Oligopolistic vs. monopolistic competition: Do intersectoral effects matter?*

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Abstract

Recent extensions of the standard Dixit–Stiglitz (Am Econ Rev 67:297–308, 1977) model, that go beyond the CES sub-utility assumption while maintaining monopolistic competition, have mainly emphasized the role of intrasectoral substitutability. We argue that introducing oligopolistic competition can be an alternative extension, still tractable, allowing to restore the role of intersectoral substitutability and reinforcing the general equilibrium dimension of the model. For this purpose, we define a comprehensive concept of oligopolistic equilibrium to give account of a large set of competition regimes with varying competitive toughness. For two particular regimes, price competition and quantity competition, we show how, with strategic interactions, pro-competitive or anti-competitive effects now depend on the intersectoral elasticity of substitution as compared to the intrasectoral elasticity of substitution.

Keywords: Oligopolistic and monopolistic competition · Intra- and intersectoral substitutability · Pro- and anti-competitive effects

JEL Classification: D43 · D51 · F12 · L13

1 Introduction

It is remarkable how a simple, and now classical, general equilibrium model has been (and still is) useful across several fields in economics. This model, which is the focus of our paper, is the two-sector model introduced by Dixit and Stiglitz (1977), with an imperfectly competitive sector producing differentiated goods under increasing returns, and a perfectly competitive sector producing a homogenous good under constant returns. A main ingredient of Dixit–Stiglitz (DS) model is given by the separability assumption on the representative consumer preferences, allowing to define a sub-utility function on the bundles of differentiated goods, which aggregates the quantities of the differentiated goods into the quantity of a composite good, and implying the analysis "to depend on the intra- and intersector elasticities of substitution" (DS, p. 297). This has the advantage of capturing the two sides of firm competitive behavior: competition within the sector (to gain market share) and competition with the rest of the economy (to increase market size). However, it is a restricted DS model that has been extensively applied to deal with many

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issues in industrial organization, international trade and macroeconomics: It assumes monopolistic competition in the differentiated goods sector with a CES sub-utility and full symmetry. More generally, combining the monopolistic competition assumption with others, such as assuming symmetric additive sub-utility and identical firms (or trivially if assuming only one sector), drastically reduces intersectoral effects. It amounts in fact to confine the analysis of the whole economy to the differentiated goods sector and to take as sole (or main) ingredients those that are linked to consumer preferences for the different goods within the sector. In particular, preferences across sectors are immaterial to fix the equilibrium price.

Therefore, the Dixit–Stiglitz model, as it has been mainly applied, seems too restrictive and none of the attempts to go beyond this model really puts forward the importance of the relationship between intra- and intersectoral substitution. We argue in this paper that this relationship becomes crucial as soon as strategic interactions are introduced, and that this is true even if we restrict the sub-utility to be CES. To use the CES has the advantage of making the analysis tractable, even in the (fully) non-symmetric case (i.e., a non-symmetric sub-utility and a finite number of firms having individualized unit and fixed costs). But our main argument is not limited to the CES case, and the model can be made interestingly tractable in several ways. We start with the most general version of DS separable utility function [formula (1) in Dixit and Stiglitz 1977] and define a comprehensive model of firm oligopolistic behavior, which is shown to nest a multiplicity (a continuum) of competition regimes, including price competition and quantity competition, as well as monopolistic competition as an extreme case. This is based on the concept of oligopolistic equilibrium introduced in d'Aspremont et al. (2007) and d'Aspremont and Dos Santos Ferreira (2010), integrating the two dimensions of competition mentioned above, competition within the sector for market share and competition with the rest of the economy to increase the sector market size. The main instrument for applications is provided by the markup formula for each firm derived from the equilibrium first-order conditions: Each firm equilibrium markup is equal to the weighted harmonic mean of the reciprocals of the intrasectoral and intersectoral elasticities of substitution. The weights involve different elasticities, as well as an essential parameter measuring the competitive toughness of the firm. In this general formula, the two elasticities of substitution are given leading roles.[1]

To illustrate the usefulness of this formula in establishing the importance of the two elasticities of substitution when firms are non-negligible within their own sector, and have consequently to take into account the interaction of their sector with the rest of the economy, we take two particular oligopolistic equilibria (with different competitive toughness parameters), one corresponding to the price equilibrium concept and the other to the quantity equilibrium concept. Allowing for non-symmetric firms but restricting to the CES sub-utility case, and referring to the Herfindahl index of concentration at equilibrium, we show that less concentration is pro-competitive if and only if intrasectoral substitutability is larger than intersectoral substitutability under both competition regimes.[2] In the symmetric case, we can also apply our markup formula to monopolistic competition. This is a limit case where all the weight is put on the intrasectoral elasticity

[1] Notice that Dixit and Stiglitz (1977) have a specific case in mind, the one where the differentiated goods are "good substitutes among themselves, but poor substitutes for the other commodities in the economy" (DS, p. 297). However, this is only a particular case, since in their model a "sector" is defined by a sub-utility, and one cannot exclude some complementarities among goods in the same sector.

[2] Under price competition, we also show that a more productive firm obtains a higher budget share for its differentiated good whenever the intrasectoral elasticity of substitution is large enough, in any case larger than the intersectoral elasticity of substitution.

of substitution. But there are two ways to reach this limit case. One, traditional, is to make the Chamberlinian large group assumption. The other, still possible with a small number of firms, is to assume that each firm has maximal competitive toughness instead of being insignificant. This gives another view of monopolistic competition, somewhat like Bertrand competition gives another view of perfect competition.

Related literature The main attempts to go beyond the restrictive version of Dixit–Stiglitz have been either to relax the CES assumption or to relax symmetry, while keeping monopolistic competition. For instance, by relaxing the CES assumption (without relaxing symmetry) to additive but not homothetic preferences (as in section II of Dixit and Stiglitz 1977) and postulating a decreasing-demand elasticity, Krugman (1979) obtains (in a one-sector model) an elasticity of substitution decreasing in differentiated goods consumption and, as a pro-competitive effect, a decreasing equilibrium price in the number of active firms and in the size of the market. Behrens and Murata (2007) get the same implication by postulating constant absolute risk-averse utility. These results have been recently generalized in a two-sector model by Zhelobodko et al. (2012), still keeping additive preferences but allowing for decreasing or increasing intrasectoral elasticity of substitution in consumption and thereby generating both pro- and anti-competitive effects. However, because the sub-utility is still additive, the determination (through the sole intrasectoral elasticity of substitution) of the equilibrium price and quantity of each differentiated good remains independent of the determination of the number of firms. Allowing for non-additive preferences in a multi-sector model, Feenstra (2003) with homothetic translog utility and Melitz and Ottaviano (2008) with quasi-linear utility and quadratic sub-utility obtain an elasticity of intrasectoral substitution varying both in consumption and in the number of firms, and generating pro-competitive effects. This is further analyzed in Bertoletti and Epifani (2014). As regards the symmetry assumption, to relax it under monopolistic competition (with or without additive preferences), the only available models seem to be extensions of Melitz (2003), where there is a continuum of potential firms, and each firm marginal cost is drawn from a continuous distribution (after paying an entry cost, the same for all). Fixed costs are identical, and the sub-utility has to remain symmetric.

However, in all these models, the role of the relationship between intra- and intersectoral substitution is obliterated by the monopolistic competition assumption. In multi-sector models with additive sub-utility, this is well known. But, even in models where preferences are non-additive, as emphasized by Parenti et al. (2014), the "primitive" of the models remains the elasticity of intrasectoral substitution and the competitive effects are mainly determined by the properties of the sub-utility.[3] This remains true of models which share with the present model the purpose of introducing oligopolistic competition, such as Bertoletti and Epifani (2014, subsection 3.3), Bertoletti and Etro (2014), which consider a single sector, or Atkeson and Burstein (2008), Neary (2009) and d'Aspremont et al. (2010), where the assumption of a continuum of sectors results in negligible intersectoral effects.

The rest of the paper is organized as follows. In Sect. 2, the concept of oligopolistic equilibrium is defined in the two-sector DS model with general separable preferences. The equilibrium first-order conditions are derived allowing for a parameterization of the set of equilibria for varying competitive toughness. This is illustrated in a duopoly example. In

[3]For example, as mentioned by Bertoletti and Epifani concerning the Melitz–Ottaviano model, "the pro-competitive effect of an increase in market size delivered by this type of preferences in monopolistic competition is entirely driven... by the fact that, just as in Krugman (1979), the elasticity of substitution is decreasing in the level of individual consumption" (2014, section 3.1.1).

Sect. 3, for three particular competition regimes, price competition, quantity competition and monopolistic competition, we show how, with strategic interactions (in the first two regimes by contrast to the third), pro-competitive or anti-competitive effects depend on the intersectoral elasticity of substitution as compared to the intrasectoral elasticity of substitution. An appendix is devoted to our definition, in this context, of these two elasticities. In Sect. 4, we conclude.

2 A general equilibrium model for a two-sector economy

Following Dixit and Stiglitz (1977), we consider a two-sector economy, with an imperfectly competitive sector producing under non-decreasing returns a group of differentiated goods and a perfectly competitive sector producing under constant returns a homogeneous numéraire good, which represents the rest of the economy. The technologies of both sectors are described by constant unit labor costs (normalized to 1 in the perfectly competitive sector) plus fixed costs in the imperfectly competitive sector. This simple general equilibrium model allows us to distinguish two types of strategic interactions between producers of the differentiated goods, one intrasectoral, when competing for their market shares, and the other intersectoral, when competing for the size of their market relative to the size of the market for the homogeneous good.

2.1 A representative consumer with general separable preferences

We suppose a representative consumer inelastically supplying L units of labor at a wage equal to 1 (the labor productivity in the competitive sector) and receiving a profit Π from the imperfectly competitive sector (the equilibrium profit of the other sector being necessarily zero). He chooses a basket $\mathbf{x} \in \mathbb{R}^n_+$ of n differentiated goods (sold at prices $\mathbf{p} \in \mathbb{R}^n_{++}$) and a quantity $z \in \mathbb{R}_+$ of the numéraire good, so as to maximize, with an income $Y = L + \Pi$ and under the budget constraint $\mathbf{px} + z \leq Y$, a separable utility function $U(X(\mathbf{x}), z)$. The utility function U and the sub-utility function X, which aggregates the quantities of the differentiated goods into the quantity of a composite good, are assumed increasing and strongly quasi-concave (except, for X, in the linear and Leontief limit cases and, for U, in the case of quasi-linearity in z).

The maximization can be performed in two stages. In the first, the consumer chooses the quantity x_i of each differentiated good i given some quantity $\underline{X}$ of the composite good (some level of sub-utility X), by solving the program

$$\min_{x \in \mathbb{R}^n_+} \{\mathbf{px} \,|\, X(\mathbf{x}) \geq \underline{X}\} \equiv e\,(\mathbf{p}, \underline{X})\,, \tag{1}$$

which defines the expenditure function e. We obtain[4]:

$$p_i = \partial_X e(\mathbf{p}, \underline{X}) \partial_i X(\mathbf{x}) \qquad \text{(first-order condition)} \tag{2}$$

$$x_i = \partial_{p_i} e(\mathbf{p}, \underline{X}) \equiv H_i(\mathbf{p}, \underline{X}) \qquad \text{(Shephard's lemma)}, \tag{3}$$

where H_i is the Hicksian demand function for good i (associated with sub-utility X). By defining the *intrasectoral elasticity of substitution* of good i as the elasticity of substitution s_i of good i for the composite good, when the bundle of differentiated goods is $\mathbf{x}$,

[4]We denote $\partial_{x_i} F(\mathbf{x}, Y) \equiv \partial F(\mathbf{x}, Y)/\partial x_i$, $\partial_Y F(\mathbf{x}, Y) \equiv \partial F(\mathbf{x}, Y)/\partial Y$ and also $\partial_i F(\mathbf{x}) \equiv \partial F(\mathbf{x})/\partial x_i$ when there is no ambiguity. Similarly, $\partial^2_{ij} F(\mathbf{x}) \equiv \partial^2 F(\mathbf{x})/\partial x_i \partial x_j$ and $\epsilon_i F(\mathbf{x}) \equiv \partial_i F(\mathbf{x}) x_i / F(\mathbf{x})$.

alternatively in terms of quantities and prices, we obtain (as shown in the "Appendix"):

$$s_i = \frac{1 - \partial_i X(\mathbf{x}) x_i / X(\mathbf{x})}{-\partial_{ii}^2 X(\mathbf{x}) x_i / \partial_i X(\mathbf{x})} = \frac{-\epsilon_{p_i} H_i\left(\mathbf{p}, \underline{X}\right)}{1 - \left[\epsilon_i X(\mathbf{x})\right]\left[\epsilon_X H_i\left(\mathbf{p}, \underline{X}\right)\right]}. \tag{4}$$

In the second stage, the consumer chooses the quantities $\underline{X}$ and z of the composite and the numéraire goods, respectively, by solving the program

$$\max_{(\underline{X}, z) \in \mathbb{R}_+^2} \left\{ U(\underline{X}, z) \,|\, e\left(\mathbf{p}, \underline{X}\right) + z \leq Y \right\}. \tag{5}$$

The solution to this program determines the Marshallian demand $\underline{X} = D(\mathbf{p}, Y)$ for the composite good and the demand $z = Y - e\left(\mathbf{p}, D(\mathbf{p}, Y)\right)$ for the numéraire good. We can then define the *intersectoral elasticity of substitution* of good i as the elasticity of substitution σ_i of good i for aggregate consumption Y in both sectors (see "Appendix"):

$$\sigma_i \equiv -\left.\frac{d(x_i/Y)}{dp_i}\right|_{X(\mathbf{x}) = D(\mathbf{p}, Y)} \frac{p_i}{x_i/Y} = -\frac{(1/Y)\partial_{p_i} D(\mathbf{p}, Y)}{\partial_i X(\mathbf{x})} \frac{p_i}{x_i/Y} = \frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\epsilon_i X(\mathbf{x})}. \tag{6}$$

It measures the intensity of the response of the quantity ratio x_i/Y to a change in the relative price $p_i/1 = p_i$ of good i, taking into account the variation of the Marshallian demand D (rather than through the mere share adjustment as in $-\epsilon_{p_i} H_i(\mathbf{p}, \underline{X})$).

2.2 Firms' competitive behavior

We consider competition among n firms, each firm i producing a single component of the composite good with a constant positive unit cost c_i and a nonnegative fixed cost ϕ_i. In order to encompass various competition regimes, we shall analyze the case where firms behave strategically in price–quantity pairs: $(p_i, x_i) \in \mathbb{R}_+^2$ for each firm $i = 1, \ldots, n$. These pairs have to satisfy two admissibility constraints.

The first is a *constraint on market share*, focusing on competition within the sector which produces the differentiated goods and referring to the first stage of the consumer's utility maximization. It bounds the quantity of good i by the corresponding Hicksian demand:

$$x_i \leq H_i\left((p_i, \mathbf{p}_{-i}), X(x_i, \mathbf{x}_{-i})\right). \tag{7}$$

The second is a *constraint on market size*, focusing on competition of the whole set of producers of the differentiated goods with the sector which produces the numéraire good. It refers to the second stage of the consumer's utility maximization and bounds the size of the market for the differentiated goods by the Marshallian demand:

$$X(x_i, \mathbf{x}_{-i}) \leq D\left((p_i, \mathbf{p}_{-i}), Y\right). \tag{8}$$

The constraint on market share emphasizes the conflictual side of competition between the oligopolists, whereas the constraint on market size translates their common interest as a sector.

2.3 Oligopolistic equilibria with varying competitive toughness

Definition 1. An *oligopolistic equilibrium* is a tuple of pairs $(p_i^*, x_i^*)_{i=1,\ldots,n} \in \mathbb{R}_+^{2n}$ such that, for any i,

$$\begin{aligned} (p_i^*, x_i^*) \in \arg \max_{(p_i, x_i) \in \mathbb{R}_+^2} & (p_i - c_i) x_i \\ \text{s.t.} \quad & x_i \leq H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right) \\ \text{and} \quad & X\left(x_i, \mathbf{x}_{-i}^*\right) \leq D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y^*\right), \end{aligned} \tag{9}$$

and such that $Y^* = L + \sum_{i=1}^n \left((p_i^* - c_i)\, x_i^* - \phi_i \right)$. In addition, we require the profits to be nonnegative, namely $(p_i^* - c_i)\, x_i^* - \phi_i \geq 0$ for each i, and the consumer to be non-rationed.[5]

Observe that we are adjusting income Y parametrically in this definition, and hence, supposing that firms neglect the so-called *Ford effects* (the no income feedback effects assumption).[6]

We next show that an oligopolistic equilibrium can be characterized by a simple expression for each firm i (*relative*) *markup* (or Lerner index for the degree of monopoly power) at that equilibrium, that is, $\mu_i^* = (p_i^* - c_i)/p_i^*$, derived from the first-order conditions of producer i's program in Definition 1. To obtain that simple expression, we refer to the intra- and intersectoral elasticities of substitution of good i, s_i^* and σ_i^*, respectively [as in Eqs. (4) and (6)], again at the considered equilibrium, and we introduce in addition simplifying notations for two additional elasticities. The elasticity $\alpha_i \equiv \epsilon_i X(\mathbf{x})$ measures the impact of a variation in the quantity of good i on the volume of the composite good. The elasticity $\beta_i \equiv \epsilon_X H_i(\mathbf{p}, \underline{X})$ measures the reverse impact of a variation in the quantity of the composite good on the demand for its component i, at given prices $\mathbf{p}$. The product of these two elasticities, which appears in the multiplier $1/(1-\alpha_i\beta_i)$ applied to the elasticity $-\epsilon_{p_i} H_i(\mathbf{p}, \underline{X})$ of the Hicksian demand in the second expression for s_i in Eq. (4), measures the intensity of the feedback originating in a variation in the quantity of good i and going through the volume of the composite good.

For easier reference, we recall the expressions for these four elasticities in the following table:

Intrasectoral substitution:	$s_i = \frac{1-\alpha_i}{-\epsilon_i \partial_i X(\mathbf{x})} = \frac{-\epsilon_{p_i} H_i(\mathbf{p}, X(\mathbf{x}))}{1-\alpha_i\beta_i}$
Intersectoral substitution:	$\sigma_i = \frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\alpha_i}$
Impact of x_i on X	$\alpha_i \equiv \epsilon_i X(\mathbf{x})$
Impact of X on x_i	$\beta_i \equiv \epsilon_X H_i(\mathbf{p}, X(\mathbf{x}))$

The markup of firm i at some equilibrium $(p_i^*, x_i^*)_{i=1,\ldots,n}$ (with positive prices and quantities) will be expressed, according to the following proposition, as the weighted harmonic mean of the reciprocals of the two elasticities of substitution s_i^* and σ_i^* at that equilibrium. The corresponding weights will involve, for each firm i, the elasticities α_i^* and β_i^* measuring the two reciprocal effects of quantity variations of good i and of the composite good, as well as a conduct parameter[7] $\theta_i^* \in [0,1]$, stemming from the first-order conditions and interpreted as a measure of the *competitive toughness* displayed by firm i toward its rival oligopolists at the equilibrium $(p_i^*, x_i^*)_{i=1,\ldots,n}$.

Proposition 1. *Let $(p_i^*, x_i^*)_{i=1,\ldots,n} \in \mathbb{R}_{++}^{2n}$ be an oligopolistic equilibrium. Then the (relative) markup $\mu_i^* = (p_i^* - c_i)/p_i^*$ of each firm i is given by*

$$\mu_i^* = \frac{\theta_i^* (1 - \alpha_i^* \beta_i^*) + (1 - \theta_i^*)\, \alpha_i^*}{\theta_i^* (1 - \alpha_i^* \beta_i^*)\, s_i^* + (1 - \theta_i^*)\, \alpha_i^* \sigma_i^*}, \tag{10}$$

[5] Non-rationing of the consumer implies that both constraints are satisfied as equalities for each i at equilibrium. It makes the equilibrium compatible with the consumer's program and the resulting demand functions.

[6] Otherwise, each firm i would take income Y in its program to be equal to

$$\mathcal{Y}_i\left((p_i, x_i), (\mathbf{p}_{-i}^*, \mathbf{x}_{-i}^*)\right) = L + \sum_{j \neq i} \left(\left(p_j^* - c_j\right) x_j^* - \phi_j \right) + (p_i - c_i) x_i - \phi_i.$$

See Hart (1985) and d'Aspremont et al. (1989, 1990).

[7] To use the terminology of the *New Empirical Industrial Organization* (see Bresnahan 1989).

for some $\theta_i^* \in [0,1]$.

Proof. We start by making dimensionally homogeneous the two constraints in the program of firm i, rewriting them in terms of the two ratios:

$$\frac{x_i}{H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right)} \leq 1 \quad \text{and} \quad \frac{X\left(x_i, \mathbf{x}_{-i}^*\right)}{D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y\right)} \leq 1. \tag{11}$$

The first-order necessary conditions for profit maximization at (p_i^*, x_i^*) under these two constraints (holding as equalities at equilibrium) can then be expressed, for nonnegative Lagrange multipliers λ_i^* and ν_i^*, as

$$\begin{aligned} x_i^* &= \lambda_i^* \left(\frac{-\partial_{p_i} H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)}{H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)} \right) + \nu_i^* \left(\frac{-\partial_{p_i} D\left(\mathbf{p}^*, Y^*\right)}{D\left(\mathbf{p}^*, Y^*\right)} \right) \\ &= \frac{\lambda_i^*}{p_i^*} \left[-\epsilon_{p_i} H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*)) \right] + \frac{\nu_i^*}{p_i^*} \left[-\epsilon_{p_i} D(\mathbf{p}^*, Y^*) \right], \end{aligned} \tag{12}$$

and

$$\begin{aligned} \left(p_i^* - c_i\right) &= \lambda_i^* \left(\frac{1 - \left[\partial_X H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right] \left[\partial_i X\left(\mathbf{x}^*\right)\right]}{H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)} \right) + \nu_i^* \left(\frac{\partial_i X\left(\mathbf{x}^*\right)}{X\left(\mathbf{x}^*\right)} \right) \\ &= \frac{\lambda_i^*}{x_i^*} \left(1 - \left[\epsilon_i X\left(\mathbf{x}^*\right)\right] \left[\epsilon_X H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right] \right) + \frac{\nu_i^*}{x_i^*} \left[\epsilon_i X\left(\mathbf{x}^*\right)\right]. \end{aligned} \tag{13}$$

We can use these two conditions to compute the markup of firm i at the equilibrium:

$$\mu_i^* = \frac{p_i^* - c_i}{p_i^*} = \frac{\lambda_i^* \left(1 - \left[\epsilon_i X\left(\mathbf{x}^*\right)\right] \left[\epsilon_X H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right]\right) + \nu_i^* \epsilon_i X\left(\mathbf{x}^*\right)}{\lambda_i^* \left[-\epsilon_{p_i} H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)\right] + \nu_i^* \left[-\epsilon_{p_i} D\left(\mathbf{p}^*, Y^*\right)\right]}. \tag{14}$$

By using the notations s_i^*, σ_i^* [as defined by (4) and (6)], $\alpha_i^* \equiv \epsilon_i X\left(\mathbf{x}^*\right)$, $\beta_i^* \equiv \epsilon_X H_i\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right)$ and $\theta_i^* \equiv \lambda_i^* / \left(\lambda_i^* + \nu_i^*\right)$, we finally obtain for μ_i^* the expression in Eq. (10). □

Observe that λ_i^* is the Lagrange multiplier associated with the constraint on market share, which emphasizes the conflictual side of competition between firm i and its rivals in the sector, whereas ν_i^* is the Lagrange multiplier associated with the constraint on market size, which reflects converging interests of the competitors in the sector. Hence, the normalized multiplier $\theta_i^* \equiv \lambda_i^* / \left(\lambda_i^* + \nu_i^*\right)$, the relative shadow price of relaxing the constraint on market share, measures the "competitive toughness" experienced by firm i at the particular equilibrium $\left(p_j^*, x_j^*\right)_{j=1,\ldots,n}$. The vector $\boldsymbol{\theta}^* = (\theta_1^*, \ldots, \theta_n^*)$ of the degrees of competitive toughness of the different firms thus specifies a regime of competition, which can be continuously modified by varying this vector in $[0,1]^n$. Inspection of Eq. (10) shows that the weight put in the reciprocal of intrasectoral elasticity of substitution s_i^* (relative to the weight put in its intersectoral homologue σ_i^*) naturally increases with the competitive toughness θ_i^* experienced by firm i at equilibrium.

Tracing the set of potential equilibria by varying $\boldsymbol{\theta}^*$ allows us in particular to retrieve standard regimes like price and quantity competition, as we will see below. Also, the limit cases $\boldsymbol{\theta}^* = \mathbf{1}$ and $\boldsymbol{\theta}^* = \mathbf{0}$ are associated with the outcomes of monopolistic competition and collusion, respectively. However, existence of the whole spectrum of potential equilibria (for all values of $\boldsymbol{\theta} \in [0,1]^n$) is generally not satisfied.

2.4 A duopoly example to illustrate the existence issue

For any specification of the utility and sub-utility functions, the existence issue consists in determining, in $[0,1]^n$ (the set of all values of $\boldsymbol{\theta}$ corresponding to all 'potential' oligopolistic equilibria), the subset of values of $\boldsymbol{\theta}$ corresponding to 'true' equilibria (say, with first- and second-order conditions satisfied). To illustrate this issue, let us give an example. It involves a symmetric duopoly and a representative consumer with preferences given by the utility function

$$U(X(\mathbf{x}),z)=\frac{b^{1/\sigma}}{1-1/\sigma}X(\mathbf{x})^{1-1/\sigma}+z,$$
$$\text{with}\quad X(\mathbf{x})=\left(\sum_{i=1}^{2}x_i^{(s-1)/s}\right)^{s/(s-1)},\quad \sigma>1\ \text{ and }\ b>0. \tag{15}$$

Utility maximization under the budget constraint $\mathbf{px}+z\leq Y$ entails, for each good i, the choice

$$x_i=H_i\left(\mathbf{p},D(\mathbf{p},Y)\right)=\left(\frac{p_i}{P(\mathbf{p})}\right)^{-s}\underbrace{bP(\mathbf{p})^{-\sigma}}_{D(\mathbf{p},Y)},\quad\text{with}\quad P(\mathbf{p})=\left(\sum_{i=1}^{2}p_i^{1-s}\right)^{1/(1-s)}. \tag{16}$$

In order to ensure a positive demand for the numéraire good, we assume that $bP(\mathbf{p})^{1-\sigma}<Y$, which results from $b<2^{\frac{\sigma-1}{1-s}}c^{\sigma-1}L$. We show in the "Appendix" that the intrasectoral and intersectoral elasticities of substitution are $s_i=s$ and $\sigma_i=\sigma$ for any i, respectively.

In Fig. 1, representing the constrained profit maximization of firm i when firm j chooses $\left(p_j^*,x_j^*\right)$, we have drawn two thick curves, one corresponding to a binding market share constraint

$$x_i=\left(\frac{p_i}{\left(p_i^{1-s}+p_j^{*1-s}\right)^{1/(1-s)}}\right)^{-s}\left(x_i^{(s-1)/s}+x_j^{*(s-1)/s}\right)^{s/(s-1)}, \tag{17}$$

the other to a binding market size constraint

$$\left(x_i^{(s-1)/s}+x_j^{*(s-1)/s}\right)^{s/(s-1)}=b\left(p_i^{1-s}+p_j^{*1-s}\right)^{-\sigma/(1-s)}. \tag{18}$$

The figure is drawn on the basis of the following parameter values: $\sigma=2.5$, $s=1.2$, $b=2^{(s-\sigma)/(s-1)}$, $c=0.6$ and $\phi=0$. Because of the chosen value of b, both constraints are satisfied as equalities when $p_i=x_i=1$, given $p_j^*=x_j^*=1$. The thin curve is the isoprofit curve going through $(x_i^*,p_i^*)=(1,1)$, the point of intersection of the two thick curves. As a matter of fact, it is tangent to the curve representing a binding market size constraint, meaning that the corresponding markup, $\mu_i=0.4=1/\sigma$, is the one entailed by the least competitive toughness $\theta_i=0$. The point $(p_i,x_i)=(1,1)$ is the solution to the constrained profit maximization problem of firm i, and by symmetry, $((p_i,x_i),(p_j,x_j))=((1,1),(1,1))$ is consequently an equilibrium corresponding to $(\theta_i,\theta_j)=(0,0)$, thus enforcing the collusive solution.

By referring to Proposition 1, it should however be clear that $((1,1),(1,1))$ is not the sole equilibrium associated with the chosen parameter values. Other configurations of price–quantity pairs, symmetric or not, in fact a continuum of them, generating modified constraints on market share and on market size, will also solve the resulting constrained profit maximization problem of both firms. In other words, they will also be

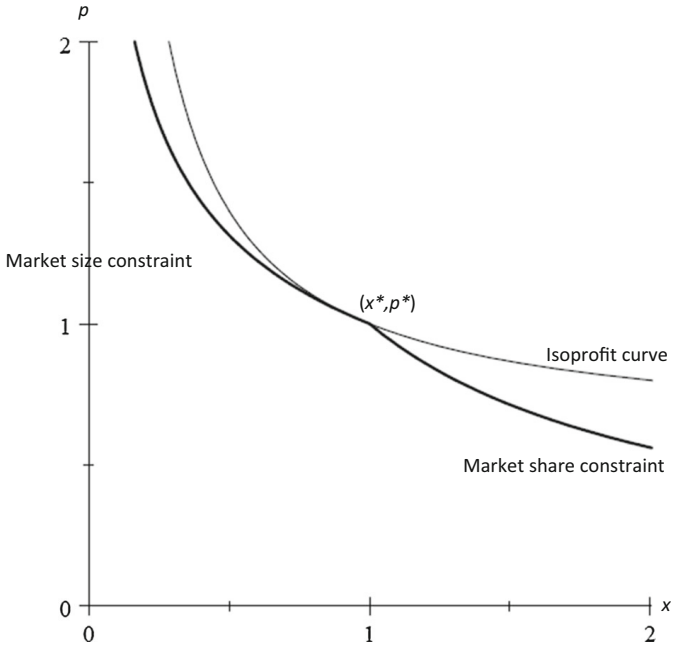


Figure 1: Profit maximization ($s < \sigma, \theta = 0$)

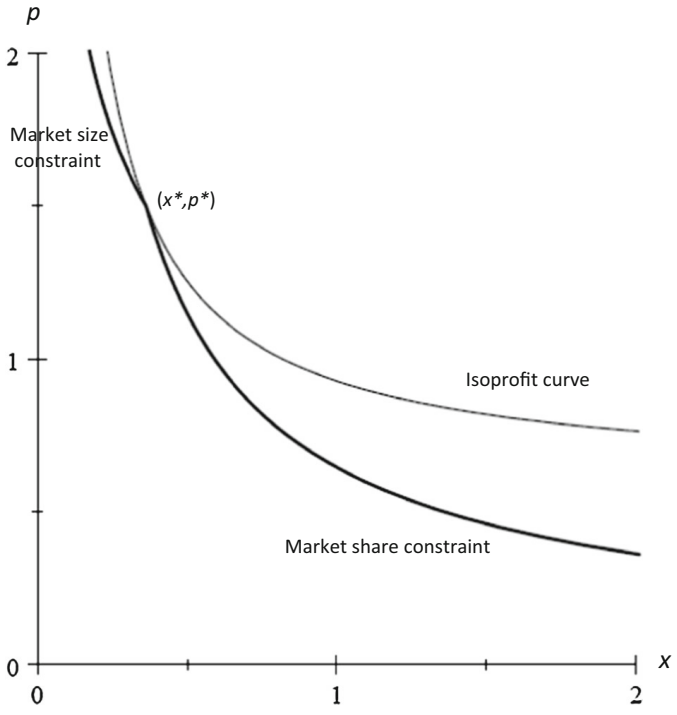


Figure 2: Profit maximization ($s < \sigma, \theta \simeq 0.64$)

oligopolistic equilibria, provided the maximum occurs at the intersection of the two constraining curves. Such a situation is exhibited in Fig. 2, with the symmetric equilibrium $(p^*, x^*) \simeq (1.5, 0.36)$ obtained for the same parameter values. The isoprofit curve is no more tangent to either of the two constraining curves, which translates in a value $\theta^* \simeq 0.64$ strictly inside the unit interval.

It is easy to show that equilibrium existence may fail for some parameter configurations if we look for extreme values of $\boldsymbol{\theta}$. A trivial remark is that too high values of θ cannot be obtained in symmetric equilibria, as soon as $s < 1$, since they would lead to values of the markup larger than one. More interesting is the case of inexistence with small values of θ. To illustrate, take again the case of symmetric potential equilibria associated with $\theta = 0$, in other words potential equilibria that would enforce collusion. In Fig. 3, with the same parameter values as above, except for s, now equal to 0.05 (very high intrasectoral complementarity), the potential equilibrium $((1, 1), (1, 1))$ is destroyed by deviations using higher prices and lower quantities, compatible with the constraint on market size. The incentive to increase the price is due to the small demand elasticity induced by high intrasectoral complementarity. Figure 4 illustrates the opposite case, with $s = 80$ (very high intrasectoral substitutability), the same potential equilibrium being now destroyed by deviations using lower prices and higher quantities, still compatible with the constraint on market size (now in an opposite position since $s > \sigma$). The incentive to decrease the price results from the high demand elasticity induced by large intrasectoral substitutability.

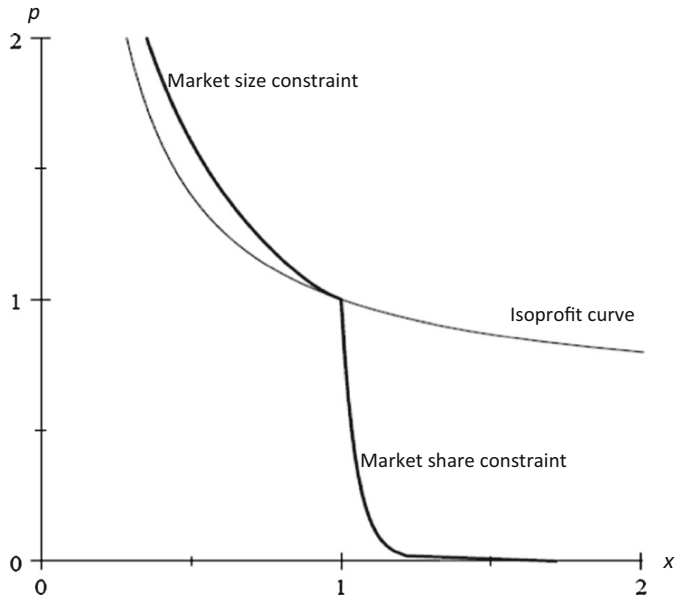


Figure 3: Upward price deviation ($s < \sigma$)

Existence of symmetric oligopolistic equilibria (for any integer n) is fully analyzed in d'Aspremont et al. (2007, Proposition 3).[8] For the present duopoly example, the resulting typology of symmetric equilibrium regimes is well summarized in Fig. 5, where intrasectoral substitutability is measured by the index $1/(1 + s)$ along the horizontal axis (increasing from perfect substitutability at zero to perfect complementarity at one), where the markup μ varies along the vertical axis, and where intersectoral substitutability is fixed at $\sigma = 2.5$.

[8]This proposition covers the case of CES sub-utility. The case of quadratic sub-utility is analyzed in d'Aspremont and Dos Santos Ferreira (2010).

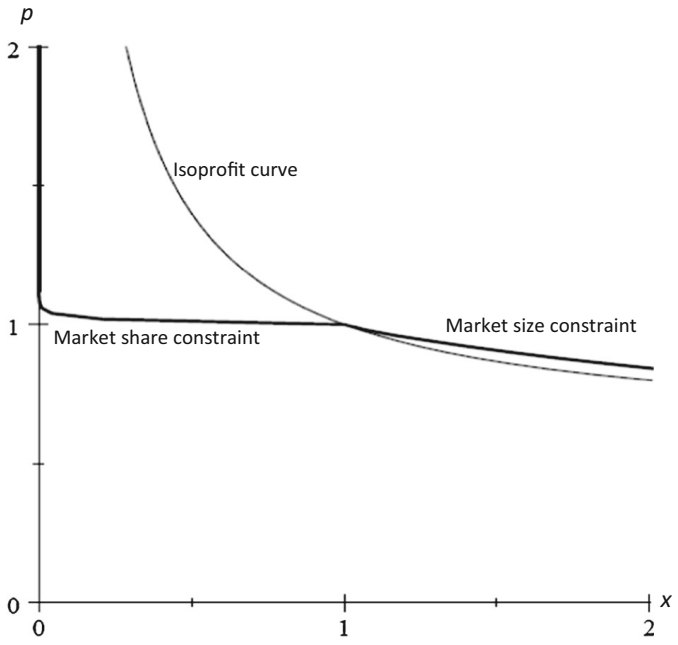


Figure 4: Downward price deviation ($s > \sigma$)

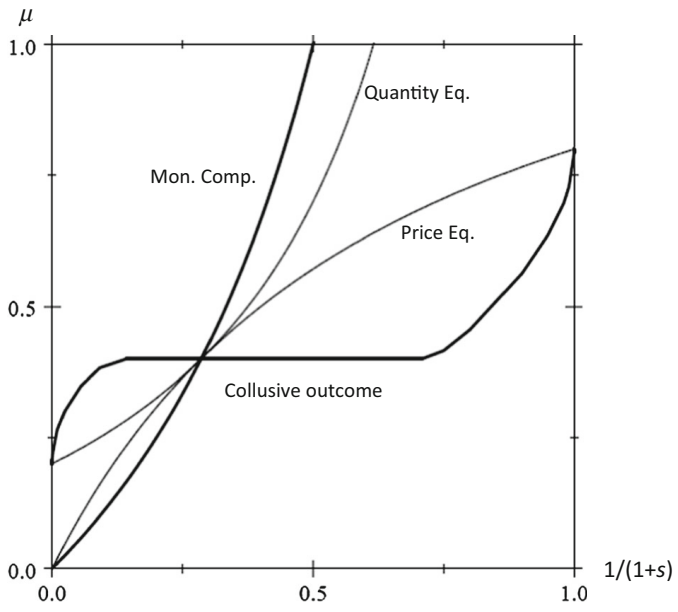


Figure 5: Equilibrium regimes

The set of equilibria is represented in this space by the region between the two thick curves, namely the *tough competition frontier* (the convex curve starting from zero, obtained by taking $\theta = 1$) and the *soft competition frontier* (the other thick curve, obtained by taking the minimum value of θ compatible with equilibrium existence). These frontiers switch their positions relative to the markup μ at a point corresponding to the equality of the two elasticities of substitution s and σ. In other words, moving vertically toward the tough competition frontier (increasing θ) corresponds to a reduction in μ (hence in price, a pro-competitive effect), if $s > \sigma$, and to an increase in μ (an anti-competitive effect) if $s < \sigma$. This frontier, along which $\mu = 1/s$, corresponds to the monopolistic competition outcome. As to the soft competition frontier, it has an intermediate flat piece (along which $\theta = 0$) giving the collusive outcome $\mu = 1/\sigma$. As we approach the two extreme values of s, the frontier moves further and further away from this outcome, because of the incentives to deviate illustrated in Figs. 3 and 4. At both ends, it converges to a Cournot solution, the quantity solution (Cournot producers' *competition*) for $s \to \infty$ or the price solution (Cournot producers' *concurrence*)[9] for $s \to 0$. The two thin curves represent the price and the quantity equilibrium outcomes (the concave and the convex curves, respectively), which will be discussed in the following section. They coincide at the point where $s = \sigma$, but they do not switch positions: The markup is always lower at a price equilibrium, whatever the ranking of s and σ.

3 Competitive effects under different regimes of competition

We discuss in this section the standard regimes of price, quantity and monopolistic competition and their competitive effects. For each regime, we start by showing that the corresponding equilibrium is a particular instance of an oligopolistic equilibrium, associated with specific values of the competitive toughness experienced by the different firms. We then show, for the first two regimes, how pro-competitive or anti-competitive effects depend on the intersectoral elasticity of substitution as compared to the intrasectoral elasticity of substitution. This is contrasted in the last subsection with the regime of monopolistic competition.

3.1 Oligopolistic equilibrium under price competition

Let us first consider price competition. It is easy to prove that price equilibria are included in the set of oligopolistic equilibria.

Denoting the Walrasian demand for good i by $Q_i(\mathbf{p}, Y) \equiv H_i(\mathbf{p}, D(\mathbf{p}, Y))$, a *price equilibrium* p^* is solution to the following program for each firm i

$$\max_{p_i \in [c_i, \infty)} (p_i - c_i) Q_i \left((p_i, \mathbf{p}_{-i}), Y^* \right), \tag{19}$$

together with $Y^* = L + \sum_{i=1}^n \left((p_i^* - c_i)\, x_i^* - \phi_i \right)$, where $x_i^* = Q_i \left((p_i^*, \mathbf{p}_{-i}^*), Y^* \right)$ and $(p_i^* - c_i)\, x_i^* - \phi_i \geq 0$ for each i.

Proposition 2. *Suppose that a price equilibrium p^* exists. Then, taking Y^* as just defined, $\left(p_i^*, Q_i \left((p_i^*, \mathbf{p}_{-i}^*), Y^* \right) \right)_{i=1,\ldots,n}$ is an oligopolistic equilibrium.*

[9] Producers' concurrence ('concours des producteurs') is the so-called c*omplementary monopoly* regime.

Proof. Suppose that, for some i and some (p_i, x_i) satisfying the two constraints, we have: $(p_i - c_i)x_i > (p_i^* - c_i)\, Q_i\left(\left(p_i^*, \mathbf{p}_{-i}^*\right), Y^*\right)$. Also, by the market size constraint, $X\left(x_i, \mathbf{x}_{-i}^*\right) \leq D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y^*\right)$ and then, by the market share constraint, $x_i \leq H_i\left(p_i, \mathbf{p}_{-i}^*, X\left(x_i, \mathbf{x}_{-i}^*\right)\right) \leq H_i\left(p_i, \mathbf{p}_{-i}^*, D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y^*\right)\right) = Q_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y^*\right)$. Thus,

$$(p_i - c_i)Q_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y^*\right) \geq (p_i - c_i)x_i > (p_i^* - c_i)\, Q_i\left(\left(p_i^*, \mathbf{p}_{-i}^*\right), Y^*\right), \tag{20}$$

and the result follows by contradiction. □

We could directly derive the first-order conditions for a price equilibrium and write them in such a way that the markup of each firm i appears to be equal to the harmonic mean of $1/s_i^*$ and $1/\sigma_i^*$, with weights, respectively, equal to $1 - \alpha_i^*\beta_i^*$ and $\alpha_i^*\beta_i^*$, *i.e.*,

$$\mu_i^* = \frac{1}{(1 - \alpha_i^*\beta_i^*)\, s_i^* + \alpha_i^*\beta_i^*\sigma_i^*}. \tag{21}$$

But the same expression for the markup μ_i^* can be obtained from the general formula (10) obtained in Proposition 1 by taking the competitive toughness to be

$$\theta_i^* = \frac{1}{1 + \beta_i^*}. \tag{22}$$

In terms of our concept of oligopolistic equilibrium, pure price competition requires the degree θ_i^* of competitive toughness of firm i to be the higher the smaller the impact β_i^* of a variation in the quantity of the composite good on the demand for good i.

In the homothetic case, with X homogeneous of degree 1, we have $\beta_i^* = 1$ for all i, so that putting all the θ_i's equal to $1/2$ entails the price equilibrium value of the μ_i's (see d'Aspremont et al. 2007). Moreover, $\sigma_i^* = \sigma^*$ for any i, in this case. If we further assume a CES sub-utility, we have $s_i^* = s$ for any i, which leads to the simplified formula

$$\mu_i^* = \frac{1}{(1 - \alpha_i^*)\, s + \alpha_i^*\sigma^*}, \tag{23}$$

namely the weighted harmonic mean of the reciprocals of the elasticities s and σ^*, with respective weights $(1 - \alpha_i^*)$ and α_i^* (α_i^* being now interpretable as the budget share of good i at equilibrium). Since it does not suppose equilibrium symmetry (even if sub-utility is symmetric, firms costs may differ), this formula is more general and more transparent than the one derived by Yang and Heijdra (1993), eq. (4), or by d'Aspremont et al. (1996), eq. (8).

3.2 Competitive effects of price competition

The analysis of the competitive effects of price competition is simple if we concentrate on the (symmetric) CES case (say, $X(\mathbf{x}) = \left(\sum_{i=1}^n x_i^{(s-1)/s}\right)^{s/(s-1)}$, $s \in (0,1) \cup (1,\infty)$) and base our conclusions on the harmonic mean of all markups weighted by their respective budget shares (the α_i's). Using (23), we get the following formula which shows the relation between the *harmonic mean markup* and the *Herfindahl index* of concentration $\sum_{i=1}^n \alpha_i^{*2}$:

$$\overline{\mu}^* = \left(\sum_{i=1}^n \frac{\alpha_i^*}{\mu_i^*}\right)^{-1} = \frac{1}{s + (\sigma^* - s)\sum_{i=1}^n \alpha_i^{*2}}. \tag{24}$$

We see that the pro-competitive effect of less concentration ($\overline{\mu}^*$ decreasing when concentration decreases) works only when there is more substitutability among the differentiated goods than between the latter and the numéraire good (i.e., an intrasectoral

elasticity of substitution larger than its intersectoral homologue). Otherwise, the effect is anti-competitive. For the symmetric case (assuming $c_i = c$ and $\phi_i = \phi$ for any i), we obtain a symmetric equilibrium with $\alpha_i^* = 1/n$ and hence the same equilibrium markup for every firm:

$$\mu^* = \frac{1}{s + (\sigma^* - s)/n}. \tag{25}$$

The equilibrium markup μ^* (with $0 < \mu^* < 1$), hence also the equilibrium price

$$p^* = \frac{c}{1 - \mu^*} = \frac{c(ns + \sigma^* - s)}{n(s-1) + \sigma^* - s}, \tag{26}$$

decreases (resp. increases) when n increases as long as $s > \sigma^*$ (resp. $s < \sigma^*$). Hence, the pro-competitive effect (on markup and price) of an increase in the number of firms depends on the dominance of intrasectoral over intersectoral substitutability. This can be summarized as a proposition.

Proposition 3. *Assume CES sub-utility. As long as $s > \sigma^*$ (resp. $s < \sigma^*$) the harmonic mean markup increases (resp. decreases) with the Herfindahl index of concentration. If in addition $c_i = c$ and $\phi_i = \phi$ for any i, the equilibrium markups and prices are identical for all firms and decreasing (resp. increasing) when n, the number of active firms, increases as long as $s > \sigma^*$ (resp. $s < \sigma^*$).*

Consider now the consequences of cost asymmetries on the relative equilibrium values of the markups, prices and budget shares of each differentiated good. Sticking to the (symmetric) CES case, where the price of the composite good can be written as $P(\mathbf{p}) = \left(\sum_i p_i^{1-s}\right)^{1/(1-s)}$, we can alternatively express the equilibrium budget share of good i as the elasticity $\epsilon_i X(\mathbf{x}^*)$ or as the elasticity $\epsilon_{p_i} e(\mathbf{p}, \underline{X}) = \epsilon_i P(\mathbf{p}^*)$, hence as

$$\alpha_i^* = \frac{x_i^{*1-1/s}}{\sum_j x_j^{*1-1/s}} = \frac{p_i^{*1-s}}{\sum_j p_j^{*1-s}}. \tag{27}$$

Consequently,

$$\frac{\alpha_i^*}{\alpha_j^*} = \left(\frac{x_i^*}{x_j^*}\right)^{1-1/s} = \left(\frac{p_i^*}{p_j^*}\right)^{1-s}, \tag{28}$$

so that $p_i^* < p_j^*$ and $x_i^* > x_j^*$ if and only if $\alpha_i^* > \alpha_j^*$ (resp. $\alpha_i^* < \alpha_j^*$) whenever $s > 1$ (resp. $s < 1$). Also, by (23), $\mu_i^* > \mu_j^*$ if and only if $\alpha_i^* > \alpha_j^*$ (resp. $\alpha_i^* < \alpha_j^*$) whenever $s > \sigma^*$ (resp. $s < \sigma^*$). We may deduce the following proposition.

Proposition 4. *Assume CES sub-utility. Then (i) if $s > \max\{1, \sigma^*\}$ (differentiated goods are highly substitutable) the more productive firms set lower prices with higher markups and obtain higher budget shares for their goods, and (ii) if $s < \min\{1, \sigma^*\}$ (differentiated goods have a low elasticity of substitution) the more productive firms, setting lower prices with higher markups, only obtain lower budget shares for their goods.*

Proof. Suppose that firm i is more productive than firm j: $c_i < c_j$. As $p_i^*(1 - \mu_i^*) = c_i$, $p_i^* < p_j^*$ or $\mu_i^* > \mu_j^*$ (or both). For (i) if $s > \max\{1, \sigma^*\}$, either inequality implies $\alpha_i^* > \alpha_j^*$, hence the other inequality. For (ii), if $s < \min\{1, \sigma^*\}$, we can formulate the same argument, but with $\alpha_i^* < \alpha_j^*$. □

Case (i), occurring when s is high ($s > \max\{1, \sigma^*\}$), can clearly be viewed as pro-competitive, whereas case (ii), occurring when s is low ($s < \min\{1, \sigma^*\}$), can be viewed as anti-competitive. Notice that case (ii) implies in fact $s < 1 < \sigma^*$, as one at least of the two elasticities of substitution must be higher than one, for the markup to be kept smaller than one. By contrast, case (i) decomposes into $s > 1 \geq \sigma^*$ and $s > \sigma^* > 1$.

3.3 Oligopolistic free-entry equilibrium under price competition

We have up to now taken as exogenous the number n of active firms. The competitive effects across symmetric price equilibria of changes in this number or in the consequent index of concentration $1/n$ have been described in Proposition 3. By assuming free entry or, more precisely, by introducing the zero-profit condition usually associated with that assumption, we can make the equilibrium number n^* endogenous and so consider more deeply the competitive effects of changes in the size or in the costs of the oligopolistic sector.

For simplicity and explicitness, we adopt the same specification of the representative consumer's preferences as in Sect. 2.4, namely

$$U(X(\mathbf{x}), z) = \frac{b^{1/\sigma}}{1 - 1/\sigma} X(\mathbf{x})^{1-1/\sigma} + z, \quad \text{with} \quad \sigma > 1 \quad \text{and} \quad b > 0, \quad \text{and}$$
$$\text{with} \quad X(\mathbf{x}) = \left(\sum_{i=1}^{n} x_i^{(s-1)/s} \right)^{s/(s-1)}, \quad s \in (0,1) \cup (1, \infty). \tag{29}$$

Recall that the Marshallian demand for the composite good is, under this specification,

$$D(\mathbf{p}, Y) = bP(\mathbf{p})^{-\sigma}, \quad \text{with} \quad P(\mathbf{p}) = \left(\sum_{i=1}^{n} p_i^{1-s} \right)^{1/(1-s)}. \tag{30}$$

It does not in fact depend upon the income $Y^* = L$ (when $\Pi = 0$), but the size of the oligopolistic sector can still be exogenously modified by perturbations of the parameter b.

At a symmetric oligopolistic equilibrium, given the number n of active firms, the profit per firm (gross of fixed costs) $\mu p D(\mathbf{p}, Y)/n$ can be written, since $p = c/(1-\mu)$,

$$\frac{\mu b P(\mathbf{p})^{1-\sigma}}{n} = bc^{1-\sigma} \left[\mu (1-\mu)^{\sigma-1} n^{(\sigma-s)/(s-1)} \right] \equiv bc^{1-\sigma} \pi \left(\widetilde{\mu}(n), n \right), \tag{31}$$

with $\widetilde{\mu}(n)$ given by (25). The zero-profit condition can thus be formulated as $bc^{1-\sigma} \pi \left(\widetilde{\mu}(n^*), n^* \right) = \phi$. Thus, under the zero-profit condition, an exogenous increase in the oligopolistic market size (through an increase in b) and a decrease in the variable unit cost c, or in the fixed cost ϕ, depress the equilibrium gross profit per firm $\pi \left(\widetilde{\mu}(n^*), n^* \right)$.

Clearly, the number of active firms must correspondingly increase. Indeed, the zero-profit condition makes only sense, as the expression of a long run equilibrium, if a progressive decrease in profit per firm results from more and more firms becoming active. Now, according to Proposition 3, the function $\widetilde{\mu}$ can be decreasing (if $s > \sigma$) or increasing (if $s < \sigma$). In other words, entry induced by positive net profits can either decrease or increase the markup, provided $\pi \left(\widetilde{\mu}(\cdot), \cdot \right)$ remains a decreasing function. As shown in the following proposition, this holds under two cases.

Proposition 5. *Take the utility function defined by* (29)*, entailing constant elasticities of substitution s and σ. Assume either (i) $1 < \sigma < s$ or (ii) $s < 1 < \sigma$. In an oligopolistic free-entry equilibrium, under the zero-profit condition, the number n^* of active firms is*

increasing in the oligopolistic market size, as resulting from a higher b, and decreasing in the variable and fixed costs c and ϕ. Variations of n^* will be accompanied by the kind of competitive effects indicated in Proposition 3: pro-competitive in case (i), anti-competitive in case (ii). More explicitly, to a higher b will respond lower prices p^* of each differentiated good and P^* of the composite good and higher corresponding quantities x^* and X^* in case (i). The sense of the response will be reversed in case (ii).

Proof. To ensure that $\pi\left(\widetilde{\mu}(\cdot),\cdot\right)$ remains a decreasing function, we must have:

$$\begin{aligned}\frac{d\pi}{dn}\frac{n}{\pi} &= \underbrace{\left(1-\frac{(\sigma-1)\widetilde{\mu}(n)}{1-\widetilde{\mu}(n)}\right)}_{\epsilon_\mu \pi}\epsilon\widetilde{\mu}(n) + \underbrace{\frac{\sigma-s}{s-1}}_{\epsilon_n \pi} \\ &= \frac{(1-1/n)(s-\sigma)}{(1-1/n)s+(1/n)\sigma}\frac{\epsilon\widetilde{\mu}(n)}{1-\widetilde{\mu}(n)} - \frac{s-\sigma}{s-1} < 0.\end{aligned} \tag{32}$$

As $\sigma > 1$, if either $s > \sigma$ or $s < 1$, the direct effect $\epsilon_n\pi$ is negative, and so is the indirect effect $\epsilon_\mu\pi\epsilon\widetilde{\mu}(n)$, since we know by Proposition 3 that $\widetilde{\mu}$ is decreasing (resp. increasing) when $s > \sigma$ (resp. $s < \sigma$). The two cases (i) $1 < \sigma < s$ and (ii) $s < 1 < \sigma$ are both admissible.

In case (i), an increase in the oligopolistic market size, inducing entry and depressing the markup, will decrease the prices $p^* = c/(1-\mu)$ of each differentiated good and $P^* = n^{*1/(1-s)}p^*$ of the composite good and will increase the corresponding quantities $x^* \approx \phi/(p^*-c)$ (by the zero-profit condition) and $X^* = bP^{*-\sigma}$. In case (ii), an increase in the oligopolistic market size, inducing entry and now enhancing the markup, will increase the prices $p^* = c/(1-\mu)$ of each differentiated good and $P^* = n^{*1/(1-s)}p^*$ of the composite good and will decrease the corresponding quantities $x^* \approx \phi/(p^*-c)$ and $X^* = bP^{*-\sigma}$. □

3.4 Oligopolistic equilibrium under quantity competition and competitive effects

To pursue our review of standard competition regimes, let us next consider quantity competition. Denoting Q^{-1} the inverse of the Walrasian demand function $Q(\mathbf{p},Y) \equiv H(\mathbf{p},D(\mathbf{p},Y))$, a *quantity equilibrium* is a solution x^* to the following program for each firm i

$$\max_{x_i\in\mathbb{R}_+}\left\{Q_i^{-1}\left(x_i,\mathbf{x}_{-i}^*,Y^*\right)x_i - c_i x_i\right\}, \tag{33}$$

where $Y^* = L + \sum_{i=1}^n\left(\left(p_i^*-c_i\right)x_i^* - \phi_i\right)$, with $p_i^* = Q_i^{-1}(\mathbf{x}^*,Y^*)$ and $\left(p_i^*-c_i\right)x_i^* - \phi_i \geq 0$ for each i. By a similar argument as in Proposition 2, we get:

Proposition 6. *Suppose that a quantity equilibrium $\mathbf{x}^*$ exists. Then, taking Y^* as just defined, $(Q^{-1}(\mathbf{x}^*,Y^*),\mathbf{x}^*)$ is an oligopolistic equilibrium.*

Proof. Suppose that the quantity equilibrium $\mathbf{x}^*$ with the associated price vector $\mathbf{p}^* = Q^{-1}(\mathbf{x}^*,Y^*)$ is not an oligopolistic equilibrium: for some i and some pair (p_i,x_i) satisfying the two constraints in (9), *i.e.,*

$$x_i \leq H_i\left(p_i,\mathbf{p}_{-i}^*,X\left(x_i,\mathbf{x}_{-i}^*\right)\right) \quad \text{and} \quad X\left(x_i,\mathbf{x}_{-i}^*\right) \leq D\left(p_i,\mathbf{p}_{-i}^*,Y^*\right), \tag{34}$$

we have $(p_i-c_i)x_i > Q_i^{-1}(\mathbf{x}^*,Y^*)x_i^* - c_i x_i^*$. First, observe that by duality, the two constraints can be reformulated (in terms of prices rather than quantities) as

$$\begin{aligned} p_i &\leq \partial_X e\left(p_i,\mathbf{p}_{-i}^*,X\left(x_i,\mathbf{x}_{-i}^*\right)\right)\partial_i X\left(x_i,\mathbf{x}_{-i}^*\right) \quad \text{and} \\ \partial_X e\left(p_i,\mathbf{p}_{-i}^*,X\left(x_i,\mathbf{x}_{-i}^*\right)\right) &\leq \partial_X e\left(Q^{-1}\left(x_i,\mathbf{x}_{-i}^*,Y^*\right),X\left(x_i,\mathbf{x}_{-i}^*\right)\right). \end{aligned} \tag{35}$$

The dual expression of the constraint on market share stems from the first-order condition (2). It states that the market price p_i of good i, relative to the shadow price $\partial_X e\left(p_i, \mathbf{p}^*_{-i}, X\left(x_i, \mathbf{x}^*_{-i}\right)\right)$ of the composite good, should not exceed the marginal utility $\partial_i X\left(x_i, \mathbf{x}^*_{-i}\right)$. As to the dual expression of the constraint on market size, it states that the level of the shadow price of the composite good, in quantity $X\left(x_i, \mathbf{x}^*_{-i}\right)$, when evaluated at market prices $\left(p_i, \mathbf{p}^*_{-i}\right)$, should not exceed its level when evaluated at the prices corresponding to the inverse demand $Q^{-1}\left(x_i, \mathbf{x}^*_{-i}, Y^*\right)$ (and so reflecting the second stage of utility maximization). Now, the two constraints together imply

$$p_i \leq \partial_X e\left(Q^{-1}\left(x_i, \mathbf{x}^*_{-i}, Y^*\right), X\left(x_i, \mathbf{x}^*_{-i}\right)\right) \partial_i X\left(x_i, \mathbf{x}^*_{-i}\right) = Q_i^{-1}\left(x_i, \mathbf{x}^*_{-i}, Y^*\right), \tag{36}$$

so that x_i is a profitable deviation from the quantity equilibrium, in contradiction with $\mathbf{x}^*$ being a quantity equilibrium. □

It is easy to derive the first-order conditions for the quantity equilibrium if we assume a homothetic sub-utility (implying $\beta_i^* = 1$ and $\sigma_i^* = \sigma^*$ for any i). We obtain:

$$\mu_i^* = \left(1 - \alpha_i^*\right)\left(1/s_i^*\right) + \alpha_i^*\left(1/\sigma^*\right), \tag{37}$$

The equilibrium markup is the *arithmetic* mean of $1/{s_i}^*$ and $1/\sigma^*$, with weights, respectively, equal to $1 - \alpha_i^*$ and α_i^* (with α_i^* interpretable as the budget share of good i). Coming back to formula (10) of Proposition 1, this amounts to put

$$\theta_i^* = 1/\left(1 + s_i^*/\sigma^*\right), \tag{38}$$

where s_i^*/σ^* is the ratio between *elasticities of intra*-and *intersectoral substitution* for good i.

Since the arithmetic mean of two numbers is always higher than the corresponding harmonic mean, the quantity equilibrium markup is higher than the price equilibrium markup in the symmetric case with constant elasticities ($\alpha_i^* = 1/n$, $s_i^* = s$ and $\sigma^* = \sigma$), as illustrated by Fig. 5. Notice however that firms display at quantity equilibrium a degree of competitive toughness lower than at price equilibrium (where $\theta = 1/2$) if and only if $s/\sigma > 1$. In quantity competition, high substitutability within the oligopolistic sector (relative to the sector of the numéraire good) is so to say compensated by mild competitiveness: The degree θ of competitive toughness decreases as the ratio s/σ increases.

To analyze the competitive effects of quantity competition, it is convenient to use the arithmetic mean of all markups weighted by their respective budget shares. Assuming a CES sub-utility, we get the following alternative formula [to (24)], now establishing the relation between the *arithmetic mean markup* and the *Herfindahl index* of concentration $\sum_{i=1}^n \alpha_i^{*2}$:

$$\overline{\mu}^* = \sum_{i=1}^n \alpha_i^* \mu_i^* = \frac{1}{s} + \left(\frac{1}{\sigma^*} - \frac{1}{s}\right) \sum_{i=1}^n \alpha_i^{*2}, \tag{39}$$

which reduces in the symmetric case to the uniform markup

$$\mu^* = \frac{1}{s} + \left(\frac{1}{\sigma^*} - \frac{1}{s}\right) \frac{1}{n}. \tag{40}$$

We thus get a result analogous to Proposition 3.

Proposition 7. *Assume CES sub-utility. As long as $s > \sigma^*$ (resp. $s < \sigma^*$), the arithmetic mean markup increases (resp. decreases) with the Herfindahl index of concentration. If in addition $c_i = c$ and $\phi_i = \phi$ for any i, the equilibrium markups and prices of all firms are identical and decreasing (resp. increasing) when n, the number of active firms, increases as long as $s > \sigma^*$ (resp. $s < \sigma^*$).*

3.5 Monopolistic competition

The last application of Proposition 1 that we will consider is monopolistic competition. This will be a limit case, assuming a symmetric equilibrium (with a symmetric sub-utility function X, as well as uniform costs $c_i = c$ and $\phi_i = \phi$ for any i), so that the index i can be omitted in the following. But this limit case can be reached in two ways. A first way, the traditional one, consists in making the *large group assumption* (in Chamberlin's sense) by supposing $\alpha^* \approx 0$ in the expression (10). Then we obtain the same

$$\mu^* \approx 1/s^* \tag{41}$$

as the markup for each firm. Also, with symmetry (with $x_i = x^*$ for all i) and $\alpha^* \approx 0$, formula (4) boils down to

$$s^* = \frac{\partial_i X(\mathbf{x}^*)}{-\partial^2_{ii} X(\mathbf{x}^*) x^*} \equiv \frac{1}{r_X(x^*)} \tag{42}$$

where $r_X(x^*)$ is the value taken by the Arrow-Pratt formula for relative risk aversion when applied to the sub-utility function X along the diagonal. This value can be interpreted as the *relative love for variety* (see Zhelobodko et al. 2012).

In this case, the following fact is well known about a symmetric monopolistic equilibrium (seen here as the limit of an oligopolistic equilibrium). Since the intersectoral elasticity of substitution vanishes in formula (10) of Proposition 1, the properties of U are immaterial for the value of the equilibrium price p^*.

Also well known is the fact that, when X is a CES function (so that s^* is a structural parameter $s \in (1, \infty)$), the equilibrium price of the composite good is constant and equal to

$$p^* = c/(1 - \mu^*) = c/(1 - 1/s) \tag{43}$$

which is independent of the number of active firms. In order to obtain richer results concerning the price variations, the variability of s^* must consequently be allowed for. Using a translog utility function as the specification of X, Feenstra (2003) showed that such variability can be secured, in the case of homothetic preferences, so as to obtain pro-competitive effects of an increase in the mass of active monopolists. Zhelobodko et al. (2012) showed that both pro-competitive and anti-competitive effects can be obtained, in the case of an additive sub-utility function X (as introduced by Krugman 1979, in a one-sector model), according to the way the relative love for variety responds to changes in the level of consumption. In particular, the pro-competitive effect of an increase in the number (or the mass) of firms depends on the increasingness of love for variety with the consumption level (rather than on the dominance of intrasectoral over intersectoral substitutability, as in our price and quantity equilibria). They also use the example of quadratic sub-utility to show that additivity can be dispensed with, but they only obtain pro-competitive effects in that example.

Under free entry, still assuming X to be a CES function, the equilibrium quantity of each differentiated good,

$$x^* \approx (s - 1)\phi/c, \tag{44}$$

is entirely determined by the elasticity of substitution s and the unit cost c/ϕ normalized in terms of the fixed cost. Switching to the additive utility function used by Zhelobodko et al. (2012) does not considerably change the situation if one sticks to the regime of monopolistic competition, with $\alpha^* \approx 0$, and $\mu^* \approx 1/s^*$ equal to the relative love for variety $r_X(x^*)$, a function of x^* which does not depend upon n^* (here interpretable as

the *mass* of active firms). Thus, x^* is still determined independently of the equilibrium number n^* of active firms, which is itself determined via the utility function U. For instance, if this function is Cobb–Douglas with exogenous weights γ for the composite good and $1-\gamma$ for the numéraire good, we obtain:

$$n^* = \frac{\gamma L}{s\phi} \tag{45}$$

in the CES case, or $n^* = \gamma(L/\phi)r_X(x^*)$ in the case examined by Zhelobodko et al. (2012). When U is not Cobb–Douglas, we may still use this expression for n^*, but with an endogenous equilibrium share γ^* of the composite good in the total budget. More generally, when the sub-utility is not additive, the intrasectoral elasticity of substitution depends (along the diagonal) both on the quantity consumed per differentiated good and on the number of active firms, implying that, at the symmetric free-entry equilibrium, prices and quantities are not determined independently from the equilibrium number of firms (see Parenti et al. 2014). Still, the main ingredient remains the intrasectoral elasticity of substitution.

But, as mentioned above, there is another way to obtain $\mu^* = 1/s^*$ (the formula characterizing monopolistic competition) which has the advantage of allowing to keep a limited number n of firms, and thus to let α^* be significantly away from zero: It suffices to take $\theta^* = 1$, meaning that firms have maximal competitive toughness, instead of being insignificant.[10] In this case, Eq. (4) becomes

$$s^* = \frac{1-\alpha^*}{r_X(x^*)} = \frac{1-1/n}{r_X(x^*)}. \tag{46}$$

introducing the dependence on n. For instance, if the relative love for variety is constant, then the intrasectoral elasticity of substitution becomes an increasing function of n, and we get the associated results (see Bertoletti and Epifani 2014). Or, when X is a CES, the relative love for variety has to be a function of n, implying that the price and the quantity vary with n.

Again this means that, at the symmetric free-entry equilibrium, prices and quantities are not determined independently from the equilibrium number of firms. But this is not sufficient to re-introduce the role of intersectoral substitution, as it appears in formula (10) of Proposition 1, and the full general equilibrium dimension of the Dixit–Stiglitz model.

4 Conclusion

The purpose of this note is to argue that going beyond the basic version of the Dixit–Stiglitz model can take other paths than relaxing the CES assumption and/or introducing heterogeneity à la Melitz (2003) while maintaining monopolistic competition. One is to introduce oligopolistic competition among a finite number of non-negligible firms. This seems realistic, at least for some sectors, and was worth exploring in a general equilibrium perspective with multiple sectors (here only two) and various possible degrees of substitutability–complementarity among goods within the sectors and among composite goods. Symmetry can be straightforwardly relaxed, and by adopting some specific sub-utility like the CES, the model can be kept tractable. In our analysis of pro-competitive

[10] This is related to the so-called *Bertrand paradox*: Tough competition *à la* Bertrand leads to the competitive outcome even in a (symmetric) duopoly, where each firm gets half of the market.

or anti-competitive effects, the main benefit that we have obtained is to stress the role of intersectoral characteristics, mainly intersectoral substitutability as compared to intrasectoral substitutability, a role which is obliterated under monopolistic competition. Much further work is required of course, but one may hope that Neary (2004) was right when, on the occasion of the twenty-fifth anniversary of Dixit–Stiglitz paper, he considered the relevance (and the obstacles) in going toward a general oligopolistic equilibrium theory but concluded that "the pay-off to even modest progress in this direction would be enormous."

Appendix

The elasticities of substitution

The concept of elasticity of substitution was introduced by Hicks (1932) to measure the response of the input quantity ratio to a variation in the corresponding price ratio, under price taking and cost minimization by the producers. The original concept concerned the case of the sole two inputs, capital and labor, of some production function, but it was later generalized in several ways to the case of an arbitrary number of inputs (see e.g., Blackorby and Russell 1989).

The intrasectoral elasticity of substitution

In this paper, while referring to the sub-utility function X, we consider the elasticity of substitution s_i of good i for the composite good (rather than the elasticity of substitution s_{ij} between two goods i and j). It is defined, for a bundle of differentiated goods $\mathbf{x} = (x_1, \ldots, x_i, \ldots, x_n)$, as the absolute value of the elasticity of the ratio $x_i/X(\mathbf{x})$ with respect to the marginal rate of substitution $\partial_i X(\mathbf{x})$, which coincides here with the marginal (sub-)utility of x_i:

$$s_i \equiv -\frac{d\left(x_i/X(\mathbf{x})\right)}{d\left(\partial_i X(\mathbf{x})\right)} \frac{\partial_i X(\mathbf{x})}{x_i/X(\mathbf{x})}. \tag{47}$$

By differentiating $x_i/X(\mathbf{x})$ and $\partial_i X(\mathbf{x})$ with respect to x_i, we obtain the following expression for the *intrasectoral elasticity of substitution* of good i:

$$s_i = \frac{1 - \partial_i X(\mathbf{x}) x_i/X(\mathbf{x})}{-\partial^2_{ii} X(\mathbf{x}) x_i/\partial_i X(\mathbf{x})}. \tag{48}$$

Notice that s_i can be seen as the reciprocal of the degree of curvature of the graph $(x_i/X(\mathbf{x}), X(\mathbf{x}))$.

The elasticity of substitution between goods i and j is:

$$\begin{aligned} s_{ij} &\equiv -\frac{d(x_i/x_j)}{d\left(dx_j/dx_i\right)} \frac{dx_j/dx_i}{x_i/x_j} \\ &= -\frac{\partial_i X(\mathbf{x}) \partial_j X(\mathbf{x}) \left(x_i \partial_i X(\mathbf{x}) + x_j \partial_j X(\mathbf{x})\right)}{x_i x_j \left(\partial^2_{ii} X(\mathbf{x}) \left(\partial_j X(\mathbf{x})\right)^2 - 2\partial^2_{ij} X(\mathbf{x}) \partial_i X(\mathbf{x}) \partial_j X(\mathbf{x}) + \partial^2_{jj} X(\mathbf{x}) \left(\partial_i X(\mathbf{x})\right)^2\right)} \end{aligned} \tag{49}$$

(see Nadiri 1982). If sub-utility X is symmetric and if we consider any pair (i, j) of goods such that $x_i = x_j = x$,

$$s_{ij} = \frac{1}{-\partial^2_{ii} X(\mathbf{x}) x/\partial_i X(\mathbf{x}) + \partial^2_{ij} X(\mathbf{x}) x/\partial_i X(\mathbf{x})}, \tag{50}$$

which converges to the same value as our elasticity s_i as n tends to infinity, implying that $\partial_i X(\mathbf{x}) x_i / X(\mathbf{x}) \to 0$ and $\partial^2_{ij} X(\mathbf{x}) x / \partial_i X(\mathbf{x}) \to 0$ (cf. Parenti et al. 2014, where the set of the differentiated goods is a continuum). Also, symmetry and constancy of the elasticity s_{ij} ($s_{ij} = s$ for any $\mathbf{x}$ and any pair (i,j) of goods), corresponding to the CES sub-utility function $X(\mathbf{x}) = \left(\sum_{i=1}^n a_i x_i^{(s-1)/s}\right)^{s/(s-1)}$, naturally lead to the coincidence of both elasticities: $s_i = s_{ij} = s$. By applying (48), we indeed obtain at $\mathbf{x}$:

$$s_i = \frac{1 - a_i \left(x_i / X(\mathbf{x})\right)^{(s-1)/s}}{\left(1 - a_i \left(x_i / X(\mathbf{x})\right)^{(s-1)/s}\right) / s} = s. \tag{51}$$

Thus, the elasticity s_i of intrasectoral substitution to which we refer in this paper coincides with the standard elasticity s_{ij} in two of the most used cases in the literature, namely the case of the CES sub-utility function and the case of a "large group" of differentiated goods (justifying the monopolistic competition approach), when the sub-utility function X and the observed basket $\mathbf{x}$ are both symmetric.

The intrasectoral elasticity of substitution s_i can alternatively be computed in terms of prices (under consumer price taking and expenditure minimizing). To do that, denoting $\underline{X} = X(\mathbf{x})$, we refer to the shadow price of the composite good $\underline{P} \equiv \partial_X e\left(\mathbf{p}, \underline{X}\right)$, and use the expression of x_i given by Shephard's lemma (3):

$$s_i = -\frac{d\left(x_i / \underline{X}\right)}{d\left(p_i / \underline{P}\right)} \frac{p_i / \underline{P}}{x_i / \underline{X}} = -\frac{d\left(\partial_{p_i} e\left(\mathbf{p}, \underline{X}\right) / \underline{X}\right)}{d\left(p_i / \underline{P}\right)} \frac{p_i / \underline{P}}{\partial_{p_i} e\left(\mathbf{p}, \underline{X}\right) / \underline{X}}. \tag{52}$$

By differentiating $\partial_{p_i} e\left(\mathbf{p}, \underline{X}\right) / \underline{X}$ and $p_i / \underline{P}$ with respect to p_i, we obtain

$$s_i = \frac{-\partial^2_{p_i p_i} e\left(\mathbf{p}, \underline{X}\right) p_i / \partial_{p_i} e\left(\mathbf{p}, \underline{X}\right)}{1 - \partial^2_{p_i X} e\left(\mathbf{p}, \underline{X}\right) p_i / \partial_X e\left(\mathbf{p}, \underline{X}\right)} \tag{53}$$

and, by using the definition of the Hicksian demand H_i in (3), together with the first-order condition (2), we finally get

$$s_i = \frac{-\epsilon_{p_i} H_i\left(\mathbf{p}, \underline{X}\right)}{1 - \left[\epsilon_i X(\mathbf{x})\right] \left[\epsilon_X H_i\left(\mathbf{p}, \underline{X}\right)\right]}. \tag{54}$$

The intersectoral elasticity of substitution

To define the intersectoral elasticity of substitution of good i, we refer to the quantity ratio x_i / Y (instead of $x_i / \underline{X}$),[11] implicitly taking into account the variation of the Marshallian demand D (rather than sticking to the mere share adjustment as measured by $-\epsilon_{p_i} H_i\left(\mathbf{p}, \underline{X}\right)$). The corresponding price ratio is of course $p_i / 1 = p_i$. So, the *intersectoral elasticity of substitution* σ_i of good i can be expressed as follows:

$$\sigma_i \equiv -\left.\frac{d(x_i/Y)}{dp_i}\right|_{X(\mathbf{x}) = D(\mathbf{p}, Y)} \frac{p_i}{x_i / Y} = -\frac{(1/Y)\partial_{p_i} D(\mathbf{p}, Y)}{\partial_i X(\mathbf{x})} \frac{p_i}{x_i / Y} = \frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\epsilon_i X(\mathbf{x})}. \tag{55}$$

To illustrate, take the case of the utility function introduced in Sect. 2.4:

$$U(X(\mathbf{x}), z) = \frac{b^{1/\sigma}}{1 - 1/\sigma} X(\mathbf{x})^{1 - 1/\sigma} + z, \tag{56}$$

$$\text{with} \quad X(\mathbf{x}) = \left(\sum_{i=1}^n x_i^{(s-1)/s}\right)^{s/(s-1)}, \quad \sigma > 1 \ \text{ and } \ b > 0.$$

[11] The reference to the quantity ratio x_i / Y, rather than x_i / z, when defining the intersectoral elasticity of substitution, is consistent with the choice of $x_i / \underline{X}$, rather than x_i / x_j, when defining the corresponding intrasectoral concept.

The corresponding Marshallian demand for the composite good is the isoelastic function

$$D(\mathbf{p}, Y) = bP(\mathbf{p})^{-\sigma}, \quad \text{with} \quad P(\mathbf{p}) = \left(\sum_{i=1}^{n} p_i^{1-s} \right)^{1/(1-s)}, \tag{57}$$

so that we obtain the following expression for the intersectoral elasticity of substitution, as defined by (55):

$$\sigma_i = \sigma \frac{(p_i/P(\mathbf{p}))^{1-s}}{(x_i/X(\mathbf{x}))^{(s-1)/s}} = \sigma, \tag{58}$$

since $x_i = (p_i/P(\mathbf{p}))^{-s} X(\mathbf{x})$ when the x_i's are chosen optimally. Thus, the intersectoral elasticity of substitution is symmetric and constant in this case ($\sigma_i = \sigma$ for any i), equal to the elasticity of the demand for the composite good.

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