Product differences and prices*

Claude d'Aspremont, Jean Jaskold Gabszewicz and Jacques-François Thisse†

Abstract

Under assumptions following Hotelling's 1929 paper 'Stability in Competition', the possibility of constructing an example in which equilibrium prices exist everywhere and the two merchants display tendencies to agglomerate, is ruled out.

1. In his celebrated paper 'Stability in Competition', Hotelling (1929) considered a particular case of the following general problem. Let 1 and 2 be two merchants selling distinct, substitute goods to a given market at prices p_1 and p_2. Assume that the specification ('characteristics') of each of these products is fully described by a one-dimensional parameter, denoted by a for product 1 and b for product 2, with $a \geq 0, b \geq 0, a \leq \ell, b \leq \ell$. For each pair of values $(a, b) \in [0, \ell]^2$, this situation gives rise to a two-person game in which the players are the two merchants, and the strategies their respective prices. Denote the payoff functions (revenue functions) of this game respectively by $\pi_1(p_1, p_2; a, b)$ and $\pi_2(p_1, p_2; a, b)$, and by $[p_1^*(a, b), p_2^*(a, b)]$ a non-cooperative price equilibrium of this game (if it exists), for a given a and b. The question is then: in a parametric analysis where both products characteristics a and b are fixed, and where sellers choose prices $p_1^*(a, b)$ and $p_2^*(a, b)$ in a non-cooperative manner, how the payoffs $\pi_i(p_1^*(a, b), p_2^*(a, b); a, b), i = 1, 2$ do react to unilateral parametric variations of a, or b? This question is undoubtedly interesting: to the extent that the revenue of each seller would change

*Reprinted from *Economics Letters*, 11, 19–23, 1983 (CORE Reprint no. 508).

†CORE, Université catholique de Louvain, 1348 Louvain-la-Neuve, Belgium

in a monotonic direction when he increases or decreases 'product differences' w.r.t. the existing product specification of his rival, possibly general statements on product variety could be obtained.

In Hotelling's example, the parameters a and b are the locational positions of two sellers of a homogeneous product on a line of length ℓ. Hotelling intended to show that under the particular assumptions of this example, the parametric variations described above necessarily reveal a general tendency of both sellers to agglomerate at the same location: the revenue of each seller, he thought, always increases when he gets closer to his rival. Furthermore, extending by analogy this spatial competition results to the general problem of product differentiation formulated above, Hotelling and his successors [e.g. Boulding (1966, p. 484)] conjectured the existence of a so-called *'Principle of Minimum Differentiation'* applying to all cases covered by this general formulation: there would exist natural forces which would tend to reduce product variety in imperfectly competitive industries.

In a recent paper d'Aspremont et al. (1979), the present authors have shown that there was a flaw in Hotelling's argumentation,[1] and more importantly, they considered a slightly modified version of Hotelling's example, restoring continuity, and showed that, for this version, there was a tendency for both sellers to *maximize* their product differences. This example thus constitutes a counterexample to the Principle of Minimum Differentiation.[2] The present note goes a set further: we show that under mild assumptions, the 'Principle' *never* holds, so that given the product specification of one of the sellers, it can never become advantageous to the other one to choose his own specification arbitrarily close to it, if prices adapt themselves at a non-cooperative

[1] It consists in the 'implicit' introduction of a concept of equilibrium different from Cournot's and based on a wrong evaluation of the best replies of the players (what Hotelling calls the 'locus of maximum profit' in footnote 8). This occurs when the parameters do not verify the existence conditions of a Cournot equilibrium given in d'Aspremont et al. (1979).

[2] In fact, Hotelling explicitly recognizes that this principle does not hold in his example since, for his alternative concept of equilibirum (see our footnote 1), the profit of a merchant becomes decreasing when he approaches too closely his competitor so that an optimal distance between sellers would exist (see footnote 9). Surprisingly, however, he claims (in the same footnote) that 'this optimal distance ... is an adventitious feature of our problem resulting from a discontinuity which is necessary for simplicity'.

equilibrium.

The above property is technically somewhat trivial to establish since it turns out to be an easy corollary of Bertrand's (1883) analysis. Consider indeed the following intuitive argument. From Bertrand, we know that if two merchants sell at no cost an *homogeneous* product (i.e., $\overline{a} = \overline{b}$), then the corresponding non-cooperative price strategies are unique and given by $p_1^*(\overline{a}, \overline{b}) = p_2^*(\overline{a}, \overline{b}) = 0$. Accordingly, for all points of the 'diagonal' Δ of $[0, \ell]^2$, equilibrium prices are equal to zero; zero prices entail zero revenues so that, for $\overline{a} = \overline{b}$,

$$\pi_i(p_1^*(\overline{a}, \overline{b}), p_2^*(\overline{a}, \overline{b}); \overline{a}, \overline{b}) = 0, \qquad i = 1, 2.$$

Assume now that, for any (a, b), there exists a unique price equilibrium $[p_1^*(a, b), p_2^*(a, b)]$ and that the functions $\pi_i(p_1^*(a, b), p_2^*(a, b); a, b)$, $i = 1, 2$, are continuous in a and b on the whole $[0, \ell]^2$. If both revenues π_i are identically equal to zero on the whole set $[0, \ell]^2$, then the problem has no interest, since revenues are invariant w.r.t. product differences. But if, for each product specification of one of the sellers, there exists at least one product specification for the other, leading to strictly positive revenue for him, then, by continuity, *it must be that this revenue starts to decrease when the latter reduces 'too much' product differences*: we know indeed that this revenue is equal to zero when product differences vanish, i.e., when the diagonal D is reached.

The above argument is made more rigorous in the following section.

2. We find it useful to give an abstract formulation of our main proposition in order to cover as many cases as possible. Let a, b be any two vectors of a compact subset C of $\mathbb{R}^n_+$ (C is the *space of 'characteristics'*) which define the specification of two substitute products. Product a (resp. b) is sold by *player* 1 (resp. 2). Consider the *two-person game* with $i = 1, 2$ where the *strategy set* of player i is $S_i = \{p_i; 0 \leq p_i \leq M\}$,[3] and where his *payoff function* is given by

$$\pi_i(p_1, p_2; a, b) = p_i \cdot D_i(p_1, p_2; a, b)$$

[3] In Hotelling's case, the sets S_i are not compact as assumed here. The model could easily be accommodated so as to include Hotelling's example, either by introducing an alternative assumption, or by assuming the existence of a very high 'reservation price' for the consumers.

(p_i is the price of player i, and D_i his contingent demand.) For any pair (a, b) and any p_j we denote by $P_i(p_j, a, b)$ the set arg max $_{p_i}\pi_i(p_1, p_2; a, b)$ whenever it is non-empty (P_i is the 'best reply' correspondence for player i). A *Nash-Cournot Equilibrium* is a pair of strategies $[p_1^*(a, b), p_2^*(a, b)]$ such that $p_i^*(a, b) \in P_i(p_j^*(a, b), a, b), i = 1, 2, i \neq j$. We denote by $p^*(a, b)$ the set of Nash-Cournot Equilibirum of this game and by E the subset of pairs (a, b) for which $P^*(a, b)$ is non-empty. The following two assumptions are now introduced.

A.1. $$\forall\,(a, b)\,\text{such that}\,a = b, P^*(a, b) = (0, 0).$$

Comment. If $a = b$, both merchants sell an homogeneous product. Then $D_i(p_1, p_2; a, b)$ coincides with the market demand function for that product as long as $p_i < p_j$. Consequently, using Bertrand's argument shows that A.1 is satisfied, provided that this market demand function is decreasing in price.

A.2. $$\lim_{p_i \to 0} \pi_i(p_1, p_2; a, b) = 0,\ \forall\, i \neq j,\ \forall\, a, b, p_j.$$

Comment. This assumption means that the elasticity of the contingent demand function of merchant i is smaller than one in the vicinity of $p_1 = 0$. For instance, A.2 holds when D_i crosses the quantity axis.

We then have the following result:

Proposition 1 *If the best reply correspondences $P_i(p_j; a, b), i = 1, 2,$ are upper hemicontinuous on $S_j \times E$, then one and only one of the following alternatives holds for any sequence $\{(a_n, b_n) \in C^2 : a_n \neq b_h\}_0^\infty$ converging to $(\bar{a}, \bar{b}) \in \Delta$:*

1. *There exists N such that for all $n \geq N$, $(a_n, b_n) \notin E$*

2. *For any subsequence $\{(a_m, b_m)\}^\infty$ in E,*

$$\lim_{n\to\infty} \pi_i(p_1^*(a_m, b_m), p_2^*(a_m, b_m); a_m, b_m) = \pi_i(p_1^*(\bar{a}, \bar{b}), p_2^*(\bar{a}, \bar{b}); \bar{a}, \bar{b}) = 0.$$

Proof. Suppose (1) does not hold. Then there is a subsequence $\{(a_m, b_m)\}_0^\infty$ in E converging to $(\bar{a}, \bar{b})$. Also, by upper hemicontinuity, each set $G_i = \{(a, b, p_1, p_2); p_i \in P_i(p_j; a, b)\}$ is closed as well as the graph G of P^* (which is simply $G_1 \cap G_2$). As $S_1 \times S_2 \times C^2$ is bounded, we may extract from the sequence $\{p_{1m}, p_{2m}, a_m, b_m) : (p_{1m}, p_{2m}) = (p_1^*(a_m, b_m), p_2^*(a_m, b_m)) \in P^*(a_m, b_m), (a_m, b_m) \in C^2)^\infty$ a subsequence converging to, say, $(\overline{p}_1, \overline{p}_2, \overline{a}, \overline{b})$.

Since G is closed $(\overline{p}_1, \overline{p}_2) \in P^*(\overline{a}, \overline{b}) = (0,0)$, where the last equality follows from A.1. The result is then an immediate consequence of A.2. ∎

In Hotelling's example, the diagonal Δ is defined by the set of location pairs (α, β) such that $\beta = \ell - \alpha$, where α and β are two scalars measured from the endpoints of the line of length ℓ. It has been shown in d'Aspremont et al. (1979) that there exists no price equilibrium when β is close to $\ell - \alpha$, and we are in case 1. By contrast, the authors have considered a slight modification of this example for which there exists a unique price equilibrium for any pair (a, b): to that effect it is sufficient to assume quadratic transportation costs (cf. d'Aspremont et al. (1979, p.1148)]. Again our assumptions are satisfied and, by the above proposition, we are in case 2. Another illustration of this proposition is provided in Jaskold Gabszewicz and Thisse (1979, p.356).

References

Bertrand, J. Théorie mathématique de la richesse sociale. *Journal des Savants*, 48, 499–508, 1883.

Boulding, K. *Economic Analysis. Vol. 1, Microeconomics, 4th edition.* Harpers, New York, 1966.

d'Aspremont, C., Jaskold Gabszewicz, J. and J.-F. Thisse. On Hotelling's stability in competition. *Econometrica*, 47, 1145–1150, 1979.

Hotteling, H. Stability in competition. *Economic Journal*, 39, 41–57, 1929.

Jaskold Gabszewicz, J. and J.-F. Thisse. Price competition, quality and income disparities. *Journal of Economic Theory*, 20, 340–359, 1979.