General Equilibrium Concepts under Imperfect Competition: A Cournotian Approach*

Claude d'Aspremont[†], Rodolphe Dos Santos Ferreira[‡] and Louis-André Gérard-Varet[§]

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Abstract

In a pure exchange economy we propose a general equilibrium concept under imperfect competition, the "Cournotian Monopolistic Competition Equilibrium", and compare it to the Cournot-Walras and the Monopolistic Competition concepts. The advantage of the proposed concept is to require less computational ability from the agents. The comparison is made first through a simple example, then through a more abstract concept, the P-equilibrium based on a general notion of price coordination, the pricing-scheme.

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[†]CORE, Université catholique de Louvain, 34 voie du Roman Pays, 1348-Louvain-la-Neuve, Belgium

[‡]BETA, Université Louis Pasteur, 38 boulevard d'Anvers, 67000-Strasbourg, France

[§]GREQE, Ecole des Hautes Etudes en Sciences Sociales, F-13002-Marseille, France.

1 Introduction

The formal simplicity of general equilibrium theory under perfect competition and of its assumptions on the individual agents' characteristics, can easily be contrasted with the complexity and the ad hoc assumptions of the general analysis of imperfect competition. However the simplicity of general competitive analysis is all due to a single behavioral hypothesis, namely that all sellers and buyers take prices as given. This price-taking hypothesis allows for a clear notion of individual rationality, based on the simplest form of anticipations (rigid ones) and an exogenous form of coordination (the auctioneer). Once the price-taking hypothesis is discarded and strategic considerations introduced, these three issues - individual rationality, anticipations and coordination – raise fundamental questions. The attempts to deal with these issues in a general equilibrium approach to imperfect competition have taken different routes, corresponding to different traditions in oligopoly theory.[1]

First, in the Cournot tradition only a subset of the agents (the firms) are supposed not to follow the price-taking hypothesis and to affect strategically the competitive price mechanism through quantity-settings. For instance, the Cournot-Walras general equilibrium concept, defined by Gabszewicz and Vial [16], presupposes the existence of a unique Walrasian equilibrium associated with every choice of quantities made by the strategic agents and, then, in the resulting game, takes as a solution the noncooperative (Cournot-Nash) equilibrium. The strategic agents are given the ability to anticipate correctly, and for every move, the result of the market mechanism, not only in a market in which they act strategically, as in Cournot partial equilibrium approach, but in all markets simultaneously. This approach is much more exacting, of course, than Negishi's [29] "subjective" approach which, following Triffin's [41] suggestion, presupposes that strategic agents conjecture simply "subjective inverse demand (or supply) functions" in their own markets.[2] But it has the advantage of leading to an "objectively" defined and well-determined solution (at least when existence is ensured).

In a second tradition, that of Bertrand, Edgeworth, and the monopolistic competition of

[1]Surveys are given by Hart [19] and Bonanno [6]. See also Gary-Bobo [17].

[2]For the subjective approach see, for example, Arrow and Hahn [2], Silvestre [39] and Bénassy [3].

Robinson and Chamberlin, as well as the spatial competition of Laundhardt and Hotelling, again only firms are supposed to behave strategically, but the strategic variables are the prices. There the difficulty, well discussed in Marschak and Selten [25], is to model how the quantities adjust to any system of chosen prices and to construct an "effective demand function" in the sense of Nikaido [30], taking into account both the direct effects of a change in prices and the indirect effects through dividends or wage income, and having properties ensuring the existence of an equilibrium in prices.[3]

A third, more recent tradition, relying even more on noncooperative game theory and initiated by the work of Shubik [38], Shapley [36], Shapley and Shubik [37],[4] consists in viewing the whole economy as a "market game", where all agents behave strategically and send both quantity and price signals in all markets. Such a market game is defined by introducing a strategic "outcome function" that determines the actual transactions and the prices actually paid by the agents as a function of the signals they send. An outcome function can be viewed as a coordination mechanism, which, depending on the properties that it satisfies, produces more or less efficient outcomes as Nash equilibria, and can even be constructed so as to reduce the set of Nash equilibria to the competitive outcomes.[5] This coordinating outcome function can also be stochastic, using extraneous random variables such as "sunspots", analogously to the game-theoretic notion of "correlated equilibrium."[6]

This paper is an attempt to reconcile these three traditions and to present a general equilibrium concept under imperfect competition which combines features of all three. First, it is an alternative generalization of Cournot's partial equilibrium concept, following the implications introduced by Laffont and Laroque [23] and by Hart [19],[7] which consist in supposing that the subset of strategic agents take as given (or fixed) a large number of the variables indirectly influenced by their decisions. This avoids the presumption that strategic agents make full gen-

[3]See Stahn [40]. For recent contributions based on fix-price models, see Benassy [5] and Roberts [33].

[4]In Mas-Colell [27], both the market game approach and the Cournot-Walras approach are studied.

[5]See Wilson [42], Hurwicz [21], Dubey [12], Schmeidler [35], Mas-Colell [26] and Bénassy [4].

[6]See Cass and Shell [7], Peck and Shell [31] and Forges [14].

[7]See also d'Aspremont, Dos Santos Ferreira and Gérard-Varet [10].

eral competitive equilibrium calculations before taking their decisions. Also, it will make clear that, in Cournot's approach, both quantities and prices may be taken as strategic variables, and that the proposed general equilibrium concept, the "Cournotian Monopolistic Competition Equilibrium", is a generalization both of the Cournot's solution and of the monopolistic competition partial equilibrium. Indeed, as we have stressed elsewhere in a partial equilibrium setting with production,[8] the Cournot solution can be viewed as the coordinated optimal decisions of a set of monopolists, each maximizing profit in price and in quantity, while facing a "residual" demand. Likewise, the Cournotian Monopolistic Competition Equilibrium may be viewed as the solution to a (coordinated) juxtaposition of monopoly problems, each monopolist facing a demand function contingent both on the equilibrium quantities in its own sector and on the equilibrium prices in the other sectors. Finally, the coordination involved may be related to the market game approach more explicitly. This relation is based on the definition of what we call a pricing scheme,[9] that is a formal representation of the way in which the price-making agents coordinate their pricing decisions, and on the definition of an associated concept of P-equilibrium. Pricing-schemes are defined sector by sector. Each introduces coordination by associating a vector of "market prices" to every vector of price signals sent by the strategic agents in that sector. In game-theoretic terminology, it is a deterministic communication system with input-signals only (by contrast, a correlation device is a stochastic communication system with output-signals only). However, although appearing as coordination mechanisms, pricing-schemes do not constitute a complete outcome function, transforming the whole economy into a single noncooperative market game. They define games sector by sector, and intersectoral interaction is modeled competitively. Moreover, they may differ in their degree of manipulability, leading to various types of P-equilibrium and hence to alternative general equilibrium concepts under imperfect competition. The Cournotian Monopolistic Competition Equilibrium is one of them and corresponds to an extreme form of manipulability. We will examine an alternative.

[8] d'Aspremont, Dos Santos Ferreira and Gérard-Varet [11].

[9] It was introduced in d'Aspremont, Dos Santos Ferreira and Gérard-Varet [11], in relation to the industrial organization literature on "facilitating practices" and the role of trade associations.

In Section 2, we start by defining, and comparing in a simple pure exchange economy, the three concepts of Cournot-Walras, Monopolistic Competition and Cournotian Monopolistic Competition Equilibrium. In Section 3, the abstract notions of pricing-scheme and P-equilibrium are introduced and used to reconsider the three types of equilibria. They all rely on fully manipulable pricing-schemes. Finally in Section 4, introducing a less manipulable but meaningful pricing-scheme, we obtain another type of equilibrium, which includes the Cournotian Monopolistic Competition Equilibrium and the Walrasian Equilibrium among its outcomes.

2 General equilibrium concepts under imperfect competition

As a first step we shall introduce a pure exchange economy and restate for such an economy two basic concepts of general equilibrium under imperfect competition, the Cournot-Walras Equilibrium and the Monopolistic Competition Equilibrium, as well as a "simplified combination" of these, the Cournotian Monopolistic Competition Equilibrium. This will be done to express the main difficulties and illustrated through a simple example.

Let us consider a set of m *consumers* $I = \{1, 2, \cdots, i, \cdots, m\}$, exchanging a set of $\ell + 1$ *goods* $H = \{0, 1, \cdots, h, \cdots, \ell\}$, in which good 0 will be interpreted as money. As we shall see, money in itself will play a coordinating role in the economy and will allow us eventually to limit the introduction of coordination to a sector by sector way. Normalizing the price of good 0 accordingly, a *price system* is a vector in $\mathbb{R}^H_+$ of the form $p = (1, p_1, \cdots, p_\ell)$. With each consumer $i \in I$ is associated a *consumption set* $X_i \subset \mathbb{R}^H$, a vector of *initial resources* $\omega_i \in X_i$ and a real-valued *utility function* $U_i(x_i)$ defined on X_i. Assumptions on these consumers' characteristics will be given later (often implicitly when these assumptions are standard). They will however always include nonsatiation and strict quasi-concavity of the utility functions. Also, for simplicity, we let $X_i = \mathbb{R}^H_+$.

Imperfect competition is introduced by assuming a set $H^* \subset H$ of monopolistic markets such that, for every $h \in H^*$, the set of consumers is partitioned into two *nonempty* subgroups, the set I_h of consumers having some monopoly power and the set $\hat{I}_h \equiv I \setminus I_h$ of consumers behaving competitively in market h. A consumer i may have monopolistic power in none or several markets. We will denote by H_i the set of such markets (H_i may be empty), so that $H^* = \cup_{i \in I} H_i$. Also, letting $\hat{H} \equiv H \setminus H^*$, we will suppose that $0 \in \hat{H}$, i.e., money is a competitive good. Finally we denote by I^* the set $\cup_{h \in H^*} I_h$ of *strategic consumers* and by $\hat{I} \equiv I \setminus I^*$ the set of *competitive consumers*.

2.1 The Cournot-Walras equilibrium

The Cournot-Walras Equilibrium can now be defined. The original definition due to Gabszewicz and Vial [16] was given in an oligopolistic framework with producers as strategic agents, choosing production levels. However it can be stated for a pure exchange context as shown in Codagnato and Gabszewicz [8] and Gabszewicz and Michel [15]. The two-stage procedure can be specified as follows. First strategic consumer $i \in I^*$ chooses a vector of orders $q_i \in \mathbb{R}^H$ representing the quantities of goods in H he wants to offer ($q_{ih} > 0$) or to bid for ($q_{ih} < 0$) in the respective markets. This vector is restricted[10] to belong to an admissible set Q_i. In particular, for $h \in H \setminus H_i$, q_{ih} is constrained to be zero. We let $Q = \times_{i \in I^*} Q_i$. Then, given $q \in Q$, the Walrasian mechanism is put in place in the standard way, except that every strategic consumer $i \in I^*$ has his consumption x_{ih} of every good h in H_i constrained by his signalling decision q_{ih}. More precisely, for each $i \in I^*$, there is a *net demand function* $\zeta(p, q_i)$ which, for every price system p, may be defined by the program

$$\zeta_i(p, q_i) + \omega_i = \arg \max_{x_i} U_i(x_i)$$

under the constraints

$$p(x_i - \omega_i) \leq 0,$$

and, for all $h \in H$,

$$q_{ih}(\omega_{ih} - x_{ih} - q_{ih}) \leq 0.$$

[10]Codognato and Gabszewicz [8] and Gabszewicz and Michel [15] only consider positive signals, upper bounded by ω_i.

Notice that, for a competitive consumer $i \in \hat{I}$, the net demand function $\zeta_i(p, 0)$ coincide with the usual competitive net demand.

Finally, in order for the oligopolistic game in the quantities q_i to be well-defined, the following assumption has to hold, ensuring the existence of a unique Walrasian price-system relative to q.

Assumption 1. For all $q \in Q$, there is a unique price system $\tilde{p}(q)$ such that

$$\sum_{i \in I} \zeta_i(\tilde{p}(q), q_i) = 0.$$

A *Cournot-Walras Equilibrium* is a pair of prices and quantities (p^{CW}, q^{CW}) in $\mathbb{R}_+^H \times Q$ such that

$$p^{CW} = \tilde{p}(q^{CW}) \text{ and } \forall\, i \in I^*, q_i^{CW} \in \arg\max_{q_i \in Q_i} U_i(\zeta_i(\tilde{p}(q_i, q_{-i}^{CW}), q_i) + \omega_i)$$
$$\text{with } q_{-i}^{CW} = (q_j^{CW})_{\substack{j \in I^* \\ j \neq i}}.$$

In other terms, once all the functions ζ_i are well-defined and Assumption 1 holds, the Cournot-Walras quantity q^{CW} is the Nash-Equilibrium of the game with players in I^*, strategies in Q and payoffs given by $\{U_i(\zeta_i(\tilde{p}(q), q_i) + \omega_i)\}$.

In the special case $H_i = \emptyset$ for all consumers i, then the first stage becomes trivial and the equilibrium reduces to the *Walrasian Equilibrium*, characterized by a price-system p such that

$$\sum_{i \in I} \zeta_i(p^W, 0) = 0.$$

We shall now consider an example with 2 strategic agents, allowing to compare the Cournot-Walras Equilibrium with the Walrasian Equilibrium. The example is chosen so that the other equilibrium concepts, introduced next, will lead to different allocations.[11] The reader not interested in the computations of this example should only note the computed Cournot-Walras equilibrium price (quantity) of the monopolistic goods is higher (lower) than the Walrasian price (quantity). This reflects the effect of having each strategic consumer "cornering the market" for the good he owns initially.

[11]This is why we cannot take the simpler example analyzed by Codognato and Gabszewicz [8].

Example (Part 1). Suppose that there are 3 goods ($h = 0, 1, 2$) and $n + 2$ consumers ($m = n + 2, n \geq 1$) and that the initial resources and utility functions are, respectively

$$\begin{array}{ll} \omega_1 = (0,1,0), \qquad \omega_2 = (0,0,1), \qquad \omega_i = \left(\frac{1}{n}, 0, 0\right), & i = 3, \cdots, n+2, \\ U_i(x_i) = \alpha(1-\beta)\ln x_{i0} + \alpha\beta \ln x_{ij} + (1-\alpha)\ln x_{ii}, i \neq j, & \text{for } i = 1, 2, \\ \qquad = (1-2a)\ln\left(x_{i0} + \frac{b}{n}\right) + a\ln\left(x_{i1} + \frac{b}{n}\right) + a\ln\left(x_{i2} + \frac{b}{n}\right) & \text{for } i = 3, \cdots, n+2. \end{array}$$

That is, for $i = 1, 2$, the utility function U_i of the Cobb-Douglas type with parameters $\alpha, \beta \in (0, 1)$, and, for $i = 3, \cdots, n + 2$, it is of the Stone-Geary type with parameters $a \in (0, \frac{1}{2})$ and $b/n > 0$. For $i = 1, 2$, the competitive net demand functions are well-known to be (recall that $p_0 = 1$)

$$\begin{array}{rcccl} \zeta_1(p,0) & = & \left(\alpha(1-\beta)p_1, (1-\alpha), \alpha\beta\frac{p_1}{p_2}\right) - (0,1,0) & = & \alpha p_1 \left(1-\beta, -\frac{1}{p_1}, \frac{\beta}{p_2}\right), \\ \zeta_2(p,0) & = & \left(\alpha(1-\beta)p_2, \alpha\beta\frac{p_2}{p_1}, (1-\alpha)\right) - (0,0,1) & = & \alpha p_2 \left(1-\beta, \frac{\beta}{p_1}, -\frac{1}{p_2}\right). \end{array}$$

Also, since ζ_i is symmetric and must satisfy the budget constraint, we get for $i = 3, \cdots, n + 2$,

$$\zeta_i(p, 0) = \frac{1}{n}(-p_1 f(p_1, p_2) - p_2 f(p_2, p_1), f(p_1, p_2), f(p_2, p_1)),$$

where $f(p_i, p_j) = -b + (a/p_i)(1 + b(1 + p_1 + p_2))$, $i \neq j$, $i, j = 1, 2$. Hence at the symmetric competitive equilibrium (with prices $p_1^W = p_2^W = \overline{p}^W$) one has, for $h = 1, 2$,

$$\sum_{i=1}^{n+2} \zeta_{ih}(p, 0) = g(\overline{p}^W) - \alpha + \alpha\beta = 0 \quad (\text{with } g(\overline{p}) \equiv f(p_1, p_2) \text{ for } p_1 = p_2 = \overline{p})$$

or

$$\frac{\alpha(1-\beta)}{g(\overline{p}^W)} = 1. \tag{W}$$

For the Cournot-Walras Equilibrium, we assume that consumers are price-takers in all markets except for consumers 1 and 2 who are strategic agents in the markets for goods 1 and 2 respectively, i.e., $H_1 = \{1\}, H_2 = \{2\}$ and $H_i = \emptyset$ for $i = 3, \cdots, n+2$. Taking $q = (q_1, q_2) \in (0, \alpha]^2$ as given,[12] the net demand functions conditional on q can be easily computed (fixing $x_{ii} = 1 - q_i$, for $i = 1, 2$):

$$\begin{array}{rcccl} \zeta_1(p,q_1) & = & \left((1-\beta)p_1 q_1, 1-q_1, \frac{\beta}{p_2}p_1 q_1\right) - \omega_1 & = & \left(1-\beta, -\frac{1}{p_1}, \frac{\beta}{p_2}\right) p_1 q_1 \\ \zeta_2(p,q_2) & = & \left((1-\beta)p_2 q_2, \frac{\beta}{p_1}p_2 q_2, 1-q_2\right) - \omega_2 & = & \left(1-\beta, \frac{\beta}{p_1}, -\frac{1}{p_2}\right) p_2 q_2. \end{array}$$

[12] Indeed, by the definition of $\zeta_i, \zeta_i(p, q_i) = \zeta_i(p, 0)$ for $q_i \geq \alpha$ or $q_i \leq 0$.

Then the market-clearing price function $\tilde{p}(q)$ is the solution to the system

$$\begin{aligned} q_1 &= \tfrac{\beta}{p_1} p_2 q_2 + f(p_1, p_2) \\ q_2 &= \tfrac{\beta}{p_2} p_1 q_1 + f(p_2, p_1), \end{aligned}$$

leading, with the particular definition of the function f, to

$$\tilde{p}_i(q) = \frac{a(1+b)[(1+\beta)q_j + b]}{[q_j + b(1-a)][q_i + b(1-a)] - [\beta q_j + ab][\beta q_i + ab]}, \quad i \neq j, i, j = 1, 2.$$

Hence, at the first stage, consumer i, $i = 1, 2$, should solve the problem

$$\begin{aligned} \max_{q_i \in (0,\alpha]} \alpha(1-\beta)\ln((1-\beta)\tilde{p}_i(q)q_i) \quad &+ \quad \alpha\beta \ln\left(\frac{\beta}{\tilde{p}_j(q)}\tilde{p}_i(q)q_i\right) \\ &+ \quad (1-\alpha)\ln(1-q_i), \qquad i \neq j, \quad i = 1, 2. \end{aligned}$$

To find a symmetric solution $q_1^{CW} = q_2^{CW} = \overline{q}$, one may simply solve in $\overline{q}$ the equation derived from the first-order conditions, which, after simplifications, is given by

$$\frac{\alpha - \overline{q}}{\alpha\overline{q}(1-\overline{q})} = (1-\beta)\frac{\overline{q} + b(1-a) - \beta(\beta\overline{q} + ab)}{[\overline{q} + b(1-a)]^2 - [\beta\overline{q} + ab]^2} + \frac{(1+\beta)\beta}{(1+\beta)\overline{q} + b}$$

or equivalently,

$$\frac{\alpha}{\overline{q}} - 1 = \frac{\alpha(1-\overline{q})}{(1+\beta)\overline{q} + b}\left\{2\beta + \frac{(1-\beta)^2(\overline{q} + b(1-a))}{(1-\beta)\overline{q} + b(1-2a)}\right\}.$$

Letting

$$\varphi(\overline{q}) = \frac{\overline{q}}{(1+\beta)\overline{q} + b}\left\{2\beta + \frac{(1-\beta)^2(\overline{q} + b(1-a))}{(1-\beta)\overline{q} + b(1-2a)}\right\}$$

and since

$$\frac{\alpha}{\overline{q}} - 1 = \frac{\alpha(1-\overline{q})}{\overline{q}}\varphi(\overline{q}) \text{ if and only if } \frac{\alpha}{\overline{q}} - 1 = (1-\alpha)\frac{\varphi(\overline{q})}{1-\varphi(q)},$$

we get, after other simplifications,

$$\frac{\alpha}{\overline{q}} - 1 = \frac{(1-\alpha)\overline{q}}{b}\left\{\frac{\overline{q}(1-\beta^2) + b[a(1-\beta)^2 + (1-2a)(1+\beta^2)]}{\overline{q}(1-\beta^2)(1-a) + b(1-2a)}\right\}. \qquad \text{(CW)}$$

This equation determines the Cournot-Walras quantity vector q^{CW} and hence the Cournot-Walras Equilibrium. It may be compared to the solution of equation (W) determining the Walrasian Equilibirum. Indeed, using the feasibility condition,

$$\overline{q} = g(\tilde{p}(\overline{q})) + \beta\overline{q}\left(\text{or } \overline{q} = \frac{1}{1-\beta}g(\tilde{p}(\overline{q}))\right),$$

one can rewrite equation (W) in a simpler form, to compare to equation (CW),

$$\frac{\alpha}{\overline{q}} - 1 = 0. \tag{W'}$$

The left-hand side of this equation can be interpreted as measuring the "degree of cornering" the monopolistic markets, which is of course nil at the Walrasian equilibrium and can be shown to be positive at the Cournot-Walras equilibrium. Indeed the right hand side of (CW) is null for $\overline{q} = 0$. It can be verified that it is increasing (and concave) in $\overline{q}$. So the value $\overline{q}^{CW}$ solving equation (CW) gives the Cournot-Walras quantity vector $q^{CW} = (\overline{q}^{CW}, \overline{q}^{CW})$. That value is smaller than the solution of equation (W'), $\overline{q}^{W} = \alpha$, thus leading to a positivie "degree of cornering". Also: $\overline{p}^{CW} = \tilde{p}_1(q^{CW}) = \tilde{p}_2(q^{CW}) > \overline{p}^{W}$, so that the Cournot-Walras equilibrium price $\overline{p}^{CW}$ is higher than the Walrasian price $\overline{p}^{W}$.

2.2 A monopolistic competition equilibrium

Another kind of general equilibrium concept under imperfect competition is given by generalizing the concept of Monopolistic Competition Equilibrium. As mentioned in the introduction, what is meant here is still to keep an objective approach and to define an "effective demand" as developed for example by Marschak and Selten [25] and Nikaido [30]. For simplicity we keep the same pure exchange model, but assume that no two consumers control the price of the same good: $H_i \cap H_j = \emptyset$, for all $i, j \in I, i \neq j$. For every price-system $p \in \mathbb{R}_+^H$ and every vector of quantities $q \in Q$, the same net demand functions $\{\zeta_i(p, q_i)\}$ are assumed to be well-defined and a two-stage procedure is again considered. The difference is that at the first stage price-making consumers are supposed to choose (unilaterally) the prices they control in some admissible subset $\mathcal{P}$ of $\mathbb{R}_+^{H^*}$, taking as given the other monopolists' prices, and anticipating that, at the second stage, the offered quantities $q \in Q$ and the competitive prices $\hat{p} \in \mathbb{R}_+^{\hat{H}}$ should adjust to clear all markets (with of course the price p_0 remaining identically 1). To take into account all these induced effects, we shall here introduce the following strong assumption:

Assumption 2. For all prices $\tilde{p} \in \mathcal{P}$, there exist a unique quantity vector $\hat{q}(\tilde{p}) \in Q$ (with $\hat{q}_{ih}(\tilde{p}) \neq 0$

for $h \in H_i$) and a unique vector of competitive prices $\hat{p}(\tilde{p}) \in \mathbb{R}_+^{\hat{H}}$ such that

$$\sum_{i \in I} \zeta_i(\tilde{p}, \hat{p}(\tilde{p}), \hat{q}_i(\tilde{p})) = 0,$$

and all constraints defining ζ are binding.

This assumption guarantees the existence of "effective" net demands which are functions of the strategically chosen prices, $\tilde{p} \in \mathcal{P}$, and are given by the composite function $\zeta_i(\cdot, \hat{p}(\cdot), \hat{q}_i(\cdot))$ and ensures the global feasibility of all exchanges for every vector of admissible strategic prices.

Given such functions, a *Monopolistic Competition Equilibrium* is a vector of prices and quantities (p^{MC}, q^{MC}) in $\mathbb{R}_+^H \times Q$, with the price-system p^{MC} identified to the vector $(\tilde{p}^{MC}, \hat{p}(\tilde{p}^{MC})) \in \mathcal{P} \times \mathbb{R}_+^{\hat{H}}$, and the quantity vector $q^{MC} = \hat{q}(\tilde{p}^{MC})$, and such that $\forall\, i \in I^*$,

$$\begin{gathered} p_i^{MC} \in \arg\max_{p_i} U_i(\zeta_i(p_i, p_{-i}^{MC}, \hat{p}(p_i, p_{-i}^{MC}), \hat{q}_i(p_i, p_{-i}^{MC})) + \omega_i), \\ \text{s.t. } (p_i, p_{-i}^{MC}) \in \mathcal{P} \text{ and } (p_i^{MC}, p_{-i}^{MC}) \equiv \tilde{p}^{MC}. \end{gathered}$$

Hence, once the functions $\zeta_i, \hat{q}_i$ and $\hat{p}$ are well-defined and satisfy Assumption 2, the Monopolistic Competition prices p^{MC} form a (generalized) Nash-Equilibrium of the game involving the price-setting consumers.

To illustrate this definition and compare the equilibrium obtained with the Cournot-Walras equilibrium we come back to the previous example. It will appear that the Monopolistic Competition Equilibrium price of the monopolistic goods is higher than the Walrasian price. However the comparison with the Cournot-Walras Equilibrium is ambiguous and depends on the value of the parameters. The important observation to be drawn from this example is that the main difficulty involved in computing not only the Cournot-Walras Equilibrium but also the Monopolistic Competition Equilibrium results from the obligation to solve a whole demand system (in order to obtain the function $\tilde{p}(\cdot)$ in the first case, or the functions $\hat{q}(\cdot)$ and $\hat{p}(\cdot)$ in the second). In the Cournot-Walras case it consists in a backward induction argument of the "subgame perfect" type, with the difference that it is the Walrasian Equilibrium, and not the Nash-Equilibrium of a subgame, which is anticipated by the strategic agents for the second stage. In the monopolistic competition case it is the computation of something like an effective demand which is imposed on the strategic agents. In both cases a sophisticated computation ability and anticipation capacity is given to the strategic agents, requiring strong existence and uniqueness assumptions for any deviation they might consider.

Example (Part 2). Taking the same utility and endowment specification as in example (1), and assuming $H_1 = \{1\}, H_2 = \{2\}, H_i = \emptyset$ for $i = 3, \cdots, n+2$, we start from the same expressions for the net demand functions $\zeta_1(p, q_1), \zeta_2(p, q_2)$ and $\zeta_i(p, 0)$ for $i = 3, \cdots, n+2$. But now the market clearing equations

$$p_1 q_1 - \beta p_2 q_2 = p_1 f(p_1, p_2)$$
$$-\beta p_1 q_1 + p_2 q_2 = p_2 f(p_2, p_1)$$

have to be solved in q (and not in p) to determine the function $\hat{q}$. We obtain, for $i, j = 1, 2$, $i \neq j$,

$$\hat{q}_i(p) = \frac{p_i p_j f(p_i, p_j) + \beta p_j^2 f(p_j, p_i)}{p_i p_j (1 - \beta^2)}.$$

Hence consumer i should solve the program (for $i = 1, 2, j \neq i$)

$$\max_{p_i} \alpha(1-\beta)\ln((1-\beta)p_i \hat{q}_i(p)) + \alpha\beta \ln\left(\frac{\beta}{p_j} p_i \hat{q}_i(p)\right) + (1-\alpha)\ln(1 - \hat{q}_i(p)).$$

Taking the first-order conditions and looking for a symmetric equilibrium in prices $p_1^{MC} = p_2^{MC} = \overline{p}^{MC}$, we get the equation

$$\alpha + \frac{\alpha - \overline{q}}{(1-\overline{q})(1+\beta)}\left(-1 - \frac{b(1-a)}{\overline{q}(1-\beta)} - \beta + \frac{\beta ab}{\overline{q}(1-\beta)}\right) = 0,$$

where $\overline{q} = \hat{q}_i(\overline{p}, \overline{p}) = g(\overline{p})/1 - \beta$.

Equivalently, to find a symmetric monopolistic equilibrium it is enough to solve the following equation in $\overline{q}$ (comparable to (W') and (CW)), where the left-hand side is again the "degree of cornering"

$$\frac{\alpha}{\overline{q}} - 1 = \frac{(1-\alpha)(1-\beta)}{b(1/(1+\beta) - a)}\overline{q}. \tag{MC}$$

Given the solution $\overline{q}^{MC}$, we let $\overline{p}^{MC}$ be such that

$$g(\overline{p}^{MC}) = (1-\beta)\overline{q}^{MC}.$$

Since $g(\overline{p})$ is decreasing in $\overline{p}$ and the right-hand side of (MC) is positive and linear increasing in $\overline{q}$, we have $\overline{p}^{MC} > \overline{p}^{W}$. Also $\overline{p}^{MC} > \overline{p}^{CW}$ if and only if $\overline{q}^{MC} < \overline{q}^{CW}$. This last inequality may or may not hold according to the values of the parameters. See the illustration for particular parameters values, given by Figures 1 and 2, with d denoting the degree of cornering.

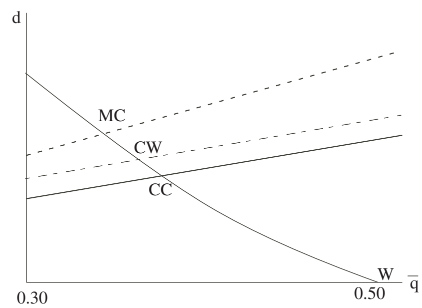

Figure 1: $\alpha = 0.5, \beta = 0.6, a = 0.48, b = 1$

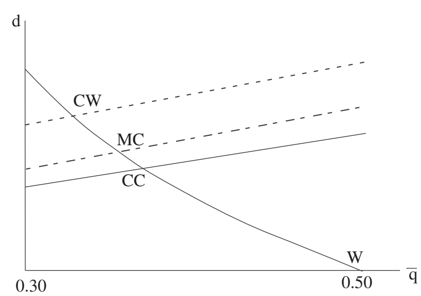

Figure 2: $\alpha = 0.5, \beta = 0.2, a = 0.48, b = 1$

2.3 The Cournotian Monopolistic Competition Equilibrium

To avoid some complexities and simplify the anticipations of agents, we now define a third general equilibrium concept under imperfect competition, the Cournotian Monopolistic Competition Equilibrium. In d'Aspremont, Dos Santos Ferreira and Géard-Varet [11], this concept was defined for a partial equilibrium model allowing for several productive sectors. Here we shall keep the same pure exchange model as above and assume that, for every price-system $p \in \mathbb{R}_+^H$ and every vector of quantities $q \in Q$, the net demand functions $\{\zeta_i(p, q_i)\}$, as given previously, are well-defined.

With respect to the previous concepts there are two main modifications. First, as a pure monopolist in partial equilibrium, each strategic consumer is supposed to have both price and quantity strategies for every market in which he has some oligopolistic power. Secondly, he is supposed to face a "residual" demand and to take into account transaction feasibility constraints only for those markets. As in the Walrasian Equilibrium, the global feasibility of all exchanges is ensured only at equilibrium. When considering a deviation a strategic consumer takes as fixed and given the prices of the goods he does not control, behaving with respect to these prices as a competitive consumer, whether these prices are competitive prices (parametrically determined) or strategic prices (strategically determined by other consumers). This is an important simplification with respect to the Cournot-Walras or the Monopolistic Competition concepts where the vector of all prices was supposed to always clear all markets. This simplification is already used by Laffont and Laroque [23, Assumption 10, page 288] in their definition of a general equilibrium concept under imperfect competition.

Let us now consider the definition. Notice that we do allow for $H_i \cap H_j \neq \emptyset$ for some $i, j \in I^*$, $i \neq j$, and that we denote by p_{-i} the vector of prices of the noncompetitive goods not in $H_i : (p_h)_{h \in H^* \setminus H_i}$.

A *Cournotian Monopolistic Competition Equilibrium* is a vector of prices and quantities (p^{CC}, q^{CC}), with the price-system p^{CC} in $\mathbb{R}_+^H$ and the quantities q^{CC} in Q, such that for all $i \in I^*$, and denoting $p_i^{CC} = (p_h^{CC})_{h \in H_i}$, $p_{-i}^{CC} = (p_h^{CC})_{h \in H^* \setminus H_i}$ and $\hat{p}^{CC} = (p^{CC})_{h \in \hat{H}}$,

$$(p_i^{CC}, q_i^{CC}) \in \arg \max_{(p_i, q_i) \in \mathbb{R}_+^{H_i} \times Q_i} U_i(\zeta_i(p_i, p_{-i}^{CC}, \hat{p}^{CC}, q_i) + \omega_i) \qquad (a.1)$$

subject to

$$\forall\, h \in H_i, \zeta_{ih}(p_i, p_{-i}^{CC}, \hat{p}^{CC}, q_i) \left\{ \zeta_{ih}(p_i, p_{-i}^{CC}, \hat{p}^{CC}, q_i) + \sum_{j \neq i} \zeta_{jh}(p_i, p_{-i}^{CC}, \hat{p}^{CC}, q_j^{CC}) \right\} \leq 0 \quad (a.2)$$

and

$$\sum_{i \in I} \zeta_i(p^{CC}, q_i^{CC}) = 0. \qquad (b)$$

It is Condition (a.1) that requires each strategic consumer to maximize his utility by choosing both the price p_i and the quantity orders q_i for the goods he controls (all h in H_i) taking as given the prices of other goods and the quantity orders of other consumers. This is a direct generalization of Cournot's concept: each agent behaves as a monopolist optimizing against a "residual" demand. But the market price for each good is "necessarily the same" for all agents (to use the words of Cournot) and so, at an equilibrium, any two consumers who are "controlling" the price of the same good should (optimally) choose the same value (for all i and j in I^* and $h \in H_i \cap H_j$, the price p_h^{CC} should be an optimal price for both i and j). There is implicit pricing coordination among strategic agents in the same market.[13] In the next section, formal pricing coordination mechanisms will be introduced explicitly.

Condition (a.2) restricts deviations by a strategic consumer. In the markets in which he is strategic, deviations should be on the nonrationed side: the deviating consumer i's net demand should have an opposite sign to, and be bounded by, the resulting total net demand of the others. This is a feasibility restriction, but it is not market-clearing and it is only imposed on the markets in H_i. Condition (b) goes further. It requires market-clearing in all markets at equilibrium.

Returning to our example, we will see that the concept just defined leads to a solution different from the solutions given by the two previous concepts and much simpler to compute. It will be shown that, for all values of the parameters, the Cournotian Monopolistic Competition Equilibrium price of each monopolistic good is higher than the Walrasian one, but lower than the Cournot-Walras one. Again, comparison with the Monopolistic Competition equilibrium is ambiguous depending upon the values of the parameters (see Figures 1, 2 and 3).

[13] This is somewhat analogous to the Lindahl solution for public goods, where each consumer has to choose independently the same quantity of public goods.

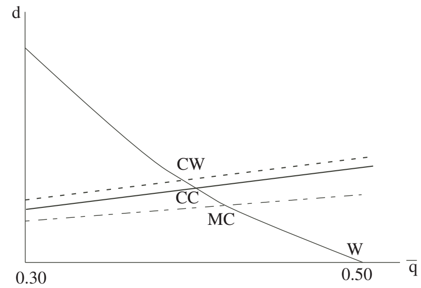

Figure 3: $\alpha = 0.5, \beta = 0.6, a = 0.1, b = 1$

Example (Part 3). Taking the same simplification as in parts 1 and 2, we may again start from the same expressions for the net demand functions $\zeta_1(p, q_1), \zeta_2(p, q_2)$ and $\zeta_i(p)$, for $i = 3, \cdots, n+2$. We assume, as before, $H_1 = \{1\}, H_2 = \{2\}$ and $H_i = \emptyset$ for $i = 3, \cdots, n+2$. For $i, j = 1, 2, i \neq j$, we have to solve the following program:

$$\begin{aligned}
&\max_{(p_i, q_i)} U_i(\zeta_i(p, q_i) + \omega_i) \\
&\quad \text{s.t. } q_i - \zeta_{ji}(p, q_j) - \sum_{k=3}^{n+2} \zeta_{ki}(p, 0) \leq 0, \\
&\quad p_i \geq 0, 0 < q_i \leq \alpha.
\end{aligned}$$

That is, for $i, j = 1, 2, i \neq j$,

$$\max_{(p_i, q_i)} \alpha(1-\beta) \ln[(1-\beta) p_i q_i] + \alpha\beta \ln \tfrac{\beta}{p_j} p_i q_i + (1-\alpha) \ln(1 - q_i)$$
$$\text{s.t. } \beta p_j q_j + p_i f(p_i, p_j) - p_i q_i \geq 0, \quad p_i \geq 0,\ 0 < q_i \leq \alpha.$$

The first order conditions give an equation in p_i and q_i:

$$\frac{\alpha - q_i}{q_i(1 - q_i)} = \frac{\alpha}{q_i - f - p_i f_i'}.$$

Since $f_i' = -a[1 + b(1 + p_j)]/p_i^2$ and $-p_i f_i' = f + b(1 - a)$ we get, looking for a symmetric solution $q_1 = q_2 = \overline{q}$, and after some simplifications, a simple equation in $\overline{q}$:

$$\frac{\alpha}{\overline{q}} - 1 = \frac{1 - \alpha}{b(1 - a)} \overline{q}. \qquad (CC)$$

Solving this equation gives $\overline{q}^{CC}$ and by the feasibility solution $g(\overline{p}) = \overline{q}(1 - \beta)$ we can compute $\overline{p}^{CC}$. Clearly we have $\overline{q}^{CC} < \overline{q}^{W}$ so that again we get a price $\overline{p}^{CC}$ greater than the competitive price. Notice that for $\beta = a/(1-a)$, the right hand sides of (CC), (MC) and even of (CW) coincide: they are linear with the same slope equal to $(1-\alpha)/b(1-a)$. However, more interestingly, it may be verified that

- for $\beta > \frac{a}{1-a}$: $\overline{q}^{CW} < \overline{q}^{CC} < \overline{q}^{MC} < \overline{q}^{W}$
- for $\beta < \frac{a}{1-a}$: $\overline{q}^{MC} < \overline{q}^{CC}$, $\overline{q}^{CW} < \overline{q}^{CC}$ and $\overline{q}^{CC} < \overline{q}^{W}$.

In the second case it is not possible to specify the relationship between $\overline{q}^{CW}$ and $\overline{q}^{MC}$ (see the examples given by Figures 1 and 2).

The clearest conclusion, true in both cases, is that the Cournotian Monopolistic Competition Equilibrium quantity, $\overline{q}^{CC}$ is closer to the Walrasian quantity $\overline{q}^{W}$ than the Cournot-Walras one $\overline{q}^{CW}$. But, and this is important to notice, it does not determine a Pareto-optimal allocation. Indeed, supposing the contrary we should have, at that allocation, the equality of consumers 1 and 2 marginal rates of substitution between good 1 and good 2:

$$\frac{\partial U_1/\partial x_{11}}{\partial U_1/\partial x_{12}} = \frac{1-\alpha}{\alpha\beta}\frac{x_{12}}{x_{11}} = \frac{\alpha\beta}{1-\alpha}\frac{x_{22}}{x_{21}} = \frac{\partial U_2/\partial x_{21}}{\partial U_2/\partial x_{22}}.$$

Let x_1 and x_2 be determined by the net demand functions ζ_1 and ζ_2 (as it should be for either $\overline{q}^{CC}$ or $\overline{q}^{MC}$), that is, for $i, j = 1, 2$ $(i \neq j)$,

$$x_{ii} = \zeta_{ii}(p, q_i) + \omega_{ii} = 1 - q_i; \quad x_{ij} = \zeta_{ij}(p, q_i) + \omega_{ij} = \beta\frac{p_i}{p_j}q_i.$$

Then we should have

$$\frac{1-\alpha}{\alpha\beta}\frac{\beta p_1 q_1}{(1-q_1)p_2} = \frac{\alpha\beta}{1-\alpha}\frac{(1-q_2)p_1}{\beta p_2 q_2}$$

which, for $q_1 = q_2 = \overline{q}$ (and $p_1 = p_2 = \overline{p}$), reduces to

$$\alpha = \overline{q}.$$

In other words, $\overline{q}$ can neither be $\overline{q}^{CW}$, nor $\overline{q}^{MC}$, nor $\overline{q}^{CC}$. It can only be the Walrasian quantity $\overline{q}^{W}$, which, among all four, is the only Pareto-optimal equilibrium quantity.

3 P-equilibria: a general Cournotian approach to general equilibrium

In this section we introduce a more abstract concept of general equilibrium under imperfect competition in a pure exchange economy. This concept is a generalization of the Cournotian Monopolistic Competition Equilibrium, but is explicitly based on a formal mechanism of price coordination and involves both price signals and quantity orders as strategic variables. In that respect our approach gets closer to the (abstract) market game approach.[14] However some important differences are maintained. First we keep[15] a "strong partial equilibrium flavor" by restricting the strategic game specification to a sector by sector formulation, even though the definition of a sector remains quite general and flexible. Secondly, an abstract strategic outcome function is limited here to represent price formation. Transacted quantities are still specified in Cournot's way, by computing for each market its residual demand. Moreover the price outcome function (called a pricing-scheme) is Cournotian, by assuming a single "market price" for each good and for any vector of price signals in the sector. In spite of these limitations the concept of P-equilibrium will be shown to encompass several alternative definitions of a general equilibrium with imperfect competition, by varying the notion of sector and the properties imposed on pricing-schemes.

3.1 Definition of a P-equilibrium

The basic idea is to have a two-stage procedure as before, and to give each strategic consumer the possibility to send, at the first stage, both price signals and quantity orders. In some sense there is a "planning" stage and an "implementation" stage. At the second stage, the transactions and the trading prices are implemented for all consumers in each sector. The notion of sector is determinant since it fixes the class of goods for which a number of consumers realize their

[14] It is abstract in the sense that the outcome function is not fully specified but described by some general properties or axioms. See for example Bénassy [4].

[15] To use Hart's [19] words.

strategic interdependence and coordinate (more or less) their strategic decisions by sending price signals. Formally, we suppose that the set of goods H is partitioned into a set of sectors $S^0, S^1, \cdots, S^T$. The first sector S^0 is identified to the competitive sector $\hat{H}$. For each of the other sectors $S^t, t \geq 1$, and each $h \in S^t$, the set of strategic consumers I^t, concerned by pricing decisions in that sector, i.e. $I^t = \cup_{h \in S^t} I_h$, is supposed to coordinate the market price formation by using a *pricing-scheme*. This is a function defined for admissible sets of price signals Ψ_i^t and for every $i \in I^t$,

$$P_h : \bigtimes_{i \in I^t} \Psi_i^t \to \mathbb{R}_+, \ h \in S^t,$$

associating with each vector $\psi^t = (\psi_i^t)_{i \in I^t}$ in $\Psi^t \equiv \bigtimes_{i \in I^*} \Psi_i^t$ the market price of good h, $P_h(\psi^t)$. We distinguish now the set H_i of goods for which consumer i is strategic and may send nonzero quantity orders from the set

$$S_i = \{h \in H^* : h \in S^t, t \geq 1 \text{ and } S^t \cap H_i \neq \emptyset\}$$

of goods belonging to sectors in which strategic consumer i acts strategically. Also, we denote by $\psi_i \equiv (\psi_i^t)_{t|i \in I_t}$ the vector of price signals chosen by consumer i and by $\psi_{-i} \equiv (\psi_j)_{j \in I^* \setminus \{i\}}$ the vector of price signals ψ_i sent by all other strategic consumers. The vector of price signals ψ_i should be admissible in the sense that it should belong to $\Psi_i \equiv \bigtimes_{t|i \in I^t} \Psi_i^t$, and similarly for all ψ_j. As a last piece of notation, we define for every $\psi \in \Psi = \bigtimes_{i \in I^*} \Psi_i$

$$P(\psi) = (P_h(\psi^t))_{h \in S^t, t \geq 1}.$$

We may now define our general equilibrium concept.

A P-equilibrium is a vector of prices and quantity orders $(p^*, q^*) \in \mathbb{R}_+^H \times Q$ such that: For every h in every sector $S^t, p_h^* = P_h(\psi^{t*})$ for some $\psi^{t*} \in \Psi^t$, and

$$\forall\, i \in I^*, \quad U_i(\zeta_i(p^*, q_i^*) + \omega_i) \geq U_i(\zeta_i(P(\psi_i, \psi_{-i}^*), \hat{p}^*, q_i) + \omega_i) \qquad (a.1)$$

subject to, $\forall\, h \in S_i$,

$$\zeta_{ih}(P(\psi_i, \psi_{-i}^*), \hat{p}^*, q_i) \times \left\{ \zeta_{ih}(P(\psi_i, \psi_{-i}^*), \hat{p}^*, q_i) + \sum_{j \neq i} \zeta_{jh}(P(\psi_i, \psi_{-i}^*), \hat{p}^*, q_j^*) \right\} \leq 0 \qquad (a.2)$$

for any ψ_i and any $q_i \in Q_i$, and also

$$\sum_{i \in I} \zeta_i(p^*, q_i^*) = 0. \qquad (b)$$

Expressions (a.1), (a.2) and (b) are very close to the corresponding expressions for the definition of a Cournotian Monopolistic Equilibrium and can be interpreted similarly. In fact the two definitions are equivalent whenever the class of pricing-schemes is further specified.

For any sector S^t, $t \geq 1$, a pricing-scheme $P^t = (P_h)_{h \in S^t}$ is *fully individually manipulable* if, for any $i \in I^t$ and $\psi_{-i}^t \in \times_{j \in I^t, j \neq i} \Psi_j^t$, the function $P^t(\cdot, \psi_{-i}^t)$ defined on Ψ_i^t has full range, i.e. $P^t(\Psi_i^t, \psi_{-i}^t) = \mathbb{R}_+^{S_t}$.

Proposition 3.1 *Let, for any $i \in I^*, S_i = H_i$. If all pricing-schemes are fully individually manipulable, then P-equilibrium and Cournotian Monopolistic Competition Equilibrium coincide.*

Proof: Since $P^t(\cdot, \psi_{-i}^t)$ has full range for any i and t such that $i \in I^t$, it is equivalent for a strategic consumer i to choose a signal ψ_i^t leading to values $p_h = P_h(\psi_i^t, \psi_{-i}^t)$ or to choose directly the price p_h itself for any $h \in S^t$. The result follows. ■

Notice that the restriction $S_i = H_i$, for all $i \in I^*$, is not essential and is due to the restricted definition of a Cournotian Monopolistic Competition Equilibrium that was given in the preceding section. Following the argument of Proposition 1, *we shall from now on define more generally a Cournotian Monopolistic Equilibrium as a P-equilibrium based on fully individually manipulable pricing-schemes.*

The main benefit of considering the class of fully individually manipulable pricing-schemes is that the equilibrium concept can be defined without explicit reference to a particular mechanism. The agents succeed in coordinating their pricing strategies through some process which has not to be completely specified.

3.2 P-equilibrium and Cournot-Walras equilibrium

It is important to observe that a P-equilibrium is not only relative to the kind of pricing-schemes on which it is based, but also to the sector structure that is postulated. One extreme case consists in assuming that the economy as a whole, except for the money market, forms a single sector and that all market prices (keeping $p_0 = 1$) can be individually manipulated by all strategic consumers. This amounts to assume that $S_i = H \setminus \{0\}$ for all $i \in I^*$. Notice that this does not mean that strategic consumers send both price-signals and nonzero quantity orders in all nonmoney markets. For many goods, those belonging to the set $S_i \setminus H_i$, consumer i will only be free to choose price-signals, the corresponding quantity orders being restricted to zero. The interesting fact, shown in the next proposition, is that a P-equilibrium based on fully individually manipulable pricing-schemes (i.e., a Cournotian Monopolistic Competition Equilibrium in its most general definition) is reduced to a Cournot-Walras Equilibrium.

Proposition 3.2 *Consider a pure exchange economy satisfying Assumption 1. For any fully individually manipulable pricing-scheme P, under the sector structure $S_i = H \setminus \{0\}$, for all $i \in I^*$, any P-equilibrium (p^*, q^*) is a Cournot-Walras Equilibrium.*

Proof: If (p^*, q^*) is a P-equilibrium and $S_i = H \setminus \{0\}$ for all $i \in I^*$, we have, for any $h \in H \setminus \{0\}$, $P_h(\psi^*) = p^*_h$, for some $\psi^* \in \Psi$, and we have $q^* \in Q$ with (by (b)) $\sum_{i \in I} \zeta_i(p^*, q^*_i) = 0$. Therefore by Assumption 1, $\tilde{p}(q^*) = p^*$. So if (p^*, q^*) is not a Cournot-Walras Equilibrium, this means that, for some $i \in I^*$ and $q_i \in Q_i$,

$$U_i(\zeta_i(\tilde{p}(q_i, q^*_{-i}), q_i) + \omega_i) > U_i(\zeta_i(p^*, q^*_i) + \omega_i).$$

But, by Assumption 1,

$$-\sum_{j \neq i} \zeta_{jh}(\tilde{p}(q_i, q^*_{-i}), q^*_j) = \zeta_{ih}(\tilde{p}(q_i, q^*_{-i}), q_i)$$

and, hence, (a.2) is satisfied. Since P is fully individually manipulable, we can find $\psi_i \in \Psi_i$ satisfying, for all $h \in H \setminus \{0\}, P_h(\psi_i, \psi^*_{-i}) = \tilde{p}_h(q_i, q^*_{-i})$ so that the inequality above contradicts (a.1). ■

To get the converse we need to reinforce Assumption 1. The problem comes from the fact that a deviation from a candidate P-equilibrium need not satisfy the market-clearing condition in all markets. A possibility is then to have the deviant agent start a tâtonnement process (which he can control since he can manipulate all prices) leading to another deviation where all markets clear. This procedure should be monotone so that the agent does not get worse off along the path. Assumption 3 ensures the existence, the monotonicity and the convergence to the unique equilibrium of such a process. Since we always assume $p_0 = 1$, it should be a "numéraire tâtonnement process", that is a process where the price of the numéraire good does not change. We do not recall all definitions but refer the reader to the survey by Hahn [18].

Assumption 3. For all $q \in Q$ the net demand functions ζ_i are continuously differentiable and the unique $\tilde{p}(q)$ such that $\sum_{i\in I} \zeta_i(\tilde{p}(q), q_i) = 0$ is perfectly globally stable for some numéraire tâtonnement process.

This assumption requires not only global stability but perfect stability in the sense of Hicks. This is required to ensure the monotonicity of the process. As shown in McFadden [28], we could restrict the net demand functions to satisfy the differential form of the gross substitutability condition, in which case both Assumptions 1 and 3 are satisfied.

Proposition 3.3 *Consider a pure exchange economy satisfying assumptions 1 and 3. Then the set of Cournot-Walras Equilibria coincides with the set of P-equilibria based on fully individually manipulable pricing-schemes, under the sector structure $S_i = H \setminus \{0\}$, for all $i \in I^*$.*

Proof: Suppose (p^{CW}, q^{CW}) is a Cournot-Walras Equilibrium but not a P-equilibrium. Since $p^{CW} = \tilde{p}(q^{CW})$ condition (b) is satisfied. Also, by full individual manipulability of P, $\tilde{p}(q^{CW})$ can be chosen by any agent in I^* and, hence, there is some $i \in I^*$ who can choose (p', q'_i) with $p' \in \mathbb{R}^H_+$, $p'_0 = 1$ and $q'_i \in Q_i$, such that

$$U_i(\zeta_i(p', q'_i) + \omega_i) > U_i(\zeta_i(p^{CW}, q_i^{CW}) + \omega_i)$$

while satisfying (a.2). Also one should have that

$$\zeta_i(p', q'_i) + \sum_{j\in I\setminus\{i\}} \zeta_j(p', q_j^{CW}) \neq 0,$$

for otherwise $p' = \tilde{p}(q'_i, q_{-i}^{CW})$ and we get a contradiction to (p^{CW}, q^{CW}) being a Cournot-Walras Equilibrium. But then, using Assumption 3, there exists a numéraire tâtonnement process starting from p' and leading to $\tilde{p}(q_i', q_{-i}^{CW})$ without making agent i worse off. Indeed, by perfect stability, there is a hierarchy of markets according to the speed of adjustment in each market. So we can take all markets sequentially, following this ranking, and for each market h that is taken (and which is not already balanced) we know by (a.2) that if i is a net demander, $\zeta_{ih} > 0$ (resp. supplier, $\zeta_{ih} < 0$) for good h, then this market is in excess aggregate supply, i.e. $\zeta_{ih} \leq -\sum_{j \neq i} \zeta_{jh}$), (resp. excess aggregate demand, i.e. $\zeta_{ih} \geq -\sum_{j \neq i} \zeta_{jh}$), so that the price p_h will decrease (resp. increase). Since this means that what i wants to buy gets cheaper (resp. what i wants to sell gets more expensive), his utility cannot decrease. So,

$$U_i(\zeta_i(\tilde{p}(q_i', q_{-i}^{CW}), q_i') + \omega_i) \geq U_i(\zeta_i(p', q_i') + \omega_i) > U_i(\zeta_i(p^{CW}, q_i^{CW}) + \omega_i)$$

contradicting that (p^{CW}, q^{CW}) is a Cournot-Walras Equilibrium. So under Assumptions 1 and 3, a Cournot-Walras Equilibrium is a P-equilibrium based on fully individually manipulable pricing-schemes. The converse is proved in Proposition 3.2. ■

3.3 P-equilibrium and Monopolistic Competition

The last comparison to make is between the P-equilibrium and the Monopolistic Competition Equilibrium. Here an equivalence result is more difficult to obtain, making clear that the P-equilibrium concept is more in the Cournot tradition than in the Bertrand-Chamberlin tradition. Such a result can be obtained in the special case of unilateral monopoly, that is, when strategic consumers are alone in controlling some markets and, further, do not sell to each other. Again we need to have only one competitive market.

Proposition 3.4 *Consider a pure exchange economy satisfying Assumption 2 with $S^t = S_i = H_i$, for $t = i \in I^*$, and $\cup_{i \in I^*} H_i = H \setminus \{0\}$ and such that: for all $i, j \in I^*$, $i \neq j$, and for $h \in H_j$, $U_i(x_i)$ is constant in x_{ih} and $\omega_{ih} = 0$, but $\hat{I}_h \neq \emptyset$. For fully individually manipulable pricing-schemes, any P-equilibrium (p^*, q^*) is a Monopolistic Competition Equilibrium.*

Proof: If (p^*, q^*) is a P-equilibrium but not a Monopolistic Equilibrium, then there is some $i \in I^*$, and $p_i \in \mathbb{R}_+^{H_i}$ such that

$$U_i(\zeta_i(p_i, p_{-i}^*, \hat{p}_0, \hat{q}_i(p_i, p_{-i}^*)) + \omega_i) > U_i(\zeta_i(p^*, \hat{q}_i(p_1^*, \cdots, p_\ell^*)) + \omega_i)$$

with $\hat{p}_0 = p_0^* = 1$, and $p_h = P_h(\psi_i)$ for some $\psi_i \in \Psi_i$ and all $h \in H_i$ (since P is fully individually manipulable). Considering the definition of a P-equilibrium and Assumption 2, we see that we must have $q_j^* = \hat{q}_j(p_1^*, \cdots, p_\ell^*)$ for all $j \in I^*$, and that we may take $q_i = \hat{q}_i(p_i, p_{-i}^*)$ satisfying constraints (a.2). Indeed, since we have unilateral monopoly no $j \in I^* \setminus \{i\}$ is either a buyer or a seller of any good $h \in H_i$, so that, for any $h \in H_i$,

$$\zeta_{ih}(p_i, p_{-i}^*, 1, \hat{q}_i(p_i, p_{-i}^*)) + \sum_{j \in \hat{I}} \zeta_{jh}(p_i, p_{-i}^*, 1, 0) = 0$$

by Assumption 2. Hence the above strict inequality contradicts (a.1) and the result follows. ■

To get equivalence we need again more restrictions. Assumption 3 is still applicable.

Proposition 3.5 *Consider a pure exchange economy satisfying Assumptions 2 and 3, with $S^t = S_i = H_i$ for $t = i \in I^*$, and $\cup_{i \in I^*} H_i = H \setminus \{0\}$, and such that: for all $i, j \in I^*, i \neq j$, and for $h \in H_j$, $U_i(x_i)$ is constant in x_{ih} and $\omega_{ih} = 0$, but $\hat{I}_h \neq 0$. Then the set of Monopolistic Competition Equilibria coincide with the set of P-equilibria based on fully individually manipulable pricing-schemes.*

Proof: Take a Monopolistic Competition Equilibrium (p^*, q^*), with $p_0^* = \hat{p}_0 = 1$ and $q^* = \hat{q}(p_1^*, \cdots, p_\ell^*)$, and suppose that it is not a P-equilibrium. Since condition (b) in the definition is satisfied, condition (a) should be violated: there are $i \in I^*$, $p_i \in \mathbb{R}_+^{H_i}$ and $q_i \in Q_i$ such that, for every $h \in S_i$, $p_h = P_h(\psi_i)$, for some $\psi_i \in \Psi_i$ and

$$U_i(\zeta_i(p_i, p_{-i}^*, 1, q_i) + \omega_i) > U_i(\zeta_i(p^*, q_i^*) + \omega_i)$$

under the constraints (a.2), which can be simply written (by the unilateral monopoly conditions), as, for all $h \in H_i$,

$$\zeta_{ih}(p_i, p_{-i}^*, 1, q_i) \left\{ \zeta_{ih}(p_i, p_{-i}^*, 1, q_i) + \sum_{j \in \hat{I}_h} \zeta_{jh}(p_i, p_{-i}^*, 1, 0) \right\} \leq 0.$$

By Assumption 3, consumer i can adjust p_i so as to verify, for each $h \in H_i$, the constraint as an equality, without incurring a loss of utility. This he does by engaging in a partial tâtonnement adjustment limited to the set of markets H_i. The argument is the same as in the proof of Proposition 3.3. This tâtonnement converges to some $\tilde{p}_i$ such that, for every $h \in H_i$,

$$\zeta_{ih}(\tilde{p}_i, p^*_{-i}, 1, q_i) + \sum_{j \in \hat{I}_h} \zeta_{jh}(\tilde{p}_i, p^*_{-i}, 1, 0) = 0,$$

implying, by Assumption 2, that there exists $\hat{q}(\tilde{p}_i, p^*_{-i}) \in Q$ such that $\zeta_{ih}(\tilde{p}_i, p^*_{-i}, 1, \hat{q}_i(\tilde{p}_i, p^*_{-i})) = \zeta_{ih}(\tilde{p}_i, p^*_{-i}, 1, q_i)$, and all markets clear. Also,

$$U_i(\zeta_i(\tilde{p}_i, p^*_{-i}, 1, q_i) + \omega_i) \geq U_i(\zeta_i(p_i, p^*_{-i}, 1, q_i) + \omega_i) > U_i(\zeta_i(p^*, q^*_i) + \omega_i),$$

in contradiction to (p^*, q^*) being a Monopolistic Competition Equilibrium. So any Monopolistic Competition Equilibrium must be a P-equilibrium for fully individually manipulable pricing-schemes. The converse is given by Proposition 3.4. ■

Notice that here again, since the pricing-schemes involved are fully individually manipulable, all P-equilibria are Cournotian Monopolistic Competition Equilibria. However it is not clear that in practice actual coordination mechanisms have such a degree of manipulability given to each individual agent. This is examined in the next section.

4 Existence of equilibria and nonfully manipulable pricing-schemes

As we have just seen, the definition of a P-equilibrium is very general indeed. With different specifications it may become either three of the general equilibrium concepts under imperfect competition that we have considered. It may even reduce to the general competitive equilibrium by putting $I = \hat{I}$ (or $I^* = \emptyset$). However, except in this last case, we don't know which assumptions to impose on the primitives of the economy in order to guarantee the existence of a general equilibrium. Our purpose here is not to investigate this existence problem, except for studying one possibility, namely that the competitive equilibrium be itself contained in the set of P-equilibria.

A first observation is that such a possibility is generally excluded by fully individually manipulable pricing-schemes. To see this let us suppose that the net demand functions and the fully individually manipulable pricing-schemes are all continuously differentiable. Let us further suppose that there is one good per noncompetitive sector: $S^t = \{h\}$, for $t = h \in H^*$. Take as given the prices $\hat{p} \in \mathbb{R}_+^{\hat{H}}$ in the competitive sector and define a strategic outcome function in consumption, for every $i \in I^*$ and $h \in H \setminus \{0\}$,

$$X_{ih}(\psi, q_i) = \zeta_{ih}(P(\psi), \hat{p}, q_i) + \omega_{ih}$$

associating the consumption of good $h \neq 0$ by the strategic consumer i with any vector of price signals $\psi \in \Psi$ and quantity orders $q_i \in Q_i$. At a P-equilibrium (p^*, q^*) (such that $p_h^* = \hat{p}_h$ for $h \in \hat{H}$ and $p_h^* = P_h(\psi^{h^*}), \psi^{h^*} \in \mathbb{R}_+^{I_h}$), the corresponding utility can be written

$$U_i(\zeta_i(P(\psi^*), \hat{p}, q_i^*) + \omega_i) = U_i(\omega_{i0} + \sum_{h\neq 0} p_h^*(\omega_{ih} - X_{ih}(\psi^*, q_i^*)), X_i(\psi^*, q_i^*)),$$

with $X_i(\psi^*, q_i^*)$ denoting the equilibrium consumption of nonnuméraire goods. Letting $\pi_{ih}^* = (\partial U_i/\partial x_{ih})/(\partial U_i/\partial x_{i0})$ denote the equilibrium marginal rate of substitution between each nonnuméraire good h and money and assuming an interior equilibrium, necessary conditions are

$$\sum_{h\neq 0}[p_h^* - \pi_{ih}^*]\frac{\partial X_{ih}}{\partial \psi_i^k} = \frac{\partial P_k}{\partial \psi_i^k}(\omega_{ik} - X_{ik}(\psi^*, q_i^*)),$$

for all $i \in I^*$ and $k \in S_i$. If this P-equilibrium were Walrasian, one would have for every $h \in H \setminus \{0\}$, $p_h^* - \pi_{ih}^* = 0$.

But the combination of these two sets of equalities will not hold in general (if some noncompetitive goods are actually transacted) for locally individually manipulable pricing-schemes (meaning $\partial P_k/\partial \psi_i^k > 0$, for all $k \in S_i$), unless some functions X_{ih} exhibit non-differentiabilities. Notice that an analogous conclusion holds if one were to require that a P-equilibrium be Pareto optimal. This fact is well known from the market game literature. In particular,[16] Aghion [1] and Bénassy [4] introduce general strategic outcome functions, generating both market transactions and market prices from individual signals, and hence determine the "Bertrand-like non-differentiabilities" that characterize almost all market games delivering the Walrasian outcome

[16] See also DUbey [12] and Dubey and Rogawski [13].

as a Nash equilibrium. For instance Bénassy's theorem giving a set of axioms sufficient to get the Walrasian equilibrium, relies essentially on the possibility for a trader to undercut or overcut infinitesimally market prices and thus to attain all trades in some interval.

We adopt a somewhat different route in this section by introducing another kind of pricing-scheme, with limited individual manipulability. Indeed, it is not clear that the kind of coordination mechanisms that are used in practice has full manipulability given to each individual agent. Empirical as well as theoretical studies of pricing strategies in some industries have concentrated on a number of "facilitating practices," or conventional norms of conduct among competitors, implying limited manipulability but leading to a market price well above its pure competitive level.[17] In many selling contracts, for instance, there are particular clauses allowing in fact competitors to coordinate their pricing strategies more efficiently than by tacit collusion: The "meet-or-release" clause, whereby a seller should meet a lower offer made to a customer or release him from the contract, or the "most-favored-customer" clause, whereby a seller engages not to sell to another customer at a lower price. As argued in the literature the introduction of such clauses amounts to use a pricing-scheme which consists in having the market price equal to the minimum of all announced prices, i.e., in a sector for a single homogeneous good h,

$$P_h^{\min}(\psi^h) = \min_{i \in I_h}\{\psi_i^h\}.$$

Indeed the best-pricei provisions imply that any seller should be informed (directly or through some trade association) of any price reduction by a competitor and follow it. Moreover, as remarked by Holt and Scheffman [20], combining the use of the meet-or-release clause with the possibility of discounting ensures that any discount made by a seller can be matched by the other sellers, thereby maintaining their sales quantities, so that the highest attainable price should, in this case, be the Cournot price. Of course, since the result of using the min-pricing-scheme on a market is to create a kinked demand curve many other prices are also attainable at some equilibrium. In general, one should expect that the set of $P^{\min}$-equilibria be larger than the set of Cournotian Monopolistic Equilibria. Let us consider the following:

[17] See Salop [34], Kalai and Satterthwaite [22], Cooper [9], Holt and Scheffman [20] and Logan and Lutter [24].

EXAMPLE. Consider an economy with two goods ($h = 0, 1$), two strategic consumers ($i = 1, 2$) in market 1 and $n \geq 1$ competitive consumers. Initial endowments and utility functions are, respectively,

$$\omega_1 = \omega_2 = (0, 1)$$
$$\omega_i = \left(\tfrac{1}{n}, 0\right), \; i = 3, 4, \cdots, n+2$$
$$U_i(x_i) = x_{i0}x_{i1}.$$

Letting $p_0 = 1$ and $p_1 = p$, we easily compute the net demand functions, competitive and noncompetitive.

For $i = 1, 2, p \in \mathbb{R}_+$ and $0 < q_i \leq \frac{1}{2}$,

$$\zeta_{i0}(p, 0) = \tfrac{p}{2}, \qquad \zeta_{i1}(p, 0) = -\tfrac{1}{2},$$
$$\zeta_{i0}(p, q_i) = pq_i, \quad \zeta_{i1}(p, q_i) = -q_i,$$

and for $i = 3, 4, \cdots, n+2$,

$$\zeta_{i0}(p, 0) = -\frac{1}{2n}, \quad \zeta_{i1}(p, 0) = \frac{1}{2np}.$$

The Walrasian price p^w is such that

$$\sum_{i=1}^{n+2} \zeta_{i0}(p^w, 0) = 0 \quad \text{or} \quad p^w = \frac{1}{2}.$$

To find a Cournotian Monopolistic Competition Equilibrium (p^{CC}, q^{CC}), we have to solve the following program: for $i, j = 1, 2$, $i \neq j$,

$$\max_{(\psi_i, q_i)} U_i(\zeta_i(\psi_i, q_i) + \omega_i)$$

subject to

$$\zeta_{i1}(\psi_i, q_i) \geq -\zeta_{j1}(\psi_i, q_j^{CC}) - \sum_{k=3}^{n+2} \zeta_{k1}(\psi_i, 0), \quad 0 < q_i \leq \frac{1}{2}, \quad \psi_i > 0,$$

or, in other terms,

$$\max_{\psi_i > 0, 0 < q_i \leq 1/2} \psi_i q_i (1 - q_i)$$

subject to

$$q_i \leq \frac{1}{2\psi_i} - q_j^{CC}.$$

Since the constraint must be binding, we may put $\psi_i = 1/[2(q_i + q_j^{CC})]$ and derive the first order conditions in q.

$$q_j - 2q_i q_j - q_i^2 = 0 \quad i, j = 1, 2, i \neq j$$

leading to the solution (unique in the interval $(0, \frac{1}{2})$),

$$q_1^{CC} = q_2^{CC} = \frac{1}{3},\ p^{CC} = \frac{3}{4}.$$

Now consider the P-equilibrium based on the pricing-scheme

$$p = P^{\min}(\psi) = \min\{\psi_1, \psi_2\}.$$

For every price p^* in the interval $[\frac{1}{2}, \frac{3}{4}]$, there exists such a P-equilibrium (p^*, q^*) with $q_1^* = q_2^* = 1/(4p^*)$. Indeed it solves the programs: $i, j = 1, 2, i \neq j$,

$$\max_{\psi_i > 0, 0 < q_i \leq 1/2} \psi_i q_i (1 - q_i)$$

subject to

$$q_i + q_j^* \leq \tfrac{1}{2\psi_i}$$
$$\psi_i \leq p^*$$

with the two constraints binding for $p^* \in [\frac{1}{2}, \frac{3}{4}]$.

Therefore, in this example, based on the min-pricing-scheme, there is a continuum of P-equilibria with, at one extreme, the Walrasian equilibrium price, and at the other, the Cournotian Monopolistic Competition Equilibrium.[18]

The fact that the Walrasian equilibrium can be attained as a P-equilibrium based on the min-pricing-scheme can be generalized somewhat, while keeping the framework of an exchange economy with undifferentiated noncompetitive sectors: $S^t = \{h\}$ for $t = h \in H^*$. For that purpose, we introduce two kinds of pricing-schemes, namely the min-pricing scheme and, its dual, the max-pricing scheme

$$\begin{aligned} P_h^{\min}(\psi^h) &= \min_{i \in I_h}\{\psi_i^h\} \\ P_h^{\max}(\psi^h) &= \max_{i \in I_h}\{\psi_i^h\}, \end{aligned}$$

[18]As suggested in Peck, Shell and Spear [32], p. 274, this indeterminacy "captures the idea that outcomes can be affected by the 'optimism' or 'pessimism' of the economic actors."

and apply each of these pricing-schemes to two different sets of strategic consumers, "natural sellers" and "natural buyers". Strategic consumer i is called a *natural seller* (resp. a *natural buyer*) with respect to good $h \in H_i$ if, for every $p \in \mathbb{R}_+^H$, and $q_i \in Q_i$,

$$\zeta_{ih}(p, q_i) \leq 0 \quad (\text{resp. } \zeta_{ih}(p, q_i) \geq 0).$$

We denote by I_h^S (resp. I_h^B) the set of natural sellers (resp. natural buyers) with respect to good h. A natural seller (or buyer) for good h is a strategic agent who is selling (buying) good h whatever the values of the prices and other variables in its net demand function. It implies restrictions on tastes and endowments for the concerned consumers.

We assume that each strategic consumer i is either a natural seller or a natural buyer in any market $h \in H_i$, excluding however bilateral monopoly. This allows us to apply the min-pricing-scheme to a market with natural sellers and, symmetrically, to apply the max-pricing-scheme to a market with natural buyers. The interesting fact is that, for an economy where pricing-schemes are limited in this way, all Walrasian Equilibria are included in the set of P-equilibria, under the condition that there are at least two strategic agents in each noncompetitive market. Formally we have

Assumption 4. For all $h \in H^*$, either $I_h = I_h^B$ or $I_h = I_h^S$ and $|I_h| \geq 2$.

Proposition 4.1 *Under Assumption 4, if p^w is the price-system characterizing a Walrasian Equilibrium, then for some quantity orders $q^* \in Q$, (p^w, q^*) is a P-equilibrium implying the same transactions and based on the min-pricing-scheme (resp. the max-pricing-scheme), for each market h involving natural sellers $I_h = I_h^S$ (resp. involving natural buyers $I_h = I_h^B$). Moreover, Cournotian Monopolistic Equilibria also belong to this set of P-equilibria.*

Proof: Suppose a Walrasian Equilibrium at prices p^w cannot be obtained as a P-equilibrium as described. Then there exists a consumer $i \in I^*, p_i \in \mathbb{R}_+^{H_i}$ and $q_i \in Q_i$ such that

$$U_i(\zeta_i(p_i, p_{-i}^w, \hat{p}^w, q_i) + \omega_i) > U_i(\zeta_i(p^w, 0) + \omega_i)$$

with, for any $h \in H_i$, and some $\psi_i^h \in \mathbb{R}_+$,

$$\begin{aligned} p_h &= \min\{\psi_i^h, p_h^w\} \quad \text{if} \quad i \in I_h^S \\ p_h &= \max\{\psi_i^h, p_h^w\} \quad \text{if} \quad i \in I_h^B, \end{aligned}$$

the other prices being held at their Walrasian Equilibrium value, and Condition (a.2) holding for all $h \in H_i$.

Consumer i would reach an even higher level of utility by disregarding Constraints (a.2) on q_i, i.e.

$$\begin{aligned} U_i(\zeta_i(p_i, p_{-i}^w, \hat{p}^w, 0) + \omega_i) &\geq U_i(\zeta_i(p_i, p_{-i}^w, \hat{p}^w, q_i) + \omega_i) \\ &> U_i(\zeta_i(p^w, 0) + \omega_i). \end{aligned}$$

Hence, by a revealed preference argument,

$$p^w \zeta_i(p_i, p_{-i}^w, \hat{p}^w, 0) > 0 = (p_i, p_{-i}^w, \hat{p}^w)\zeta_i(p_i, p_{-i}^w, \hat{p}^w, 0),$$

implying

$$\sum_{h \in H_i} (p_h^w - p_h)\zeta_{ih}(p_i, p_{-i}^w, \hat{p}^w, 0) > 0.$$

However, if $i \in I_h^S$, $\zeta_{ih}(p_i, p_{-i}^w, \hat{p}^w, 0) \leq 0$ and, by the min-pricing-scheme, $p_h^w \geq p_h$. Also, if $i \in I_h^B$, $\zeta_{ih}(p_i, p_{-i}^w, \hat{p}^w, 0) \geq 0$ and, by the max-pricing-scheme, $p_h^w \leq p_h$. We have a contradiction and the Walrasian Equilibrium can be obtained as a P-equilibrium based on the min or max pricing-schemes. Finally, that a Cournotian Monopolistic Equilibrium belongs to the set of P-equilibria follows by definition. ■

It should be emphasized that in an economy as described by Assumption 4, one should expect, as it was the case in the previous example, multiple P-equilibria based on the $P^{\min}$ or $P^{\max}$ pricing-schemes. However by Proposition 4.1, whenever there exists a Walrasian Equilibrium, it is one of them. In that sense, Proposition 4.1 may be interpreted as an existence result. From a policy point of view, the fact that the Walrasian Equilibrium belongs to the set of P-equilibria, based on this type of pricing-schemes, may provide an argument in the defense of "facilitating practices" or contractual clauses leading to such kind of pricing coordination.

5 Conclusion

For exchange economies with imperfect competition, we have thus introduced the general concept of P-equilibrium, which belongs to the Cournot tradition and relies on some sector by sector coordination device, the so-called pricing-scheme. Under different kinds of manipulability properties, including the extension given to the notion of sector, this concept leads to different general equilibrium definitions. Actually, when the pricing-scheme is fully individually manipulable, the concept of P-equilibrium coincides with the Cournotian Monopolistic Competition Equilibrium and when, in addition, the notion of sector is given full extension (the whole economy is one sector), it coincides with the Cournot-Walras Equilibrium. On the contrary, to reduce it to the Monopolistic Competition Equilibrium, we need to restrict the sector structure to the unilateral monopoly case, with a single competitive market.

By analogy with the Walrasian Equilibrium two main issues are raised by all these concepts. One is the existence issue: What conditions could be imposed on the primitives of the economy to ensure that there is at least one equilibrium ? The other is the efficiency issue: Are some of these equilibria Pareto-optimal ? In the market game literature these issues are treated simultaneously by admitting the competitive solution as a Nash Equilibrium. With full individual manipulability, this generally can only be obtained by allowing for Bertrand-like discontinuities in the transaction outcome function. In some sense, this is a particular way to contest individual power, by giving to competitors the ability to undercut (or overcut) the market price. To admit the competitive solution among P-equilibria, we have adopted another route for limiting individual market power. In fact, in our analysis, this has been achieved in two ways. First, for the Cournotian Monopolistic Competition Equilibrium concept, we have reduced sectoral extension by restricting the number of markets in which a strategic agent can be a price-maker. The advantage thus obtained has been illustrated by the example developed in Section 2, where computing the Cournotian Monopolistic Equilibrium appears less complex than computing either the Cournot-Walras Equilibrium or the Monopolistic Competition Equilibrium. A second way we have used to restrict market power, is to introduce min or max pricing-schemes, respectively limiting price manipulability upwards or downwards for each individual. This captures already some feature of the Walrasian Equilibrium concept, for which individual manipulability is limited both upwards and downwards. It is then less surprising, as shown by Proposition 4.1, that a Walrasian Equilibrium belongs to the set of P-equilibria based on min or max pricing-schemes. By the same token, Proposition 4.1 provides, for a rather general class of economies (with strategic agents limited to be natural buyers or sellers), an existence result under the conditions required for the Walrasian Equilibrium. These are typically weaker than those required for the Cournot-Walras or the Monopolistic Competition Equilibria. But this result also shows how limiting the manipulability of pricing-schemes results in a loss of determinateness of P-equilibria. In the example we have provided, there is a continuum of P-equilibria going from the Walrasian Equilibrium (the only efficient one) to the Cournotian Monopolistic Competition Equilibrium.

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