Pricing Schemes and Cournotian Equilibria *

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There are, essentially, two basic models for studying oligopolistic competition: the Cournot model, with quantity-setting firms in a market for a single homogeneous good, and the monopolistic competition model, with price-setting firms, each in a different market of a differentiated good. In both models, the analysis starts by assuming a large set of consumers who adjust in a perfectly competitive way. Only the firms are supposed to behave strategically and to privilege one strategic variable, the quantity produced or the selling price. This paper is an attempt – among others[1] – to integrate the two models and to admit the interaction of the to kinds of strategic variables.

This integration will be based on a reinterpretation of Augustin Cournot's approach. In the famous chapter in which Cournot (1838) introduced his oligopoly theory, he does mention, from the start that the selling price should "necessarily" be the same for each producer. However, he does not elaborate much on price formation, except to say that it is convenient to use the inverse demand function. One may argue that the lack of explanation for price determination in the Cournot model is analogous to the lack of one in the perfect-competition model, and

*Reprinted from *The American Economic Review*, 81(3), 666–673, 1991.

We thank Jean Gabszewicz, Ehud Kalai, Philippe Michel, Louis Phlips, Yaïr Taumann, and Xavier Vives for their comments and suggestions. Support from the Commissariat Général du Plan and the Ministère de la Recherche is also gratefully acknowedged.

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[1]See, for example, Sanford J. Grossman (1981), David Kreps and José A. Scheinkman (1983), Jean Frayssé (1986), Xavier Vives (1986), James W. Friedman (1987), and Paul Klemperer and Margaret Meyer (1989).

a Walrasian auctioneer might be hypothetically introduced. Of course, one difference is that the auctioneer may only act on one side of the market, the demand side. Supply is dictated to him by the firms, and so they themselves integrate into their computations the influence of their supply on the resulting market price. However, the lack of a price-adjustment theory and the idea that such a theory should be rooted on a price-adjustment process under monopoly (one firm adjusting both price and quantity), as advocated by Kenneth J. Arrow (1959) for the perfect-competition model, apply as well to the Cournot oligopoly model. To invoke the "necessity" that the producers of a homogeneous good charge the same price is not different from invoking the "impersonal forces of the market" to justify one competitive equilibrium price that all agents take as given. The argument that, out of equilibrium, producers behave as monopolists and do consider the influence of the price they charge on the (imperfectly elastic) demand they face should also be maintained. The main contrast, finally, is that this argument is still valid at equilibrium in the Cournot oligopoly model. Even there, the producers do not take the price they charge as given. Moreover, they know that, if they were charging different prices, some adjustment process would take place leading again to a single market price. Out of equilibrium, there is a discrepancy between the price they want to charge and the market price (or the average price[2]), and this triggers the adjustment. At equilibrium the two coincide.

In this paper, we shall not introduce a theory of price adjustment under conditions of imperfect competition. We shall insist instead that the result of any such theory would be different from the result of a theory of price adjustment under perfect competition: at equilibrium each firm should be fully aware of its own influence on the market price. To formalize this conclusion, we shall introduce explicitly a concept of "pricing scheme" associating with a vector of price announcements the resulting market price. It will appear that, if the pricing scheme (which is nothing else than a coordination device) is sufficiently responsive to individual price signals, then we get the Cournot equilibrium. This leads to the interpretation of a Cournot equilibrium as the coordinated optimal decisions of a set of monopolists, each facing some (imperfectly elastic)

[2]To quote Arrow (1959, p. 48), "However the "price" whose movements are explained by the law must be thought of as the average price."

residual demand. In the original Cournot model, the same coordination is ensured by the use of the inverse demand function. Formally, pricing schemes have the same status as auctions or bidding mechanisms. They could be assimilated to what is known in industrial organization as "facilitating practices" (see Steven C. Salop, 1985): these are more or less explicit customs established in some industries to allow for price coordination. Examples are the best-price guarantee given to a customer, either with respect to other sellers ("meet-or-release" clause) or with respect to other customers ("most-favored-customer" clause) and the practice of public advance notification of price increases.[3] However, we do not introduce pricing schemes here as corresponding to well-specified price-formation mechanisms used in particular industries. They are seen as an explicit but formal representation of the coordination of pricing decisions that is reached in an industry for a homogeneous product (maybe after a long process) and which is implicit in the use of the inverse demand function in Cournot's traditional approach.

Furthermore, by introducing pricing schemes, we allow for a more natural definition of Cournotian oligopolistic competition with several sectors, each containing several producers of the same homogeneous good, the market price in any one sector being fixed through its own respective pricing scheme. We thus get, in each sector, a well-defined juxtaposition of monopoly problems, one for each firm in the sector, contingent on the total quantity produced in the same sector and on the prices set in the other sectors. The benefit is to avoid assuming that the producers are able to carry over in their computation the inversion of a complete demand system, taking all cross-sectoral effects into account. In a general equilibrium model, this would lead to an alternative to the Cournot-Walras approach (as developed by Jean J. Gabszewicz and Jean-Philippe Vial [1972]) and to the market game approach (see e.g., Lloyd S. Shapley and Martin Shubik, 1977; Pradeep Dubey, 1981; Leo K. Simon, 1984).

The paper is organized as follows. In Section 1, a formal concept of equilibrium with pricing

[3]The main result of the facilitating-practive literature is to show how such clauses can implement prices above the competitive price. For example Ehud Kalai and Mark A. Satterthwaite (1986) and Christopher Doyle (1988) get the implementation of the collusive price. However, by introducing discount possibilities below list prices in a second stage, Charles A. Holt and David T. Scheffman (1987) get the Cournot price as the maximal implementable price. For an experimental approach, see David M. Grether and Charles R. Plott (1984).

schemes is compared to the Cournot equilibrium and then, in Section 2, it is illustrated for a particular scheme, the min-pricing scheme. In Section 3, it is extended to the multisectoral case generalizing both the Cournot and the monopolistic competition concepts.

1 Pricing schemes in the Cournot model

In this section, we consider the market for a single homogeneous good with an extended real-valued demand function D, defined on $\mathbb{R}_+$, and a set $N = \{1, \cdots, i, \cdots, n\}$ of firms. Each firm $i \in N$ can produce any quantity $y_i \geq 0$, at a nonnegative cost $C_i(y_i)$, and choose any price signal $\psi_i \geq 0$. Each C_i is a continuous increasing function on $\mathbb{R}_+$. As discussed in the introduction, the market price is supposed to be determined by a pricing scheme P, a continuous nondecreasing function from $\mathbb{R}^n_+$ to $\mathbb{R}_+$, associating with each vector of price signals $\boldsymbol{\psi} = (\psi_1, \cdots, \psi_i, \cdots, \psi_n)$ a single price $P(\boldsymbol{\psi})$. For a given pricing scheme P, we thus obtain a game involving the n firms, the strategies of firm i being the set of nonnegative quantity-price paris (y_i, ψ_i) and, for any vector $(\mathbf{y}, \boldsymbol{\psi})$ of such strategies, the payoff of firm i being given by the profit function

$$\Pi_i(\mathbf{y}, \boldsymbol{\psi}) \equiv y_i P(\boldsymbol{\psi}) - C_i(y_i).$$

In addition, a feasibility constraint is imposed on $(\mathbf{y}, \boldsymbol{\psi})$ in the strategy set:

$$Y \equiv \sum_{i=1}^{n} y_i \leq D(P(\boldsymbol{\psi})).$$

Then, letting $\mathbf{y}_{-i} \equiv (y_1, \cdots, y_{i-1}, y_{i+1}, \cdots, y_n) \in \mathbb{R}^{n-1}_+$ (and defining $\boldsymbol{\psi}_{-i}$ similarly) and also $Y_{-i} \equiv \sum_{j \neq i} y_j$, we define a P-equilibrium as a vector $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ in $\mathbb{R}^{2n}_+$, such that $Y^* = D(P(\boldsymbol{\psi}^*))$ and, for every $i \in N$, (y_i^*, ψ_i^*) is a solution to

$$\max_{(y_i, \psi_i)} y_i P(\psi_i, \boldsymbol{\psi}^*_{-1}) - C_i(y_i)$$

such that $y_i \leq D(P(\psi_i, \boldsymbol{\psi}^*_{-i})) - Y^*_{-i}, y_i \geq 0$, $\psi_i \geq 0$. To find the Cournot equilibrium in this way, it is clear that pricing schemes have to be more precisely specified. Three classes will be discussed, varying according to the degree of control each firm has on the market price. They are schemes such that (1) all firms together exercise complete control, (2) each individual firm has local control, or (3) each individual firm has complete control on the market price. Property (1) is minimal and amounts to requiring that P has full range: $P(\mathbb{R}_+^n) = \mathbb{R}_+$. However, the main feature distinguishing Cournotian competition from perfect competition is the influence an individual firm may have on the market price. Hence, property (2) requires that the pricing scheme P be strictly increasing in each variable ψ_i. Abstract pricing schemes satisfying these first two properties are, for example, the "arithmetic mean" $\overline{P}(\boldsymbol{\psi}) = (\sum_{i=1}^n \psi_i)/n$, and the "harmonic mean" $\hat{P}(\boldsymbol{\psi}) = n/[\sum_{i=1}^n (1/\psi_i)]$ if $\boldsymbol{\psi} \gg 0$ ($\hat{P}(\boldsymbol{\psi}) = 0$, otherwise). By contrast, the "min-pricing scheme" $P^{\min}(\boldsymbol{\psi}) = \min_j\{\psi_j\}$ and the "max-pricing scheme" $P^{\max}(\boldsymbol{\psi}) = \max_j\{\psi_j\}$, which have full range, are not strictly increasing in ψ_i. One may go further and consider as property (3) that, for every $i \in N$ and $\boldsymbol{\psi}_{-i} \in \mathbb{R}_+^{n-1}$, $P(0, \boldsymbol{\psi}_{-i}) = 0$ and, for $\boldsymbol{\psi}_i \gg 0$, $P(\cdot, \boldsymbol{\psi}_{-i})$ has full range. This property is not satisfied by the previous examples: the arithmetic mean and the max-pricing scheme give complete control to the individual firms only "upwards"; the harmonic mean and the min-pricing scheme give complete control only "downwards". A pricing scheme satisfying all three properties is the "geometric mean" $\tilde{P}(\boldsymbol{\psi}) = [\Pi_{i=1}^n \psi_i]^{1/n}$. We shall denote by $\mathcal{P}_1, \mathcal{P}_2$, and $\mathcal{P}_3$ the sets of pricing schemes satisfying respectively the first, the first two, and all three properties. The interest in considering these classes of pricing schemes is that they imply a close relationship between P-equilibria and Cournot equilibria. This is exhibited in Proposition 1 below.

Recall that, whenever the inverse demand function D^{-1} is well defined [i.e., continuous, decreasing on $\mathbb{R}_+$, and such that, for $0 < Y < D(0), D^{-1}(Y) = p$ if and only if $D(p) = Y$] and nontrivial [i.e., $D^{-1}(Y) > 0$ for some $Y > 0$] a Cournot equilibrium is a quantity vector $\mathbf{y}^c \in \mathbb{R}_+^n$ such that, for every $i \in N$, y_i^c is a solution to

$$\max_{y_i \geq 0} y_i D^{-1}(Y_{-i}^c + y_i) - C_i(y_i) \text{ with } Y_{-i}^c = \sum_{j \neq i} y_j^c.$$

We may then show the following.

Proposition 1 *Let the inverse demand function D^{-1} be well defined and nontrivial. We have:*

(a) For any pricing scheme $P \in \mathcal{P}_1$, if $\mathbf{y}^c$ is a Cournot equilibrium, then $(\mathbf{y}^c, \boldsymbol{\psi}^c)$ is a P-equilibrium where $\boldsymbol{\psi}^c$ is chosen so that $P(\boldsymbol{\psi}^c) = D^{-1}(Y^c)$.

(b) For any pricing scheme $P \in \mathcal{P}_3$, if $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ is a P-equilibrium, then $\mathbf{y}^*$ is a Cournot equilibrium.

(c) Assume that the profit $[y_i D^{-1}(Y_{-i} + y_i) - C_i(y_i)]$ is a strictly quasi-concave function of y_i for every $Y_{-i} \in \mathbb{R}_+$ and every $i \in N$. For any pricing scheme $P \in \mathcal{P}_2$, if $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ is a P-equilibrium, then $\mathbf{y}^*$ is a Cournot equilibrium.

Proof:

(a) Consider a Cournot equilibrium $\mathbf{y}^c$ and take any $P \in \mathcal{P}_1$. Because P has full range, we can pick $\boldsymbol{\psi}^c \in \mathbb{R}^n_+$ such that $P(\boldsymbol{\psi}^c) = D^{-1}(Y^c)$. Suppose $(\mathbf{y}^c, \boldsymbol{\psi}^c)$ is not a P-equilibrium; that is $y_i^0 P(\psi_i^0, \boldsymbol{\psi}^c_{-i}) - C_i(y_i^0) > y_i^c P(\boldsymbol{\psi}^c) - C_i(y_i^c)$ and $y_i^0 \leq D(P(\psi_i^0, \boldsymbol{\psi}^c_{-i})) - Y^c_{-i}$, for some $i \in N$, $\psi_i^0 \geq 0$ and $y_i^0 \geq 0$. Since D^{-1} is well defined, the second inequality implies

$$D^{-1}(Y^c_{-i} + y_i^0) \geq P(\psi_i^0, \boldsymbol{\psi}^c_{-i})$$

so that,

$$y_k^0 D^{-1}(Y^c_{-i} + y_i^0) - C_i(y_i^0) > y_i^c D^{-1}(Y^c) - C_i(y_i^c)$$

which is a contradiction to y^c being a Cournot equilibrium.

(b) For $P \in \mathcal{P}_3$, suppose $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ is a P-equilibrium but $\mathbf{y}^*$ is not a Cournot equilibrium; that is, for some $y_i^0 \geq 0$ and $i \in N$,

$$y_i^0 D^{-1}(Y^*_{-i} + y_i^0) - C_i(y_i^0) > y_i^* D^{-1}(Y^*) - C_i(y_i^*)$$

and

$$D^{-1}(Y^*) = P(\psi^*) > 0.$$

Since i has complete control, there is $\psi_i^0 \geq 0$ such that $P(\psi_i^0, \boldsymbol{\psi}^*_{-i}) = D^{-1}(Y^*_{-i} + y_i^0)$ and

$$y_i^0 P(\psi_i^0, \boldsymbol{\psi}^*_{-i}) - C_i(y_i^0) > y_i^* P(\boldsymbol{\psi}^*) - C_i(y_i^*)$$

which contradicts that $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ is a P-equilibrium.

(c) If $P \in \mathcal{P}_2$, the problem with the previous argument is that one cannot be sure to find $\psi_i^0 \geq 0$ such that $P(\psi_i^0, \boldsymbol{\psi}_{-i}^*) = D^{-1}(Y_{-i}^* + y_i^0)$. However, by strict quasi-concavity, one can find some y_i^0 satisfying the above strict inequality arbitrarily close to y_i^* and, hence, some ψ_i^0 such that $P(\psi_i^0, \boldsymbol{\psi}_{-i}^*) = D^{-1}(Y_{-i}^* + y_i^0)$, since $P(\cdot, \boldsymbol{\psi}_{-i}^*)$ is strictly increasing. The result follows. ■

Now, given a pricing scheme P and a P-equilibrium $(\mathbf{y}^*, \boldsymbol{\psi}^*)$, we denote by B_i^* the set of prices

$$\arg\sup_{p \geq 0} \{p[D(p) - Y_{-i}^*] - C_i(D(p) - Y_{-i}^*)\}$$

subject to

$$D(p) - Y_{-i}^* \geq 0.$$

This set may include $p = \infty$. We see that B_i^* is the set of prices among which firm i would choose if it were a monopolist facing the residual demand $[D(p) - Y_{-i}^*]$. We shall call firm i a "P-leader" at $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ whenever $P(\boldsymbol{\psi}^*) \in B_i^*$. A firm i that is not a P-leader at $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ will be called a "P-follower." Then, y_i^* is a solution to

$$\max_{y_i \geq 0} P(\boldsymbol{\psi}^*) y_i - C_i(y_i)$$

subject to

$$y_i \leq D(P(\boldsymbol{\psi}^*)) - Y_{-i}^*.$$

We see that a P-follower[4] can only be a price-taker with respect to the price $P(\boldsymbol{\psi}^*)$. The next proposition shows how the Cournot equilibrium can be interpreted. At a Cournot equilibrium, each firm behaves as a monopolist facing the residual demand.

Proposition 2 *If the inverse demand function D^{-1} is well defined and nontrivial, then, for any full-range pricing scheme ($P \in \mathcal{P}_1$) and any P-equilibrium $(\mathbf{y}^*, \boldsymbol{\psi}^*), \mathbf{y}^*$ is a Cournot equilibrium if and only if all firms are P-leaders at $(\mathbf{y}^*, \boldsymbol{\psi}^*)$.*

[4]If i is a P-leader, then y_i^* is also a solution to this program; otherwise $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ would not be a P-equilibrium. The difference is that the leader chooses an optimal price.

Proof: Suppose first that $\mathbf{y}^*$ is not a Cournot equilibrium. Then, for some $i \in N$ and $y_i^0 \geq 0$,

$$y_i^0 D^{-1}(Y_{-i}^* + y_i^0) - C_i(y_i^0) > y_i^* D^{-1}(Y^*) - C_i(y_i^*)$$
$$\text{with } D^{-1}(Y^*) = P(\boldsymbol{\psi}^*).$$

Then, taking $p^0 = D^{-1}(Y_{-i}^* + y_i^0)$, we see immediately that i cannot be a P-leader. Now, if $\mathbf{y}^*$ is a Cournot equilibrium, then $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ is a P-equilibrium (in the first part of Proposition 1 we need only that P be of full range). Thus, if i is not a P-leader, for some $p^0 \geq 0$, we must have,

$$\begin{aligned} & p^0[D(p^0) - Y_{-i}^*] - C_i(D(p^0) - Y_{-i}^*) \\ > \quad & P(\boldsymbol{\psi}^*)[D(P(\boldsymbol{\psi}^*)) - Y_{-i}^*] - C_i(D(P(\boldsymbol{\psi}^*)) - Y_{-i}^*) \end{aligned}$$

or, letting $y_i^0 = D(p^0) - Y_{-i}^* \geq 0$ [and $p^0 = D^{-1}(Y_{-i}^* + y_i^0)$], we obtain the same strict inequality as above [since $y_i^* = D(P(\boldsymbol{\psi}^*)) - Y_{-i}^*$], contradicting that $\mathbf{y}^*$ is a Cournot equilibrium. ■

This leads to a typology of P-equilibria. Besides the P-equilibrium in which all firms are P-leaders and which is of a Cournot type, we have P-equilibria in which all firms are P-followers and which include the Bertrand-type equilibrium, and we also have those in which some firms are P-leaders and some are P-followers. This is clarified in the following example, in which all types of P-equilibria can be exhibited for a particular pricing scheme, the min-pricing scheme $P^{\min}$.

2 The special case of the min-pricing scheme

Consider two firms, with a linear demand function,

$$D(p) = \max\{0, a - bp\} \quad a > 0, b > 0$$

and for each firm i, the same twice continuously differentiable cost function C, with $C' > 0, C'' > 0$, and $C(0) = C'(0) = 0$. Let $\eta(p) \equiv -p[D'(p)/D(p)]$ denote the price elasticity. Here $\eta(p) = bp/(a - bp)$. The first-order conditions for a Cournot equilibrium $\mathbf{y}^c$ are well known to be

$$p^c \left[1 - \frac{y_i^c}{yc_i + y_j^c} \frac{1}{\eta(p^c)}\right] = C'(y_i^c)$$

with $y_i^c + y_j^c = D(p^c), i \neq j$. Here, this clearly implies $y_1^c = y_2^c = y_c$ and is simply given by the solution of

$$\frac{1}{b}(a - 3y_c) = C'(y_c).$$

This can be interpreted as a symmetric $P^{\min}$-equilibrium in which both firms are P-leaders (by Proposition 2), by letting $\psi_1^* = \psi_2^* = p^c$ and $y_1^* = y_2^* = y_c$. This symmetric $P^{\min}$-equilibrium appears as point C in Figure 1.

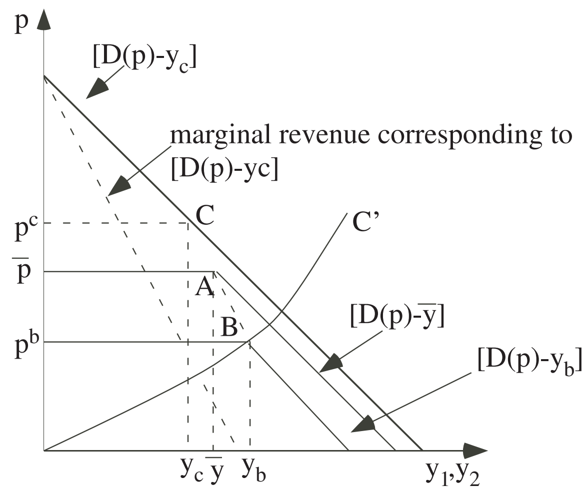

Figure 1: Symmetric $P^{\min}$-equilibria

There are other symmetric $P^{\min}$-equilibria for which, on the contrary, all firms are P-followers: each would prefer to increase the price but cannot individually do so. Therefore, each produces as much as possible considering the residual demand, as long as the market price is higher than the marginal cost. It is as if firms were facing "kinked" demand curves, with the equilibrium corresponding to the "kink," the point at which there is a discontinuity in the marginal revenue. Two such equilibria are illustrated by points A and B in Figure 1. Point A corresponds to the symmetric $P^{\min}$-equilibrium with $\psi_1^* = \psi_2^* = \overline{p}$ and $y_1^* = y_2^* = \overline{y}$ and satisfies the conditions

$$\overline{p}\left[1 - \frac{1}{2}\left(\frac{1}{\eta(\overline{p})}\right)\right] < C'(\overline{y}) < \overline{p}.$$

Point B corresponds to the symmetric $P^{\min}$-equilibrium with $\psi_1^* = \psi_2^* = p^b$ and $y_1^* = y_2^* = y_b$ and satisfies the conditions

$$p^b\left[1-\frac{1}{2}\left(\frac{1}{\eta(p^b)}\right)\right] < C'(y_b) = p^b.$$

It is in fact a Bertrand equilibrium. There is a continuum of symmetric $P^{\min}$-equilibria of the type given by point A between points B and C, corresponding to prices between p^b and p^c. No symmetric $P^{\min}$-equilibrium corresponds to prices below p^b, since then the equilibrium condition $D(\min\{\psi_1^*, \psi_2^*\}) = Y^*$ cannot be satisfied. Also there are no $P^{\min}$-equilibria corresponding to prices above the Cournot price p^c.[5]

There are also non symmetric $P^{\min}$-equilibria. These would consist of vectors $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ such that, for $p^* = \min\{\psi_1^*, \psi_2^*\}$,

$$\begin{aligned} p^*\left[1-\frac{y_i^*}{Y^*}\left(\frac{1}{\eta(p^*)}\right)\right] &= C'(y_i^*) && \text{for some } i\in\{1,2\}\\ p^* \geq C'(y_j^*) &> p^*\left[1-\frac{y_j^*}{Y^*}\left(\frac{1}{\eta(p^*)}\right)\right] && \text{for some } j\neq i. \end{aligned}$$

This implies that $y_j^* > y_i^*$ and is illustrated in Figure 2. This is a "price leadership" type of equilibrium: i is the price-maker, j is the price-taker (as such, firm j could even be at a point where $\min\{\psi_1^*, \psi_2^*\} = C'(y_j^*)$, but not further).

3 A simple multisectoral extension

The advantage of the approach of oligopoly via pricing schemes is to allow straightforward extension of a Cournot-type equilibrium concept to the multisector case. Indeed, the usual definition of the Cournot equilibrium requires the definition of the inverse demand function to reduce the strategy space of each firm to the choice of a quantity. However, in the multisector case, the demand in each sector depends on the price in the other sectors, and so one has

[5] In their two-stage procedure Holt and Scheffman (1987) reach the same conclusions for the symmetric case (see their proposition 2).

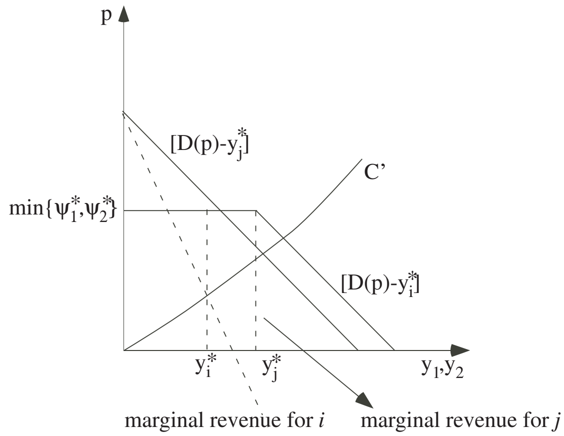

Figure 2: Nonsymmetric $P^{\min}$-equilibria

to invert a complete demand system as in the Cournot-Walras approach (see Gabszewicz and Vial, 1972). This implies that even stronger conditions have to be imposed before defining the equilibrium concept. Moreover, the inverse demand system thus obtained might not facilitate the interpretation of the additional restrictions needed later to prove existence. Using pricing schemes, we can define directly the equilibrium concept and postpone the introduction of specific assumptions to the time when the existence problem would have to be examined.

Suppose we have m goods, each one homogeneous, and that the market for each good k is characterized by an extended real-valued demand function D_k, defined for any vector of nonnegative prices $\mathbf{p} \equiv (p^1, \cdots, p^k, \cdots, p^m) \in \mathbb{R}^m_+$. Also, in each sector k, we have a set N^k of n^k firms. Each firm $i \in N^k$ can produce a quantity $y_i \in \mathbb{R}_+$ of good k and has a continuous increasing cost function $C_i(y_i)$. In the market for good k, the price is determined by a pricing scheme P^k, a continuous nondecreasing function of the price signals of all firms in sector k; say $\boldsymbol{\psi}^k = (\psi^k_i)_{i \in N^k} \in \mathbb{R}^{n^k}_+$. Letting $Y^k \equiv \sum_{i \in N^k} y_i$, $Y^k_{-i} \equiv \sum_{j \in N^k \setminus \{i\}} y_j$, for $k = 1, \cdots, m$, and $\mathbf{P} = (P^1, \cdots, P^k, \cdots, P^m)$, we define a P-equilibrium as a vector $(\mathbf{y}^*, \boldsymbol{\psi}^*)$ in $\mathbb{R}^n_+ \times \mathbb{R}^n_+$ such that, for $k = 1, \cdots, m, Y^{k^*} = D^k(P(\boldsymbol{\psi}^*))$ with $P(\boldsymbol{\psi}^*) = (P^1(\boldsymbol{\psi}^{1^*}), \cdots, P^m(\boldsymbol{\psi}^{m^*}))$, and for every $i \in N^k$, (y^*_i, ψ^*_i) is a solution to

$$\max_{(y_i, \psi_i)} y_i P^k(\psi_i, \boldsymbol{\psi}^{k^*}_{-i}) - C_i(y_i)$$

subject to

$$y_i \le D^k(P(\psi_i, \boldsymbol{\psi}^*_{-i})) - Y^{k^*}_{-i}, y_i \ge 0, \psi_i \ge 0.$$

From the preceding section, we know that to get Cournot-type equilibria we have to require at least that $P_k \in \mathcal{P}_2$ for every k. In fact, the equilibrium we obtain then can be described without referring to any pricing scheme. Indeed, it amounts to having a pair of prices and quantities $(\mathbf{p}^*, \mathbf{y}^*)$ in $\mathbb{R}^m_+ \times \mathbb{R}^n_+$ such that each firm i in each sector k is (at least locally) a P^k-leader: it behaves as a price-setting monopolist facing the demand function $[D^k(\cdots, \mathbf{p}^{-k^*}) - Y^{k^*}_{-i}]$ which is contingent on the other equilibrium prices $\mathbf{p}^{-k^*}$ and the equilibrium total quantity of good k, $Y^{k^*}_{-i}$, produced by the other firms of sector k. This equilibrium concept can be defined independently from pricing schemes and may be called a *Cournotian monopolistic competition equilibrium.* Thus, for this equilibrium concept, pricing schemes appear simply as formal devices allowing one to model the coordination between producers who are all conscious of their own influence on their market price, without computing the inverse of the complete demand system.

4 Conclusion

In this paper, we have proposed a reinterpretation of Cournot equilibrium based on a general and formal coordination device, the pricing scheme, satisfying some global and individual responsiveness properties. The exact determination of this device is not required. For the multi-sectoral case, we get an equilibrium concept that generalizes both the Cournot equilibrium and the monopolistic competition equilibrium (i.e., if there is one sector, it generalizes Cournot; if there is only one firm per sector, it generalizes monopolistic competition). The more general equilibrium concept to which it leads, the Cournotian monopolistic competition equilibrium, can be described as the solution to the juxtaposition of many monopoly problems, each monopolist facing a demand function contingent on the equilibrium quantities produced in its sector and on the equilibrium prices set in the other sectors. We leave for future work the existence problem for such a concept in a general-equilibrium framework.

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