Unemployment in an Extended Cournot Oligopoly Model*

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1 Introduction

Attempts to explain unemployment[1] begin with the labour market. Yet, by attributing it to deficient demand for goods, Keynes questioned the use of partial analysis, and stressed the need to consider interactions between the labour and product markets. Imperfect competition in the labour market, reflecting union power, has been a favoured explanation; but imperfect competition may also affect employment through producers' oligopolistic behaviour. This again points to a general equilibrium approach such as that by Negishi (1961, 1979).

Here we propose an extension of the Cournot oligopoly model (unlike that of Gabszewicz and Vial (1972), where labour does not appear), which takes full account of the interdependence between the labour market and any product market. Our extension shares some features with both Negishi's conjectural approach to demand curves, and macroeconomic non-Walrasian equi-

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1See the recent survey by Lindbeck and Snower (1985).

librium theories, usually associated with the fix-price approach.[2] For Negishi, economic agents base decisions on perceived demand (or supply) curves which are purely "subjective" except at equilibrium; in contrast our conjectured demand curves are, in Nikaido's (1975) terminology, "objective".[3]

We start by defining a Cournot-Nash equilibrium for a single sector (viewed as a "representativie industry") and at a money wage taken as given by all agents. If one follows temporary general equilibrium theory (see Grandmont (1977)) with its particular view of money, and admits the possibility of quantity rationing in the labour market, one can view this as a "short-run" equilibrium and suppose that the wage is chosen at some preliminary stage. The model also has a "static" interpretation, where money is a nonproduced good and the nominal wage is a parameter which is competitively adjusted. In the following we shall always maintain the two interpretations.

This paper aims to give sufficient conditions first for the existence of a Cournot-Nash equilibrium in an objective demand framework and then for the existence of unemployment at any positive wage level (what we call "involuntary unemployment"). The idea can be described simply for a single produced good. In the labour market we assume an inelastic labour supply at any positive wage level. First we construct an oligopoly model which allows for the labour supply constraint and the indirect effects of producers' decisions on the demand they face. Sufficient conditions are given for the existence of such an extended Cournot equilibrium at any given money wage. Then we want to show that, at all positive wage levels, equilibrium entails unemployment. For that, allowing some wealth to be in the form of money and assuming that the elasticity of demand is low for small prices, we find that the prices associated with the Cournot equilibrium do not tend to zero even when the money wage tends to zero (just as in Cournot's own model with zero marginal costs). Demand is positive if the money stock is. But if money is low enough, the equilibrium price level and elasticity of demand are then small enough to equate the marginal revenue product of labour with the money wage (possibly zero) at some

[2]For a general survey see Drazen (1980).

[3]The survey of Hart (1985) is organized around this distinction.

small equilibrium level of employment. This could not happen under perfect competition. As the money wage tends to zero, competitive prices vanish and full employment is reached unless demand is saturated or productive employment is bounded.

Our definition of involuntary unemployment is stronger than the usual one: "at the prevailing real wages, workers unsuccessfully seek jobs for which they are just as qualified as the current job holders" (type 1 involuntary unemployment, Lindbeck and Snower (1985)). We require unemployed workers willing to work not only at the prevailing wage (whatever this may be) but at any positive money wage that could have been preliminary fixed. This, we feel, is closer to Keynes' idea that unemployment is involuntary when there is "*no* method available to labour as a whole" for attaining full employment "by making revised *money* bargains with the entrepreneurs" (Keynes (1936), p. 13).

This definition also differs from the idea that involuntary unemployment occurs, when "workers unsuccessfully seek work at real wages which fall short of their potential contribution to society" (type 2 involuntary unemployment, Lindbeck and Snower (1985)). This is really a notion of "underemployment", in the sense either that there are multiple oligopolistic equilibria, with more or less employment, that may be Pareto-ranked (see e.g. Heller (1986) or Roberts (1986)), or that the level of employment is less at the oligopolistic equilibrium than at the competitive equilibrium (see e.g. Hart 1982)). All markets, including labour, clear. Yet the distinction between these notions of underemployment and our concept is less important than might appear. In the static interpretation of our model we recover market-clearing with perfectly flexible wages (either by allowing a zero wage or by having a supply of labour that is not perfectly inelastic), and then our involuntary unemployment result can be reinterpreted accordingly as an underemployment result of one of the above types. Our Cournot-Nash equilibrium, with full-employment at a zero wage, is compatible with a competitive equilibrium with full employment at a positive wage, and the different oligopolistic equilibria obtained by varying the wage level could be Pareto-ranked. The essence of our unemployment result is this: despite assuming neither demand saturation nor bounded productive employment, and allowing for correctly perceived feedback effects, the total labour demand correspondence, generated by computing the different Cournot-Nash equilibria corresponding to different wage levels, is bounded. We therefore leave our model open. We do not need to specify how the wage level is fixed. We think that a plausible way to close the model is to introduce a first stage at which the wage is decided either by the firms (as in Roberts (1986)), or by the workers through their unions (as in Hart (1982) or Silvestre (1988)), or even by both through some sort of bargaining process (as in Dixon (1987)). According to the static interpretation, the model can also be closed by varying the wage level parametrically down to zero, the labour market clearing level. Involuntary unemployment is thus tantamount to a general equilibrium with a zero wage in an economy where product markets are oligopolistic but the labour market is competitive. Such a phenomenon appears in the Keynesian model with monopolistic competition of Grandmont and Laroque (1976), where it depends upon arbitrary pessimistic expectations of firms.

The problem with describing demand curves faced by producers as "objective" is essentially related to the income variable appearing in the demand function. It creates a particular kind of multiplier effect. That this effect may be allowed for by producers themselves was clear in the mind of Henry Ford:

> *"I believe in the first place that, all other considerations aside, our own sales depend in a measure upon the wages we pay. If we can distribute high wages, then that money is going to be spent and it will serve to make storekeepers and distributors and manufacturers and workers in other lines more prosperous and their prosperity will be reflected in our sales. Country-wide hide wages spell country-wide prosperity, provided, however, the higher wages are paid for higher production."* [(1922), p. 124]

The "Ford effect" complicates the definition of an oligopolistic equilibrium. One tack is to make producers take the total wage income plus the total distributed profits as given. This parameter is adjusted at equilibrium. That is the approach of Marschak and Selten (1974) and more recently Hart (1982, 1985) who calls it the "no feedback effects" assumption.[4] We make a different assumption – more plausible if firms are seen as knowing the "true" demand function – and

[4] By introducing two (types of) firms and two (types of) consumers, each one consuming the goods produced by the firm that does not employ him, Roberts (1986) and Heller (1986) are following the same assumption.

make producers take all feedback effects into account. Alternatively one could model producers as only recognizing feedback effects going through total wage income. This may seem natural since, on the one hand, for any producer, the wage as well as the employment levels of the other producers are supposed to be known by him when fixing his own employment level and since, on the other hand, the consumers know precisely whether they are employed when they decide on their consumption. But one would then retain the Marschak and Selten (1974) assumption on distributed profits: they are treated parametrically and adjusted at equilibrium. Such an intermediate approach found in d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1989) complicates the proofs. The present paper will include the feedback effects due to distributed profits.

In this framework we establish existence of an oligopolistic equilibrium under assumptions that include convexity of producers' technologies and homotheticity of (identical) consumers' preferences. This keeps the model closer to the usual partial equilibrium Cournot model – but does not require identical firms (as in Hart (1982), Heller (1986)[5]) nor, as often assumed, demand saturation at a nil price (see e.g. Novshek (1985)). This last assumption would render an "involuntary unemployment" result trivial.

The paper is organized thus. Section 2 introduces the model, defines an extended Cournot-Nash equilibrium, and presents assumptions ensuring existence. Section 3 defines involuntary unemployment and analyses its possibility. We conclude in Section 4.

2 An extended Cournot-Nash equilibrium

We analyse an economy with an oligopolistic market for a homogeneous product and a labour market. There is also a nonproduced good, money.

A finite set $N = \{1, 2, \cdots, i, \cdots, n\}$ of firms produces the homogeneous produced good. Each firm in this set is characterized by a real cost function ℓ_i which, for every level y_i of output,

[5] For the existence of a Cournot equilibrium for identical firms facing a decreasing demand, with nondecreasing marginal costs but without concave profit functions, see McManus (1964) and Roberts and Sonnenschein (1976).

indicates the minimal amount of labour $z_i = \ell_i(y_i)$ required. We assume that competition between these firms is oligopolistic "à la Cournot" and that they have a complete knowledge of the demand behaviour of the consumers. Y and Z denote total production and total employment.

The set of consumers coincides with the set of potential workers and is represented by an interval $[0, L]$. For simplicity, each individual supplies one unit of labour whatever the nonnegative money wage w. Hence L is the total inelastic supply of labour. Our main restriction concerns the demand for the produced good. We assume that the consumers differ only in income and wealth; all have the same homothetic preferences. This implies that aggregate demand is multiplicatively separable and can be written for a price $p > 0$ of the produced good, as

$$h(p)[M + (pY - wZ) + wZ],$$

where $(pY - wZ)$ denotes aggregate profits, wZ the total wage income and $M > 0$ the total monetary wealth in the economy – which is the total initial endowment of the non produced good for the static interpretation, and includes past savings as well as the present value of expected future income for the temporary equilibrium interpretation. This will keep us close to the usual partial equilibrium Cournot model. Moreover, assuming along this line that the producers serve total demand (i.e., $Y = h(p)[M + pY]$), we get

$$Y = \frac{h(p)M}{1 - ph(p)}.$$

The next two assumptions ensure existence of a differentiable inverse demand function. The first is rather standard:

[1] The function h, defined and continuous for all positive prices, is non-negative and, whenever positive, twice continuously differentiable and strictly decreasing. Also,

$$h(0) \stackrel{\text{def}}{=} \lim_{p \to 0} h(p) \in (0, \infty] \text{ and } \lim_{p \to \infty} h(p) = 0.$$

The second restricts the marginal propensity to consume $ph(p)$. It states (as in Hart (1982)) that the desired ratio of the two goods,

$$H(p) \stackrel{\text{def}}{=} \frac{h(p)}{1 - ph(p)}$$

is nonnegative, finite and decreasing in p whenever positive. More conveniently:

[2] For all $p > 0$ and whenever $h(p) > 0, ph(p) < \min\{1, \eta(p)\}$, where

$$\eta(p) \stackrel{\text{def}}{=} -\frac{h'(p)}{h(p)}p$$

denotes the (Marshallian) price elasticity of the demand function. The elasticity of substitution between the two goods

$$\sigma(p) \stackrel{\text{def}}{=} -\frac{H'(p)}{H(p)}p = \frac{\eta(p) - ph(p)}{1 - ph(p)}$$

is simply assumed to be positive and finite for any positive desired ratio of the two goods.

Letting $m = M/L$ be the per capita monetary wealth, we may define an "inverse demand function" ψ parameterized in $m > 0$ and $L > 0$, i.e., $\psi(Y, m, L) = p > 0$ if and only if $Y = H(p)Lm > 0$. From **[1]** and **[2]**, ψ exists and is equal to $H^{-1}(Y/Lm)$ for $Y \in (0, h(0)Lm)$. For $Y = 0$, either $\psi(0, m, L) = \infty$ or $\psi(0, m, L)$ is the least price p for which $h(p) = 0$ (if there is a fine reservation price). Lastly, $\psi(Y, m, L) = 0$ for $Y \geq h(0)Lm$ (if demand is saturated at a nil price).

For any firm $i \in N$, the profit associated with any chosen level of production $y_i \geq 0$, given the choices of the other firms, say $y_{-i} = (y_1, \cdots, y_{i-1}, y_{i+1}, \cdots, y_n) \in \mathbb{R}_+^{n-1}$, is the function parameterized in $w \geq 0$, $m > 0$ and $L > 0$:

$$\Pi_i(y_i, y_{-i}, w, m, L) \stackrel{\text{def}}{=} y_i \psi\left(y_i + \sum_{j \neq i} y_j, m, L\right) - w\ell_i(y_i).$$

For given values of the parameters w, m and L we get a standard Cournot oligopoly model based on an "objective" inverse demand function, incorporating all feedback effects, except that we now also need to consider the constraint from the explicit representation of the labour market, that is, the choice of the y_i's is restricted by the condition $\sum_{i=1}^{n} \ell_i(y_i) \leq L$.

An *extended Cournot-Nash equilibrium* for given possible values of the parameters w, m and L is a vector $y^* \in \mathbb{R}_+^n$ such that, for all $i \in N$, y_i^* is a solution of the program:

$$\max_{y_i \geq 0} \Pi_i(y_i, y_{-i}^*, w, m, L) \text{ subject to } \ell_i(y_i) \leq L - \sum_{j \neq i} \ell_j(y_j^*).$$

For any possible values of the parameters w, m and L we denote by $\mathcal{Y}(w, m, L) \subset \mathbb{R}_+^n$ the set of extended Cournot-Nash equilibria.

The next set of assumptions ensures the existence of a Cournot-Nash equilibrium for any value of the parameters w, m and L:

[3] $\lim_{p \to \infty} ph(p) = 0$.

[**4**] For all $p > 0$ and whenever

$$h(p) > 0, \frac{\sigma'(p)}{\sigma(p)} p > -|1 - \sigma(p)|,$$

where σ is the elasticity of substitution between the two goods, as defined above.[6]

Assumption [**3**] makes the profit functions continuous at the origin.[7] Assumption [**4**][8] implies that the ith firm's marginal revenue is decreasing in output whenever it is non-negative and whatever the value of $y_{-i} \in \mathbb{R}_+^{n-1}$. Examples of demand functions explicitly derived from consumers' utility maximizing programs, both in a static and in a temporary equilibrium context, are given at the end of the next section.

In Theorem 3 of Novshek (1985), saturation at a nil price and his condition on the inverse demand function (which would imply in the present model: $\sigma'(p)p/\sigma(p) \geq 1$) are stronger than what we have here on the demand side. However, by his Theorem 4, we know that our assumptions on demand are too weak to get existence while keeping his minimal assumptions on the production side. We must assume convex cost functions ore, more precisely:

[**5**] For any $i \in N$, the function ℓ_i, defined and twice continuously differentiable for all positive values of y_i, is positive, convex and strictly increasing. Also,

$$\ell_i(0) = \lim_{y_i \to 0} \ell_i(y_i) = 0.$$

[6] [**4**] is equivalent to having the ratio of expenses $pH(p) = H^{-1}(Y/M)(Y/M)$ concave whenever increasing, as a function both of p and of Y/M.

[7] It could be weakened by imposing that $ph(p)$ be decreasing in p for arbitrarily large values of p.

[8] Assumption [**4**] could be weakened if Ford effects were neglected, or limited to wages as in d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1989), where condition [4] is imposed on η, not on σ. As it stands, [4] implies that $\eta' p/\eta$ is larger than $-|1 - \eta|$ if $\eta \geq 1$, but larger than $-|1 - \eta| \left[1 - \frac{2ph(1-\eta)}{\eta(1-ph)}\right] > -|1 - \eta|$ if $\eta < 1$.

Given these assumptions, we can use Debreu (1952) "Social Equilibrium Existence Theorem" to obtain $\mathcal{Y}(w, m, L) \neq \emptyset$ for all values of the parameters $w \geq 0$, $m > 0$ and $L > 0$. Indeed by **[5]**, for each $i \in N$, the correspondence

$$y_{-i} \to \left\{ y_i \geq 0 \mid \ell_i(y_i) \leq \max \left\{ 0, L - \sum_{j \neq i} \ell_j(y_j) \right\} \right\}$$

from $\mathbb{R}_+^{n-1}$ to $\mathbb{R}_+$, restricting the admissible strategies, is continuous, has a nonempty compact graph and is convex-valued, so that the function

$$\max_{\{y_i \geq 0 \mid \ell_i(y_i) \leq \max\{0, L - \sum_{j \neq i} \ell_j(y_j)\}\}} \Pi_i(y_i, y_{-i}, w, m, L)$$

is continuous in y_{-i}. The function ℓ_i is convex. Using **[4]** we then find that the profit function Π_i is strictly quasi-concave in y_i: it is enough to check that, whenever

$$\frac{\partial}{\partial y_i} \Pi_i(y_i, y_{-i}, w, m, L) = 0,$$

we must[9] have

$$\frac{\partial^2}{\partial y_i^2} \Pi_i(y_i, y_{-i}, w, m, L) < 0.$$

Moreover,

$$\lim_{y_i \to 0} \frac{\partial}{\partial y_i} [\psi(y_i, m, L) y_i] > 0,$$

as $\lim\limits_{y_i \to 0} \psi(y_i, m, L) y_i = 0$ by **[3]**, and $\ell_i'(0) < \infty$ by **[5]**, so that $\lim_{w \to 0} [-w \ell_i'(0)] = 0$. Hence,

$$\frac{\partial}{\partial y_i} \Pi_i(0, 0, w, m, L) > 0$$

for sufficiently small values of w. The consequences can be summarized by

[9] For any $Y \in (0, h(0)Lm)$, the derivatives of $\psi(\cdot, m, L)$ are $\psi_Y = \frac{1}{H'M}$ and $\psi_{YY} = -\frac{H''}{(H')^3 M^2}$, where H' and H'' denote the derivatives of H at $p = \psi(Y, m, L)$. Hence, $\frac{\partial}{\partial y_i} \Pi_i(y_i, y_{-i}, w, m, L) = \psi + y_i \psi_Y - w\ell_i' = p\left(1 - \frac{y_i/Y}{\sigma(p)}\right) - w\ell_i'$ and $\frac{\partial^2}{\partial y_i^2} \Pi_i(y_i, y_{-i}, w, m, L) = 2\psi_Y + y_i \psi_{YY} - w\ell_i'' \leq \psi_Y (2 + y_i \psi_{YY}/\psi_Y) = \psi_Y \left(2 + \frac{y_i/Y}{\sigma} \frac{H''p}{H'}\right)$. By assumption **[4]**, $\frac{\sigma' p}{\sigma} = \frac{H''p}{H'} + 1 + \sigma > -|1 - \sigma|$. Hence, at $y_i > 0$, if $\sigma \geq 1$, $\frac{\partial^2}{\partial y_i^2} \Pi_i < 2\psi_Y \left(1 - \frac{y_i}{Y}\right) \leq 0$ (as $\psi_Y < 0$), and if $\sigma \leq 1$, $\frac{\partial^2}{\partial y_i^2} \Pi_i < 2\psi_Y \left(1 - \frac{y_i/Y}{\sigma}\right) \leq 0$, whenever $\frac{\partial}{\partial y_i} \Pi_i = 0$ $\left(\text{implying } 1 \geq \frac{y_i/Y}{\sigma}\right)$.

Theorem 1 *For any $m > 0$, $L > 0$ and $w \geq 0$, there exists an extended Cournot-Nash equilibrium, i.e. $\mathcal{Y}(w, m, L) \neq \emptyset$, by assumptions* **[1]**– **[5]** *Moreover, there exists $\overline{w} > 0$ such that for any $w \in [0, \overline{w})$ no equilibrium is trivial: $y^* \in \mathcal{Y}(w, m, L)$ implies $Y^* > 0$.*

We remark in addition that for $n = 1$, the equilibrium is simply the monopoly solution, which is unique.

3 Existence of involuntary unemployment

For fixed values of the parameters, and for any equilibrium $y^* \in \mathcal{Y}(w, m, L)$, unemployment obtains if (and only if) $Z^* \stackrel{\text{def}}{=} \sum_{i=1}^{n} \ell_i(y_i^*) < L$. Could a change in the wage level reduce, or even eliminate unemployment? Such a question leads to a definition of involuntary unemployment. Let

$$\mathcal{Z}(w, m, L) \stackrel{\text{def}}{=} \left\{ Z \in \mathbb{R}_+ : Z = \sum_{i \in N} \ell_i(y_i), \text{ for some } y \in \mathcal{Y}(w, m, L) \right\}.$$

For fixed values of m and L, $\mathcal{Z}(\cdot, m, L)$ gives the total labour equilibrium demand correspondence. We say that there is *involuntary unemployment given m and L* if, for any selection $\hat{Z}$ for $\mathcal{Z}(\cdot, m, L)$:

$$\inf_{w \geq 0} \{ L - \hat{Z}(w) \} > 0.$$

To illustrate this notion simply, we may suppose that the correspondence $\mathcal{Z}(\cdot, m, L)$ can be represented by a continuous curve as shown on Figure 1. The extent of involuntary unemployment is given by the distance I. One could adjust the nominal wage parametrically to zero and then talk about "underemployment" (we leave this semantic decision to the reader). Indeed Figure 1 also depicts the usual demand for labour D, in a pure competitive world, where the price of the produced good is fixed at its Walrasian equilibrium level: it will lead, in the case shown on the figure, to a positive equilibrium wage w^*.

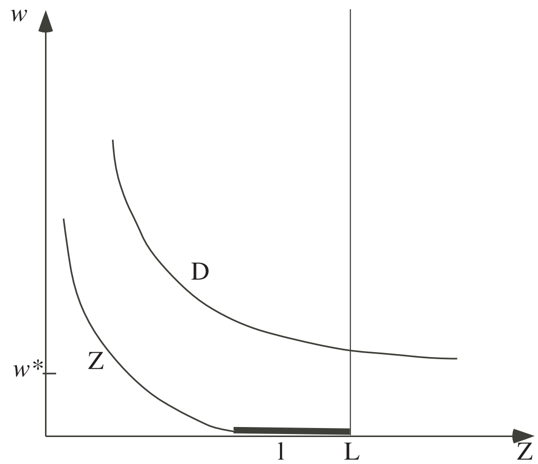

Fig. 1

What is the likelihood of involuntary unemployment? Consider the following argument by contradiction. If involuntary unemployment never occurs, then for any m, however small, given arbitrary values of L or M, there will always be some wage w (possible 0) and some equilibrium $y \in \mathcal{Y}(w, m, L)$, such that $\sum_{i=1}^{n} \ell_i(y_i) = L$. In other words, we can pick a sequence $\{m^s, L^s\}_{s=0}^{\infty}$ such that $\lim_{s\to\infty} m^s = 0$, and either $L^s = L > 0$, for all s, or $L^s m^s = M^s = M > 0$, for all s. And accordingly, for every s, there will be a wage w^s and an equilibrium $y^s \in \mathcal{Y}(w^s, m^s, L^s)$ such that $\sum_{i=1}^{n} \ell_i(y_i^s) = L^s$, i.e., full employment will always be achieved. For such a sequence in the next theorem we distinguish two cases. In the first case we suppose that $\lim_{s\to\infty} w^s > 0$ and we will be led to a contradiction by the assumptions we have already made. In the second case we suppose that $\lim_{s\to\infty} w^s = 0$ and we will need the additional assumption:

[6] $\lim_{p\to 0} \sigma(p) < 1/n$,

implying some complementarity between the two goods for large desired ratios of the produced to the non-produced good (and the more so, the larger the number of firms, i.e. the larger the degree of competition in the output market).

Our main result is then:

Theorem 2 *Under assumptions* **[1]** *to* **[5]**, *given any $L > 0$ (alternatively, given any $M = Lm > 0$) and any $\underline{w} > 0$, there is a low enough $\underline{m} > 0$ such that for all $m \in (0, \underline{m})$,*

$$\inf_{w \geq \underline{w}} \{L - \hat{Z}(w)\} > 0, \text{ for any selection } \hat{Z} \text{ for } \mathcal{Z}(\cdot, m, \ell).$$

If, in addition, Assumption **[6]** *holds, then the same inequality holds true for $\underline{w} = 0$, i.e. there is involuntary unemployment.*

The proof is given in appendix. But we can make the following observation. Consider the equilibrium total production:

$$Y^s = \frac{h(p^s)}{1 - p^s h(p^s)} L^s m^s.$$

As m^s goes to zero, Y^s must remain large enough so that $\sum_{i=1}^{n} \ell_i(y_i^s) = L^s$. Thus, whether L^s is kept constant and $M^s = Lm^s$ tends to zero, or M^s is kept constant and $L^s = M/m^s$ (and hence, by **[5]**, Y^s) tends to infinity, $h(p^s)/[1 - p^s h(p^s)]$ should go to infinity. But, by **[2]** and **[3]**, $1 - p^s h(p^s)$ is positive and bounded away from zero at least as long as p^s is bounded away from zero. Then $h(p^s)$ should tend to infinity. Therefore, using **[1]**, $p^s = \psi(Y^s, m^s, L^s)$ tends to zero anyway. The proof consists of showing that this leads to a contradiction.

How restrictive is Assumption **[6]**? It becomes more restrictive as the number of firms increases. At the limit, when the economy is perfectly competitive, it cannot be fulfilled since $\sigma(p) = 0$ implies $\eta(p) = ph(p)$ in contradiction with **[2]**. In this limit case, involuntary unemployment is still possible, but it requires demand saturation in the output market. Indeed, if this market is perfectly competitive and unemployment persists as w tends to zero, we get, by first order conditions for profit maximization, for all $i \in N$,

$$p = w\ell_i'(y_i) \text{ and } Y = \frac{h(p)Lm}{1 - ph(p)}.$$

By the same argument p tends to zero if Y tends to infinity. If Y remains finite, then by **[5]** $\ell_i'(y_i) < \infty$ and the competitive price p must go to zero as w tends to zero. So, by involuntary unemployment $\lim_{p \to 0} h(p) < \infty$, i.e. there is demand saturation. Conversely if $\lim_{p \to 0} h(p) = \infty$, we must get full employment with $w > 0$ at the competitive equilibrium. Theorem 2 shows that when there is imperfect competition and **[6]** holds, unemployment may persist at any wage level even though $\lim_{p\to 0} h(p) = \infty$. Then the price corresponding to Cournot-Nash equilibrium does not tend to zero with the money wage.

Assumption **[6]** is less restrictive than the corresponding assumption when the Ford effect is negligible or limited to wages, as in d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1989), where a similar inequality is imposed on η, the elasticity of h, not on σ, the elasticity of H. Indeed, $\eta(p) = ph(p) + (1 - ph(p))\sigma(p)$, so that $\eta < 1/n$ implies $\sigma < 1/n$, by Assumption **[2]**, whereas the converse is not true. As $ph < \eta$, again by **[2]**, the condition $\lim_{p\to 0} \eta(p) < 1/n$ implies $\lim_{p\to 0} ph(p) < 1/n$, so that the marginal propensity to consume must become very small for prices close to zero when n is large, if this condition is to be satisfied. This does not result from **[6]**. A profit income feedback effect, leading to a lower price elasticity of the relevant demand function (when demand is inelastic), makes involuntary unemployment more likely.

To shed more light here, we shall present utility functions satisfying assumptions **[1]** to **[4]** and **[6]**. We deal successively with the two basic interpretations of the model we gave at the start, the temporary equilibrium one and the static one.

First consider the static interpretation. Assumptions **[1]** and **[2]** are satisfied if all consumers have identical smooth, convex, homothetic preferences (defined on the nonnegative orthant), such that the indifference curves do not cut the produced good axis and that the marginal rate of substitution is strictly decreasing (whenever both goods are desired). Assumptions **[3]**, **[4]** and **[6]** impose additional restrictions on the elasticity of substitution between the two goods. A sufficient condition for these restrictions to be met is that the elasticity of substitution decreases from a value larger than one to a value less than $1/n$ when the ratio of the produced to the nonproduced good increases from zero to infinity. This excludes C.E.S. utility functions, though a constant elasticity of substitution larger than one leads to a demand function satisfying assumptions **[1]** to **[4]**.

An example of utility function satisfying this sufficient condition on the elasticity of substitution and, more generally, leading to a demand function fulfilling all the assumptions **[1]** to **[4]** and **[6]** is:

$$U(c, \tilde{m}) = \ln \tilde{m} - e^{-c/\tilde{m}},$$

where c and $\tilde{m}$ denote the consumed quantities of the produced and nonproduced goods, respectively. Denoting by $R(k), k = c/\tilde{m}$, the marginal rate of substitution, we find:

$$R(k) = e^k - k \text{ and } \sigma(k) \overset{\text{def}}{=} \frac{R(k)/k}{R'(k)} = \frac{(e^k/k) - 1}{e^k - 1}.$$

Thus,

$$\lim_{k \to 0} R(k) = 1, \ \lim_{k \to \infty} \sigma(k) = 0 \text{ and } \sigma'(k) = \frac{k^2 - k + 1 - e^k}{(e^k - 1)^2 k^2} e^k < 0 \text{ for } k > 0.$$

As $R'(k) = e^k - 1 > 0$ (entailing **[2]**) and given the first-order condition for an interior optimum, $R(k) = 1/p$, there is a reservation price equal to one (entailing **[3]**), and the elasticity of substitution increases in p (fulfilling **[4]**) and tends to zero when p tends to zero (fulfilling **[6]**). **[1]** is also clearly satisfied. From the first-order condition, the inverse demand function is:

$$H^{-1}(k) = \frac{1}{R(k)} = \frac{1}{e^k - k}.$$

$H^{-1}(k)$ remains positive for all k: demand is not saturated when the price is zero.

Lastly, the temporary equilibrium interpretation of the model. Now the consumer maximizes an intertemporal utility function, and the nonproduced good has an indirect utility derived from its monetary function as a store of value. Under constant expectations about future prices, similar assumptions about elasticity of substitution between present and future consumption will again lead to the fulfillment of **[1]** to **[4]** and **[6]**. But if future prices depend upon the present output price, we must introduce a suitable combination of two kinds of elasticities, for intertemporal substitution and expectations.

To make things simple, consider a two-period planning horizon and a C.E.S. intertemporal utility function:

$$U(c_1, c_2) = (c_1^{(\gamma-1)/\gamma} + \delta c_2^{(\gamma-1)/\gamma})^{\gamma/(\gamma-1)}, \text{ where } \delta > 0, \ \gamma > 0 \text{ and } \gamma \neq 1,$$

and where c_t is the tth period consumption. Maximization of U under a single intertemporal budget constraint yields:

$$c_1 = \frac{1}{p_1 + \delta^\gamma p_1^\gamma p_2^{1-\gamma}} W,$$

where W is the present value of consumer wealth. Assuming that the future expected price is a positive nondecreasing function of the (positive) present price: $p_2 = P(p_1)$, and omitting the subscript for the first period, we have:

$$h(p) = \frac{1}{p + \delta^\gamma p^\gamma P(p)^{1-\gamma}} \text{ and } H(p) = \frac{1}{\delta^\gamma p^\gamma P(p)^{1-\gamma}}$$

and also

$$\sigma(p) \stackrel{\text{def}}{=} -\frac{H'(p)}{H(p)} p = \gamma + (1-\gamma)\varepsilon(p)$$

where

$$\varepsilon(p) \stackrel{\text{def}}{=} \frac{P'(p)}{P(p)} p$$

is the elasticity of expectations and γ the elasticity of intertemporal substitution.

Here assumptions [**1**] to [**4**] and [**6**] are satisfied if:

(i) $\quad \gamma < 1, \ \lim\limits_{p\to\infty} \frac{P(p)}{p} = \infty, \quad \varepsilon'(p) \geq 0$ and $\lim\limits_{p\to 0} \varepsilon(p) < \dfrac{(1/n) - \gamma}{1-\gamma}$

or (ii) $\quad \gamma > 1, \ \lim\limits_{p\to\infty} \frac{P(p)}{p} = 0, \quad \varepsilon'(p) \leq 0$ and $\dfrac{\gamma - (1/n)}{\gamma - 1} < \lim\limits_{p\to 0} \varepsilon(p) \leq \left(\frac{\gamma}{\gamma-1}\right)$.

In the static case, assumption [**6**], leading to involuntary unemployment, required some complementarity between the two goods. In the present temporary equilibrium context, the same assumption is compatible with an elasticity of substitution between present and future consumption larger than one, provided expectations of the future output price are elastic enough. A fall in the current price could even lower current demand if it led to a disproportionately large fall in expected future prices, and intertemporal substitution were sufficiently strong, a case that we have excluded to avoid demand saturation. Assumption [**6**] becomes increasingly difficult to satisfy, as the elasticity of intertemporal substitution and the number of firms increase. As a last remark, we observe that in case (i), and as long as $\lim\limits_{p\to 0} \varepsilon(p) < \gamma/(\gamma-1)$, and in case (ii), assumption [**6**] is compatible in our example with the absence of demand saturation at a nil price.[10]

[10] Indeed, by the restrictions imposed on ε, entailing that H is decreasing, $p^\gamma P(p)^{1-\gamma}$ is increasing in p, and it is easy to show that it tends to zero with p. Suppose that it does not. Then $P(p)^{\gamma-1}$ tends to zero and, by l'Hospital's rule, $\lim\limits_{p\to 0}\{p^\gamma / P(p)^{\gamma-1}\} = \lim\limits_{p\to 0}\left\{\frac{\gamma}{\gamma-1} p^\gamma / \varepsilon(p) P(p)^{\gamma-1}\right\} \neq 0$ implies $\lim\limits_{p\to 0} \varepsilon(p) = \gamma/(\gamma-1)$.

4 Conclusion

This paper has studied the possibility of a type of unemployment which is not associated with a misperception of the producers' environment and which appears at any positive wage. Leaving aside the trivial cases where there is a capacity ceiling to productive employment and where consumers' current needs can be satiated, we have seen that such a type of unemployment is likelier the weaker the degree of competition. Under the same assumptions, a perfectly competitive general equilibrium exists at a non zero wage.

As a first step in this investigation, we proposed an extension of Cournot equilibrium, allowing for the interaction between the product and the labour markets, at a given money wage. Adopting an objective viewpoint, producers were assumed here to be able to conjecture the "true" demand functions they face. We have formulated conditions sufficient for the existence of the defined equilibrium, implying neither concave profit functions nor identical firms but including convex technologies. The analysis was simplified by assuming a particular type of homothetic preference.

The next step showed that unemployment emerges at any money wage for a sufficiently low value of the individual wealth, with an additional assumption of a low enough elasticity of substitution at prices close to zero (Assumption [**6**]). This is weaker than assuming demand saturation at a nil price, but becomes more and more stringent as the number of firms increases (i.e. as competition becomes less imperfect). In the limit case of perfect competition, only demand saturation can lead to unemployment when the money wage can be chosen arbitrarily small.

When unemployment arises whatever the money wage level, there is genuine "involuntary" unemployment, in Keynes' sense that no expedient exists by which labour as a whole can achieve full employment "by making revised money bargains with the entrepreneurs". Under the temporary equilibrium interpretation of the model, the possibility of such unemployment turns on the weakness of the Pigou effect. This is partly the consequence of a low money value of the individual wealth, itself a result of past performance on one side, and of pessimistic consumers' expectations about future income on the other side. But the weakness of the Pigou effect is also the consequence of the assumed small elasticity of demand at prices close to zero, itself a result of a suitable combination of intertemporal complementarity and inelastic price expectations or of intertemporal substitutability and elastic price expectations. Such a combination might induce demand saturation at a nil price, and hence involuntary unemployment whatever the degree of competition. But even without demand saturation the Pigou effect can be blocked in an imperfectly competitive economy short of full employment, because the equilibrium price does not decrease to zero along with the money wage. Thus a lower degree of competition (a smaller number of producers in our extended Cournot oligopoly model) renders involuntary unemployment more likely.

Appendix

Proof of Theorem 2. Take, as above, a sequence $\{m^s, L^s\}_{s=0}^{\infty}$ such that $\lim_{s\to\infty} m^s = 0$ and, for all s, either $L^s = L > 0$ or $M^s = L^s m^s = M > 0$. Then considering the associated sequences of monetary wages $\{w^s\}$, of equilibrium total quantities $\{Y^s\}$ and prices $\{p^s\}$, we have seen that $\lim_{s\to\infty} p^s = 0$, under the hypothesis that full-employment is always preserved. Then there should be at least one firm $i \in N$ for which, for s large enough, the equilibrium production y_i^s is not smaller than $Y^s/n > 0$ and such that $\frac{\partial \Pi_i}{\partial y_i}(y_i^s, y_{-i}^s, w^s, m^s, L^s) \geq 0$ (otherwise y_i^s would not be an equilibrium quantity). But, in the first case, we suppose $\lim_{s\to\infty} w^s \geq \underline{\mathrm{w}}$ for some $\underline{\mathrm{w}} > 0$ and letting

$$\psi_Y^s \stackrel{\text{def}}{=} \frac{\partial}{\partial Y}\psi(Y^s, m^s, L^s)$$

we get (by definition of Π_i),

$$\lim_{s\to\infty} \frac{\partial \Pi_i}{\partial y_i}(y^s, w^s, m^s, L^s) = \lim_{s\to\infty}[p^s + y_i^s \psi_Y^s - x^s \ell_i'(y_i^s)].$$

Since

$$\lim_{s\to\infty} p^s = 0,\ \lim_{s\to\infty} y_i^s \psi_Y^s \leq 0 \text{ by } \mathbf{[1]}, \text{ and } \lim_{s\to\infty}[-w^s \ell_i'(y_i^s)] < 0 \text{ by } \mathbf{[5]},$$

we get a contradiction. In the second case, we suppose $\lim_{s\to\infty} w^s = 0$ and we get, for s large enough,

$$\frac{\partial \Pi_i}{\partial y_i}(y^s, w^s, m^s, L^s) = p^s \left[1 + \frac{y_i^s}{Y^s}\frac{\psi_Y^s Y^s}{p^s} - \frac{w^s}{p^s}\ell_i'(y_i^s)\right] \geq 0.$$

But

$$\frac{\psi_Y^s}{p^s} = -\frac{1}{\sigma(p^s)} \text{ and } \frac{y_i^s}{Y^s} \geq \frac{1}{n}$$

for all s, so that, for s large enough,

$$1 - \frac{y_i^s}{Y^s}\frac{1}{\sigma(p^s)} \geq \frac{w^s}{p^s}\ell_i'(y_i^s),$$

or, equivalently,

$$\frac{y_i^s}{Y^s} - \sigma(p^s) \leq -\sigma(p^s)\frac{w^s}{p^s}\ell_i'(y_i^s) \leq 0,$$

implying

$$\sigma(p^s) \geq \frac{y_i^s}{Y^s} \geq \frac{1}{n},$$

in contradiction with **[6]**. The result follows. ■

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