Measuring the power to monopolize
A simple-game-theoretic approach*

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Abstract

This paper proposes a measure of the power to monopolize based on the ability of firms to induce a shift from a competitive to a non-competitive regime, through coalition formation. First, in the framework of monotonic simple games, we show how to use the Shapley value in terms of a power index for individual firms within an industry. Second, we construct an aggregate power index for industries. It is based on axioms allowing to make explicit the respective roles of the number and size of the minimal winning coalitions and aggregates additively non-normalized individual power indices.

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1 Introduction

If one is interested in having some indications about the degree of realized monopoly power in a given industry, then conventional concentration indices may be computed from the current distribution of market shares. Moreover, these may be explained theoretically as the result of some achieved oligopoly equilibrium. However, there is also an interest of working out a measure of the potential ability of firms to monopolize the market, in the sense of inducing a move from a competitive to a non-competitive regime. Antitrust legislations are mainly concerned with the prevention of market conducts that appear to contain a significant potential for harm. The various means – including mergers – by which the activities of a group of firms can be brought under a centralized control may result in a more or less irreversible change in the structure of the industry and it would be useful to elaborate a measure based upon the probability that this kind of event could occur.

This is the purpose of the present paper. Section 2 questions the relevance of concentration indices for such an ex-ante analysis and proposes the simple-game representation of an industry for examining the distribution of power. The Shapley-Shubik index is used to measure individual power inside an industry. Then, in order to allow interindustrial comparisons, Section 3 gives an axiomatic justification for a class of aggregate measures of the power to monopolize. Finally simple examples are discussed.

2 Market shares and individual power indices for simple games

2.1 The measure of monopoly

The measure of the degree of monopoly in market economies has been extensively studied both theoretically and empirically. A first approach, starting with the work of Lerner (1933-34), is based on the performances of firms in a market. The basic measure used is the profit margin of each firm, i.e., the relative difference between the price and the marginal cost of each firm. The degree of monopoly should increase with these differences. Hence the Lerner index for the industry is an increasing function $\mathcal{L}(L_1, L_2, \cdots, L_n)$ where n is the number of firms in the industry and each $L_i = (p - c_i)/p$ the relative difference between the price p and the marginal cost c_i of the ith firm. A second approach, based on existing market structures, uses a concentration index on the market shares of the firms. Two phenomena are taken into account by this kind of measure: the number of firms in the industry and the degree of symmetry of the distribution of market shares. Most classical measures can be expressed as weighted sums of the market shares, i.e. a function $C^n(\omega_1, \cdots, \omega_n) = \sum_{i=1}^n h(\omega_i)\omega_i$ where ω_i is the ith firm market share and h is some weighting function.

Several studies have been made [see, among others, Cowling and Waterson (1976), Encaoua and Jacquemin (1980)] to construct equilibrium relationships between Lerner type measures and concentration indices in the framework of static and dynamic models of oligopoly. For example, if the firms in the industry are supposed to behave non-cooperatively and choose the Nash-Cournot quantities, then the arithmetic average of the profit margins is proportional to the Herfindhal index of concentration. Two important aspects of these models must be underlined [see Donsimoni, Geroski and Jacquemin (1984)]. First the derived index depends both on explicit beliefs about industry conducts (Cournot-Nash, Stackelberg or any other solution concept), and on normative judgements about weights adopted to aggregate individual firm Lerner indices. Hence there is an unavoidable degree of arbitrariness in the selection of such concentration measures. A second implication of these models is that both the index of seller concentration and the measure of market performance are jointly determined in the equilibrium solution; it is then erroneous to consider that there is a causal relationship, from the 'ex-ante' concentration to the 'ex-post' price-cost margin. The endogeneity of these two variables, both determined by market conduct and by cost and demand conditions in the given industry, clearly implies that the actual degree of concentration used in empirical studies is not as such a measure of the potential exercise of market power: the Lerner index and the concentration index are both determined by ex-post equilibrium identity.

2.2 Analytical tools

Instead of limiting the analysis to ex-post equilibrium situations, it seems to us useful to also have analytical tools able to give quantitative indications on the probability of market monopolization.

At least two types of questions illustrate the need for such a kind of measure, the first one concerning the link between the firm's existing market share and its power to contribute to a monopolization, the other concerning the size and number of coalitions allowing monopolization. The first question is based on the fact that all the firms in the industry do not have the capability to contribute to a monopolization process and that this capability is not adequately measured by their current market shares. This is related to the generally admitted fact that the addition of small firms to a market which is essentially dominated by large firms does not affect the degree of monopoly on this market.[1] It can be argued that if a firm is of no utility in reaching an agreement in order to elevate price above cost, it has indeed a null power, even though its market share is positive. Such a firm should be considered as a 'null' firm. An implication is that contrary to what happens with the conventional measures of concentration, a variation in the number of firms should not affect the power of monopolization existing on a market as long as these firms are 'null' firms. On the other hand, one can imagine a firm with small market share but still indispensable to achieve a coalition sufficient to control the market. In that last case its power should be considered as much larger than simply given by its market share. This situation is especially clear in the case of a merger or a divestiture: the prevention (the encouragement) of any structural change which is likely to induce a switch into (from) an effectively coordinated oligopolistic regime becomes crucial from a public policy point of view, whatever the dimension of the market share of the firm in question.[2]

[1]Thus, in contrast to inequality measures, concentration measures are assumed to be relatively insensitive to the number of small firms and to be mainly influenced by the size and number of the largest firms. As stated by Hannah and Kay (1977, p. 50) 'imagine an economy dominated by a small number of giants of similar size. Now suppose many small firms enter, but enjoy little success, so that even in aggregate their market share is very low. Then concentration has not been significantly affected although the degree of inequality in firms' sizes has greatly increased'.

[2]An application of this view exists in Antitrust Policy: not only in industries where concentration is already

A second problem concerns the size of coalitions required to induce in the industry a change of regime. Considering the difficulty of enforcing coalitions between a larger number of firms and the role of the corresponding costs, monopolization should be easier on markets where the number of colluding firms required to adopt a non-competitive behavior is lower: the smaller is the number of firms which must (tacitly/explicitly) collude in order to elevate price above cost, the greater is the danger of non-competitive behavior. This leads to the idea of including in the evaluation of the power to monopolize the 'minimal' number of firms which should collude to apply a non-competitive price.

Following this idea even further a switch from a competitive to a non-competitive regime may result from several alternative coalitions, grouping different firms in the industry. The larger the number of such feasible combinations, the greater the probability that such a coalition might form and that competition might be reduced. A measure of the power to monopolize should therefore take into account all the possibilities of combination offered to the existing firms.

2.3 An ex-ante measure

At this stage, it must be underlined that our objective is now to work out an ex-ante measure of the power to monopolize, at the firm and the industry level, that incorporates the previous aspects in order to allow an extension of the usual comparison and empirical studies based on ex-post measures of concentration. It has no normative implications as such. From a policy point of view however, let us recall that in order to forbid any attempt to monopolize, most antitrust authorities have adopted thresholds expressed by market shares or degrees of concentration above which it is presumed that a monopoly, a dominant position or a harmful cartel will be created. One objective of these legislations is to prevent the attainment of such thresholds and to control more closely firms capable of inducing it [see O.E.C.D. (1979)].

high, should a slight increase be prevented but in those where there is a strong trend toward oligopoly, further tendencies in that direction are to be curbed in their 'incipiency'. A U.S. application is the Brown Shoe case (1962): the Brown Shoe Company which manufactured 4.0 percent of the nation's output of shoes was not allowed to acquire the Kinney Company with manufactured 0.5 percent, in the light of the trends in this industry.

For example, the European Commission, in its 1977 Notice, considers that agreements between undertakings engaged in the production or distribution of goods do not fall under the prohibition of Article 85(1) of the EEC Treaty 'if the products which are the subject of the agreement and other products of the participating undertakings considered by consumers to be similar ... do not represent more than 5% of the total market for such products.' Concerning specialization agreements, they are not condemned if the combined market shares of the participating undertakings do not exceed 15% of the market.[3] In the German law concerning market dominating enterprises, a firm is presumed to be market dominating if it has a market share of one-third. Three or less enterprises with no substantial competition existing between them shall be deemed market dominating if they have a combined market share of 50% or more. Similarly, in the United States, the standards most often applied by the Department of Justice in determining whether to challenge horizontal mergers can be stated in terms of the sizes of the merging firms' market shares.

Hence the antitrust authorities held preferences expressed by their selection of thresholds. Given the complexities of each case and the difficulty of holding a set of consistent preferences over all possible configurations of industries, these authorities have realistically adopted a simplified rule of thumb reflecting some strong a priori judgements.[4] Simply, the policymaker considers as unacceptable a proposed merger or coalition leading to a configuration that reaches the critical threshold: above this threshold, it is presumed that a new oligopoly equilibrium will emerge, characterized by a sharp decline in the level of social welfare.

Instead of assuming a continuous variation in the degree of collusion so that every small increase in the level of concentration is translated into a small increase in the degree of collusion, these policies assume that for some increases in concentration, market behavior will not be

[3] In the Commission proposal for a regulation on the control of concentration (1973), the control would be applied if the goods or services concerned by the concentration account in any Member State for more than 25% of the turnover in identical or similar goods or services.

[4] More generally, one could adopt an approach assuming the existence of a set of consistent preferences held by the authorities over all possible configuration of output vectors, for a particular industry. For such an approach see Blackorby, Donaldson and Weymark (1982).

notably affected and that only above a certain level, there will be a recognition of mutual interdependence and a change to a non-competitive regime.

Such a view can be better appreciated after recent empirical works showing that the relationship between the degree of concentration of market shares and price-cost margins is not continuous.[5] Numerous studies have established that below some level of concentration there is no significant relationship between concentration and performance and that above a breaking point (or some critical small interval on the concentration scale) profit rates are significantly higher than for industries below this mark.[6]

As our approach does not depend on the existence of a precise threshold, it is not important to analyze here how it is determined, either by empirical studies or by legal criteria based on past historical experiences and case studies, but to admit that, at the present stage, it constitutes a widely used rule of thumb for antitrust implementation.

2.4 Simple games

It has often been stressed that 'simple games' provide a valuable tool to describe the distribution of power and the structure of control in political organizations. Due to the importance we have given to the notion of potential power to monopolize and to the presumed existence of critical thresholds, it seems that simple games can also provide an adequate representation of an industry

[5] According to a recent survey by Plott (1982, p. 1523), laboratory experimentation applied to industrial organization also suggests that 'market performance is very fragile with respect to underlying structural and institutional variables and that "slight" changes can switch a market from "competitive" to "collusive" or vice versa'.

[6] One of the most valuable research has been made by White (1976). Instead of an ad hoc determination of this critical level, he has viewed it as a parameter to be estimated and has determined it by applying the econometric technique of the 'switching of regimes'. Most of these works are based on a large sample of industries and use a four-firm concentration ratio. The exhibited non-linearity is sudden rather than smooth. Some studies use other indices. For example Marvel (1978) applies the Herfindhal index $H^n = \sum_{i=1}^{n} \omega_i^2$ and uses the price level as the dependent variable. It is clear that the precise critical concentration ratio at which jumps occur will vary among industries, as many other variables besides seller concentration affect the ability to collude (barriers to entry, homogeneity of products, turnover among buyers, concentration of customers ...).

for our purposes. Indeed, very generally, a simple game is a formal description of the family of subsets of players which can be called 'winning coalitions' in some specific situation. For the purpose of analyzing the distribution of power, resulting from different industrial configurations, the concept of winning coalitions applies to those potential combinations of firms which could allow the industry to reach the threshold at which there is a switch of regime. The determination of these potentially winning coalitions of firms can be based on the current market shares of all firms and should take into account the way the threshold has been estimated.

It is only after introducing such a representation of the industry, integrating all monopolizing possibilities, that we shall return to the problem of measuring the distribution of the power.

For formal simplicity we shall adopt the following (usual) presentation of simple games. A *simple game* is defined to be a pair (N, v), where $N = \{1, 2, \cdots, n\}$ represents the set of firms and v is a function from the set of subsets of N, called *coalitions*, to the set $\{0, 1\}$ such that[7]

$$v(\emptyset) = 0 \text{ and } \forall\, S, T \subset N, S \subset T,\ v(S) \leq v(T).$$

A *winning coalition* is then defined as coalition S such that $v(S) = 1$. In other terms, the function v determines the set of winning coalitions. Another important definition is the notion of *minimal winning coalition* which is a coalition M such that no proper subcoalition is winning, i.e., $\forall\, S \underset{\neq}{\subset} M,\ v(S) = 0$. A minimal winning coalition is thus a combination of particular firms, which cannot be reduced without losing its ability to induce a switch from a competitive to a non-competitive market situation. We shall denote by Γ the set of all simple games. For any two games (N, v) and (N, w) in Γ, their *union* $(N, v \vee w)$ and their *intersection* $(N, v \wedge w)$ in Γ are, respectively, defined by

$$\begin{aligned} \forall\, S \subset N, \quad (v \vee v)(S) &= \max\{v(S), w(S)\}, \\ (v \wedge w)(S) &= \min\{v(S), w(S)\}. \end{aligned}$$

It is possible to give an interpretation of these two operations in our context of industrial organization. Indeed imagine a set of firms which are involved on two markets. Then, in the

[7]An alternative specification would be to impose superadditivity, i.e. $\forall\, S_1, S_2 \subset N, S_1 \cap S_2 = \phi$, $v(S_1 \cup S_2) \geq v(S_1) + v(S_2)$.

case of a union, realizing a switch of regime in *one* of the two markets is sufficient to induce a switch of regime in the combined markets. In the case of an intersection the switch has to be in *both* markets. Finally we shall denote by (N, v_M) the simple game having M as its unique minimal winning coalition, i.e.,

$$\begin{aligned} v_M(S) &= 1 \quad \text{if } M \subset S, \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

Such a game is usually called an M-unanimity game.

The formal presentation we have given of simple games corresponds to the representation we have announced above since it is easy to verify that any simple game is characterized by the set of its minimal winning coalitions [see Dubey (1975)]. Indeed if $\{M_1, M_2, \cdots, M_n\}$ is the set of minimal winning coalitions in $(N, v) \in \Gamma$, then we have, for all S in N,

$$v(S) = (v_{M_1} \vee v_{M_2} \vee \cdots \vee v_{M_k})(S).$$

To construct such a representation, for a given industry, we shall assume that the following data concerning the n firms in this industry are available:

(i) a given distribution of market shares $\omega_1, \omega_2, \cdots, \omega_n$,

(ii) a given value q of the critical threshold corresponding to a (weighted or unweighted) sum of market shares.

As an illustration, consider an industry made of three firms and the distribution of market shares $\omega = (0.10, 0.40, 0.50)$. Then for $q = 0.65$, we may define the game

$$\begin{array}{lll} v(\{1\}) = v(\{2\}) = v(\{3\}) = 0 & \text{since} & \forall\, i, \omega_i < 0.65, \\ v(12) = 0 & \text{since} & \omega_1 + \omega_2 < 0.65, \\ v(13) = 0 & \text{since} & \omega_1 + \omega_3 < 0.65, \\ v(23) = 1 & \text{since} & \omega_2 + \omega_3 > 0.65, \\ v(123) = 1 & \text{since} & \displaystyle\sum_{i=1}^{3} \omega_1 > 0.65. \end{array}$$

More generally, letting N denote the set of n firms, the following simple game can be defined. For any coalition S, of cardinal $s \geq 1$, one may consider the corresponding index $C^{n-s+1}[\cdot]$ defined on the space where the shares of these s firms are grouped, and denote by ω^s the $(n-s+1)$-vector having as one of its components the sum of the s components ω_i, $i \in S$, and as other components all $\omega_j, j \notin S$. We then write

$$\begin{aligned} v(S) &= 1 \quad \text{if } C^{n-s+1}(\omega^S) \leq q, \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

In our context, the game is thus determined by a family of subsets of the n firms in the industry capable of inducing a switch of regime.

It is then possible to measure the power of each firm on the basis of its potential contribution to a monopolization of the industry. To make the point clear, let us recall that a well-known application of simple games concerns the formalization of weighted voting systems (e.g., simple majority rule, voting by stockholders in corporations, voting by political parties in parliaments, etc). In all these cases the discrepancy between the real distribution of influence and the distribution of weights in the decision process has been brought forward by various authors.[8] For instance, a voter with small numbers of votes (i.e., small weight) may have great voting power if his votes are necessary to reach a majority; reciprocally a voter with some non-negligible number of votes may have no power whatsoever. This kind of phenomena is well captured by applying to simple games the Shapley-Shubik index for measuring individual power [see Shapley (1953) and Shapley and Shubik (1954)]. More precisely, for any simple game (N, v) and any individual $i \in N$, the Shapley-Shubik value of this individual is given by the formula

$$\varphi_i(N, v) = \sum_{\{S \subset N : i \in S\}} \frac{(s-1)!(n-s)!}{n!} [v(S) - v(S \setminus \{i\}].$$

This approach can obviously also be used for the kind of simple games we have described to analyze the distribution of power among firms in an industry and given the limited amount of data usually available. For instance, in the above example, there is only one minimal winning

[8]See Lucas (1974) who provides several illustrations of the fact that power is not adequately measured by the number of votes.

coalition, namely $\{2,3\}$. The Shapley values for the three firms are: $\varphi = (\varphi_1, \varphi_2, \varphi_3) = (0, \frac{1}{2}, \frac{1}{2})$. This reflects the fact that firm 1 is a null player in spite of its non-null market share. It also shows that the two other firms have an identical power, in spite of their different market shares. Indeed the contribution of firm 1 to reach the critical threshold is null and the contributions of firms 2 and 3 to the minimal winning coalitions are the same.

2.5 Axioms

Finally the above formula, for measuring individual power, has some fundamental justification in the sense that it has been shown to be the only formula satisfying a certain number of conditions.

It is therefore useful to interpret in the context of an industry the axioms characterizing the Shapley-Shubik value applied[9] to all games that can be generated as above (i.e., all monotonic simple games). The first axiom says that if an industry is represented by a single game then the individual firm power index should not vary according to the name (i.e., a number between 1 and n) received by each firm in this representation. It is called the

Symmetry axiom. For any permutation π of N (i.e., a bijection from N to N) and any $(N, v) \in \Gamma$, if (N, w) is such that

$$\forall\, S \subset N, \quad w(\pi(S)) = v(S)$$

then

$$\varphi_i(N, v) = \varphi_{\pi(i)}(N, w) \quad \forall\, i \in N.$$

The second axiom concerns the 'null' firms. The power index of a null firm should be equal to zero. Formally we call *null player* in a game (N, v) a player i which is such that

$$\forall\, i \in N,\ \forall\, S \subset N, \quad v(S \cup \{i\}) = v(S).$$

In other terms, a null player has a null impact on any coalition.

Null player axiom. If i is a null player in a game $(N, v) \in \Gamma$, then $\varphi_i(N, v) = 0$.

[9] For a general characterization for monotonic simple games see Dubey (1975).

The third axiom imposes an additivity property which is related to the two operations of union and intersection of simple games defined above.

Additivity axiom. For any (N, v) *and* (N, w) *in* Γ *and any* $i \in N$

$$\varphi(N, v) + \varphi_i(N, w) = \varphi_i(N, v \vee w) + \varphi_i(N, v \wedge w).$$

A possible interpretation is the following.[10] In the situation of firms having some control on two different markets, the total power of a firm is the sum of its power limited to each single market and its power when its control is exercised on both markets simultaneously. An example is the situation of two imperfectly substitutable products: there is some power resulting from the control of one of the two products, because of product differentiation, and there is an additional power when the control extends to the two products, because of the existing degree of substitution.

The last axiom is simply a

Normalization axiom. For any $(N, v) \in \Gamma$, $\sum_{i \in N} \varphi_i(N, v) = 1$.

This concludes our analysis of individual firms' power index allowing *intra-industrial* comparisons. The next step is to construct an aggregate monopoly power index which would permit *inter-industrial* comparisons.

3 Aggregate power indices for simple games

3.1 Index of the overall power to monopolize

Measures of concentration try to express the existing degree of monopoly power in a given industry by a one-parameter index; similarly we need an index for measuring overall power to monopolize an industry.

In contrast with the measures of concentration which are based on a priori judgements allowing to compute some weighted sums of the individual indices, we want, however, to impose

[10] Some alternative justification of Axiom 3 – in terms of marginal power or in terms of lotteries – are suggested in Dubey and Shapley (1979).

conditions for such an aggregation, expressing the purpose of our measure.[11] This purpose is to evaluate the danger of monopolization in a given industry on the basis of a given family of 'minimal winning coalitions' for firms. It seems natural then to try to disentangle the respective influences of the number and of the sizes of these minimal winning coalitions on this evaluation. More specifically, we must take into account the view that, ceteris paribus, a decrease in the number of firms necessary for a minimal winning coalition and an increase in the number of feasible minimal winning coalitions, enhance the probability of a switch from a competitive to a non-competitive type of regime.

To that effect, we shall now define *axiomatically* a function F, on Γ, to be interpreted as a satisfactory evaluation of the danger of monopolization in any industry represented by (N, v).

The first four axioms are similar to the axioms for the Shapley value both formally and in terms of their interpretation.

I. Symmetry axiom. For any permutation π of N and any $(N, v) \in \Gamma$, if (N, w) is such that: $\forall\, S \subset N,\ w(\pi(S)) = v(S)$, then $F(N, v) = F(N, w)$.

II. Null player axiom. If (N, v) and (N', v') are such that $N' = N \cup \{n'\}$, with n' a null player and if, $\forall\, S \subset N,\ v'(S) = v(S)$, then $F(N', v') = F(N, w)$.

III. Additivity axiom. For any (N, v) and (N, w) in Γ,

$$F(N, v) + F(N, w) = F(N, v \vee w) + F(N, v \wedge w).$$

IV. Normalization axiom. For any $(N, v) \in \Gamma$, $0 \leq F(N, v) \leq 1$.

[11] For example, we could construct a Herfindhal-like aggregate measure of monopolization power by computing the sum of the squared individual indices:

$$C_H^n(N, v) \stackrel{\text{def}}{=} \sum_{i \in N} [\varphi_i(N, v)]^2.$$

For the illustration given in Section 2, 2.4, $C_H^n = 0.50$, while the application of the Herfindhal formula to market shares gives a lower value, namely

$$H = \sum_{i=1}^{n} \omega_i^2 = 0.42.$$

The last axiom will introduce explicitly a property concerning the size of the minimal winning coalitions. It is a generalization of the following idea. Compare two situations each characterized by a single minimal winning coalition. If the minimal winning coalition in the first situation is contained in the minimal winning coalition in the other situation, then the second one exhibits a lower level of aggregate power to monopolize.

V. Minimal winning coalition axiom. Let (N, v) *and* (N, v') *be two games having, except for one player* $j \in N$*, the same family of* p *disjoint minimal winning coalitions, respectively* $\{M_1, M_2, \cdots, M_p\}$*and* $\{M_1 \setminus \{j\}, M_2, \cdots, M_p\}$ *where* $M_1 \neq M_1 \setminus \{j\} \neq \emptyset$*, then*

$$F(N, v) < F(N, v').$$

Hence the previous idea is generalized[12] in the sense that once the minimal winning coalition M_1 is strictly reduced, the aggregate power to monopolize increases if the other minimal winning coalitions $M_2, M_3, \cdots, M_p$ remain the same. These other minimal winning coalitions are required to be disjoint from each other and from M_1 in order to focus on *the pure size effect.* The reduction in M_1 should affect aggregate monopolization power whatever the fixed number of other disjoint minimal winning coalitions.

3.2 Theorem

The following theorem can then be proved:

Theorem 1 *An aggregate power index* F *on* Γ*, satisfies the Axioms I-V if and only if for some positive Borel measure* μ *on* $(0, 1]$ *which is bounded by but not concentrated on 1, it satisfies the following integral formula: for any* $(N, v) \in \Gamma$*, with minimal winning coalitions*

[12]This axiom coincides exactly with the previous idea if we restrict our attention to the class of superadditive games. Indeed in that case there cannot be more than one minimal winning coalition.

$\{M_1, M_2, \cdots, M_p\}$, $p \geq 1$

$$\begin{aligned} F(N,v) &= \int_0^1 \left\{ \textstyle\sum_{h=1}^{p} \alpha^{|Mh|} - \displaystyle\sum_{h=1}^{p-1} \sum_{k=h+1}^{m} \alpha^{|M_h \cup M_k|} \right. \\ &+ \sum_{h=1}^{p-2} \sum_{k=h+1}^{p-1} \sum_{\ell=k+1}^{p} \alpha^{|M_h \cup M_k \cup M_\ell|} - \dots \\ &+ \left. (-1)^{p-1} \alpha^{|M_1 \cup M_2 \cup \cdots \cup M_p|} \right\} d\mu(\alpha). \end{aligned} \tag{1}$$

This theorem, the proof of which is given in appendix,[13] determines in fact the class of all functions satisfying conditions I–V in T. A representative element of this class is given by specifying a certain value for α in the open interval (0,1) and then applying the formula for the measure μ concentrated on this value of α. Any other function F in the class is some linear combination of such representative elements.

This formula is obtained by first showing that if there is an F satisfying Axioms I and II, then there exists a function f defined on the set of positive integers such that, for any game (N, v_M), $M \subset N$,

$$F(N, v_M) = f(m), \quad m = |M|.$$

Using the other axioms, it is then possible to specify the function f.

3.3 An example

As an illustration, we may take the case of an industry made of three firms, and use $f(m) = \alpha^m$, with $\alpha = \frac{1}{2}$. Assuming that the minimal winning coalitions for (N, v) are $\{1,2\}, \{1,3\}, \{2,3\}$, the value of the aggregate index is

$$\begin{aligned} F &= f(|M_1|) + f(|M_2|) + f(|M_3) - f(|M_1 \cup M_2|) - f(|M_1 \cup M_3|) \\ &- f(M_2 \cup M_3|) + f(|M_1 \cup M_2 \cup M_3|) \\ &= 3(\tfrac{1}{2})^2 - 3(\tfrac{1}{2})^3 + (\tfrac{1}{2})^3 = 0.50. \end{aligned}$$

Table 1 gives the whole structure of possible minimal winning coalitions, given alternative possible thresholds, and the corresponding values of F, given three different choices of μ:

[13] We thank J.-F. Mertens for pointing out an incomplete argument in a previous version of this paper.

concentrated on $\frac{1}{2}$, on $\frac{1}{4}$, and the Lebesgue measure on $[0,1]$. Two general aspects may be underlined. First, for a given number of minimal winning coalitions, a decrease of the number of firms contained in them increases the value of F. In particular, in the case of a single minimal winning coalition, our index obtains a maximum when the coalition reduces to a single firm and decreases as it contains more firms. Second, to the extent that an increase in the number of disjoint minimal winning coalitions is considered as augmenting the probability of a shift of regime, it is natural that the value of F tends to increase with the number of minimal coalitions. In our example, the absolute maximum of F is obtained when each single firm in the industry constitutes a minimal winning coalition. The results will depend, however, on the strength of the effect of the number of minimal winning coalitions relative to the effect of the number of firms in each minimal winning coalition. As illustrated in the table this tendency will be affected by the selected value of μ.

As an alternative illustration, let us assume an industry made of four firms, the respective market shares being $\omega = (0.10, 0.20, 0.30, 0.40)$, with a critical threshold of 0.55, so that

$$\begin{aligned} v(S) &= 1 \quad \text{if} \sum_{i \in S} \omega_i \geq 0.55, \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

Table 1

Structure of minimal winning coalitions	F with $f(m) = (\frac{1}{2})^m$	F with $f(m) = (\frac{1}{4})^m$	F with $f(m) = 1/(m+1)$
$\{1,2,3\}$	1/8	1/64	3/12
$\{1,2\}$	2/8	4/64	4/12
$\{1,2\},\{1,3\}$	3/8	7/64	5/12
$\{1,2\},\{1,3\},\{2,3\}$	4/8	9/64	6/12
$\{1\}$	4/8	16/64	6/12
$\{1\},\{2,3\}$	5/8	19/64	7/12
$\{1\},\{2\}$	6/8	28/64	8/12
$\{1\},\{2\},\{3\}$	7/8	37/64	9/12

The minimal winning coalitions are therefore

$$M_1 = \{2, 4\}, M_2 = \{3, 4\}, M_3 = \{1, 2, 3\}.$$

The aggregate power to monopolize, for $f(m) = (\frac{1}{2})^m$ is

$$F = (\frac{1}{4} + \frac{1}{4} + \frac{1}{8}) - (\frac{1}{8} + \frac{1}{16} + \frac{1}{16}) + \frac{1}{16} = 0.4375.$$

If we now assume a realized merger, say between the first and the second firm so that the current distribution of market shares is $\omega = (0.30, 0.30, 0.40)$, the aggregate index, for the same threshold, rises to $F = 0.50$: indeed the number of minimal winning coalitions is identical but for each of them, two firms are now sufficient. However it must be underlined that, contrary to the measures of concentration, a merger does not automatically increase the value of our index if the number of minimal winning coalitions varies.

4 Conclusion

This paper has proposed a measure of the power to monopolize, that could usefully complement the conventional indices of concentration. When these indices give a picture of an ex-post equilibrium situation, our measure constitutes an ex-ante tool of analysis, allowing to estimate the probability of a market monopolization.[14]

First, in the framework of monotonic simple games, we have shown how to use the Shapley value in terms of a power index for individual firms within an industry. Second, we have constructed an aggregate power index for industries. The resulting measure may be viewed as a weighted sum of individual power indices and varies within $[0, 1]$. It satisfies axioms allowing to make explicit the respective influence of the number and the size of the minimal winning coalitions on the level of the aggregate power to monopolize. In this respect, the proposed measure is not more fully specified than the conventional indices of concentration, in the sense that the measure μ remains to be chosen. Further investigation would be required if such measure had to be specified on an axiomatic basis.

[14] This probabilistic interpretation is fully analyzed in d'Aspremont, Jacquemin and Mertens (1984).

It must also be underlined that our approach is in its principle quite general and can be applied to any monotonic simple games, the basic question being to determine what are the minimal winning coalitions, whatever the precise measure, based or not on market shares.

Finally, from a policy joint of view, most antitrust authorities have adopted, for applying their policy, a system of thresholds used as rules of thumb for signalling when projected coalitions and mergers could result in more or less irreversible shift, from a competitive to a non-competitive regime. In this respect, our approach appears to be a more adequate instrument than the conventional indices, to give indications on the corresponding potential power to monopolize.

5 Appendix

Proof of the theorem

As a preliminary step we notice that, for any subset $\{M_{k_1}, M_{k_2}, \cdots, M_{k_s}\} \subset \{M_1, M_2, \cdots, M_p\}$, we have

$$v_{M_{k_1}} \wedge v_{M_{k_2}} \wedge \cdots \wedge v_{M_{k_s}} = v_{M_{k_1} \cup M_{k_2} \cup \cdots \cup M_{k_s}}$$

and that, using Axiom III recurrently and using the distributive lattice structure of Γ with the two operations $\vee$ and $\wedge$, one gets

$$\begin{aligned}
F(N, v) &= F(N, (v_{M_1} \vee v_{M_2} \vee \cdots \vee v_{M_p})) \\
&= \sum_h F(N, v_{M_h}) - \sum_h \sum_{k>h} F(N, v_{M_h \cup M_k}) \\
&+ \sum_h \sum_{k>h} \sum_{\ell>k} F(N, v_{M_h \cup M_k \cup M_\ell}) - \cdots \\
&+ (-1)^{p-1} F(N, v_{M_1 \cup M_2 \cup \cdots \cup M_p}).
\end{aligned}$$

Now, we shall first show that if there is an F satisfying Axioms I and II there exists a function f defined on the set of positive integers such that, for any game $(N, v_M), M \subset N$,

$$F(N, v_M) = f(m), \quad m = |M|.$$

Second, using the other axioms, we shall specify the function f and the theorem will be established.

That such a function exists is clear from the fact that, for any two games (N, v_M) and $(N', v_{M'})$, with $M \subset N \subset N'$ and $M' \subset N'$, $F(N, v_{M'}) = F(N', v_{M'})$, whenever $m = m'$. Indeed there exists a permutation π of N such that $M = \pi(M')$ and so, by Axiom I, $F(N', v_M) = F(N', v_{M'})$. Also all the players in $N \setminus M$ and in $N' \setminus M$ are null players in the games (N, v_M) and (N', v_M), respectively. Thus, by Axiom II, $F(N, v_M) = F(N', v_M) = F(N', v_{M'})$.

To specify the function f, take any two games (N, v) and (N, v') in Γ having, except for one player $j \in N$, the same particular family of p disjoint minimal coalitions, respectively $\{M \cup \{1\}, \{2\}, \cdots, \{p\}\}$ and $\{M \cup \{1, j\}, \{2\}, \cdots, \{p\}\}$. Let $m = |M|$. The, for such games, with $p \geq 1$ and $m = 0, 1, 2, \cdots$,

$$\begin{aligned}
F(N,v) - F(N,v') &= \sum_{t=0}^{p-1} (-1)^t \begin{pmatrix} p-1 \\ t \end{pmatrix} [f(m+1+t) - f(m+2+t)] \\
&= f(m+1) - \sum_{t=1}^{p-1} (-1)^t \left[\begin{pmatrix} p-1 \\ t \end{pmatrix} + \begin{pmatrix} p-1 \\ t-1 \end{pmatrix} \right] f(m+1+t) \\
&- (-1)^{p-1} f(m+1+p) \\
&= \sum_{t=0}^{p} (-1)^t \begin{pmatrix} p \\ t \end{pmatrix} f(m+1+t) \text{ since } \begin{pmatrix} p-1 \\ t \end{pmatrix} + \begin{pmatrix} p-1 \\ t-1 \end{pmatrix} = \begin{pmatrix} p \\ t \end{pmatrix}.
\end{aligned}$$

By Axioms IV and V we get that

$$\sum_{t=0}^{p} (-1)^t \begin{pmatrix} p \\ t \end{pmatrix} f(m+1+t) \geq 0, \quad m, p = 0, 1, 2, \cdots$$

This means that the sequence $\{f(m+1)\}_0^\infty$ is completely monotonic (i.e., its elements are non-negative and its successive differences are alternatively non-positive and non-negative). Therefore by Hausforff's theorem we may write f as the integral

$$f(m+1) = \int_0^1 \alpha^m dv(\alpha), \quad m = 0, 1, 2, \cdots,$$

where v is a bounded non-negative measure on $[0, 1]$.

Or defining the positive measure μ on $(0, 1]$ such that

$$d\mu(\alpha) = \frac{d\tilde{v}(\alpha)}{\alpha} \text{ with } \int_0^1 \alpha \, d\mu(\alpha) < +\infty,$$

and where $\tilde{v}(A) \stackrel{\text{def}}{=} v(A \setminus \{0\})$, for any measurable set $A \subset [0,1]$. Hence letting $v_0 = v(\{0\})$ we get

$$\begin{aligned} f(m) &= v_0 + \int_0^1 \alpha \, d\mu(\alpha) \quad \text{if } m = 1, \\ &= \int_0^1 \alpha^m d\mu(\alpha) \qquad \text{if } m = 2, 3, \cdots \end{aligned}$$

Then, for any given N and for the game (N, v^n) with minimal winning coalitions $\{1\}, \{2\}, \cdots, \{n\}$, we may write

$$\begin{aligned} F(N, v^n) &= \sum_{t=1}^{n} (-1)^{t-1} \begin{pmatrix} n \\ t \end{pmatrix} f(t) = nv_0 + \int_0^1 \left\{ \sum_{t=1}^{n} (-1)^{t-1} \begin{pmatrix} n \\ t \end{pmatrix} \alpha^t \right\} d\mu(\alpha) \\ &= nv_0 + \int_0^1 d\mu(\alpha) - \int_0^1 \sum_{t=0}^{n} \begin{pmatrix} n \\ t \end{pmatrix} (-\alpha)^t d\mu(\alpha) \\ &= \int_0^1 \{1 - (1-\alpha)^n\} d\mu(\alpha) + v_0 n. \end{aligned}$$

Since the last integrand generates an increasing sequence of μ-integrable functions for $n = 1, 2, \cdots$, converging to $\int_0^1 1 d\mu(\alpha) > 0$ and since, for all $N, F(N, v^n) \leq 1$ (by Axiom IV) we must have: $v_0 = 0$ and $\mu(0, 1] \leq 1$. In addition, using Axiom V with $p = 1$, μ cannot be concentrated on 1. This gives all the necessary conditions on μ stated in the theorem.

The other direction, i.e., that formula (1) satisfies the axioms, is implied by the following lemma:

Lemma 1 *If, for some positive Borel measure μ on $(0, 1]$ which is bounded by but not concentrated on 1, $F(N, v)$ is defined according to formula (1), then the function G defined by*

$$G(N, v) = \sum_{i \in N} \left\{ \sum_{\substack{S \subset N \\ i \in S}} \left[\sum_{r=s}^{n} (-1)^{r-s} \begin{pmatrix} n-s \\ r-s \end{pmatrix} \int_0^1 \frac{\alpha^r}{r} d\mu(\alpha) \right] [v(S) - v(S \setminus \{i\})] \right\} \tag{2}$$

satisfies the Axioms I–V and

$$G(N, v) = F(N, v).$$

The proof of this lemma is given in d'Aspremont and Jacquemin (1981). ∎

The interest of formula (2) is to show that the aggregate power index F, satisfying the Axioms I–V, may be seen as the sum of individual power indices. This suggests some relationship between this aggregate index and the concept of semi-value [see Dubey, Neyman and Weber (1981), Dubey and Shapley (1979)]. This is discussed in d'Aspremont, Jacquemin and Mertens (1984) which gives an extension of this result to a larger class of games.

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