On Monopolistic Competition and Involuntary Unemployment *

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Abstract

In a simple temporary general equilibrium model, it is shown that, if the number of firms is small, imperfect price competition in the markets for goods may be responsible for the existence of unemployment at any given positive wage. In our examples involving two firms facing their "true" demand curves, total monopolistic labor demand remains bounded as the wage rate goes to zero, and unemployment prevails for a sufficiently large inelastic labor supply. In the competitive case total labor demand would go to infinity and intersect labor supply at a positive wage.

1 Introduction

In a period and in a region where unemployment persists unwillingly at a very high rate, it might seem paradoxical that economists are still looking for an adequate definition of, and even for the theoretical possibility of, involuntary unemployment. Of course, such a possibility goes

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against the predictions of a perfect competition theory with complete markets in which agent is completely informed, takes as given all prices including wages, and knows that he will not be rationed. By contraposition, in an imperfect world with unemployment each of these conditions might be violated. No wonder that the theoretical explanations of involuntary unemployment are so many and incomplete, and that the resulting policy recommendations are so basically controversial.[1]

Classical explanations of unemployment are based upon various sources of downward wage rigidities – in particular, the market power of unions. In a well-known paper Hart [1982] argues again in favor of the idea that imperfect competition in the labor market is responsible for the existence of unemployment. The originality of his argument lies in his general equilibrium approach to imperfect competition in all markets. In his approach if the wage rate were to go to zero, the supply of goods would increase to infinity (as by assumption total revenue is always increasing in output), and so would the labor demand. Hence unemployment is due to the unions preventing the wage rate from falling.

More recent policy recommendations by Weitzman [1984, 1985] are based on a similar diagnosis. It is proposed to cure unemployment by adjusting the wage rate down to the positive level at which full employment is reached, and meanwhile by compensating the workers through some profit sharing. The approach is a general temporary equilibrium one, with monopolistic competition, using simple parameterized utility functions and a linear technology. The short-run equilibrium employment is shown to be a decreasing function of the wage rate, cutting the perfectly inelastic supply of labor at some positive wage.

Here we shall introduce a similar model, again taking prices as the strategic variables. The class of economies considered will appear to contain those analyzed by Weitzman [1985]. But, and this is our main point, it will also contain another set of economies for which Weitzman's policy recommendation does not fully work. No positive wage ensuring full employment at equilibrium will exist. In other words, only a zero wage could, possibly, clear the labor market. This

[1] For example, see the recent evaluation (with many references) by Lindbeck and Snower [1985].

we have called[2] a situation of "involuntary unemployment" in the spirit of Keynes, according to whom unemployment is involuntary when there is "*no* method available to labor as a whole" for attaining full employment "by making revised *money* bargains with the entrepreneurs" [Keynes, 1973, p. 13]. Moreover, such a situation is compatible with the existence of a Walrasian competitive equilibrium at which the labor market clears at a positive wage level and at a higher level of employment. As well commented on by Silvestre [1988, p. 1], involuntary unemployment in our sense corresponds to a severe exploitation of workers in the neoclassical sense[3] that the real wage is lower than the physical margin product of labor (at zero wage productive labor becomes a free good). The situation is well illustrated by Figure 1.

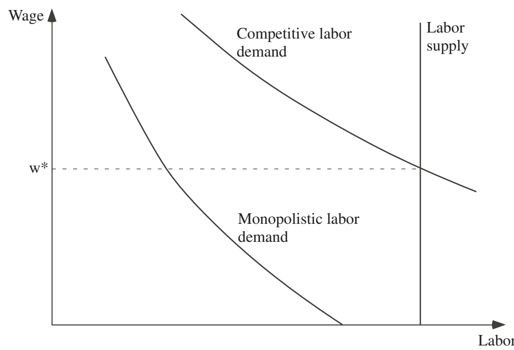

Figure 1:

We see that there are two "Labor Demand Curves" or, more precisely, two curves of equilibrium employment levels associated with all possible wages: the Competitive one, leading to a positive competitive equilibrium wage w^*; and the Monopolistic one, which does not intersect the Labor Supply at a positive wage. Our result will be to determine a set of economies in which such a figure is a true possibility. It will owe much to the fact that we abandon Hart's and Weitzman's assumption that the total revenue of each producer is an increasing function of output,

[2]See d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1984] and the subsequent development of a monopoly example by Dehez [1985]. Also see d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1989].

[3]See Pigou [1920], p. 51 or pp. 813–14. Robinson [1933] has well emphasized that such exploitation may occur, even with wage-taking behavior of the firm, because of monopolistic power in the output market [Ch. 25].

however large this output. Instead we shall assume, as in our previous work on a Cournot model (see also Dehez [1985] and Silvestre [1988]), that total revenue becomes decreasing in output as output becomes large. However, we shall keep the "objective" approach used by Hart and Weitzman to describe the demand for goods faced by the producers. As discussed in Nikaido's [1975] book and Hart's [1985] survey, this "objective" approach[4] (as opposed to the "subjective" approach of Negishi [1961]) supposes that the producers know the "true" demand curve they face. This implies that the indirect effects (or "feedback effects" in Hart's terminology) of the producers' decisions on their own demand, through aggregate wealth, be explicitly taken care of by some specific assumption. Several possibilities will emerge from our discussion.

The present paper is organized as follows. In Section 2 we describe the basic model and define equilibrium concepts. A class of examples is developed in Section 3. Then, in Section 4 the possibility of involuntary unemployment is demonstrated and contrasted with the conclusion of Weitzman [1985]. Different interpretations or extensions of the model are discussed in Section 5.

2 The model

We consider an economy with four goods: two produced goods, labor, and money[5] in a temporary equilibrium framework. There are two firms, each one specialized in producing, out of labor and with constant productivity, one of the two consumption goods. All prices are nonnegative monetary prices: we denote by w the wage rate, by p the price of one good (henceforth called the "Latin" good) and by π the price of the other (called the "Greek" good). Labor productivity in the Latin sector is denoted by $1/\ell$, $\ell > 0$, and, in the Greek sector, $1/\lambda$, $\lambda > 0$.

[4] Also see Gabszewicz and Vial [1972], Marschak and Selten [1972], Laffont and Laroque [1976], Roberts [1986, 1989], Jones and Manuelli [1987], and Benassy [1988].

[5] Although in the presentation of our results we use a temporary equilibrium framework (as described, for example, in Grandmont [1983]), we consider in Section 5 an alternative framework similar to the one in Hart [1982], where there is only one period (still divided into two stages) and where money is taken as a "nonproduced good".

There is a continuum of consumers with identical homothetic preferences. Labor has no disutility. Thus, every consumer θ in $[0,1]$ is described, for a given wealth $\omega_\theta(p,\pi,w) \geq 0$ varying with prices and the wage, by two multiplicatively separable continuous demand functions, respectively, denoted $h(p,\pi)\omega_\theta(p,\pi,w)$ for the Latin good and $\chi(p,\pi)\omega_\theta(p,\pi,w)$ for the Greek good. For a given distribution of consumers ν, aggregate wealth is given by the function $\Omega(p,\pi,w) = \int_0^1 \omega_\theta(p,\pi,w)\nu(d\theta)$, and the two aggregate demands are simply $h(p,\pi)\Omega(p,\pi,w)$ and $\chi(p,\pi)\Omega(p,\pi,w)$. In addition, every consumer θ is assumed to supply one unit of labor at every positive wage.[6] Hence the total supply of labor is $L = \int_0^1 \nu(d\theta) > 0$, whenever w is positive.

From a strategic viewpoint, the (Latin and Greek) producers have to base their decisions on ex ante conjectures about the consumers' demand for their product. We want that these conjectures be, in some sense, objectively founded. In our homothetic preference case, we assume that the functions h and χ are correctly perceived. As to the aggregate wealth function Ω, several specifications are possible, however, and have been used in the literature. They differ according to the degree in which producers are assumed to take into account the impact of their own decisions upon the value of Ω. To cover these different cases, let $r(p,\pi,w)$ and $\rho(p,\pi,w)$ denote the wealth functions conjectured, respectively, by the Latin and Greek producers.

Moreover, let us divide r and ρ into an autonomous wealth part, say $A > 0$, taken as given by the producers, and an induced part, depending upon the producers' decisions. The decomposition into these two parts relies on alternative behavioral assumptions. Here are three examples.

1. A first (extreme) specification is to take the whole wealth as autonomous, i.e., for all $p,\pi,w > 0$,

$$r(p,\pi,w) = \rho(p,\pi,w) = A,$$

[6] At a zero wage we can consider different types of behavior leading to different interpretations. In the examples introduced below we assume that every individual is indifferent between working or not at a zero wage. In this limit case unemployment may still arise in the sense that only part of the labor force is employed – identical individuals being unequally treated.

and to adjust A parametrically so that conjectures are fulfilled in equilibrium. This is one of the solutions proposed by Marschak and Selten [1972] and used, for instance, in Hart [1982] and Silvestre [1988]. For each producer, it amounts to neglecting all the effects of his decisions on aggregate wealth.

2. Another specification (also proposed by Marschak and Selten [1972] and used by d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1984]) is to suppose that the producers take into account the effects of their decisions on the total wage bill, but not on the distributed profits, which are included in the autonomous part.[7] Accordingly, the conjectured wealth function is the sum of the total wage bill and the autonomous wealth; i.e.,

$$r(p, \pi, w) = w[\ell h(p, \pi) + \lambda \chi(p, \pi)]r(p, \pi, w) + A$$

and

$$\rho(p, \pi, w) = w[\ell h(p, \pi) + \lambda \chi(p, \pi)]\rho(p, \pi, w) + A$$

or, with $w[\ell h(p, \pi) + \lambda \chi(p, \pi)] < 1$,

$$r(p, \pi, w) = \rho(p, \pi, w) = A/[1 - w(\ell h(p, \pi) + \lambda \chi(p, \pi))].$$

Again A can be adjusted parametrically to get fulfillment of conjectures in equilibrium.

3. A third specification (at another extreme) is to consider that the producers take into account the effects of their decisions both on the total wage bill and on the distributed profits (assuming that all profits are distributed). We thus get

$$\begin{aligned} r(p, \pi, w) &= w[\ell h(p, \pi) + \lambda \chi(p, \pi)]r(p, \pi, w) \\ &\quad +(p - w\ell)h(p, \pi)r(p, \pi, w) \\ &\quad +(\pi - w\lambda)\chi(p, \pi)r(p, \pi, w) + A \\ &= [ph(p, \pi) + \pi \chi(p, \pi)]r(p, \pi, w) + A \end{aligned}$$

and, similarly,

$$\rho(p, \pi, w) = [ph(p, \pi) + \pi \chi(p, \pi)]\rho(p, \pi, w) + A,$$

[7] This amounts to considering a conjectured demand including what we have called "Ford effects", following the idea developed by Henry Ford [1922].

leading to

$$r(p,\pi,w) = \rho(p,\pi,w) = A/[1 - ph(p,\pi) - \pi\chi(p,\pi)]. \tag{1}$$

To write this last expression, we need to assume that the marginal propensity to consume in the present period $[ph(p,\pi) + \pi\chi(p,\pi)]$ is less than one.

Also, given our temporary equilibrium framework, equation (1) will apply in equilibrium with $A = M + \hat{I}$, where M is the supply of money and $\hat{I}$ stands for the total expected future income. We thus have A positive and adjusted to fulfill conjectures in equilibrium. Notice that there is in this framework a "money market" which, by Walras' law, always clears when the goods markets clear.

Whatever the adopted specification of the conjectured wealth functions r and ρ, we see that they can always be written as a product,

$$r(p,\pi,w) = \rho(p,\pi,w) = AK(p,\pi,w),$$

where $K(p,\pi,w)$ is a multiplier, the definition of which varies accordingly. For any positive wage, we can construct a game between the two producers with prices as strategies ($p \geq w\ell$ and $\pi \geq w\lambda$, respectively), and payoff functions given by

$$\begin{aligned} F(p,\pi,w) &= (p - w\ell)h(p,\pi)K(p,\pi,w)A \\ \Phi(p,\pi,w) &= (\pi - w\lambda)\chi(p,\pi)K(p,\pi,w)A. \end{aligned}$$

In the following, we shall concentrate mainly on the game obtained from the third specification of the wealth functions. Also, we shall put aside the way in which the wage is determined. Actually, there are several ways to close the model. One is to consider the labor market as competitive with the producers as wage-takers and to adjust the wage parametrically for equilibrium.[8] Another way is to introduce two stages. In a first stage the labor market is organized, and the wage w is determined.[9] Then, at the second stage the goods prices are selected by the producers, taking the wage as given.

[8]Then, of course, the behavior of the consumers at a zero wage must be determined, since the wage may have to be adjusted at zero.

[9]This includes the way proposed by Hart [1982], in which the workers fix the nominal wage through their union.

Anyway, we can define a pair of nonnegative prices (p^*, π^*) as an *equilibrium in the goods markets at a given wage* w if

$$\text{(i)} \qquad p^* \in \arg\sup_{p \geq w\ell} F(p, \pi^*, w) \text{ and } \pi^* \in \arg\sup_{\pi \geq w\lambda} \Phi(p^*, \pi, w);$$

$$\text{(ii)} \qquad AK(p^*, \pi^*, w) = \Omega(p^*, \pi^*, w).$$

The first property is a Nash-equilibrium property for the game, and the second property imposes that conjectures be fulfilled. This definition, however, is incomplete, since it neglects a feasibility constraint. Indeed by letting the total employment required by the producers be

$$Z(p, \pi, w) = [\ell h(p, \pi) + \lambda \chi(p, \pi)] AK(p, \pi, w),$$

we need to impose that $Z(p, \pi, w) \leq L$, for $w \geq 0$. In fact, in the objective approach we have adopted, it is more reasonable to impose this feasibility constraint on the producers' strategy spaces. Consider the following correspondences:

$$P(\pi, w) = \{p \geq w\ell : Z(p, \pi, w) \leq L\}$$
$$\Pi(p, w) = \{\pi \geq w\lambda : Z(p, \pi, w) \leq L\}.$$

We call a pair of nonnegative prices (p^*, π^*) a *multisectoral equilibrium at given wage* $w \geq 0$ if

$$\text{(i)} \qquad p^* \in \arg\sup_{p \in P(\pi^*, w)} F(p, \pi^*, w) \text{ and } \pi^* \in \arg\sup_{\pi \in \Pi(p^*, w)} \Phi(p^*, \pi, w);$$

$$\text{(ii)} \qquad AK(p^*, \pi^*, w) = \Omega(p^*, \pi^*, w).$$

Notice that an equilibrium in the goods markets (p^*, π^*) at a given w is a multisectoral equilibrium given the same wage whenever

$$Z(p^*, \pi^*, w) \leq L.$$

In the next section we shall study these equilibrium concepts in a set of examples. For that purpose it will be useful to simplify notation. Aggregate objective demand conjectures will be written as multiplicative forms, namely, $H(p, \pi, w)A$ and $\chi(p, \pi, w)A$, where

$$H(p, \pi, w) = h(p, \pi) K(p, \pi, w)$$
$$X(p, \pi, w) = \chi(p, \pi) K(p, \pi, w).$$

Moreover, since we shall be essentially restricted to the third specification, as given by (1), the w argument will be omitted in H and X.

3 A class of examples

Let us illustrate the model by a class of economies arising in a two-period world. Suppose that each consumer θ has a C^2 strongly quasi-concave utility function that is homogeneous and intertemporally separable. For consumer θ, let $(c_{\theta t}, \gamma_{\theta t})$ be his consumptions of the Latin good and the Greek good in period $t(t = 1, 2)$, m_θ and $\tilde{m}_\theta$ be his initial money balance and his savings, i_θ and $\hat{i}_\theta$ be his current income and his expected future income, respectively. Also let $(\hat{p}, \hat{\pi})$ be the expected future prices. The consumer's program can now be written as

$$\max_{c_\theta, \gamma_\theta, \tilde{m}_\theta)} U(u_1(c_{\theta 1}, \gamma_{\theta 1}), u_2(c_{\theta 2}, \gamma_{\theta 2}))$$

subject to

$$pc_{\theta 1} + \pi \gamma_{\theta 1} + \tilde{m}_\theta \leq m_\theta + i_\theta$$

with

$$c_{\theta 1}, \gamma_{\theta 1} \geq 0$$

and

$$\hat{p} c_{\theta 2} + \hat{\pi} \gamma_{\theta 2} \leq \tilde{m}_\theta + \hat{i}_\theta$$

with

$$c_{\theta 2}, \gamma_{\theta 2} \geq 0.$$

We shall assume in the following that the function u_t is C.E.S. and constant over time:

$$u(c_{\theta t}, \gamma_{\theta t}) = (c_{\theta t}^{(s-1)/s} + \gamma_{\theta t}^{(s-1)/s})^{s/(s-1)},$$

with an intersectoral elasticity of substitution $s > 0$ $(s \neq 1)$.

Maximizing in two stages, first the two arguments of U, conditional on $\tilde{m}_\theta$, then[10] over $\tilde{m}_\theta$ and defining the marginal propensity to consume in period 1:

$$a = (pc_{\theta 1} \pi \gamma_{\gamma 1})/(m_\theta + i_\theta + \hat{\imath}) = (m_\theta + i_\theta - \tilde{m}_\theta)/(m_\theta + i_\theta + \hat{\imath}_\theta),$$

[10]Here we neglect the nonnegativity constraint on $\tilde{m}_\theta$. An individual's savings might be negative (he is then borrowing at zero interest rate).In any case the total savings $\tilde{M}$ are equal to the total money supply $M > 0$.

one easily gets the following solution for present consumption:

$$\begin{aligned} c_{\theta 1}^{*} &= \frac{1}{2(p/P)^{s}}\frac{a}{P}(m_{\theta}+i_{\theta}+\hat{\imath}_{\theta}) \\ \gamma_{\theta 1}^{*} &= \frac{1}{2(\pi/p)^{s}}\frac{A}{P}(m_{\theta}+i_{\theta}+\hat{\imath}_{\theta}), \end{aligned} \tag{2}$$

where a becomes simply a function of $P/\hat{P}$, with P and $\hat{P}$ denoting the price averages:

$$P = \left(\frac{p^{1-s}+\pi^{1-s}}{2}\right)^{1/(1-s)} \quad \text{and} \quad \hat{P} = \left(\frac{\hat{p}^{1-s}+\hat{\pi}^{1-s}}{2}\right)^{1/(1-s)}.$$

To be more specific, we may take U to be a C.E.S. function with elasticity of intertemporal substitution $\sigma > 0$ and relative weight of current utility $\delta \in]0,1[$, getting

$$a = [1+(\delta/(1-\delta))^{-\sigma}(P/\hat{P})^{\sigma-1}]^{-1}.$$

Considering the solution values $c_{\theta 1}^{*}$ and $\gamma_{\theta 1}^{*}$ for $m_{\theta}+i_{\theta}+\hat{\imath}_{\theta}=1$, we obtain the functions (identical for each θ):

$$\begin{aligned} h(p,\pi) &= \frac{1}{2p(p/P)^{s-1}[1+(\delta/(1-\delta))^{-\sigma}(P/\hat{P})^{\sigma-1}]} \\ \chi(p,\pi) &= \frac{1}{2\pi(\pi/P)^{s-1}[1+(\delta/(1-\delta))^{-\sigma}(P/\hat{P})^{\sigma-1}]}. \end{aligned}$$

Taking the third specification of producers' conjectures as given by (1), we finally get, for $\alpha = \frac{1}{2}(\delta(1-\delta))^{\sigma}$,

$$\begin{aligned} H(p,\pi) &= \alpha p^{-s}P^{s-\sigma}\hat{P}^{\sigma-1} \\ X(p,\pi) &= \alpha \pi^{-s}P^{s-\sigma}\hat{P}^{\sigma-1}. \end{aligned} \tag{3}$$

We need also to choose some class of price expectations. For simplicity, we assume the price expected in one sector to depend only on the current price in that sector. Let these price expectation functions $\hat{p}(p)$ and $\hat{\pi}(\pi)$ be twice continuously differentiable, strictly increasing, and such that $\lim_{p\to 0}\hat{p}(p) = \lim_{\pi\to 0}\hat{\pi}(\pi) = 0$ and $\lim_{p\to\infty}\hat{p}(p) = \lim_{\pi\to\infty}\hat{\pi}(\pi) = \infty$. Denote, respectively, by $b(p)$ and $\beta(\pi)$ the elasticities of $\hat{p}(p)$ and $\hat{\pi}(\pi)$. We shall actually restrict our attention to two cases:

(a) Pure Substitutability. $1 < \sigma \leq s$, with $b(p)$ and $\beta(\pi)$ nonincreasing, upper-bounded by $\sigma/(\sigma-1)$, and taking values below $1+(s-1)/(\sigma-1)$.

(b) Pure Complementarity. $s \le \sigma < 1$, with $b(p)$ and $\beta(\pi)$ nondecreasing and taking values above $1 + ((1-s)/(1-\sigma))$.

These restrictions are put on the parameters of our example in order to get some general properties of the functions H and X that will be used to prove the existence of an equilibrium in the goods markets. Further restrictions on the parameters will be added to demonstrate the possibility of involuntary unemployment in Section 4.

We now describe these general properties.

Property A. (1) The function H (respectively, X) is positive, decreasing, and twice continuously differentiable in p (respectively, in π). It is continuous in both variables. (2) Both functions H and X are asymptotically finite for any sequence of prices bounded away from the axes.

In the example, positivity, continuity, and twice-differentiability of H are obvious (see (2) and (3). Also, in the pure complementarity case (b), where $(s-\sigma) \le 0$ and $(\sigma - 1) < 0$, we see that H and X are decreasing in both prices so that Property A is verified. To treat the pure substitutability case (a), let us compute the price elasticity of H:

$$e(p,\pi) = -\frac{H'_p(p,\pi)}{H(p,\pi)}p = s - \frac{s-\sigma}{1+(p/\pi)^{s-1}} - \frac{(\sigma-1)b(p)}{1+(\hat{p}/\hat{\pi})^{s-1}}. \tag{4}$$

(The price elasticity of X is computed similarly and denoted $\eta(p,\pi)$.) Using (4), we have

$$H'_p(p,\pi) = (H(p,\pi)/p)(-e(p,\pi)) < 0,$$

since $-e(p,\pi) < -s + (s-\sigma) + (sigma-1)b(p) \le 0$, according to the upper bound imposed on b. Hence (A1) holds in case (a) As for (A2), notice that

$$\begin{aligned} \sup_{\pi>0} H(p,\pi) &= \sup_{\pi>0} \alpha p^{-s}\left[\frac{2}{p^{1-s}+\pi^{1-s}}\right]^{(s-\sigma)/(s-1)}\left[\frac{2}{\hat{p}^{1-s}+\hat{p}^{1-s}}\right]^{(\sigma-1)/(s-1)} \\ &= 2\alpha p^{-\sigma}\hat{p}^{\sigma-1} \quad \text{(setting } \pi = \infty\text{)}. \end{aligned}$$

Clearly, this last expression is finite for every $p > 0$. The same is true when p tends to infinity if

$$\lim_{p\to\infty} \ln(p^{-\sigma}\hat{p}^{\sigma-1}) = \lim_{p\to\infty} -\sigma \ln p \left[1 - \frac{(\sigma-1)\ln \hat{p}}{}\sigma \ln p\right] \le 0 < \infty.$$

But this is implied by

$$\lim_{p\to\infty}\frac{(\sigma-1)\ln\hat{p}(p)}{\sigma\ln p}=\frac{\sigma-1}{\sigma}\lim_{p\to\infty}b(p)\leq 1.$$

A similar argument applies to X.

Property B. The profit function $F(p,\pi,w)=(p-w\ell)H(p,\pi)A$ (respectively, $\Phi(p,\pi,w)=(\pi-w\lambda)\,X(p,\pi)A$) is strictly quasi-concave in p (respectively, in π).

In the example, we have for $p>w\ell$:

$$F'_p(p,\pi,w)=AH(p,\pi)[(p-w\ell)/p](p/(p-w\ell))-e(p,\pi)], \tag{5}$$

where $e(p,\pi)$ is the price elasticity given by (4).

Since $b'(p)\leq 0$ in case (a) and $b'(p)\geq 0$ in case (b), $[(p/(p-w\ell))-e(p,\pi)]$ is strictly decreasing in p, and F'_p remains negative once it becomes so. Property B is thus verified.

Property C. For any sequence of positive price vectors such that some price tends to infinity, the corresponding total revenue function becomes decreasing in this price for at least one producer.

First, let $(p^\tau,\pi^\tau)_{\tau\geq 1}$ be a sequence such that $\lim_{\tau\to\infty}\pi^\tau<\lim_{\tau\to\infty}p^\tau=\infty$. We have to check that $e(p^\tau,\pi^\tau)$ is larger than one infinitely often (or, equivalently, that $p^\tau H(p^\tau,\pi^\tau)$ is decreasing in p). Clearly, by (4) and the upper-boundedness of b, $\lim_{\tau\to\infty}e(p^\tau,\pi^\tau)=s>1$, in case (a). Using the restriction $\lim_{p\to\infty}b(p)>1+((1-s)/(1-\sigma))$, we get, in case (b),

$$\lim_{\tau\to\infty}e(p^\tau,\pi^\pi)=\sigma-(\sigma-1)\lim_{\tau\to\infty}b(p^\tau)>1+1-s>1.$$

Second, supposing that both prices tend to infinity, it is enough to check that $e(p^\tau,\pi^\tau)+\eta(p^\tau,\pi^\tau)>2$ infinitely often. Now,

$$\begin{aligned} e(p,\pi)+\eta(p,\pi) &= s+\sigma-(\sigma-1)[(\hat{p}^{1-s}/(\hat{p}^{1-s}+\hat{\pi}^{1-s}))b(p) \\ &+ (\hat{\pi}^{1-s}/(\hat{p}^{1-s}+\hat{\pi}^{1-s}))\beta(\pi)]. \end{aligned} \tag{6}$$

The conclusion is obtained under the restrictions,

$$\lim_{p\to\infty}b(p)<1+\frac{s-1}{\sigma-1}\quad\text{and}\quad\lim_{\pi\to\infty}\beta(\pi)<1+\frac{s-1}{\sigma-1},$$

in case (a), and the reverse restrictions in case (b).

These first three properties, satisfied by our class of examples, are enough to derive a preliminary result, namely, the existence, at any positive wage, of two continuous "reaction curves" that intersect at some pair of prices, thus establishing existence of an equilibrium in the goods markets.

Proposition 1 *Under Properties A1, B, and C, there exists an equilibrium in the goods markets at any positive wage.*

Proof: First, using standard arguments, it is possible to show that the solution to $\max_{p \geq w\ell} F(p, \pi, w)$ is a well-defined continuous function $\pi(p, w)$ (for a detailed proof, see CORE DP 8635). We can define in a symmetric way the function $\tilde{\pi}(p, w)$. Second, suppose for some positive w that no $(p, \pi) \geq (w\ell, w\lambda)$ satisfies $p\tilde{p}(\pi, w)$ and $\pi = \tilde{p}(p, w)$; i.e., there is no equilibrium in the goods markets. Then it is possible to find a sequence of prices $(p^\tau, \pi^\tau)_{\tau \geq 1}$ such that, for any $\tau, w\ell \leq p^\tau < \tilde{p}(\pi^\tau, w)$ and $w\lambda \leq \pi^\tau \leq \tilde{\pi}(p^\tau, w)$ and such that at least one of the two prices, say p^τ, tends to infinity. By Property C, the corresponding total revenue function becomes decreasing for at least one producer, say the Latin. Hence the profit function,

$$F(p^\tau, \pi^\tau, w) = (1 - (w/ell/p^\tau))p^\tau H(p^\tau, \pi^\tau)A,$$

which tends to the total revenue function as p^τ tends to infinity, becomes decreasing, a contradiction to $p^\tau \leq \tilde{p}(\pi^\tau, w)$, by Property B. ■

Notice that the proposition does not ensure the existence of a multisectoral equilibrium at a given positive wage, since the equilibrium in the goods markets may violate the labor market constraint. However, it is possible to demonstrate the existence of a multisectoral equilibrium at any positive wage by requiring in addition that the set of admissible prices,

$$\tilde{Z}(w) = \{(p, \pi) : p \geq w\ell, \pi \geq w\lambda, [\ell H(p, \pi) + \lambda X(p, \pi)]A \leq L\},$$

be nonempty and strictly biconvex (or, $\ell H + \lambda X$ strictly quasi-convex in each price separately).[11]

[11] In the pure complementarity case this is always true because $\ell H + \lambda X$ is decreasing in both arguments

In the case where for some positive wage the two reaction curves intersect outside the admissible region, we find full employment multisectoral equilibria on the boundary of $\tilde{Z}(w)$, the labor constraint being binding for at least one producer. One possible case is given by point E in Figure 2, where $\tilde{Z}(w)$ is the hatched region, and $\tilde{p}(\pi, w)$ and $\tilde{\pi}(p, w)$ are the reaction curves of the two producers. Arrows at point E indicate the directions of increasing profits. It is interesting to observe that this Figure 2, with the type of equilibria it illustrates, is similar to the one (Figure 10) presented by Cournot [1838] in Chapter VIII analyzing price competition between two producers of complementary inputs.

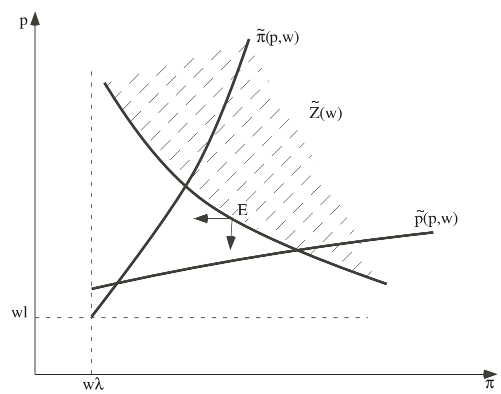

Figure 2:

We now turn to the main objective of this paper, and that is the possibility of involuntary unemployment.

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separately and goes to zero as at least one of the two prices goes to infinity. In the pure substitutability case nonemptiness of $\tilde{Z}(w)$ is entailed by the restrictions, $\lim_{p\to\infty} \hat{p}(p)/p < \infty$ and $\lim_{z\to\infty} \hat{\pi}(\pi)/\pi < \infty$, as again $\ell H + \lambda X$ goes to zero when both prices go to infinity. For strict quasi convexity of $\ell H + \lambda X$ in each price separately, a sufficient condition, for the simple case where $s = \sigma$, is that

$$-\frac{b'(p)p}{b(p)} < (\sigma - 1)\left[\frac{\sigma}{\sigma - 1} - b(p)\right] \text{ for any } p$$

and correspondingly for the greek sector, as shown in CORE DP 8635.

4 The possibility of involuntary unemployment

The class of examples that we have introduced is related to the class used by Weitzman [1985] in his analysis of the impact of profit sharing on unemployment. Weitzman considers, as we do, an economy where the goods markets are imperfectly competitive. Then he shows that, for such economies, it is always possible to find a positive wage at which full employment obtains at equilibrium. We challenge this conclusion by exhibiting economies where, because of the oligopolistic features of the goods markets, there is unemployment at any positive wage.

Let us first take the case of a given wage w and consider Weitzman's model. It starts with a composite utility function which is simply the particular specification of the above taking $\sigma = 1$, namely, an intertemporal Cobb-Douglas function of parameter $\delta \in]0, 1[$ but still an intersectoral CES-function. Restricting his model to the case $n = 2$ and using our notation, we get

$$h(p, \pi) = \frac{1}{2}\delta p^{-s} P^{s-1} \quad \text{and} \quad \chi(p, \pi) = \frac{1}{2}\delta \pi^{-s} P^{s-1}.$$

It is readily checked that the marginal propensity to consume $a = [ph(p, \pi) + \pi\chi(p, \pi)]$ is constant and equal to δ Hence, in this case, the two extreme specifications – completely autonomous versus completely endogenous wealth – are formally equivalent. Indeed if, for the sake of our comparison, we equalize to zero all the government and overhead-labor[12] variables introduced by Weitzman as well as our variable $\hat{I} = \int_0^1 \hat{\imath}_\theta \nu(d\theta)$, we find for both specifications that

$$AK(p, \pi) = M/(1 - \delta),$$

so that we simply take $A = M$ and

$$H(p, \pi) = \frac{1}{2}\frac{\delta}{1 - \delta} p^{-s} P^{s-1} \quad \text{and} \quad X(p, \pi) = \frac{1}{2}\frac{\delta}{1 - \delta} \pi^{-s} P^{s-1}.$$

Computing the equilibrium in the goods markets given w, (p^*, π^*), we get (taking $\ell = \lambda$ as in Weitzman):

$$p^* = \pi^* = ((s + 1)/(s - 1))w\ell.$$

[12] The overhead labor variables here do not play the essential role they played in Weitzman [1982].

Correspondingly, total labor demanded at equilibrium as a function of w is

$$Z^*(w) = (s-1)\delta M/(s+1)(1-\delta)w.$$

It is strictly decreasing and unbounded.[13] Now this fact is justly seen by Weitzman as a way of reducing unemployment as much as wanted. Weitzman suggests doing it by introducing a pay function whereby, in addition to the base wage rate w, the workers should receive a share of the profit. More precisely, the payoff functions of the two producers become, for some profit-sharing parameter $\tau \in]0,1[$:

$$\begin{aligned} F_\tau(p,\pi,w) &= (1-\tau)(p-w\ell)H(p,\pi)M, \qquad \text{for } p \geq w\ell, \\ \Phi_\tau(p,\pi,w) &= (1-\tau)(\pi-w\lambda)X(p,\pi)M, \qquad \text{for } \pi \geq w\lambda. \end{aligned}$$

Since these are positive linear transformations of the previous payoff functions, we get the same equilibrium prices as before and, hence, the same labor demand as a function of w. Therefore, a possible policy to reach full employment, without affecting the workers' income (in a world without uncertainty), is to decrease the base wage w enough, and increase the profit-sharing parameter accordingly.

We want to challenge this conclusion. It is crucially based on the fact that total labor demanded at equilibrium is a function of w and that this function goes to infinity when w vanishes. This is a particular case. In general, Z^* is a correspondence, and it may be bounded, or it may contain bounded selections. This leads to our formal definition of involuntary unemployment in a strong or in a weak sense.

For a given economy, we say that there is *weak involuntary unemployment* whenever there exists $u_0 \in]0,1[$ such that for each positive wage w there is a multisectoral equilibrium (p^*, π^*),

[13] Weitzman [1985] speaks of a "symmetric Nash equilibrium in prices", and since he supposes a large number of firms, he computes it by assuming "that each firm i is justified in regarding its demand ... as a true function of only its own price p_i, with aggregate variables P ... parametrically fixed beyond its control". Then the equilibrium prices are $p^* = \pi^* = (s/(s-1))w\ell$ and $Z^*(w) = (s-1)\delta M/s(1-\delta)w$ is simply a linear transformation of the above expression. This means that Weitzman's argument still holds when using the regular Nash equilibrium concept.

depending upon w, verifying

$$L - Z(p^*, \pi^*, w) \geq u_0 L. \tag{7}$$

Alternatively, we say that there is strong *involuntary unemployment* whenever the same conclusion holds for *every* multisectoral equilibrium at any positive wage.

To get weak involuntary unemployment, we need to further restrict our class of examples. It must verify:

Property D. For any sequence of positive price vectors such that some price tends to zero, the corresponding total revenue function becomes increasing in this price for at least one producer.

To ensure this property, we simply introduce, by symmetry with C, the following restrictions on the parameters of the example:

$$\lim_{p \to 0} b(p) > 1 + \frac{s-1}{\sigma - 1} \text{ and } \lim_{\pi \to 0} \beta(\pi) > 1 + \frac{s-1}{\sigma-1}, \text{ in case(a),}$$

and the reverse inequalities in case (b).

We may now prove

Proposition 2 *Under Properties A to D there exists $\varepsilon > 0$ such that, for every economy with mean autonomous wealth A/L less than ε, there is weak involuntary unemployment.*

Proof: By contradiction, assume that, however small the mean autonomous wealth A/L may be, there exists an economy which does not display weak involuntary unemployment. Then there exists a sequence $(u_0^\tau, L^\tau, A^\tau, w^\tau, p^\tau, \pi^\tau)_{\tau \geq 1}$ such that u_0^τ and A^τ/L^τ both tend to zero, and, for all τ, (p^τ, π^τ) is an equilibrium in the goods markets at w^τ, violating inequality (7) or, equivalently, verifying the inequality:

$$1 - [\ell H(p^\tau, \pi^\tau) + \lambda X(p^\tau, \pi^\tau)](A^\tau/L^\tau) < u_0^\tau.$$

This implies that, at least for one good, demand is unbounded along that sequence. Hence by Property A2 some price must vanish. By Property D the corresponding demand elasticity becomes less than one for at least one sector, leading to an increasing profit function (see (5) and thus contradicting the fact that (p^τ, π^τ) is always an equilibrium in the goods markets. ■

In fact, the argument we have used to prove Proposition 2 entails a stronger result. It proves that, for a mean autonomous wealth small enough and any positive wage, *all* equilibria in the goods markets satisfy inequality (7). So, in order to get strong involuntary unemployment, it suffices to exclude multisectoral equilibria such that the feasibility constraint is binding for some producer (making an increasing profit function compatible with equilibrium). An example of such an equilibrium is given by the point E in Figure 3, where we have represented the limiting case of a zero wage.[14] The set of admissible prices $\tilde{Z}(0)$ is the hatched region, and $\tilde{p}(\pi, 0)$ and $\tilde{\pi}(p, 0)$ are the two producers' reaction curves (which intersect in the interior of $\tilde{Z}(0)$ at an unemployment multisectoral equilibrium point). Arrows at point E indicate the directions of increasing profits.

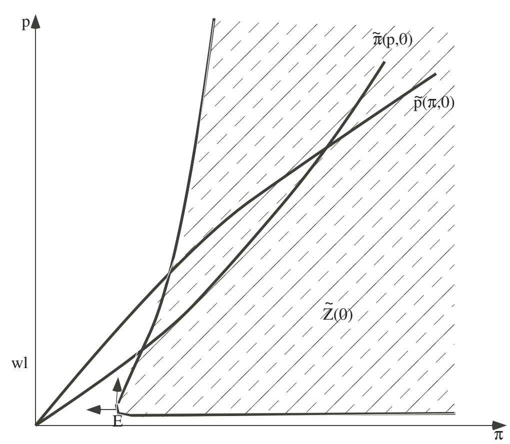

Figure 3:

In order to rule out multisectoral equilibria such as point E in Figure 3, still an additional property is needed.

Property E. For any sequence of positive price vectors such that some price tends to zero, the function $\ell H + \lambda X$ becomes decreasing in the price of a sector where the total revenue function

[14]The curves in Figure 3 have been calculated for the following values of the parameters: $\ell = 1, \lambda = 3$, and $s = \sigma = 3/2$. The price expectation functions were assumed to be identical and given by $\hat{p}(p) = (p^{-1} + p^{-5/2})^{-1}$.

becomes increasing in this price.

This property leads immediately to our main result.

Proposition 3 *Under Properties A to E there exists $\varepsilon > 0$ such that, for every economy with mean autonomous wealth A/L less than ε, there is strong involuntary unemployment.*

Proof: By contradiction, again take a sequence $(u_0^\tau, L^\tau, A^\tau, w^\tau, p^\tau, \pi^\tau)_{\tau \geq 1}$, as defined above, but where (p^τ, π^τ) is now a multisectoral equilibrium such that the labor constraint is binding for at least one producer. Since some demand is unbounded along that sequence, some price must tend to zero, and the profit function must become increasing in the corresponding sector, as seen in the proof of Proposition 2. By Property E the function $\ell H + \lambda X$ becomes decreasing in the price of such a sector. Thus, the producer will want to increase his price and will not be constrained by the labor supply, so that, for τ large enough (p^τ, π^τ) cannot be a multisectoral equilibrium. ■

It remains to verify that Property E is satisfied for some values of the parameters of our class of examples.

In case (b) this is simply because H and X are decreasing in both prices. Property E holds immediately. In case (a) we assume that the price expectations satisfy the following additional restrictions: $b(0) = \beta(0) = B$ and for $0 < c < \infty$, $0 < \gamma < \infty$, $\lim_{p \to \infty}[\hat{p}(p)/p^B] = c$ and $\lim_{\pi \to 0}[\hat{p}(\pi)/\pi^B] = \gamma$.

Consider the elasticity of $\ell H + \lambda X$ in p:

$$\frac{\ell H'_p + \lambda X'_p}{\ell H + \lambda X} p = \frac{(s-\sigma)\pi^{s-1}}{p^{s-1} + \pi^{s-1}} + \frac{(\sigma - 1)b(p)\hat{\pi}^{s-1}}{\hat{p}^{s-1} + \hat{\pi}^{s-1}} - \frac{s\ell\pi^s}{\lambda p^s + \ell\pi^s}. \tag{8}$$

The elasticity of $\ell H + \lambda X$ in π is defined analogously.

If only one price tends to zero, then the corresponding elasticity tends to $[-\sigma + (\sigma - 1)B]$ which has been assumed to be nonpositive. In fact, the case where it is zero can be neglected since

$$\lim_{p \to 0}(\sup_{\pi > 0} H(p, \pi)) = \lim_{p \to 0}(2\alpha(\hat{p}^{\sigma-1}/p^\sigma)) = 2\alpha c^{\sigma-1} < \infty,$$

and similarly,

$$\lim_{\pi \to 0} (\sup_{p > 0} X(p, \pi)) < \infty,$$

so that we have demand saturation at a nil price, leading trivially to strong involuntary unemployment. The case where it is negative gives what we need by Property D.

Therefore, to verify Property E, the difficult case is when we have a sequence $(p^\tau, \pi^\tau)_{\tau \geq 1}$ of prices converging both to zero with, say, $\lim_{\tau \to \infty} (p^\tau / \pi^\tau) = q$. Since, by Property D, at least one of the total revenue functions becomes increasing, (i.e., e or η becomes less than one), it is enough to show that $\eta(p^\tau, \pi^\tau) \geq 1$ implies $\ell H_p' + \lambda X_p' < 0$, for infinitely many τ (and the symmetric implication). By (4) and (8) this amounts to showing, assuming $\sigma = s$ for simplicity, that

$$\frac{\beta(\pi^\tau)[p^\tau(\pi^\tau)/\hat{\pi}(\pi^\tau)]^{s-1}}{1 + [\hat{p}(p^\tau)/\hat{\pi}(\pi^\tau)]^{s-1}} \leq 1$$

implies

$$\frac{(s-1)b(p^\tau)}{1 + [\hat{p}(p^\tau)/\hat{\pi}(\pi^\tau)]^{s-1}} < \frac{s}{1 + (\lambda/\ell)(p^\tau/\pi^\tau)^s}$$

for infinitely many τ. Since $\lim_{\tau \to \infty} [\hat{p}(p^\tau)/\hat{\pi}(\pi^\tau)] = (c/\gamma)q^B$, the above implication leads, at the limit, to

$$q \leq \left(\frac{\gamma}{c}\right)^{1/B} [B-1]^{-1/B(s-1)}$$

implies

$$\frac{1 + (\gamma/\ell)q^s}{1 + (c/\gamma)^{s-1} q^{B(s-1)}} < \frac{s}{B(s-1)}.$$

The last inequality is satisfied for $q = 0$ (if not, demand is saturated at a nil price and strong involuntary unemployment is a trivial result). As its left-hand side is a quasi-convex function of q, it suffices to impose that it also be satisfied when q is equal to its maximum admissible value. By symmetry this finally leads to the required condition,

$$\left[1 + \frac{s}{s-1}) - B\right]^{-1} [B-1]^{1-(s/B(s-1))} < \frac{\lambda}{\ell} \left(\frac{\gamma}{c}\right)^{s/B} < \left[1 + \frac{s}{s-1} - B\right] [B-1]^{(s/B(s-1))-1}. \quad (9)$$

The admissible range for $(\lambda/\ell)(\gamma/c)^{s/B}$ is decreasing in s. As s increases from 1 to 2, more and more symmetry is imposed upon unit labor costs and price expectations at zero. Without this symmetry one cannot exclude the persistence of equilibria in which the feasibility constraint is binding for at least one producer (such as the one represented by point E in Figure 3) as A/L tends to zero.

5 Possible extensions

In order to give an idea of the robustness of the preceding results about involuntary unemployment, let us consider some natural extensions (or modifications) of our model.

1. First, we may increase the number of sectors, without invalidating Propositions 1 to 3. But, in order that Properties C and D hold, we need the elasticities of price expectation to take values below and above $1 + (n-1)((s-1)/(\sigma-1))$. This implies that the elasticity of price expectation takes, for some range of prices, higher and higher values as n increases. In the complementarity case Property C (and hence existence) becomes more and more difficult to obtain. In the substitutability case Property D becomes more restrictive and, combined with Property A, leads to

$$1 + (n-1)\frac{s-1}{\sigma-1} < \frac{\sigma}{\sigma-1} \text{ implying that } 1 < s < \frac{n}{n-1}.$$

Clearly, the admissible interval decreases as n increases. More competition makes the occurrence of involuntary unemployment more unlikely.

2. In the class of examples we have discussed, we have maintained a constant elasticity of intertemporal substitution and imposed strong restrictions on price expectations. These restrictions could be weakened by allowing a variable elasticity of intertemporal substitution. For instance, take the case of rigid price expectations, i.e., $\hat{p}$ and $\hat{\pi}$ exogenously given. By (1) and (2) we may write

$$H(p,\pi) = \frac{1}{2(p/P)^s P} \cdot \frac{a}{1-a} = \frac{1}{2(p/P)^s \hat{P}} \cdot \frac{a/P}{(1-a)/\hat{P}},$$

where a is a function of $P/\hat{P}$.

Since the term

$$\frac{a/P}{(1-a)/\hat{P}}$$

is the ratio of present real consumption to future real consumption, its elasticity with respect to $\hat{P}/P$ is nothing else than the elasticity of intertemporal substitution σ, which is now a function of $P/\hat{P}$. From this (or using equation (4) with $b(p) \equiv 0$) it is easy to check that

$$e(p,\pi) = s - s\frac{\partial P}{\partial p}\frac{p}{P} + \sigma\frac{\partial P}{\partial p}\frac{p}{P} = \frac{\pi^{1-s}}{p^{1-s}+\pi^{1-s}}s + \frac{p^{1-s}}{p^{1-s}+\pi^{1-s}}\sigma;$$

i.e., the price elasticity of H is a convex combination of the (constant) elasticity of intersectoral substitution s and the elasticity of intertemporal substitution σ. Therefore, imposing that σ be a differentiable nondecreasing function of $P/\hat{P}$, taking values below and above $2-s$ (in the general case: $n-(n-1)s$), and upperbounded by s if $s>1$, we can verify Properties A, B, C, and D. As for Property E, this is still the case for $s<1$. If $s>1$, a longer calculation along the lines of the preceding section shows that, denoting by σ_0 the limit of σ when $P/\hat{P}$ tends to zero, the condition for Property E to hold is,

$$\left[1+\frac{\sigma_0}{s-1}\right]^{-1}\left[\frac{1-\sigma_0}{s-1}\right]^{-1/(s-1)} < \frac{\lambda}{\ell} < \left[1+\frac{\sigma_0}{s-1}\right]\left[\frac{1-\sigma_0}{s-1}\right]^{1/(s-1)}.$$

3. A further extension of our results is to apply them to a purely atemporal version of our model, thereby avoiding the use of any kind of arbitrary price expectations. This consists in taking, as in Hart [1982], a utility function depending for every *theta* on the consumption of the produced goods $(c_\theta, \gamma_\theta)$ and of a nonproduced good k_θ. If this function is of the form $U(u(c_\theta,\gamma_\theta),k_\theta)$, with U and u having the same properties as before, the preceding derivations can be reinterpreted straightforwardly: let $\hat{P}$ be the price of the nonproduced good normalized to one and, for consumer θ, m_θ be the endowment of consumer θ in the nonproduced good, $\tilde{m}_\theta$ be replaced by k_θ (the consumption of the nonproduced good), and of course, $\hat{\imath}\theta$ be zero. Reinterpreting σ as the elasticity of substitution between the produced goods and the nonproduced one, we are immediately led to Properties A-E by the same restrictions. An example[15] of such a utility function is given by Silvestre [1988], where U is a C.E.S. function in u and k_θ modified by a linear term in k_θ.

[15] Another example, not derived from a C.E.S. function, is given in d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1989].

4. Finally, one can use a modified version of this model to derive results analogous to those in Weitzman [1982]. The modification consists of suppressing the nonproduced good, of adding fixed costs for each producer in the form of a given quantity of overhead labor needed to produce a positive amount, and introducing a zero-profit condition. One way to treat this modified model is to use the second specification of the feedback effects, namely those limited to the wage income (as discussed in Section 2 above), with A equal to the sum of fixed labor costs.

In the absence of a nonproduced good, labor can be taken as the numeraire and w put equal to one. Also the propensity to consume the produced goods is now one. Hence we get

$$h(p, \pi) = \frac{1}{2p(p/P)^{s-1}}.$$

Using a similar expression for χ, we may compute the multiplier to be

$$\frac{1}{1-(\ell h + \lambda\chi)} = \frac{2P^{1-s}}{(p-\ell)p^{-s} + (\pi-\lambda)\pi^{-s}},$$

leading to

$$H(p, \pi) = \frac{\pi^s}{(p-\ell)\pi^s + (\pi-\lambda)p^s}$$

and

$$e(p, \pi) = \frac{(p-\ell)\pi^s}{(p-\ell)\pi^s + (\pi-\lambda)^s}\frac{p}{p-\ell} + \frac{(\pi-\lambda)\pi^s}{(p-\ell)\pi^s + (\pi-\lambda)p^s}s.$$

Properties A1, B, and C are readily verified if $s > 1$, so that existence of an equilibrium in the goods markets is ensured. Taking A/L low enough, we get unemployment.

6 Conclusion

This work has presented a simple general equilibrium model of imperfect competition in prices. The purpose was to find a class of examples in which involuntary unemployment occurs and can be unambiguously attributed to oligopolistic competition in the goods markets. The exercise has been made difficult in several respects. First, trivial cases, due to bounded productive capacities or due to saturated demand, have been excluded. Second, the different producers have been assumed to conjecture objective demand curves and, unlike previous work of this kind,[16] to allow both for cross-sectoral price effects and for all kinds of income feedback effects. Third, we have stuck to utility functions with a constant elasticity of intersectoral substitutions. When the elasticity of intertemporal substitution is also constant, sensitivity is obtained by varying the elasticity of expectation of future prices. Alternatively, allowing for a variable elasticity of intertemporal substitution, while keeping the elasticity of expectation null, permits the same results to be reached, which therefore apply even to an atemporal economy (where future consumption is replaced by a nonproduced good). A clear distinction has been made between the complementarity case and the substitutability case. It is in this latter case, where the cross-sectoral price effect is positive, that the difficulty is greatest.

Among the properties that carry the results through, the main one is Property D. It ensures that the total revenue of some producer becomes increasing in price (or decreasing in quantity) when this or both prices go to zero.[17] Hence, as the wage vanishes, the equilibrium prices (in the goods markets) will not go to zero, thus excluding a complete Pigou effect and the achievement of full employment. This is enough to get weak involuntary unemployment, i.e. unemployment at some multisectoral equilibrium given any positive wage; and even, in the case of complementary goods, strong involuntary unemployment, i.e., unemployment at all multisectoral equilibria given any positive wage. In contrast, under perfect competition, where prices equal marginal costs, the Walrasian equilibrium would realize full employment at some positive equilibrium wage. To obtain strong involuntary unemployment in the substitutability case, one needs the additional Property E, implying some kind of symmetry (and the more so, the greater is the substitutability) in order to exclude multisectoral equilibria where some producer is off his reaction curve because the labor supply constraint is binding.

It is worth mentioning that these results are still meaningful when the total labor supply is not perfectly inelastic and is not positive (or undetermined) at zero wage. Because of imperfect

[16] We are thinking of Hart [1982]; d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1984, 1989]; Dehez [1985]; and Silvestre [1988].

[17] As well stressed by Silvestre [1988], Hart [1982] makes the opposite assumption that total revenue should always be increasing in quantity. The same property is implied by a Cobb-Douglas intertemporal utility function, combined with a constant elasticity of intersectoral substitution larger than one, as in Weitzman [1985].

competition, the real wage, adjusted to clear the labor market, would still be inferior to labor marginal productivity, and to the Walrasian equilibrium wage, and any multisectoral equilibrium employment would be less than the Walrasian employment level. Moreover, as we have seen when discussing our weak involuntary unemployment concept, we could get at the same adjusted positive wage different multisectoral equilibria, some implying full employment but (and this is the point) others not. The market failure would then result from the multiplicity of equilibria (as in Heller [1986] or Roberts [1989]) leading, by lack of coordination, to persistent underemployment.

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