = 1$) or minimal ($\theta_1 = \theta_2 = 0$) competitive aggressiveness. We shall now show that the pure quantity competition regime is also nested, as an intermediate case. In addition, we shall prove that different Stackelberg equilibria (in prices, in quantities or in price-quantity pairs) can be easily computed as equilibria of our θ-game for appropriate values of (θ_1, θ_2).

II.4.1. The Pure Quantity Competition Regime

Firm i chooses the quantity q_i in order to maximize its profit, while abiding by the announced sales target q_j of firm j. This amounts to solve

$$\begin{aligned} &\max_{(p_i, p_j) \in [0,v]^2} p_i D_i(p_i, p_j) \\ &\text{s.t. } D_j(p_j, p_i) = q_j. \end{aligned} \tag{13}$$

[11] Increasing the distance between the two firms, hence the size of the contested segment, would increase the equilibrium profit $\Pi_i(\theta_i, \theta_j)$ (see footnote 9). It would however not change the value θ_i at which $\Pi_i(\cdot, \theta_j)$ is maximized.

The quantity equilibrium prices (p_1^q, p_2^q) are accordingly defined by the two first-order conditions (for $i, j = 1, 2,\ i \neq j$)

$$D_i(p_i^q, p_j^q) + p_i^q \frac{\partial D_i(p_i^q, p_j^q)}{\partial p_i} - \frac{\partial D_i(p_i^q, p_j^q)/\partial p_j}{\partial D_j(p_j^q, p_i^q)/\partial p_j} p_i^q \frac{\partial D_j(p_j^q, p_i^q)}{\partial p_i} = 0. \tag{14}$$

Comparing with equations (5) (with $c_i = 0$) and using equations (6), we see that equations (14) define the equilibrium of the θ-game for

$$\theta_i^q = 1 + \frac{\partial D_i(p_i^q, p_j^q)/\partial p_j}{\partial D_j(p_j^q, p_i^q)/\partial p_j} = \frac{\tau_j}{\tau_j + 1/2}, \tag{15}$$

again for $i, j = 1, 2,\ i \neq j$. Inserting these values in equations (10), we obtain, at the corresponding equilibrium of the θ-game, the quantity equilibrium prices $(p_1^q, p_2^q) = (p_1^*(\theta_1^q, \theta_2^q),\ p_2^*(\theta_2^q, \theta_1^q))$ and then, using equations (6), the corresponding quantities $(q_1^q, q_2^q) = (D_1(p_1^q, p_2^q),\ D_2(p_2^q, p_1^q))$.

Observe that, as travel cost t on the contested segment tends to zero, that is, as the two brands become perfectly substitutable, $(\tau_1, \tau_2) \to (0, 0)$, so that $(\theta_1^q, \theta_2^q) \to (0, 0)$: pure quantity competition and Cournot-like target accomodating competition coincide at the limit, in the homogeneous duopoly, as we should expect.

II.4.2. Stackelberg Equilibria and First Mover (Dis)advantage

In the three competition regimes we have specified (Bertrand-Hotelling price competition, Cournot-like target accomodating competition and pure quantity competition) firms are supposed to move simultaneously. These regimes were shown to be nested in our general formulation, the equilibria corresponding to each regime being obtained as equilibria of our θ-game for appropriate, exogenously given, symmetric expressions of θ. In the case of Stackelberg competition, where firms move sequentially (one firm is the leader, the other the follower), we can still obtain the Stackelberg equilibria as equilibria of our θ-game for appropriate, exogenously given but now asymmetric, expressions of θ. It will in addition be shown that this procedure allows to simplify the computation of Stackelberg equilibria.

Take firm 1 as the leader and firm 2 as the follower. Suppose first that the Stackelberg firms compete in prices. Thus, the follower sets p_2 so as to maximize $p_2 D_2(p_2, p_1)$, which amounts to taking $\theta_2 = 1$ in the managerial objective function (4). The leader sets p_1 so as to maximize $p_1 D_1(p_1, \arg\max_{p_2}(p_2 D_2(p_2, p_1)))$. This is the same as maximizing $p_1 D_1(p_1, p_2)$ in both prices p_1 and p_2, under the constraint resulting from the first order condition of the follower's program (see (5)), a sufficient condition because of the concavity in p_2 of $p_2 D_2(p_2, p_1)$. In order to prepare a generalization of the procedure, we may write as follows the leader's program:

$$\begin{aligned} &\max_{(p_1, p_2) \in \mathbb{R}^2_+} p_1 D_1(p_1, p_2) \\ &\text{s.t. } D_2(p_2, p_1) + p_2 \frac{\partial D_2(p_2, p_1)}{\partial p_2} + (1 - \theta_2) p_2 \frac{\partial D_1(p_1, p_2)}{\partial p_2} = 0, \end{aligned} \tag{16}$$

where θ_2 is here to be taken equal to 1.

The leader's program can be equivalently formulated under an additional constraint

and with an additional decision variable, as follows:

$$\max_{(p_1,p_2,\theta_1)\in\mathbb{R}^2_+\times\mathbb{R}} p_1 D_1(p_1,p_2) \tag{17}$$
$$\text{s.t.} \begin{cases} D_1(p_1,p_2) + p_1\frac{\partial D_1(p_1,p_2)}{\partial p_1} + (1-\theta_1)p_1\frac{\partial D_2(p_2,p_1)}{\partial p_1} = 0 \\ D_2(p_2,p_1) + p_2\frac{\partial D_2(p_2,p_1)}{\partial p_2} + (1-\theta_2)p_2\frac{\partial D_1(p_1,p_2)}{\partial p_2} = 0 \end{cases}.$$

Notice that the additional constraint is in fact not restrictive provided θ_1 can be any real number, so that the programs (16) and (17) are indeed equivalent. The program (17) can be solved in two stages, the first of which consists in solving the system of two equations for $\theta_2 = 1$ and for any given θ_1. When $\theta_1 \in [0,1]$, these equations express the (sufficient) first order conditions for maximization of the managerial objective functions (4) of the two firms in the corresponding θ-game (see (5)), so that this stage corresponds then to the determination of an equilibrium of the θ-game. The second stage consists in determining the value of θ_1 which maximizes, for $i = 1$ and $\theta_2 = 1$, the function defined in (11), namely

$$\begin{aligned} \theta_1^S(\theta_2) &\equiv \arg\max_{\theta_1\in[0,1]} p_1^*(\theta_1,\theta_2) D_1(p_1^*(\theta_1,\theta_2), p_2^*(\theta_2,\theta_1)) \\ &\equiv \arg\max_{\theta_1\in[0,1]} \Pi_1(\theta_1,\theta_2) = \frac{4\tau_2+\theta_2}{1+4\tau_2+\theta_2}. \end{aligned} \tag{18}$$

It is true that the maximization of $\Pi_1(\cdot,\theta_2)$ is here performed over $[0,1]$ whereas θ_1 can be taken as any real number in program (17). But $\Pi_1(\cdot,\theta_2)$, as given by (11), is unimodal in the interval where it is positive, with a maximum in the open interval $(0,1)$, so that the solutions to the two maximization problems do coincide. In the present case, under price competition, the follower's competitive aggressiveness is $\theta_2^{Sp} = 1$ and, by (18), the leader's competitive aggressiveness is

$$\theta_1^{Sp} = \theta_1^S(1) = \frac{1+4\tau_2}{2+4\tau_2}. \tag{19}$$

We observe that the leader is in this case in the worst possible position, since the competitive aggressiveness of the follower is at its maximum and Π_1 is a decreasing function of θ_2. The more moderate competitive aggressiveness of the leader benefits more to the follower, who may end up with a larger profit (and certainly does if there is not too much asymmetry benefitting to the leader). This is a situation of first mover *disadvantage*, well known in the context of strategic complementarity (see Gal-Or (1985)). This disadvantage is the larger the smaller the value of θ_1^{Sp}, hence the smaller the ratio $\tau_2 = t/t_2$. Thus, a redistribution of market power from the contested segment to the follower's captive segment (a reduction of τ_2) will induce a smaller competitive aggressiveness and so a larger first mover disadvantage for the leader (a conclusion that can be generalized, by (18), to any other competition regime characterized by θ_2).

The same procedure can be applied to the regime of quantity competition, where the follower solves an optimization program in q_2 (given q_1), the solution of which can however be obtained, according to equation (15), by making the competitive aggressiveness of firm 2 be equal to $\theta_2^q = \tau_1/(1/2+\tau_1)$. Hence, the leader's choice of q_1 can be retrieved from the maximization in θ_1 of $\Pi_1(\theta_1,\theta_2^q)$, that is, according to (18),

$$\theta_1^{Sq} = \theta_1^S\left(\frac{\tau_1}{1/2+\tau_1}\right) = 1 - \frac{1/2+\tau_1}{1/2+2\tau_1+2\tau_2+4\tau_1\tau_2}. \tag{20}$$

In contrast with the previous case of price competition, $\theta_1^{Sq} > \theta_2^q$ if $\tau_2 > \tau_1^2/(1+2\tau_1)$, which is always true if $\tau_2/\tau_1 = t_1/t_2 \geq 1/2$ (if the ratio of market powers in the two captive segments is not too unfavorable to the leader). The leader's profit is then higher than the follower's (first mover *advantage*). Moreover, as $(\theta_1^{Sq}, \theta_2^{Sq}) < (\theta_1^{Sp}, \theta_2^{Sp})$, both profits are higher in a Stackelberg quantity equilibrium than in a Stackelberg price equilibrium.

More generally, we may consider the follower's optimizing price-quantity choice under any competition regime parameterized by some given degree θ_2 of competitive aggressiveness, given any arbitrary price-quantity choice of the leader. The optimizing choice of the latter will be the one resulting from competitive aggressiveness $\theta_1^S(\theta_2) = \arg\max_{\theta_1} \Pi_1(\theta_1, \theta_2)$.

To summarize these results we have:

Proposition 2. Stackelberg equilibria, whether in prices, in quantities, or more generally in price-quantities under a regime parameterized by the follower's competitive aggressiveness θ_2, can be determined as equilibria of the θ-game, by equating the leader's competitive aggressiveness θ_1 to the one which maximizes $\Pi_1(\cdot, \theta_2)$, namely $\theta_1^S(\theta_2)$, as given by (18).

This result illustrates the fact that introducing managerial attitudes in competition makes the debate on sequentiality vs. simultaneity of decisions somewhat irrelevant. In particular, the first mover advantage (resp. disadvantage) in a sequential game translates into a more aggressive attitude of the leader (resp. the follower) in the simultaneous game.

II.5. Competitive Aggressiveness and Intensity of Competition

In this subsection we extend our model by introducing a preliminary stage where the θ parameters are chosen strategically by the two firms. This extension allows us to consider the influence on firms competitive aggressiveness of changes in their environment, in particular in the intensity of competition.

In a context of separation between ownership and management, the selection of the manager on the basis of his personality traits, experience or accepted policy orientation is a way for the owners of setting the competitive aggressiveness of their firm.[12] This generates a two-stage *delegation game*. In the first stage, the owners of each firm i hire a manager with aggressiveness θ_i and, in the second stage, these managers play the corresponding θ-game. The first-stage payoffs $\Pi_i(\theta_i, \theta_j)$, $i = 1, 2$, are given by (11).

Using the equations given by (12), the equilibrium competitive aggressiveness $\widehat{\theta}_i(\tau_i, \tau_j)$ of each firm i can be computed:

$$\widehat{\theta}_i(\tau_i, \tau_j) = \sqrt{\frac{2(\tau_i + \tau_j + 4\tau_i\tau_j)}{(1+2\tau_j)} + 4\tau_i^2} - 2\tau_i, \ i, j = 1, 2, \ i \neq j. \tag{21}$$

The equilibrium value $\widehat{\theta}_i$ is an increasing function of both its arguments, although more responsive to the competitor's ratio τ_j. A redistribution of market power from the contested segment to any one of the two captive segments (a reduction of τ_i or τ_j) will induce less competitive aggressiveness of both firms in equilibrium. If this redistribution is the consequence of a higher intensity of competition (a decrease in t, the two brands becoming more substitutable), the adjustment in competitive aggressivenes will have a moderating effect on profits, slowing down their decline. If this redistribution concerns

[12] Of course, as mentioned in the introduction, the same objective can be attained through an appropriate monetary compensation, independently of the personality of the manager. Our model is valid for the two interpretations.

firm i alone (an increase in t_i, enhancing its market power on the captive segment), this firm will benefit from the reduction of its rival's competitive aggressiveness, θ_j, which will decrease more than its own, θ_i.

Considering the standard case where $t_1 = t_2 = t$, the Nash equilibrium at the first stage is $\theta_1^* = \theta_2^* = 2(\sqrt{2} - 1) \simeq 0.828$. Thus, when competing in aggressiveness, firms are led to a (subgame perfect) equilibrium outcome between the quantity and the price equilibria, since $2/3 = \theta^q < \theta_i^* < \theta^p = 1$, with a profit $\Pi_i(\theta^*)$ lower than the quantity competition profit but higher than the price competition one.

These equilibria can all be represented in the $\theta_1 \times \theta_2$ space, as depicted in Figure 1. With any point $\theta \in [0,1] \times [0,1]$ we associate the pair of prices $p^*(\theta)$ (given by (10)) of the equilibrium of the θ-game. The relations (12) determine the reaction functions $\theta_i = R_i(\theta_j)$, $i = 1, 2$, $i \neq j$, of the first stage of the delegation game: they are represented by the curves R_1 and R_2, the intersection of which gives the equilibrium θ^* of this game.

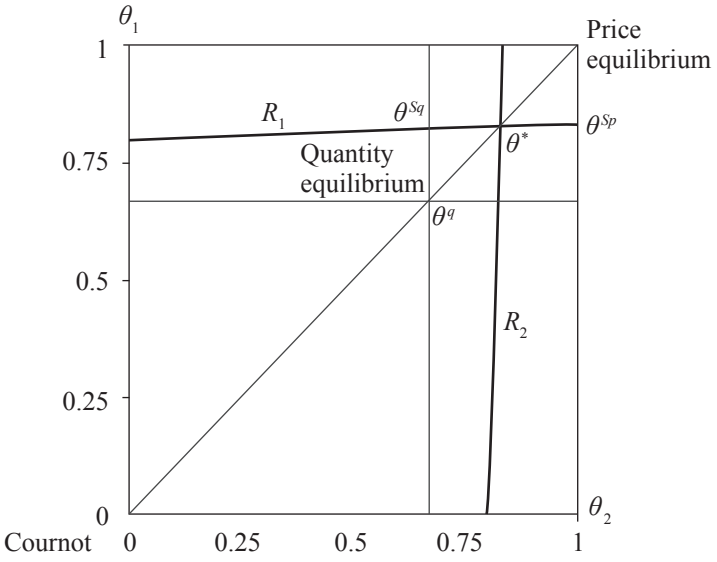


Figure 1: The Set of θ-Equilibria under Uniform Travel Costs

The competitive aggressiveness pairs θ leading in the family of θ-games to the Cournot solution[13] as well as to the price and quantity equilibria are represented by the points 0, 1 and θ^q of the first diagonal, respectively. Also, the competitive aggressiveness pairs associated with Stackelberg equilibria when firms compete in prices or in quantities are represented by the points θ^{Sp} and θ^{Sq}, respectively, on the reaction curve R_1 of the leader, namely firm 1.

III. Endogenizing Competitive Aggressiveness

All the preceding analysis relies on our formulation of profit maximization as a general parameterized managerial objective. Our purpose now is to introduce a more primitive

[13]It can be noted that the Cournot solution coincides with the collusive solution in the symmetric case. This can be checked by examining the first order conditions (5), where the partial derivatives of the two demand functions with respect to p_i are both multiplied by p_i^*. In the first order conditions for joint profit maximization, they are each multiplied by the own price (the derivative of D_j by p_j^*). However, in the symmetric case where the two prices are equal, this difference is immaterial.

model from which our class of managerial objectives and its parameterization can be derived.

III.1. The Discount-Pricing Game

The primitive model we are going to consider is given by the following two-firm game. The strategy of each firm i is an *announced* price-quantity pair $(p_i, y_i) \in [0, v] \times [0, 1]$. The price p_i is the unit *list price* announced by firm i for its own brand, applying as above to both the contested and the captive segments (no price discrimination). The quantity y_i is the *sales target* of firm i on the contested segment, so that the quantity targeted on the whole market is $q_i = y_i + F_i(p_i)$, with $F_i(p_i) = (v - p_i)/t_i$. The main new feature of this primitive model is the possibility given to each firm i of adjusting the list price p_i through a discount (see Holt and Scheffman (1987), and Narasimhan (1988), for similar uses of price discounting strategies). The purpose of the discount is both to meet the sales target y_i as far as possible and to protect the captive segment from undercutting by the rival. Formally, this means that firm i follows the *discount-pricing rule*:

$$P_i(p_i, y_i, p_j) = \begin{cases} p_i & \text{if} \quad y_i \leq G(p_i, p_j) \\ G^{-1}(\cdot, p_j)(y_i) & \text{if} \quad G(p_i, p_j) \leq y_i \leq G(0, p_j) \\ 0 & \text{if} \quad G(0, p_j) \leq y_i \end{cases}, \tag{22}$$

where $G(p_i, p_j) = (t + p_j - p_i)/2t$. This discount pricing rule covers three cases. In the first case ($y_i \leq G(p_i, p_j)$), the sales target y_i can be realised at the list price p_i, and no discount is required. In the second case (if $G(p_i, p_j) < y_i$), the sales target exceeds the demand for brand i on the contested segment, so that firm i charges a discount price $G^{-1}(\cdot, p_j)(y_i) = p_j + t - 2ty_i$ ensuring the realization of its target. Besides, if there is undercutting ($G(p_i, p_j) < 0$), the discount price keeps firm i *hinterland* captive ($p_j + t - P_i(p_i, y_i, p_j) = 2ty_i \geq 0$). In the third case ($G(0, p_j) \leq y_i$), the sales target can at best be reached at a zero price, but the *hinterland* can be kept captive.

Given this rule applied by both firms, and a strategy pair $((p_i, y_i), (p_j, y_j))$, the payoff $\pi_i(p_i, y_i, p_j, y_j)$ of firm i is equal to its revenue computed at the price and the quantity resulting from the discount pricing rule:

$$P_i(p_i, y_i, p_j)(F_i(P_i(p_i, y_i, p_j)) + G(P_i(p_i, y_i, p_j), P_j(p_j, y_j, p_i))).$$

This completes the definition of the *discount-pricing game*. A Nash equilibrium $((p_1^*, y_1^*), (p_2^*, y_2^*))$ of this generalized game is said *to fulfil the sales targets* of both firms if $y_i^* = G(p_i^*, p_j^*)$ for $i, j = 1, 2$ and $i \neq j$. The following lemma states that the outcome of any Nash equilibrium of the discount-pricing game can always be obtained through an equilibrium fulfilling the sales targets of both firms, hence actually with no price discounting.

Lemma 1. Let the pair of strategies $((p_1^{**}, y_1^{**}), (p_2^{**}, y_2^{**})) \in ([0, v] \times [0, 1])^2$ be a Nash equilibrium of the discount-pricing game such that $y_i^{**} \neq G(p_i^{**}, p_j^{**})$ for some i. Then there exists an equilibrium $((p_1^*, y_1^*), (p_2^*, y_2^*))$ with the same outcome (*i.e.*, $P_i(p_i^*, y_i^*, p_j^*) = P_i(p_i^{**}, y_i^{**}, p_j^{**})$, for $i, j = 1, 2$ and $i \neq j$) and such that the sales targets are fulfilled $y_i^* = G(p_i^*, p_j^*)$ for $i, j = 1, 2$ and $i \neq j$.

Proof. See Appendix. □

According to this result, we may restrict our analysis to *equilibria with fulfilled sales targets*, without loss of generality as concerns outcomes, and always refer from now on to such equilibria without always explicitly mentioning this restriction.

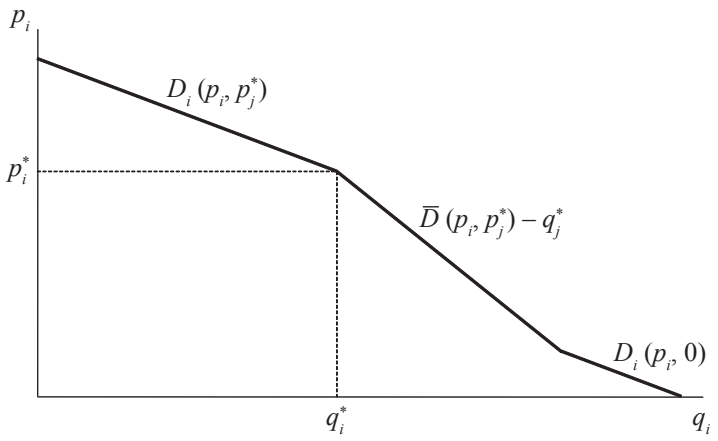


Figure 2: The Demand for Brand i

III.2. A Kinked Demand for Each Brand

In order to characterize the equilibria with fulfilled sales targets of the discount-pricing game, we shall now consider the demand D_i for brand i, as defined by the first of equations (6), when taking into account the fulfilled sales target restriction $y_i = G(p_i, p_j)$. First notice that this restriction implies, by the discount-pricing rule (22): $P_i(p_i, y_i, p_j) = p_i$. Hence, the demand for brand i we have to consider is

$$D_i(p_i, P_j(p_j, y_j, p_i)) = F_i(p_i) + G(p_i, P_j(y_j, p_j, p_i)). \tag{23}$$

As the discount-pricing rule P_j covers three different cases, we find three different pieces in the demand for brand i:

$$D_i(p_i, P_j(p_j, y_j, p_i)) = \begin{cases} D_i(p_i, p_j) & \text{if} \quad y_j \leq G(p_j, p_i) \\ F_i(p_i) + 1 - y_j & \text{if} \quad G(p_j, p_i) \leq y_j \leq G(0, p_i) \\ D_i(p_i, 0) & \text{if} \quad G(0, p_i) \leq y_j \end{cases}, \tag{24}$$

where we use $D_i\big(p_i, G^{-1}(\cdot, p_i)(y_j)\big) = F_i(p_i) + 1 - y_j$, since the rival's sales target is always fulfilled in this case through the discount-pricing rule, so that changes of p_i can only modify the loyal customers' demand. Using aggregate demand $\overline{D}$ as defined by the second of equations (6), we further have: $F_i(p_i) + 1 - y_j = \overline{D}(p_i, p_j) - (F_j(p_j) + y_j) = \overline{D}(p_i, p_j) - q_j$.

Because of non-differentiability of the discount-pricing rule $P_j(\cdot, y_j, p_i)$ at $y_j = G(p_j, p_i)$ and at $y_j = G(0, p_i)$, firm i faces a demand curve with two kinks, of which the higher one is the sole significant. Indeed, at an equilibrium $((p_1^*, q_1^*), (p_2^*, q_2^*))$ with fulfilled sales targets, $y_j^* = G\big(p_j^*, p_i^*\big)$, so that the corresponding point on the demand curve is the higher kink (see Figure 2).

Lemma 2. Let $((p_1^*, y_1^*), (p_2^*, y_2^*)) \in ([0, v] \times [0, 1])^2$ be an equilibrium of the discount-pricing game with fulfilled sales targets. Then, for $i, j = 1, 2$ and $i \neq j$, $(p_i^*, q_i^*) = (p_i^*, F_i(p_i^*) + y_i^*)$ solves the program:

$$\begin{aligned} \max_{(p_i, q_i) \in [0, v] \times [0, 1+a]} \quad & p_i q_i \\ \text{s.t.} \quad & q_i \leq D_i\big(p_i, p_j^*\big) \\ & q_i \leq \overline{D}\big(p_i, p_j^*\big) - q_j^*. \end{aligned} \tag{25}$$

Proof. See Appendix. □

The first constraint of firm i's program (25) can be seen as a *price competitiveness* condition, imposing the consistency of the planned output q_i and the list price p_i, given the list price p_j^* of firm j. The second constraint can be interpreted as a *target compatibility* condition which imposes the consistency of the planned output q_i and the list price p_i, taking into account potential price discounting by firm j so as to fulfill its sales target $y_j^* = q_j^* - F_i(p_i^*)$. Referring to the kinked demand curve represented in Figure 2, the portion above the higher kink corresponds to a binding price competitiveness constraint, whereas the portion between the kinks corresponds to a binding target compatibility constraint. We shall now use program (25) to obtain a convenient parameterization of its solutions and ultimately of the set of equilibria with fulfilled sales targets of the discount-pricing game.

III.3. Parameterization

Take a particular Nash equilibrium $(p^*, y^*) = ((p_1^*, y_1^*), (p_2^*, y_2^*))$ of the discount-pricing game with fulfilled sales targets. By Lemma 2, we know that the associated equilibrium configuration $(p^*, q^*) = ((p_1^*, p_2^*), (q_1^*, q_2^*))$, with $q_i^* = y_i^* + F_i(p_i^*)$ for $i = 1, 2$, solves the program (25), and hence satisfies the first order conditions:

$$\begin{cases} q_i^* + \lambda_i \frac{\partial D_i(p_i^*, p_j^*)}{\partial p_i} + \mu_i \frac{\partial \overline{D}(p_i^*, p_j^*)}{\partial p_i} = 0 \\ p_i^* - \lambda_i - \mu_i = 0 \end{cases}, \quad i, j = 1, 2 \text{ and } i \neq j, \tag{26}$$

where $\lambda_i, \mu_i \geq 0$ are the Lagrange multipliers respectively associated with the price competitiveness and the target compatibility constraints in firm i's program. Following the usual interpretation, the Lagrange multipliers λ_i and μ_i measure the contribution to the value of firm i of a marginal release of the price competitiveness and target compatibility constraints, respectively. The ratio $\theta_i = \lambda_i/(\lambda_i + \mu_i) \in [0, 1]$ may accordingly be seen as the relative pressure exerted on firm i by its price competitiveness constraint, when evaluated at (p^*, q^*). When scanning the set of equilibria, as θ_i increases from 0 (when the target compatibility constraint is alone to bind) to 1 (when only the price competitiveness constraint binds), the optimal strategy of firm i is chosen more and more for the sake of the latter.

Using the definition of θ_i, the two first order conditions (26) can be merged into the condition

$$q_i^* + \theta_i p_i^* \frac{\partial D_i(p_i^*, p_j^*)}{\partial p_i} + (1 - \theta_i) p_i^* \frac{\partial \overline{D}(p_i^*, p_j^*)}{\partial p_i} = 0, \ i, j = 1, 2 \text{ and } i \neq j, \tag{27}$$

which, by Lemma 2, must also be the first order condition for the maximization of the payoff of firm i in the discount-pricing game, under the fulfilled sales target restriction $y_i = G(p_i, p_j^*)$ (or, equivalently, $q_i = D_i(p_i, p_j^*)$). This first order condition leads to a complete parameterization of the set of equilibria, one for each $\theta = (\theta_1, \theta_2)$, which we call a θ-equilibrium.

Moreover, using $q_i^* = D_i(p_i^*, p_j^*) = \overline{D}(p_i^*, p_j^*) - D_j(p_j^*, p_i^*)$ and recalling that we are assuming $c_1 = c_2 = 0$, we see that condition (27) further coincides with the first order condition (5) for the maximization in (p_i, q_i) of $\pi_i^\theta(p_i, q_i, p_j^*, q_j^*)$, that is, of firm i payoff in the θ-game for the same value θ of the parameter. Because of local concavity of the payoff functions in the two games, and referring to Proposition 1, we can formulate our last proposition establishing that, for any $\theta = (\theta_1, \theta_2)$, a θ-equilibrium exists and coincides with a Nash equilibrium of the θ-game.

Proposition 3. Under Assumption 2, for any $\theta = (\theta_1, \theta_2) \in [0,1]^2$, a θ-equilibrium with fulfilled sales targets of the discount-pricing game exists and coincides with the equilibrium of the θ-game.

Proof. See Appendix. □

IV. Concluding Remarks

In this paper, competitive aggressiveness is introduced as a parameter specifying in various ways profit maximization as a managerial objective of the firm in a simple price-quantity competition model *à la* Hotelling. The managerial objective is a combination of profits under two extreme scenarios: the first one when the manager straightly acts without taking care of the rival's sales target, the second one, when he does, by fully accomodating that target. This approach is useful in modelling in a unified way the multiplicity of competition regimes and a large variety of observed managerial behaviors, facilitating comparisons and simplifying computations. It proves that very different managerial attitudes can be observed without departing from profit maximization.

Furthermore, as in the delegation literature, under the separation of ownership and management, competitive aggressiveness can be considered as a variable under the control of each firm. Firms are assumed to choose their managerial aggressiveness in the first stage of a delegation game, taking their environment into account. When in this environment competition becomes relatively more intense, threatening profitability, firms compensate by hiring less aggressive managers. Rather hire a dove than a hawk when the wind is rising.

Finally, enlarging the model by introducing the possibility for each manager to offer consumer price discounts is a way to endogenize our parameterization of various profit maximizing objectives. Price-discounting is used to meet the firm sales target and to protect its captive segment from undercutting. It leads to a constrained price-quantity duopoly game with multiplicity of equilibria. A parameterization of the set of equilibria is obtained by coefficients equal, for each firm, to the relative weight of the Lagrange multiplier associated with a competitiveness constraint. Each equilibrium of this enlarged game coincides with the equilibrium of the original managerial game with the corresponding competitive degrees of aggressiveness.

Appendix

A. Proof of Proposition 1

By the no-mill price undercutting condition, we must simply verify that there are no local deviations. Equations (10) together with the equalities (6) express the first order conditions for a solution of the programs of both firms whenever they are active competitors on the contested segment. Note that $p_i^*(\theta_i, \theta_j)$ being decreasing in both its arguments, we get the following upper bound:

$$p_i^*(\theta_i, \theta_j) \leq p_i^*(0,0) = \left(\frac{v}{t} + \frac{1 + 2\tau_j}{\tau_i + \tau_j + 4\tau_i\tau_j} \right) \frac{t}{2}.$$

For the contested segment to be covered and competition to remain active on this segment even when the prices are both equal to this upper bound, we must have $p_i^*(0,0) + t/2 < v$, which is equivalent to the left inequality in Assumption 2. This simplifying

condition, which reinforces the preceding assumption $1 < v/t$, excludes equilibria where firms are local monopolies with juxtaposed market areas on the contested segment. Also, in order for both firms to be active competitors on the contested segment, we must have $|p_i^*(\theta) - p_j^*(\theta)| < t$. The difference $p_i^*(\theta) - p_j^*(\theta)$ is maximized at $\theta_i = 0$ and $\theta_j = 1$, and we get

$$p_i^*(0,1) - p_j^*(1,0) = \frac{1 + 4\tau_j + 2\tau_i v/t - 4\tau_i}{2(1+4\tau_i)(1+2\tau_j) - 1} t.$$

When the expression on the RHS is negative, its absolute value can be computed to be smaller than t. When it is positive, it is smaller than t, provided the right inequality in Assumption 2 is satisfied.

To complete the proof, we must show that first order conditions are sufficient for a local maximum. By deriving the LHS of equation (8) with respect to p_i, we obtain a negative value, so that the quadratic objective function of firm i is indeed concave.

B. Proof of Lemma 1

Take the simple case in which $y_i^{**} < G(p_i^{**}, p_j^{**})$, implying $P_i(p_i^{**}, y_i^{**}, p_j^{**}) = p_i^{**}$. Then we can obtain a new equilibrium by taking $y_i^* = G(p_i^{**}, p_j^{**})$ and keeping the other strategies constant ($y_j^* = y_j^{**}$, $p_i^* = p_i^{**}$, $p_j^* = p_j^{**}$), without changing the equilibrium outcome. Now, suppose that $y_i^{**} > G(p_i^{**}, p_j^{**})$, so that there is price discounting by firm i at equilibrium. First, we can eliminate the case $P_i(p_i^{**}, y_i^{**}, p_j^{**}) = 0$, leading to zero profit. Indeed, even for $p_j^{**} = 0$, firm i can always obtain, at $p_i = p_j^{**} + t = t$, a positive profit $tF_i(t)$ on its captive segment. Second, observe that, for $0 < P_i(p_i^{**}, y_i^{**}, p_j^{**}) < p_i^{**}$, $P_i(p_i^{**}, y_i^{**}, p_j^{**}) = G^{-1}(\cdot, p_j^{**})(y_i^{**}) = p_j^{**} + t - 2ty_i^{**}$, so that firm i's discount price is locally independent of its list price, and of course of its rival's sales target. Thus, if both firms were in this case, firm i would be able to increase its profit by slightly increasing its list price p_i, hence its rival's discount price $P_j(p_j^{**}, y_j^{**}, p_i) = p_i + t - 2ty_j^{**}$ and ultimately its own demand $G(P_i(p_i, y_i^{**}, p_j^{**}), P_j(p_j^{**}, y_j^{**}, p_i))$. Firm i is consequently alone to give a discount (recall that $P_j(p_j^*, y_j^*, p_i^*) = 0$ is excluded). The equilibrium outcome depends then neither upon p_i^{**} nor upon y_j^{**}. So, we may take $p_i^* = P_i(p_i^{**}, y_i^{**}, p_j^{**})$ and $y_j^* = G(P_j(p_j^{**}, y_j^{**}, p_i^{**}), P_i(p_i^{**}, y_i^{**}, p_j^{**}))$, keeping constant the strategies p_j^{**} and y_i^{**} ($p_j^* = p_j^{**}$ and $y_i^* = y_i^{**}$), in order to get the required equilibrium with the same outcome, but now fulfilling the sales targets of both firms and involving no discounting.

C. Proof of Lemma 2

Since $((p_1^*, y_1^*), (p_2^*, y_2^*))$ is an equilibrium with fulfilled sales targets, we have, for $i, j = 1, 2$ and $i \neq j$,

$$p_i^* \in \arg \max_{p_i \in [0,v]} p_i D_i(p_i, P_j(p_j^*, y_j^*, p_i)) \text{ and } q_i^* = D_i(p_i^*, p_j^*),$$

which is equivalent to

$$(p_i^*, q_i^*) \in \arg \max_{(p_i, q_i) \in [0,v] \times [0, 1+a]} p_i q_i, \text{ s.t. } q_i \leq D_i(p_i, P_j(p_j^*, y_j^*, p_i)). \tag{C.1}$$

Since $\partial D_i(p_i, p_j)/\partial p_i = F_i'(p_i) + \partial G(p_i, p_j)/\partial p_i < F_i'(p_i)$, the function $D_i(\cdot, P_j(p_j^*, y_j^*, \cdot))$ restricted to its two first pieces $D_i(\cdot, p_j^*)$ and $F_i(\cdot) + 1 - y_j^* = \overline{D}(\cdot, p_j^*) - q_j^*$ (the only relevant since (p_i^*, q_i^*) is the corresponding kink) is concave (see Figure 2), so that

$$D_i(p_i, P_j(p_j^*, y_j^*, p_i)) = \min\{D_i(p_i, p_j^*), \overline{D}(p_i, p_j^*) - q_j^*\}.$$

Hence, the two programs (25) and (1), the latter with the restricted function $D_i(\cdot, P_j(p_j^*, y_j^*, \cdot))$, are equivalent, which completes the proof.

D. Proof of Proposition 3

First, we must prove existence, for any $\theta \in [0,1]^2$, of a θ-equilibrium in the discount-pricing game. Since we do not assume no-mill price undercutting, as we did for the equilibrium of the θ-game in Proposition 1, existence is now threatened by the possibility for a firm of profitably undercutting its competitor. This may occur once the rival is already setting a zero discount price, so that further price reductions are exhausted for that firm. One way of tackling this question is to verify that there is no incentive for firm i to deviate from any potential θ-equilibrium (at the higher kink of the demand curve of Figure 2), whatever the value of θ, to the lower portion of the curve (corresponding to $D_i(p_i, 0)$). Under Assumption 2 and by Proposition 1, we may apply the expression (11) of firm i profit at the potential θ-equilibrium, and verify that

$$\min_{\theta\in[0,1]^2} \Pi_i(\theta_i, \theta_j) > \max_{p_i\in[0,v]} p_i D_i(p_i, 0) = \max_{p_i\in[0,v]} p_i \left(\frac{v - p_i}{t_i} + \frac{t - p_i}{2t} \right). \tag{D.1}$$

By a straightforward computation, we get

$$\begin{aligned}
&\min_{\theta\in[0,1]^2} \Pi_i(\theta_i, \theta_j) \\
&= \left(\frac{2(1+2\tau_j)(1+2(v/t)\tau_i) + (1+2(v/t)\tau_j)}{2(1+2\tau_j)(1+4\tau_i) + 4\tau_j} \right)^2 \left(\tau_i + \frac{1+4\tau_j}{4(1+2\tau_j)} \right) t \\
&= \left(\frac{3 + 4(v/t)\tau_i + 2\tau_j((1+4\tau_i)v/t + 2)}{2(1+4\tau_i) + 8\tau_j(1+2\tau_i)} \right)^2 \left(\tau_i + \frac{1+4\tau_j}{4(1+2\tau_j)} \right) t.
\end{aligned}$$

When varying τ_j, this expression takes extreme values

$$\frac{(3 + 4(v/t)\tau_i)^2}{16(1+4\tau_i)} t \text{ for } \tau_j = 0 \text{ and } \frac{(2 + (1+4\tau_i)v/t)^2}{8(1+2\tau_i)} t \text{ for } \tau_j \to \infty.$$

It is easily checked that both values are indeed larger than

$$\max_{p_i\in[0,v]} p_i D_i(p_i, 0) = \frac{(1 + 2(v/t)\tau_i)^2}{8(1+2\tau_i)} t.$$

Finally, since first order conditions (5) for the θ-game (with $c_i = 0$) and (27) for the discount-pricing game coincide, the equilibrium of the θ-game also coincides with the θ-equilibrium of the discount-pricing game.

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