On the Dixit-Stiglitz Model of Monopolistic Competition*

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Our purpose in this note is to revisit the popular monopolistic-competition model of Avinash K. Dixit and Joseph E. Stiglitz (1977) and to stress the fact that the variant of this model used in the recent macroeconomic literature is significantly different from the original. In particular, by taking n as the number of active monopolists, the recent discussion of Dixit and Stiglitz (1993) and Xiaokai Yang and Ben J. Heijdra (1993) about the advantages of neglecting terms of the order $1/n$ in the computed elasticities, is significantly affected by the choice of the variant of the model.

The basic model, presented in Section 1, has been used from the start by Dixit and Stiglitz to study optimum product diversity. It is a simple general equilibrium model with n monopolistic goods and a numeraire good, which can be interpreted as labor (or leisure) time or as the aggregation of all the other goods in the economy. The variant of the model, analyzed in Section 2, was independently developed by several authors for different simple applications in macroeconomics.[1] It is an "enlarged model," that includes an additional good, interpreted as

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[1]See, for example, Martin J. Weitzman (1985), Olivier J. Blanchard and Nobuhiro Kiyotaki (1987) and d'Aspremont et al. (1990).

labor time but not taken as the numeraire. More importantly, the enlarged model does not lead to a general equilibrium analysis until the wage rate, taken as given in a first step, is adjusted competitively or strategically.

It is for the basic model that Yang and Heijdra (1993) (YH) give an alternative computation method taking into account the price-index effect of individual pricing decisions. This effect had been neglected in the original paper of Dixit and Stiglitz (1977) (DS), who were only concerned with the large n case (ensured by low fixed costs and imperfect substitution between the monopolistic goods). Limiting their model to the special case of a unitary elasticity of substitution between the monopolistic goods and the numeraire good, YH obtain an explicit solution. But YH's solution is still an approximation, because it neglects the indirect effects that feedback has on pricing decisions.

We will show that, in the enlarged model, taking into account this income-feedback effect allows for an explicit solution and simplifies calculations. But in the variant, some meaningful cases are incompatible with free entry and thus prohibit the use of DS's approximation. However, as we conclude in Section 3, this is not to say that their approximation should never be used. On the contrary, the approximation hypothesis is a very useful part of Dixit and Stiglitz's contribution.

1 The basic model

We briefly recall the DS model. There are $n + 1$ goods and a representative consumer choosing a consumption plan $x \in \mathbb{R}_+^{n+1}$ so as to maximize the utility function

$$U\left(x_0, \left[\sum_{i=1}^{n} x_i^{(s-1)/s}\right]^{s/(s-1)}\right) \tag{1}$$

where U is a homogeneous, strongly quasi-concave, increasing function, s is the constant elasticity of substitution inside the group of monopolistic goods $i = 1, \cdots, n$, and commodity 0 is the numeraire good. DS normalize the economy's endowment of this good at unity, and suggest that it can be interpreted as the labor (or leisure) time at the disposal of the consumer. A strict convexity of preferences implies $0 < s < \infty$. Also, if the number of goods which is actually produced in equilibrium is allowed to be less than n, monopolistic commodities must be gross substitutes: $s > 1$. Utility is maximized subject to the budget constraint

$$x_0 + \sum_{i=1}^{n} p_i x_i = I, \tag{2}$$

where p_i is the price of good i and I is the consumer's income.

We adopt a two-stage maximization procedure. First, for a given expenditure Y in monopolistic commodities, we determine the optimal consumption of good i:

$$x_i = \left(\frac{p_i}{q}\right)^{-s} \frac{Y}{q}, \quad \text{with} \quad q = \left(\sum_{j=1}^{n} p_j^{1-s}\right)^{1/(1-s)}. \tag{3}$$

In (3) q is the *price index* of the monopolistic commodities. Second, we compute the optimal level of the *quantity index*, the consumption of the monopolistic goods,[2] and the optimal level of x_0, the numeraire good:

$$y = \left(\sum_{i=1}^{n} x_i^{(s-1)/s}\right)^{s/(s-1)} = \frac{Y}{q} = \alpha(q)\frac{I}{q}, \qquad x_0 = [1-\alpha(q)]I, \tag{4}$$

where $\alpha(q)$ is the propensity to consume in the monopolistic sector.

Under the same increasing differentiable cost function C (in units of labor), each monopolistic commodity i is produced by a single firm. This firm chooses the corresponding price p_i so as to maximize the profit function $p_i x_i - wC(x_i)$, taking as given the prices of all other commodities as well as the wage w (the labor market is assumed to be competitive), and on the basis of the "true" demand function for good i. Of course, considering labor as the numeraire, w is put equal to 1. When production is positive, the first-order necessary condition for profit maximization is given by the usual equality of marginal revenue and marginal cost:

$$p_i\left(1 - \frac{1}{\varepsilon_i}\right) = wC'(x_i), \quad \text{with} \quad \varepsilon_i = -\frac{dx_i}{dp_i}\frac{p_i}{x_i}. \tag{5}$$

[2] In applications where the number of goods actually produced is fixed, an alternative, and perhaps more straightforward, convention is to refer to the mean price $P = ((1/n)\sum_{i=1}^{n} p_i^{1-s})^{1/(1-s)}$ and to the aggregate consumption $X = n((1/n)\sum_{i=1}^{n} x_i^{(s-1)/s})^{s/(s-1)} = Y/P$ (see d'Aspremont et al., 1990, 1991). When the number of goods actually produced is endogenous, DS's convention, however, proves itself more simply.

In (5), ε_i is the own-price demand elasticity of good i. Adding a condition of nonnegative profits ($p_i x_i \geq wC(x_i)$) leads to the inequality:

$$1 - \frac{1}{\varepsilon_i} \leq \frac{C'(x_i)}{C(x_i)} x_i. \tag{6}$$

In the case of gross substitutability ($s > 1$), if free entry is assumed, the number n^* of active firms in equilibrium is the largest integer in the set $\{1, \cdots, n\}$ such that the preceding inequality is satisfied.

To calculate ε_i, DS use (3) and consider only the direct effect on demand of a change in p_i, neglecting any indirect effect through q or Y. They thus get a demand elasticity:

$$\varepsilon_i^{DS} = s, \tag{7}$$

equal to the *intrasector* elasticity of substitution.

To calculate ε_i, YH use both (3) and (4), and take into account the influence of p_i on q, obtaining a demand elasticity:

$$\begin{aligned} \varepsilon_i^{YH} &= s - \left(\tfrac{p_i}{q}\right)^{1-s} \left[s - 1 + \tfrac{\alpha'(q)}{\alpha(q)} q\right] \\ &= \left[1 - \left(\tfrac{p_i}{q}\right)^{1-s}\right] s + \left(\tfrac{p_i}{q}\right)^{1-s} \times [\alpha(q) + (1 - \alpha(q))\sigma(q)], \end{aligned} \tag{8}$$

where $\sigma(q)$ is the *intersector* elasticity of substitution and where $(p_i/q)^{1-s} = 1/n^*$ in any symmetric equilibrium. Hence, demand elasticity appears now to be a weighted average of unity and both intra- and intersector elasticities of substitution.

There is however no more reason to take a priori the income I as being fixed independently of p_i, than to neglect the effect of a change of p_i, than to neglect the effect of a change of p_i on the price index q. If one insists on an *objective* approach, that is, on rational producers' conjectures about consumer's behaviour, one may as well take into account the effect upon demand of a change in price p_i through the income I. This we have called the *Ford effect* (see d'Aspremont et al., 1990, 1991), since historically Henry Ford was the first to acknowledge, as a producer, such an effect. Indeed, income is the sum of the value of endowment (equal to one unit of labor, putting $w = 1$) and distributed profits:

$$I = 1 + \sum_{j=1}^{n} (p_j x_j - C(x_j)). \tag{9}$$

Thus, income is inherently variable. As such, it is illegitimate to infer from the condition of 0 profits in a free-entry equilibrium, as YH apparently do, that, when firms deviate from equilibrium strategies, profits remain 0, and thus that income remains equal to endowment. Keeping income fixed and equal to the given endowment (its equilibrium value) is just an *approximation*, similar to the one that keeps the price index fixed at its equilibrium value.

Let us admit that the first-order condition (5) is sufficient for a global maximum of the profit function as implicitly defined by equations (3), (4) and (9). Then, by using these four equations combined with a straightforward calculation, we can express the symmetric equilibrium value of the demand elasticity ε^* as the sum of three terms:

$$\begin{aligned} \varepsilon^* &= s + \tfrac{1}{n*}[\alpha^* + (1-\alpha^*)\sigma^* - s] \\ &+ \tfrac{1}{n*}\tfrac{\alpha^*}{\varepsilon^*-\alpha^*}[\alpha^* + (1-\alpha^*)\sigma^* - \varepsilon^*], \end{aligned} \tag{10}$$

where α^* and σ^* are shorthand notations for $\alpha(q^*)$ and $\sigma(q^*)$, respectively. In this equation the first term corresponds to the DS approximation, the second to the YH partial correction (corresponding to the price-index effect) and the third to the correction introducing the Ford effect.[3] In the typical situation DS have in mind, $s > \max\{1, \sigma^*\}$ (when the substitution between the monopolistic goods is large enough, at least larger than the substitution between these goods and leisure), so that (for $n^* > 1$) both the second and the third terms must be negative,[4] leading to: $\varepsilon^* < \varepsilon^{*YH} < \varepsilon^{*DS}$. Taking successively into account the two indirect price effects neglected by the DS approximation leads to a gradual decrease in demand elasticity as perceived by the monopolists, and hence to a gradual increase in monopoly power.

Now, using equations (3), (4), (5), (9) and (10), and given n^*, it is in principle possible to calculate the symmetric equilibrium values of all the endogenous variables. On the other hand,

[3]Equation (10) can be expressed as a quadratic equation:

$$n^*\varepsilon^{*2} - [n^*\alpha^* + (n^*-1)s + (1-\alpha^*)\sigma^*]\varepsilon^* + (n^*-1)\alpha^* s = 0,$$

which has a unique solution ε^* in $[1, \infty)$ if $\sigma^* \geq 1$. The symmetric equilibrium value of the demand elasticity ε^* is thus uniquely related in this case to n^* and q^* (through $\alpha(q^*)$ and $\sigma(q^*)$), as in the YH approximation.

[4]Indeed, as it can be easily checked from (10), $\alpha^* + (1-\alpha^*)\sigma^* < \varepsilon^* < s$, with $\varepsilon^* \to \alpha^* + (1-\alpha^*)\sigma^*$ and $\varepsilon^* \to s$, as $n^* \to 1$ and $n^* \to \infty$, respectively.

assuming free entry, increasing returns and a large number n of potential products (so that profits are 0 and $I = 1$), relation (6) – taken as an equality, together with equations (3), (4) and (5) – determines the number n^* of active firms:

$$n^* \approx \frac{\alpha^*}{a\varepsilon^*}, \tag{11}$$

where $\alpha^* = \alpha(n^{*1/(1-s)}c\varepsilon^*/(\varepsilon^* - 1))$, in the case of the linear affine cost function $C(x_i) = a + cx_i, (a, c) \in \mathbb{R}^2_{++}$, considered by DS. Taking α^* as a function of n^* and ε^* given by the preceding expression, (11) implicitly defines n^* as a decreasing function of ε^* in the relevant domain. Thus, the DS approximation **under**estimates the equilibrium number of active firms (and so does the YH solution): $n^{*DS} \leq n^{*YH} \leq n^*$.

Therefore, it is clear that whenever the number of active firms is sufficiently large to make the DS approximation acceptable, their approximation should be used, being obviously the simplest one. Otherwise, if this number is too small, then one can use either the YH formula (8) or preferably, to take into account all the indirect effects of price changes using formula (10).

2 The enlarged model

The DS model has been applied in several fields.[5] However, some of these applications, like the one by Blanchard and Kiyotaki (1987), are based on a variant of the model, where labor is introduced as an additional good and where the numeraire can be seen as a nonproduced good aggregating the rest of the economy outside the monopolistic sector. The DS utility function U becomes a component of a separable overall utility function, the other component representing labor disutility. Thus the DS "market equilibrium" should be viewed as the partial equilibrium of the monopolistic sector, the equilibrium in the labor market being determined at another stage. Indeed, the "market equilibrium" parametrized by the money wage w determines the labor "demand," a relationship between aggregate employment and the money wage. Then, maximization by the consumer determines the corresponding relationship on the supply side,

[5] See the references in Yang and Heijdra (1993).

taking into account the equilibrium price associated with each value of w. Finally, as a possibility, the equilibrium value of w could be selected by equalizing labor "demand" and labor "supply."

Income I is the sum of distributed profits and the value, in monetary terms, of the representative consumer's endowment. More precisely, this value is what he is able to realize, given the money wage $w : M + w \sum_j C(x_j)$ (where M is the money endowment). Formally,

$$I = \left[M + w \sum_{j=1}^{n} C(x_j) \right] + \sum_{j=1}^{n} [p_j x_j - wC(x_j)] = M + Y. \tag{12}$$

Since $Y = \alpha(q)I$, it is then possible to express income as the product of its autonomous component M and of the *Keynesian multiplier:*

$$I = \frac{1}{1 - \alpha(q)} M. \tag{13}$$

Using (3) and (4), it is now easy to compute the demand for good i:

$$x_i = \left(\frac{p_i}{q} \right)^{-s} \frac{\alpha(q)}{[1 - \alpha(q)]q} M, \tag{14}$$

and the corresponding demand elasticity:

$$\varepsilon_i = \left[1 - \left(\frac{p_i}{q} \right)^{1-s} \right] s + \left(\frac{p_i}{q} \right)^{1-s} \sigma(q), \tag{15}$$

as a weighted average of the intra- and intersectoral elasticities of substitution (the coefficients depend on the relative weight of the individual price p_i in the price level q).[6] Notice that the expression for the demand elalsticity, given by (15) for this specification of the model, is even simpler than the one obtained by YH (see equation (8)).

We will not formulate general assumptions to ensure the existence of an equilibrium. We may, however, observe that the first-order condition (5), together with the nonnegative profits condition (6), entail existence if, for any admissible price p_i (such that profit and marginal revenue are both nonnegative), the elasticity of marginal revenue is higher than the elasticity of marginal cost (both elasticities being calculated with respect to price). This is always true if

[6] This expression for the demand elasticity is the one used by d'Aspremont et al. (1990, 1991).

demand elasticity is increasing or constant, in either the case of the linear affine cost function used by DS, or the case of the isoelastic cost function suggested by YH ($C(x_i) = ax_i^{\beta}, (a, \beta) \in \mathbb{R}^2_{++}$). Thus, in the simple case of a constant intersectoral elasticity of substitution σ, equilibrium can be characterized by conditions (5) and (6), whenever we limit ourselves to the two following clear-cut cases:[7]

(i) $s > 1$ and $s \geq \sigma$, or

(ii) $s \leq 1 < \sigma$.

Case (i) corresponds to the case examined in the preceding section, and encompasses what DS have in mind: monopolistic commodities are "good substitutes among themselves, but poor substitutes for the other commodities in the economy." With free entry, and hence an endogenous number $n^* \leq n$ of active firms, a symmetric equilibrium is characterized by equations:

$$\begin{aligned} p_i^* &= n^{*1/(s-1)} q^*, \\ x_i^* &= \tfrac{1}{n^{*s/(s-1)}} \tfrac{\alpha(q^*)}{1-\alpha(q^*)} \tfrac{M}{q^*}, \end{aligned} \tag{16}$$

for $i = 1, \cdots, n^*$. The number n^* of active firms is the largest integer in $\{1, \cdots, n\}$ such that

$$\begin{aligned} \varepsilon^* &= \left(1 - \tfrac{1}{n^*}\right) s + \tfrac{1}{n*}\sigma \\ &\leq 1 + \tfrac{cx_i^*}{a} \qquad \text{(DS cost function)} \\ &\leq \tfrac{1}{1-\beta} \qquad \text{(YH cost function).} \end{aligned} \tag{17}$$

This type of equilibrium allows comparisons with the DS and YH solutions (assuming a large number n, so that equilibrium profits are 0). To begin with, let us consider the case of the linear affine cost function used by DS. The propensity to consume derived from a constant-elasticity-of-substitution (CES) utility function parametrized by σ, the intersectoral elasticity of substitution, and by μ, the relative weight given to the numeraire good, is given by the

[7] It is also easy to check that the first-order condition (5) has a unique solution in p_i, given q, with the DS cost function, so that all equilibria are symmetric relatively to active firms in this case. The same is true with the YH cost function if $(1 - \beta)s < 1$, a condition which, however, implies that all firms are active in equilibrium.

expression: $\alpha(q) = 1/(1 + \mu^\sigma n q^{\sigma-1})$. From (16) and (17), we easily obtain

$$n^* \approx \left[\frac{M}{(\mu w)^\sigma} \frac{c^{1-\sigma}}{a} \frac{1}{\varepsilon^{*\sigma}(\varepsilon^* - 1)^{1-\sigma}} \right]^{\frac{s-1}{s-\sigma}}. \tag{18}$$

Notice that the right-hand side is decreasing in ε^* if $\varepsilon^* > \max\{\sigma, 1\}$. Excluding the case $\sigma = s$, where the DS approximation happens to coincide with the "true" solution (which takes into account all indirect effects), the elasticity ε^* can only be smaller than the approximation s (see (15)). Thus, the equilibrium number of active firms is in fact larger than predicted by the DS approximation. Actually, the same holds for the YH solution (also smaller than s), as long as $\varepsilon_i^{YH} > \sigma$, which is certainly the case in the example chosen by YH, where $\sigma = 1$. However, as an alternative approximation, the YH solution does not lead us necessarily closer to the true solution. Since this true solution is based on a demand elasticity ε_i that is a weighted average of intra- and intersectoral elasticities of substitution, s and σ, it is immediate that the DS approximation $\varepsilon_i^{DS} = s$ is good, whatever the number of active firms, when σ is close to s. The elasticity ε_i^{YH}, a weighted average of s, σ and 1, may then be a worse approximation of ε_i.

But should an approximation, however good, be preferred to the true solution? In their reply to YH, DS argue that the price paid for abandoning their approximation, namely the restriction to the special case of unitary elasticity of intrasectoral substitution, is very high. Strictly speaking, this restriction is required because if one wants to obtain explicit solutions, and is in no way inherent to the YH solution method. Unitary elasticity of intersectoral substitution is just a particularly simple special case, in which the YH approximation happens also to coincide with the true solution. The true solution, however, by its simplicity, does not depend on such restriction to get explicit results. This is especially obvious if one adopts, as YH do, the isoelastic cost function. The equilibrium number n^* of active firms is then determined by (17) independently of any reference to equations (16), since ε_i, contrary to ε_i^{YH}, is constant in prices in a symmetric situation:

$$n^* \approx \frac{s - \sigma}{s - 1/(1-\beta)}. \tag{19}$$

We thus get, for case (i) and in a free-entry equilibrium with 0 profits, a simple true solution without any drastic restriction on the parameter space of the model.[8]

If returns are nonincreasing or if the number n of potential products is small, profits are not necessarily 0. However, equations (16) still hold, even if equation (17) does not.

The same applies to case (ii). In fact, this case is always incompatible with free entry: all commodities must be produced, otherwise utility is identically 0. Moreover, the DS approximation cannot be used in this case, since $\varepsilon_i^{DS} = s \leq 1$ implies nonpositive marginal revenue. But in this situation, which cannot be ruled out in principle, the true solution still applies.

3 Concluding remarks

In all the general monopolistic-competition models that assume the firms know the "true" demand curve,[9] a central issue is to determine what effects of agents' strategic choices should be taken into account in the description of an "objective demand." Indeed, in contrast to the direct effect of its own pricing decision on demand, which should always be taken into account by a monopolistic firm, some or all indirect effects, like the effect through the price index or the effect through income, might be neglected. In fact, the income-feedback effect, or "Ford effect," can even be decomposed into two effects, the effect through the wage income and the effect through the dividends (see d'Aspremont et al. [1990] for a discussion).

In the basic model, all indirect effects are neglected by DS. This can be justified by assuming a large number of active firms in equilibrium or by invoking their limited computational ability. YH develop a partial correction of the DS approximation,[10] which neglects both income effects, but includes the price-index effect. This approximation is useful. By making the demand

[8] Of course in comparison with a linear affine cost function, adopting an isoelastic cost function makes it more difficult to get existence of an equilibrium number n^* of active firms associated with zero profits, since, in that case, the whole adjustment in the number of firms relies upon the variability of the demand elasticity ε^* as a function of n^*.

[9] This is the so-called "objective" approach, used for example by Hukukane Nikaido (1975) to construct an effective demand, as opposed to the "subjective" approach used for example by Takashi Negishi (1961).

[10] Weitzman (1985) introduces a particular version of the enlarged model, which uses another partial correction that neglects the price-index effect but not the income-feedback effects.

elasticity variable, an alternative to the case where increasing returns result from fixed costs can be explored. This alternative is the convenient case where increasing returns are associated with an isoelastic cost function, and gives simple explicit solutions.

In contrast, in the enlarged model, an explicit solution can be obtained as simply without neglecting any indirect effects.

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