Cooperative and Noncooperative R&D in Duopoly with Spillovers*

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Contrary to the usual assumption made in most oligopoly models, relations among firms are seldom of a wholly cooperative or noncooperative type: in many situations, they compete in some fields, while they cooperate in others. An important example is the case of cooperative research efforts bringing fierce competitors together.

Two types of agreement are observed. First R&D cooperation can take place at the so-called “precompetitive stage”: companies share basic information and efforts in the R&D stage but remain rivals in the market-place.¹

A second type of agreement involves an extended collusion between partners, creating common policies at the product level. The usual justifications of this extension are the difficulties of protecting intellectual property. The idea is then to allow partners who have achieved inventions together, to also control together the processes and products which embody the results of their collaboration, in order to recuperate jointly their R&D investments.²

What could be expected from these types of agreement is a reduction in R&D expenditures, because of less wasteful duplication, and a reduction of total production, because of more

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¹A well-known example is the European Strategic Program for R&D in Information Technologies (ESP-RIT). The Microelectronics and Computer Technology Corporation (MCC) in the United States, and the Very Large-Scale Integration Program (VLSI) in Japan are other examples.
²An illustration is given by the EUREKA project intended to undertake common European research with concrete objectives and leading to the joint exploitation of the results.
monopoly power. Using a two-stage approach, this paper provides an example that does not fulfill these expectations and that allows a social welfare comparison between the corresponding games. An important factor in this analysis consists in the externalities or spillovers in R&D from one firm to another.

1 The example

Consider an industry with two firms facing an inverse demand function $D^{-1}(Q)$, where $Q = q_1 + q_2$ is the total quantity produced. Each firm has a cost of production $C_i(q_i, x_i, x_j)$ which is a function of its own production, $q_i$, of the amount of research $x_i$ that it undertakes and the amount of research $x_j$ that its rival undertakes. Both $D^{-1}$ and $C$ are assumed linear, so that

$$D^{-1} = a - bQ \text{ with } a, b > 0,$$

and

$$C_i(q_i, x_i, x_j) = [A - x_i - \beta x_j]q_i, \ i = 1, 2, i \neq j$$

with $0 < A < a$, $0 < \beta < 1$; $x_i + \beta x_j \leq A$; $Q \leq a/b$.

The R&D externalities or spillovers imply that some benefits of each firm’s R&D flow without payment to other firms. In our specification the external effect of firm $j$ R&D is to lower firm $i$’s unit production cost.$^3$ The cost of R&D is assumed to be quadratic, reflecting the existence of diminishing returns to R&D expenditures.$^4$

Firms’ strategies consist of a level of research and a subsequent production strategy based on their R&D choice. We shall now analyze three different games.

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$^3$One interpretation is that successful inventions of rivals can be imitated at less cost to firm $i$ that if it were to invent the new processes itself. See J. Hartwick (1984) who presents a static Cournot model with free entry and concludes that with R&D spillovers one cannot conclude that the corresponding equilibrium is associated with excessive duplication as Dasgupta and Stiglitz (1980).

$^4$A valuable justification of this assumption is that the “technological possibilities linking R&D inputs and innovative outputs do not display any economies of scale with respect to the size of the firm in which R&D is undertaken” (Dasgupta, 1986, p. 523).
1. In the first one, firms act noncooperatively in both output and R&D. Consider the profit of firm $i$ at the second stage, conditional on $x_1$ and $x_2$:

$$\pi_i = [a - bQ]q_i - [A - x_i - \beta x_j]q_i - \gamma \frac{x_i^2}{2}, \ j \neq i, i = 1, 2.$$ 

The Nash-Cournot equilibrium can be computed to be\(^5\)

$$q_i = \frac{(a - A) + (2 - \beta)x_i + (2\beta - 1)x_j}{3b}.$$ 

At the preceding stage, in which firms choose R&D levels, profits can be written as

$$\pi^*_i = \frac{1}{9b}[(a - A) + (2 - \beta)x_i + (2\beta - 1)x_j]^2 - \gamma \frac{x_i^2}{2}, \ j \neq i, i = 1, 2.$$ 

This integrates a triple influence of the R&D levels: via the outputs, the unit production cost, and the R&D costs themselves.

There exists a unique (and symmetric) solution\(^6\) satisfying $\partial \pi^*_i / \partial x_i = 0$, for which

$$x^*_i = \frac{(a - A)(2 - \beta)}{4.5b\gamma - (2 - \beta)(1 + \beta)}, \ i = 1, 2.$$ 

$$Q^* = \frac{q_i^* + q_j^*}{3b} = \frac{2(a - A)}{3b} + \frac{2(\beta + 1)}{3b} x_i^* = \frac{2(a - A)}{3b} \left[ \frac{4.5b\gamma}{4.5b\gamma - (2 - \beta)(1 + \beta)} \right].$$

2. In the second game, we introduce cooperation in R&D, the second stage remaining noncooperative. At the first stage the firms maximize the joint profits, as a function of $x_1$ and $x_2$:

$$\hat{\pi} = \pi^*_1 + \pi^*_2 = \frac{1}{9b} \sum_{i=1}^{2} \left\{ [(a - A) + (2 - \beta)x_i + (2\beta - 1)x_j]^2 - \gamma \frac{x_i^2}{2} \right\}, \ j \neq i.$$ 

\(^5\)Notice that

$$q_1 + q_2 \leq \frac{1}{3b} [2(a - A) + 2A] \leq \frac{a}{b}.$$ 

\(^6\)Second-order conditions require that

$$\frac{2(2 - \beta)^2}{9b} - \gamma < 0$$

or $\frac{2}{3}(2 - \beta)^2 < b\gamma$. 
Considering the symmetric solution $x_1 = x_2 = \hat{x}$, we obtain the following unique solution for the equilibrium with cooperation in R&D:

$$\hat{x} = \frac{(\beta + 1)(a - A)}{4.5b\gamma - (\beta + 1)^2};$$

$$\hat{Q} = \frac{2(a - A)}{3b} + \frac{2(\beta + 1)}{3b}: \hat{x} = \frac{2(a - A)}{3b} \left[\frac{4.5b\gamma}{4.5b\gamma - (1 + \beta)^2}\right].$$

These solutions correspond to an internalization of the R&D external effects through joint decision on the levels of R&D expenditures. Contrary to what could have been expected from possible reduction in the duplication of R&D, especially in the case of large spillovers, a comparison between $\hat{x}$ and $x^*$ clearly indicates that for large spillovers, that is, $\beta > 0.5$, the level of R&D increases when firms cooperate in R&D, that is, $\hat{x} > x^*$. In the same perspective, as shown by the respective values of $\hat{Q}$ and $Q^*$, the amount of production is also higher with cooperation in R&D, than in the noncooperative situation, that is, $\hat{Q} > Q^*$.

To the extent that profits are higher in the case of cooperative research than in the noncooperative game private incentives, independently of any public policy such as subsidies, can be sufficient to lead to such a cooperation.

3. The third case deals with monopoly: Firms cooperate in both stages of the game. At the second stage, the joint profit conditional on $x_1$ and $x_2$ is given by

$$\pi = [a - bQ]Q - AQ + (x_1 + \beta x_2)q_1 + (x_2 + \beta x_1)q_2 - \gamma \sum_{i=1}^{2} x_i^2.$$ 

For $x_2 = x_1 = x$, the symmetric solution $\hat{q}_1 = \hat{q}_2$ leads to

$$Q = q_1 + q_2 = [(a - A) + (1 + \beta)x]/2b.$$ 

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7Second-order conditions require $\frac{2}{9}(1 + \beta)^2 < b\gamma$.

8Indeed, if $\beta + 1 > 2 - \beta$, $x^* < \hat{x}$.

9Indeed the noncooperative choice could always be adopted by cooperating firms, if more profitable.

10Notice that

$$\hat{Q} \leq \frac{1}{2b} [2(a - A) + 2A] = \frac{a}{b}.$$
At the preceding stage, the joint profit becomes
\[ \tilde{\pi} = \frac{1}{b} \left[ \frac{a - A + (1 + \beta)x}{2} \right]^2 - \gamma x^2. \]

The symmetric cooperative equilibrium in R&D and in production corresponds to the following unique solution\(^{11}\)
\[ \tilde{x} = \frac{(a - A)(1 + \beta)}{4b\gamma - (1 + \beta)^2}; \]
\[ \tilde{Q} = \frac{(a - A)}{2b} + \frac{(1 + \beta)}{2b} \cdot \tilde{x} = \frac{a - A}{2b} \left[ \frac{4b\gamma}{4b\gamma - (1 + \beta)^2} \right]. \]

Here as expected, the collusive output, for a given level of R&D, is smaller than the noncooperative one, but it is not necessarily so when the optimal amount of R&D is incorporated.\(^{12}\)
Similarly, the collusive amount of R&D varies with the value of \(\beta\) and is, for reasonably large spillovers, higher than in the fully noncooperative equilibrium.\(^{13}\) Furthermore the amount of R&D in the case of collusion in both output and R&D is higher than in the case of pure R&D cooperation. This stems from the fact that less competition in the product market allows the firms to capture more of the surplus created by their research and induce more R&D expenditures. But despite this larger amount of R&D, the quantity produced at the fully cooperative equilibrium is less than with a cooperation limited to the R&D stage.\(^{14}\)

\(^{11}\) According to the second-order conditions, \(\partial^2 \tilde{\pi} / \partial \tilde{x}^2 = (1 + \beta)^2/2 - 2\gamma < 0\) or \((1 + \beta)^2/4 < \beta\gamma\). Also we consider here only the symmetric cooperative equilibrium. The producers could reach higher joint profits by having different R&D expenditures and only one firm producing (the one with the lower unit cost). However, to consider this asymmetric cooperative solution would not affect our qualitative results.

\(^{12}\) We have \(\tilde{Q} < Q^*\) if \(5\beta^2 + 4\beta - 1 < 3b\gamma\). For \(\beta = 1\), \(\tilde{Q} < Q^*\) would require that \(b\gamma > \frac{5}{3}\), which is more restrictive than second-order conditions.

\(^{13}\) It appears that \(\tilde{x} < x^*\) if
\[ \frac{(1 + \beta)}{4b\gamma - (1 + \beta)^2} > \frac{(2 + \beta)}{4.5b\gamma - (2 - \beta)(1 + \beta)^2} \]
or
\[ \beta > 0.41. \]

\(^{14}\) Indeed \(\tilde{Q} < \tilde{Q}\) whenever \(b\gamma > (1 + \beta)^2/3\), a less restrictive condition than some of our second-order conditions.
2 Welfare conclusions

Given this set of results, it is not a priori clear that, from a social welfare point of view, one type of behavior is more efficient than another. Indeed more cooperation could lead to higher profits but lower consumer surplus. Less production could be compensated by more R&D. And a higher level of research could correspond to a wasteful duplication that ignores R&D externalities. In order to classify the solution obtained in the different situations, we need an efficiency standard. Let us define social welfare $W(Q)$ as the sum of the consumer’s surplus $V(Q)$ and the producer’s surplus (assuming $x_1 = x_2 = x$)

$$W(Q) = V(Q) - AQ + (1 + \beta)xQ - \gamma x^2.$$ 

Given $x$, the efficient output is the following

$$Q = \frac{1}{b}[a - A + (1 + \beta)x].$$

At the first stage social welfare is

$$W^{**} = V(Q) - AQ + (1 + \beta)xQ - \gamma x^2.$$ 

The efficient level of R&D satisfying the first-order conditions\(^\text{15}\) is

$$x^{**} = \frac{(a - A)(1 + \beta)}{2b\gamma - (\beta + 1)^2}.$$ 

And finally the socially efficient amount of production incorporating the efficient level of research can be written as

$$Q^{**} = \frac{a - A}{b} + \frac{1 + \beta}{b} \cdot x^{**} = \frac{a - A}{b} \left[ \frac{2b\gamma}{2b\gamma - (1 + \beta)^2} \right].$$ 

Therefore the solution obtained by maximizing social welfare requires not only more production but also a higher level of R&D than what is obtained with any of the previous noncooperative and cooperative equilibria.

\(^\text{15}\)The second-order condition requires that $[(1 + \beta)/2]^2 < b\gamma.$
Indeed, $x^{**} > x^*$, since
\[
\frac{1 + \beta}{2b\gamma - (1 + \beta)^2} > \frac{2 - \beta}{4.5b\gamma - (2 - \beta)(1 + \beta)}
\]
and
\[
Q^{**} > Q^*.
\]
Similarly $x^{**} > \tilde{x} > \hat{x}$ and $Q^{**} > \hat{Q} > \tilde{Q}$.

This provides us with a convenient social efficiency standard to classify our various results.

The clearest conclusion is that cooperation in R&D (but not in production) increases both expenditures in R&D and quantities of production, with respect to the noncooperative solution, that is, $\hat{x} > x^*$ and $\hat{Q} > Q^*$, whenever the spillover effect is large enough; otherwise it is the reverse.

Further, considering separately production and R&D aspects, the cases of large and small spillovers should also be distinguished. For large spillovers, such that $\beta > 0.5$, the amount of research which is the closest to the social optimum is the one achieved by firms’ cooperating in both output and research, and the most distant, the one obtained by noncooperative behavior. The complete classification is the following
\[
x^{**} > \tilde{x} > \hat{x} > x^*.
\]
Concerning the quantity of production, the closest to the social optimum is what is produced by firms’ cooperating at the “pre-competitive stage”, that is, in research. The classification is then
\[
Q^{**} > \hat{Q} > Q^* > \tilde{Q}.
\]
For small spillovers, such that $\beta \leq 0.4$, the classifications are different but the “second-best” for R&D is still obtained by a cooperative behavior in both stages.\footnote{The classifications are then $x^{**} > \tilde{x} \geq x^* > \hat{x}$; $Q^{**} > Q^* > \hat{Q} > \tilde{Q}$.}

To conclude our example has shown that cooperative behavior can play a positive role in industries having a few firms and characterized by R&D activities generating spillover effects.
This is in line with the permissive American and European antitrust regulations allowing co-operative research whereby member firms agree to share the costs and the results of a research project.\textsuperscript{17}

However, in order to compute explicitly and to classify our various types of subgame-perfect solutions, our analysis has been very partial and based on a model ignoring many crucial aspects of R&D activities.\textsuperscript{18}

**References**


\textsuperscript{17}Policy questions are analyzed in A. Jacquemin (1987, 1988).

\textsuperscript{18}For a broader analysis, see M. Katz (1986). An overview of the main aspects of R&D activities and innovations is to be found in S. Davies (1979) and M. Spence (1986). Multinational aspects are considered in R. De Bondt et al. (1988).