On Bayesian Incentive Compatible Mechanisms*

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1. Introduction: incomplete information and Bayesian incentive compatibility

In many contexts, it is a requirement that collective decision making preserve some kind of decentralization and be a function of the information that each individual agent privately controls. To the extent that the private information affects the decision, any individual agent may find it advantageous to distort the information he reveals in a way which cannot be sanctioned. However, this phenomenon may cause some global inefficiencies to the collectivity. Hence it is a relevant question to consider whether it is in general possible to construct some decision schemes which would give each individual the 'correct' incentives, namely to reveal all the information that he controls undistorted. The purpose of this paper is to show that this 'incentive' question is mainly a problem of incomplete information and that one can fruitfully apply to it the theory of games with incomplete information.

1.1 The basic collective decision problem and the communication game

We consider a set N of n *players* (or individual agents) who have to choose among a set X of *alternatives*. We assume X is some subset of $\mathbb{R}^K$ (K is any positive integer). In addition we introduce a commodity called *money* in order to allow any kind of transfers among the agents.[1] Such a transfer is an element $y = (y_1, \ldots, y_n)$ of $\mathbb{R}^n$. Thus, an *outcome* will always be an element of $X \times \mathbb{R}^n$. Also, we suppose that the players agree to delegate the choice of a particular outcome to some *Central Agency* according to some well-specified rules. In particular these rules will have to take into account in some way the characteristics α_i of each individual player. By assumption each α_i belongs to some subset A_i of $\mathbb{R}^L$ (L is any positive integer). We shall call α_i the *type* of player i. Finally, to each player $i \in N$ we associate a function $V_i(\cdot; \alpha_i)$ from $\mathbb{R}^K \times \mathbb{R}^n$ to $\mathbb{R}$ where $V_i(x, y; \alpha_i)$ denotes player i's *payoff* in the situation where $(x, y) \in X \times \mathbb{R}^n$ is the outcome selected. For the following we shall introduce a separability requirement, namely

Separability For every $i \in N$, there exists a function $U_i(\cdot; \alpha_i)$ from $\mathbb{R}^K$ to $\mathbb{R}$ such that, for every $x \in X$ and every $y \in \mathbb{R}^n$: $V_i(x, y; \alpha_i) = U_i(x; \alpha_i) + y_i$.

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1. In Section 3 we will explicitly put some feasibility restrictions on these transfers.

With the feasibility requirement on the transfers that we shall introduce in Section 3 this assumption will simply amount, in game theoretic terms, to admit unrestricted side-payments with full transferability.

To characterize the decentralized information structure in this model, we assume that the type α_i of player i is known to i but unknown to the Central Agency. However, all the sets A_i and all the functions V_i and U_i, as defined respectively on $X \times \mathbb{R}^n \times A_i$ and on $X \times A_i$, are supposed to be 'common knowledge'.[2]

Within this framework, the Central Agency must rely upon communication by the players of their private information, which is characterized by their type α_i. Specifically we assume that each agent i has to publicly announce some type $a_i \in A_i$ as being his own type α_i. We call such an announcement by agent i a *message* of agent i. However, before sending any message, each individual agent is supposed to know the mechanism ruling the agency behaviour. Letting $A = \bigtimes_{i \in N} A_i$, we call *mechanism* any function $m = (d, t)$ from A to $X \times \mathbb{R}^n$ where

(i) d is a function from A to X called a *decision rule* and giving the alternative selected $d(a) = x$ in X if $a \in A$ is the n-tuple of individual messages, and

(ii) t is a function from A to $\mathbb{R}^n$ called a *transfer scheme* and such that $t(a) = y$ is the vector of individual transfers which are designed if $a \in A$ is the n-tuple of individual messages.

We denote by M the set of all possible mechanisms. In the subsequent sections we shall put important restrictions on M. However, it is already clear that, given a mechanism $m \in M$, the existence of some kind of communication process between the agents and the Central Agency introduces strategic considerations. Indeed, a strategy of player $i \in N$ consists in announcing some particular message $a_i \in A_i$ as being his own type. Suppose that $\alpha \in A$ is the n-tuple of the individual agents' types. Then, for every n-tuple of messages $a \in A$, the payoffs are defined by

$$\begin{aligned} \forall\, i \in N, \forall\, a \in A, W_i^m(a_i; \alpha_i) &= V_i(m(a); \alpha_i) \\ &= U_i(d(a); \alpha_i) + t_i(a). \end{aligned}$$

Thus, the communication process between the individual agents and the center may be formalized as an n-person game in normal form, which is conditional to $\alpha \in A$ and is denoted

$$\Gamma^m(\alpha) = \big\{\{A_i, i \in N\}, \{W_i^m(\cdot; \alpha_i); \alpha_i \in N\}\big\}.$$

This game is called the *communication game.*

1.2 The incomplete information hypothesis

The analysis of the game $\Gamma^m(\alpha)$ is essential in the formulation of the incentive problem. Indeed, the question is to determine whether, knowing mechanism $m \in M$, it is advantageous for every player to reveal his type. Now, such a property may have different characterizations according to the alternative informational specifications which can be made relative to the game $\Gamma^m(\alpha)$.

A first specification would be to say that even though the Central Agency has to take into account only the individual messages, the individual agents themselves are completely informed about the

2. This notion is taken in the sense of Aumann (1975), i.e. no one can consciously disagree with some one else about what they are.

game $\Gamma^m(\alpha)$ at the time of the communication process. In other words, every type α_i is known to all players but, still, the Central Agency can only base its selection on the messages of the players. Hence the communication game $\Gamma^m(\alpha)$ is played as a game under complete information.[3] Such a specification leads to a first 'incentive compatibility property'. Namely a mechanism $m \in M$ is said to be *incentive compatible (under complete information)* iff the n-tuple of strategies $\alpha \in A$, where every player announces his own type, is a Nash-equilibrium for $\Gamma^m(\alpha)$.

The relevancy of this first incentive compatibility notion crucially depends on the complete information assumption. Generally, however, in decentralized contexts it seems better to assume that every agent has *incomplete information* concerning the types α_i of all other agents, i.e. every agent j does not know which type α_i in A_i is the type of agent i, for any $i \neq j$. In probabilistic forms, for every $j \in N$, the type of i, $i \neq j$, is a random phenomenon with A_i as its sample space. As a consequence given a mechanism $m \in M$, every player j has only partial knowledge of what are the other players' payoff functions. Consequently, also, although he knows that there exists, for every $\alpha \in A$, a communication game $\Gamma^m(\alpha)$, player j ignores which one exactly is to be played. Hence, when he wants to characterize the behaviour of any other player i, player j must consider not only what message a_i player i announces but also what type α_i could be player i's true type. In particular, every player j may want to characterize, for every other player, the behaviour consisting in revealing his true type.

For this reason we have to introduce a more sophisticated strategy concept. For every $i \in N$, a *normalized strategy* of player i[4] is a decision rule a_i^* associating a unique strategy choice $a_i \in A_i$ to each of his possible types $\alpha_i \in A_i$. Formally, a_i^* is simply a function from A_i to A_i. We denote by A_i^* the set of all admissible normalized strategies for i. For example, the strategy consisting in declaring the true value of his parameter in the communication process (the 'truth strategy') is a normalized strategy for each player. It is denoted $\hat{a}_i^*$ for player $i \in N$ and such that

$$\forall\, \alpha_i \in A_i, \hat{a}_i^*(\alpha_i) = \alpha_i.$$

Now, to treat the incentive problem in the incomplete information framework, we may distinguish two approaches, each one associated with a different information assumption concerning the communication process. The first approach considers that for every player $i \in N$, the other players' space of types $A_{-i} = \times_{j \neq N} A_j$ is a space of states of nature for which player i satisfies, as a decision-maker, the '*complete ignorance' postulate*.[5] In this case define, for $m \in M$,

$$G(m) \overset{\text{def}}{=} \big\{\{\Gamma^m(\alpha); \alpha \in A\}, \{A_i^*; i \in N\}\big\}.$$

According to this approach, all the games belonging to $G(m)$ have to be considered simultaneously by *all* agents.

Take now any mechanism $m \in M$ and consider the game collection $G(m)$. For notation convenience let

$$a^*(\alpha) = \big(a_1^*(\alpha_1), \ldots, a_n^*(\alpha_n)\big), a^* \in A^* = \bigtimes_{i \in N} A_i^*,$$

and

$$a_{-i}^*(\alpha_{-i}) = \big(a_1^*(\alpha_1), \ldots, a_{i-1}^*(\alpha_{i-1}), a_{i+1}^*(\alpha_{i+1}), \ldots, a_n^*(\alpha_n)\big),$$

3. It seems to us that this would be the best interpretation of the model presented by Roberts (1979), Schoumaker (1976) and Henry (1979).

4. This is Harsanyi (1967-68) terminology.

5. See Luce and Raiffa (1957), p. 294.

where $\alpha_{-i} = (\alpha_1, \ldots, \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_n)$ is an element of

$$A_{-i} = \bigtimes_{\substack{j \in N \\ j \neq i}} A_j$$

and $a^*_{-i} = (a^*_1, \ldots, a^*_{i-1}, a^*_{i+1}, \ldots, a^*_n)$ is an element of

$$A^*_{-i} = \bigtimes_{\substack{j \in N \\ j \neq i}} A^*_j.$$

We say that the mechanism $m \in M$ is *incentive compatible (under complete ignorance)* iff the n-tuple of normalized strategies $\hat{a}^* \in A^*$ (the truth strategies) is a Nash-equilibrium for every game $\Gamma^m(\alpha)$ in $G(m)$ in the sense that

$$\forall\, \alpha \in A, \forall\, i \in N, \forall\, a_i \in A_i, W_i^m \hat{a}_i, \hat{a}^*_{-i}(\alpha_{-i}), \alpha_i) \leq W_i^m(\hat{a}^*(\alpha); \alpha_i).$$

This first approach has already been extensively studied in the literature.[6] In the following section we present another approach to incomplete information which we shall use from now on, in all the rest of the paper.

1.3 Bayesian incentive compatible mechanisms

The second approach to incomplete information takes explicitly into account the 'beliefs' that any player may have concerning the other players types. Formally, we shall represent the beliefs as follows.

Let, for every $i \in N$, $\mathcal{I}_i$ be a σ-algebra on A_i and

$$\mathcal{I}_{-i} = \bigoplus_{\substack{j \in N \\ j \neq i}} \mathcal{I}_j$$

be the product σ-algebra on A_{-i}. We assume that the beliefs of players $i \in N$ are represented by a family $P_i = \{P_i(\cdot \mid \alpha_i \in A_i\}$ where, for every $\alpha_i \in A_i$, $P_i(\cdot \mid \alpha_i)$ is a probability over $(A_{-i}, \mathcal{I}_{-i})$. We also consider that all beliefs $\{P_i; i \in N\}$ are common knowledge but that in general player $i \in N$ beliefs are completely known only when his type $\alpha_i \in A_i$ is also known.

Now, to each mechanism $m \in M$ we may associate the game (with incomplete information):[7]

$$\Gamma(m) = \big\{\{\Gamma^m(\alpha); \alpha \in A\}, \{A^*_i; i \in N\}, \{P_i; i \in N\}\big\}.$$

In such a game every player $i \in N$ is going to choose a message in terms of his *expected payoff* conditional on α_i and given the choice of some normalized strategy by every other player. Hence, we

6. This concept which is primarily due to Hurwicz (1972) has been used in particular by Groves (1973), Groves and Loeb (1975) and Green and Laffont (1976). Notice that Ledyard (1979) when he refers to complete information actually studies the present complete ignorance situation. Notice also that several authors have introduced an apparently stronger concept by requiring that the n-tuple $\hat{a}^*$ be an equilibrium in dominating strategies for $G(m)$. As shown in d'Aspremont and Gérard-Varet (1979) the two concepts actually coincide. This last paper contains other results in the complete ignorance framework.

7. See Harsanyi (1967-68).

shall write the payoffs

$$\forall\, i \in N, \forall\, \alpha_i \in A_i, \forall\, a^*_{-i} \in A^*_{-i}, \forall\, a_i \in A_i,$$
$$\overline{W}^m_i(a_i, a^*_{-i}; \alpha_i) = \int_{A_{-i}} W^m_i(a_i, a^*_{-i}(\alpha_{-i}); \alpha_i) P_i(d\alpha_{-i} \mid \alpha_i).$$

In this paper we shall assume that such an expression is always well-defined.

Now, a mechanism $m \in M$ is said to be *Bayesian incentive compatible* iff the n-tuple of normalized strategies $\hat{a}^* \in A^*$ (the 'truth' strategies) is a Bayesian equilibrium for $\Gamma(m)$ in the sense that

$$\forall\, i \in N, \forall\, \alpha_i \in A_i, \forall\, a_i \in A_i,$$
$$\overline{W}^m_i(a_i, \hat{a}^*_{-i}; \alpha_i) \leq \overline{W}^m_i(\hat{a}^*_i(\alpha_i), \hat{a}^*_{-i}; \alpha_i).$$

The purpose of this article is to study admissible subsets of the set M of all possible mechanisms which are Bayesian incentive compatible.

2. Outcome efficiency and Bayesian incentive compatibility

We shall concentrate first on the subset in M of all mechanisms that are 'outcome efficient' and examine in this subset those that are Bayesian incentive compatible.

2.1 Outcome efficient mechanisms

For any mechanism $m = (d, t) \in M$ we say that it is *outcome efficient* iff we have

$$\forall\, a \in A, d(a) \in \mathcal{P}(a) \stackrel{\text{def}}{=} \left\{ x^* \in X; \sum_{i \in N} U_i(x^*; a_i) = \sup_{x \in X} \sum_{i \in N} U_i(x; a_i) \right\}.$$

In order for such a requirement to be meaningful one has to impose some regularity hypothesis. One of these is:

H_1: X is compact and for every $a \in A$, $\sum_{i \in N} U_i(\cdot; a_i)$ is upper semi-continuous on X.

Condition H_1 implies only that, for every $a \in A$, we have $\mathcal{P}(a) \neq \emptyset$. We shall, for some results, need the stronger condition:

H_2: $X \subseteq \mathbb{R}^K$ is open convex; for every $i \in N$, $A_i \subseteq \mathbb{R}^L$ is open; U_i is twice continuously differentiable on $\mathbb{R}^K \times \mathbb{R}^L$; for every $a \in A$, $\sum_{i \in N} U_i(\cdot; a_i)$ has a critical point occurring at a point interior to X and the quadratic form corresponding to the $K \times K$-matrix of second order partial derivatives of $\sum_{i \in N} U_i(\cdot; a_i)$ is negative definite for every $x \in X$.

Condition H_2 implies – by using the implicit function theorem (see Fleming, 1965, p. 116) – that we have:

Lemma 1 *Assuming H_2 there is a unique outcome efficient decision rule d which is continuously differentiable over A and such that*[8]

$$\forall\, a \in A, \sum_{i \in N} D_x U_i(d(a); a_i) = 0.$$

2.2 Subjectively discretionary distribution mechanisms

In order to study outcome efficient mechanisms which have the Bayesian incentive compatibility property, we shall consider first a particular subset of mechanisms.

We say of a mechanism $m \in M$ that it is a *distribution mechanism* iff the transfer scheme t is such that

$$\forall\, i \in N, \forall\, a \in A, t_i(a) = \sum_{\substack{j \in N \\ j \neq i}} U_j(d(a); a_j) - f_i(a),$$

where f_i is, for every $i \in N$, a real-valued function defined over A. In this case, player $i \in N$ receives from (or pays to) the Central Agency the difference between the amounts

$$\sum_{\substack{j \in N \\ j \neq i}} U_j(d(a); a_j)$$

and $f_i(a)$, both defined in terms of the declared types. The n-tuple $f = (f_1, \ldots, f_i, \ldots, f_n)$ is called a *distribution rule*.

Notice that we may, without loss of generality, restrict ourselves to distribution mechanisms, since to every mechanism $m = (d, t)$ one may always associate a distribution mechanism $m = (d, t')$ by designing the following distribution rule f':

$$\forall\, i \in N, \forall\, a \in A, f'_i(a) = \sum_{\substack{j \in N \\ j \neq i}} U_j(d(a); a_j) - t_i(a).$$

Now, we say of a distribution rule f that it is *subjectively discretionary* iff

$$\begin{aligned}
&\forall\, i \in N, \forall\, \alpha_i \in A_i, \forall\, a_i \in A_i, \forall\, a'_i \in A_i,\\
&\int_{A_{-i}} f_i(a_i, \alpha_{-i})\, P_i(d\alpha_{-i} \mid \alpha_i) = \int_{A_{-i}} f_i(a'_i, \alpha_{-i})\, P_i(d\alpha_{-i} \mid \alpha_i),
\end{aligned}$$

i.e. for every player $i \in N$, the expected value of f_i must remain constant with respect to the player's messages. A *subjectively discretionary distribution mechanism* is a distribution mechanism for which the associated distribution rule is subjectively discretionary.

We are interested by the class of distribution mechanisms which are both outcome efficient and subjectively discretionary[9] for the reason that it is included in the class of Bayesian incentive compatible mechanisms.

8. Given $Y_1 \subseteq \mathbb{R}^L$, $Y_2 \subseteq \mathbb{R}^M$, two open sets, and a function f from $\mathbb{R}^L \times \mathbb{R}^M$ to $\mathbb{R}^P$, we denote by $Df(y) = (Df^1(y), \ldots, Df^P(y))^t$ the derivative of f at a point $y = (y_1, y_2) \in Y_1 \times Y_2$, given by the Jacobian matrix of f at y. Also we denote by $D_{y1} f(y) = (D_{y11} f(y), \ldots, D_{y1L} f(y))$ the vector of partial derivatives of f with respect to the components of y_1 at a point $y = (y_1, y_2)$. Similarly for $D_{y2} f(y)$. All vectors are taken with respect to the usual basis.
9. The class of subjectively discretionary mechanisms which are outcome efficient includes the class of *Groves-mechanisms* as introduced by Groves (1973).

Theorem 1 *Under assumption H_1, any distribution mechanism $m \in M$ which is outcome efficient and subjectively discretionary is Bayesian incentive compatible.*

Proof By assumption, the individual payoffs are well-defined. We want to show that for the mechanism m:

$$\forall\, i \in N, \forall\, \alpha_i \in A_i, \forall\, a_i \in A_i, \overline{W}_i^m(a_i, \hat{a}^*_{-i}; \alpha_i) \leq \overline{W}_i^m(\alpha_i, \hat{a}^*_{-i}; \alpha_i), \tag{1}$$

since m is outcome efficient, we have

$$\begin{aligned}&\forall\, \alpha \in A, \forall\, i \in N, \forall\, a_i \in A_i,\\ &U_i\big(d(\alpha); \alpha_i\big); \alpha_i) + \sum_{\substack{j \in N\\ j \neq i}} U_j\big(d(\alpha); \alpha_j\big) \geq U_i\big(d(a_i, \alpha_{-i}); \alpha_i\big) + \sum_{\substack{j \in N\\ j \neq i}} U_j\big(d(a_i, \alpha_{-i}); \alpha_j\big),\end{aligned}$$

which implies

$$\begin{aligned}&\int_{A_{-i}} U_i\big(d(\alpha); \alpha_i\big)\, P_i(d\alpha_{-i} \mid \alpha_i) + \int_{A_{-i}} \sum_{j \neq i} U_j\big(d(\alpha); \alpha_j\big)\, P_i(d\alpha_{-i} \mid \alpha_i)\\ \geq &\int_{A} U_i\big(d(a_i, \alpha_{-i}); \alpha_i\big)\, P_i(d\alpha_{-i} \mid \alpha_i)\\ + &\int_{A_{-i}} \sum_{j \neq i} U_j\big(d(a_i, \alpha_{-i}); \alpha_j\big)\, P_i(d\alpha_{-i} \mid \alpha_i).\end{aligned} \tag{2}$$

Since m is subjectively discretionary:

$$\begin{aligned}&\forall\, i \in N, \forall\, \alpha_i \in A_i, \forall\, a_i \in A_i,\\ &\int_{A_{-i}} f_i(\alpha_i, \alpha_{-i})\, P_i(d\alpha_{-i} \mid \alpha_i) = \int_{A_{-i}} f_i(a_i, \alpha_{-i})\, P_i(d\alpha_{-i} \mid \alpha_i).\end{aligned} \tag{3}$$

In light of (3), condition (2) is equivalent to condition (1). ■

2.3 A characterization of Bayesian incentive compatibility

We are now interested in giving a characterization of the mechanisms which are simultaneously outcome efficient and Bayesian incentive compatible. For that matter we shall restrict ourselves to the mechanisms which are *continuously differentiable*. Under assumption H_2, a distribution mechanism is continuously differentiable iff the distribution rule f is continuously differentiable over A.

On the other hand we shall assume that all the players' beliefs satisfy a restriction called the *independence condition* which requires

$$\begin{aligned}&\forall\, i \in N, \forall\, \alpha_i \in A_i, \forall\, \alpha_i' \in A_i, \forall\, E \in \mathcal{I}_{-i},\\ &P_i(E \mid \alpha_i) = P_i(E \mid \alpha_i') = \Pi_i(E).\end{aligned}$$

In that case, the beliefs are represented by the family $\{\Pi_i; i \in N\}$ of probabilities, where, for every $i \in N$, Π_i is a probability over $(A_{-i}, \mathcal{I}_{-i})$ which is free with respect to the parameter $\alpha_i \in A_i$.

Considering that the beliefs $\{P_i; i \in N\}$ have been assumed common knowledge, this condition appears rather restrictive. It means not only that the private information, which each individual agent has concerning his own type, has no bearing on his information concerning the others, but also that the beliefs of any player are not subject to strategic distortions. This strong condition could be justified if the players' types were known to have been selected through a random process with given law.[10] In a parallel paper we propose to weaken this condition (see d'Aspremont and Gérard-Varet, 1979).

Furthermore, for technical matters we shall restrict ourselves to the class of beliefs satisfying, in the independent case, the following condition:

D: For every $i \in N$, A_i is open, bounded, connected and atomless (w.r.t. the Lebesgue measure) in $\mathbb{R}^L$, $\mathfrak{I}_i$ is the Borel σ-algebra, and there exists a continuous real-valued function π_i over $\mathbb{R}^{(n-1)L}$ such that

$$\forall\, E \in \mathfrak{I}_{-i}\, \Pi_i(E) = \int_E \pi(\alpha_{-i})\, d\alpha_{-i}.$$

We have now the following preliminary result.

Lemma 2 *If H_2 holds, and if the beliefs are independent and satisfy condition D, for every continuously differentiable distribution mechanism m, for every $i \in N$ and every $\alpha_i \in A_i$, $\overline{W}_i^m(\cdot; \hat{a}^*_{-i}; \alpha_i)$ is differentiable over A_i.*

Proof Take any $i \in N$ and any $\alpha_i \in A_i$:

$$\forall\, a_i \in A_i, \overline{W}_i^m(a_i, \hat{a}^*_{-i}; \alpha_i) = \int_{A_{-i}} W_i^m(a_i; \alpha_{-i}; \alpha_i)\pi_i(\alpha_{-i})d\alpha_{-i}.$$

We also have

$$\begin{aligned}&\forall\, i \in N, \forall\, \alpha_i \in A_i, \forall\, \alpha_{-i} \in A_{-i}, \forall\, a_i \in A_i\\ &W_i^m(a_i, \alpha_{-i}; \alpha_i) = \sum_{j \in N} U_j\big(d(a_i, \alpha_{-i}); \alpha_j\big) - f_i(a_i, \alpha_{-i}).\end{aligned}$$

With our assumptions, for every $\alpha_i \in A_i$ and every $i \in N$, $W_i^m(\cdot; \alpha_i)$ is continuously differentiable over $\mathbb{R}^{nL}$. Consequently,

$$\begin{aligned}&\forall\, a_i \in \mathbb{R}^L, \forall\, \alpha_{-i} \in R^{(n-1)L},\\ &D_{a_i}\, W_i^m(a_i, \alpha_{-i}; \alpha_i) = D_{a_i}\left[\sum_{j \in N} U_j(d(a_i, \alpha_{-i}); \alpha_j) - f_i(a_i, \alpha_{-i})\right]\end{aligned}$$

is continuous. In particular, $D_{a_i}\, W_i^m(\cdot; \alpha_i)$ is continuous over the product of the open set A_i by the closure $\overline{A}_{-i}$ of A_{-i} which is compact by assumption D. Also, with assumption D, we may write

$$\begin{aligned}&\forall\, i \in N, \forall\, a_i \in A_i,\\ &D_{a_i}\int_{A_{-i}} W_i^m(a_i, \alpha_{-i}; \alpha_i)\pi_i(\alpha_{-i})d\alpha_{-i} = D_{a_i}\int_{\overline{A}_{-i}} W_i^m(a_i, \alpha_{-i}; \alpha_i)\, \pi_i(\alpha_{-i})d\alpha_{-i}.\end{aligned}$$

10. For example, Green et al. (1976), assume that each of the individuals in the society believes that all of the others are drawn independently from a normal population with zero mean' (p.384). For a general discussion on this topic see Harsanyi (1967-68).

Therefore we get (by Fleming, 1965, p. 199):

$$\begin{aligned}&\forall i \in N, \forall\, a_i \in A_i,\\ &D_{a_i}\,\overline{W}_i^m(a_i,\hat{\alpha}^*_{-i};\alpha_i) = D_{a_i}\int_{A_{-i}} W_i^m(a_i,\alpha_{-i};\alpha_i)\pi_i(\alpha_{-i})d\alpha_{-i}\\ &\qquad\qquad = \int_{A_{-i}} D_{a_i}\,W_i^m(a_i,\alpha_{-i};\alpha_i)\,\pi_i(\alpha_{-i})d\alpha_{-i}.\end{aligned}$$

■

We are now in position to prove the main theorem of this section:[11]

Theorem 2 *Under assumption H_2 and for independent beliefs satisfying condition D, a distribution mechanism $m \in M$ which is continuously differentiable and outcome efficient is Bayesian incentive compatible iff it is subjectively discretionary.*

Proof Since our assumptions are now stronger, the first part simply results from Theorem 1. It remains to show that any distribution mechanism $m = (d,t)$ which is outcome efficient and Bayesian incentive compatible is subjectively discretionary which, with our assumptions, is equivalent to (Fleming, 1965, p. 42),

$$\forall\, i \in N, \forall\, a_i \in A_i, D_{a_i}\int_{A_{-i}} f_i(a_i,\alpha_{-i})\,\pi_i(\alpha_{-i})d\alpha_{-i} = 0.$$

If $m = (d,t)$ is Bayesian incentive compatible and since, by Lemma 2, for every $i \in N$ and every $\alpha_i \in A_i$, $\overline{W}_i^m(\hat{a}^*_{-i};\alpha_i)$ is differentiable over the open set A_i, we must have

$$\forall\, i \in N, \forall\, \alpha_i \in A_i, D_{a_i}\overline{W}_i^m(a_i,\hat{a}^*_{-i};\alpha_i) = 0, \quad \text{if } a_i = \alpha_i.$$

This last condition gives for every $i \in N$:

$$\begin{aligned}&\forall\, \alpha_i \in A_i, D_{a_i}\overline{W}_i^m(\alpha_i,\hat{a}^*_{-i};\alpha_i)\\ &\quad = \int_{A_{-i}} D_{a_i}\,W_i^m(\alpha_i,\alpha_{-i};\alpha_i)\,\pi_i(\alpha_{-i})d\alpha_{-i}\\ &\quad = \int_{A_{-i}}\Big[\sum_{j\in N} D_x\,U_j\big(d(\alpha_i,\alpha_{-i});\alpha_i\big)D_{a_i},d(\alpha_i,\alpha_{-i}) - D_{a_i}\,f_i(\alpha_i,\alpha_{-i})\Big]\pi_i(\alpha_{-i})\,d\alpha_{-i}\\ &\quad = 0.\end{aligned}$$

By Lemma 1, the outcome efficient decision rule d is such that

$$\sum_{j\in N} D_x\,U_j\big(d(\alpha_i,\alpha_{-i});\alpha_i\big) = 0.$$

Hence, for every $i \in N$, we have

$$\begin{aligned}\forall\, \alpha_i \in A_i, D_{a_i}\overline{W}_i^m(\alpha_i,\hat{a}^*_{-i};\alpha_i) &= -\int_{A_{-i}} D_{a_i}\,f_i(\alpha_i,\alpha_{-i})\pi_i(\alpha_{-i})d\alpha_{-i}\\ &= -D_{a_i}\int_{A_{-i}} f_i(\alpha_i,\alpha_{-i})\,\pi_i(\alpha_{-i})\,d\alpha_{-i} = 0.\end{aligned}$$

11. A proof of this theorem has been given for $n = 2$ and $X \subseteq \mathbb{R}$ in d'Aspremont and Gérard-Varet (1975).

∎

Corollary 1 *Under the same assumptions, a continuously differentiable mechanism which is outcome efficient and Bayesian incentive compatible is such that*

$$\forall\, i \in N, \forall\, a_i \in A_i,$$
$$\int_{A_{-i}} t_i(a_i, \alpha_{-i})\, \pi_i(\alpha_{-i})\, d\alpha_{-i} = \int_{A_{-i}} \sum_{\substack{j\in N \\ j\neq i}} U_j\big(d(a_i, \alpha_{-i}); \alpha_j\big)\, \pi_i(\alpha_{-i})\, d\alpha_{-i} - k_i,$$

where k_i is some constant.

3. The budget problem and the collective rationality postulate

The Bayesian incentive compatible mechanisms introduced in the preceding section imply the existence of transfers which are to be made through the budget of the Central Agency. Hence, an important consideration is to know whether the structure of the transfers make it possible for the Agency to balance its budget.

3.1 Budget balancing mechanisms

We say that a mechanism $m \in M$ is *budget balancing* iff

$$\sum_{i\in N} t_i(\cdot) = 0.$$

Define a set $Y \equiv \{y \in \mathbb{R}^n; \sum_{i\in N} y_i \leq 0\}$ of all admissible vectors of individual transfers and the function v from A to R such that

$$\forall\, a \in A, v(a) = \sup_{X\times Y} \sum_{i\in N} V_i(x, y; a_i).$$

We say of a mechanism $m \in M$ that it satisfies the *collective rationality postulate* iff we have

$$\forall\, a \in A, \sum_{i\in N} V_i(m(a); a_i) = v(a).$$

A mechanism belonging to this admissibility set is clearly such that the players do not have any collective incentive to reject it as the solution of the collective choice problem.

Under our separability assumption, the properties of outcome efficiency and budget balancing are equivalent to the collective rationality postulate, since then

$$\forall\, a \in A, v(a) = \sup_{x\in X} \sum_{i\in N} U_i(x; a_i).$$

Collective rationality in the separable case is simply an assumption of Pareto-optimality. Hence the budget balancing condition introduces both a feasibility restriction and an efficiency restriction on the transfers.

3.2 Collective rationality in the independent case

In this section we study the possibility of finding collectively rational mechanisms which satisfy Bayesian incentive compatibility in the particular case where the players' beliefs respect the independence condition. An alternative case is treated in d'Aspremont and Gérard-Varet (1979).

Theorem 3 *Under assumption H_1 and if the beliefs are independent, let m be a mechanism $m = (d,t) \in M$, where d is outcome efficient and such that $\forall\, i \in N, \forall\, a \in A, t_i(a) = g_i(a_i) - g_{-i}(a_{-i})$ for*

$$g_i(a_i) = \int_{A_{-i}} \sum_{\substack{j\in N\\ j\neq i}} U_j\big(d(a_i,\alpha_{-i});\alpha_j\big)\,\Pi_i(d\,\alpha_{-i})$$

and

$$g_{-i}(a_{-i}) = \frac{1}{n-1}\sum_{\substack{j\in N\\ j\neq i}} g_j(a_j).$$

Then m is collectively rational and Bayesian incentive compatible.

Proof Since for every $a \in A$ we have

$$\sum_{i\in N} t_i(a) = \sum_{i\in N} g_i(a_i) - \frac{1}{n-1}\sum_{i\in N}\sum_{\substack{j\in N\\ j\neq i}} g_j(a_j) = 0,$$

m is budget balancing. Moreover, m may be seen as the distribution mechanism where

$$\forall\, a \in A, f_i(a) = \sum_{\substack{j\in N\\ j\neq i}} U_j\big(d(a);a_j\big) - g_i(a_i) + g_{-i}(a_{-i}).$$

But then, $\forall\,\alpha \in A, \forall\, i \in N, \forall\, a_i \in A_i$,

$$\int_{A_{-i}} f_i(a_i,\alpha_{-i})\,\Pi_i(d\alpha_{-i}) = \int_{A_{-i}} g_{-i}(\alpha_{-i})\Pi_i(d\alpha_{-i}),$$

which is constant. Hence, the distribution mechanism m is subjectively discretionary. Therefore, by Theorem 1, m is Bayesian incentive compatible. ■

Corollary 2 *Under the same assumption and with the same notation, let $m = (d,t)$ be an outcome efficient mechanism defined by*

$$\forall\, i \in N, \forall\, a \in A, t_i(a) = g_i(a_i) - g_{-i}(a_i) + h_i(a),$$

where the functions h_i are defined on A so that $\sum_{i\in N} h_i(\cdot)$ be identically zero and $\int_{A_{-i}} h_i(\cdot,\alpha_{-i})$ $\Pi_i(d\alpha_{-i})$ be constant. Then m is collectively rational and Bayesian incentive compatible.

The proof of this corollary is immediate. It leads, together with Theorem 2, to the following, which then gives a characterization of collectively rational and Bayesian incentive compatible mechanisms.

Theorem 4 *Assuming H_2 and for independent beliefs satisfying condition D, let $m = (d, t)$ be a continuously differentiable mechanism which is collectively rational and Bayesian incentive compatible. Then there exist continuously differentiable functions h_i on A, $i \in N$, such that $\sum_{i \in N} h_i(\cdot)$ is identically zero and $\int_{A_{-i}} h_i(\cdot, \alpha_{-i})\, \pi(\alpha_{-i})\, d\alpha_{-i}$ is constant and which satisfy*

$$\forall\, i \in N, \forall\, a \in A, t_i(a) = g_i(a_i) - g_{-i}(a_{-i}) + h_i(a).$$

Proof Let, $\forall\, i \in N, \forall\, a \in A$,

$$h_i(a) = g_{-i}(a_{-i}) - g_i(a_i) + t_i(a).$$

Then using the corollary to Theorem 2, it is easy to see that the functions h_i have all the required properties. ■

It is clear now that, with sufficient differentiability assumptions, Theorem 4 together with the corollary to Theorem 3 give a complete characterization of the class of mechanisms which are both collectively rational and Bayesian incentive compatible, in the independent case.

4. Individual rationality

Let us consider a mechanism $m \in M$ which is collectively rational and Bayesian incentive compatible. A supplementary requirement to be made is that whenever each player reports to the Agency the true value of his parameter, a player's payoff (or at least expected payoff) will not be worse than what he could guarantee himself without being involved in the collective decision problem.

We shall concentrate on the class of mechanisms given by Theorem 3 which are Bayesian incentive compatible and collectively rational when the beliefs are independent. We shall examine whether any mechanism in the class satisfies some specification of the individual rationality requirement.

For that matter, let us assume that among all admissible alternatives $X = \mathbb{R}^K$, we may isolate one particular alternative $x_0 \in X$ called the *status quo point*, i.e. the alternative which would result if no mechanism were designed. For player $i \in N$, if his true type is $\alpha_i \in A_i$, the status quo payoff is: $U_i(x^0; \alpha_i)$. We shall – sometimes – restrict ourselves to *zero-normalized* payoffs, by letting for every $i \in N$, $U_i(x^0; \alpha_i) = 0$.

Let us now consider two alternative specifications of the individual rationality requirement.

4.1 Two kinds of individual rationality

We say of a mechanism $m = (d, t)$ that it is *strongly individually rational* iff

$$\forall\, i \in N, \forall\, \alpha \in A, V_i(m(\alpha); \alpha_i) \geq U_i(x^0; \alpha_i),$$

i.e. under the separability assumption

$$\forall\, i \in N, \forall\, \alpha \in A, U_i\big(d(\alpha); \alpha_i\big) + t_i(\alpha) \geq U_i(x^0; \alpha_i).$$

In that case, assuming everyone reports the truth, each player must get at least his status-quo payoff.

Alternatively, we say of a mechanism $m = (d, t)$ that it is *weakly individually rational* (or individually rational in *expected value*) iff

$$\forall\, i \in N, \forall\, \alpha \in A, \int_{A_{-i}} V_i(m(\alpha), \alpha_i)\, \Pi_i(d\alpha_{-i}) \geq U_i(x^0; \alpha_i),$$

i.e. under the separability assumption

$$\forall\, i \in N, \forall\, \alpha \in A,$$
$$\int_{A_{-i}} U_i\big(d(\alpha_i, \alpha_{-i}); \alpha_i\big)\, \Pi_i(d\alpha_{-i}) + \int_{A_{-i}} t_i(\alpha_i, \alpha_{-i})\, \Pi_i(d\alpha_{-i}) \geq U_i(x^0; \alpha_i).$$

Obviously any strongly individually rational mechanism is weakly individually rational.

We shall not in this paper give any complete answer to the problem of finding Bayesian incentive compatible mechanisms which are rational both collectively and individually (in some sense). However, the analysis in the following section seems to indicate that, unless the data of the model satisfy rather peculiar conditions, one should not in general expect strong individual rationality to hold.

As far as weak individual rationality is concerned, it is even difficult to make some conjecture. However, using Theorem 4, which applies to the differentiable independent case, it is possible to reformulate the problem in the following way: find continuously differentiable functions h_i, for all $i \in N$, for which again $\sum_{i \in N} h_i(\cdot)$ is identically zero and $\int_{A_{-i}} h_i(\cdot; \alpha_{-i})\, \pi_i(\alpha_{-i})\, d\alpha_{-i}$ is constant, and such that

$$\forall\, i \in N, \forall\, \alpha_i \in A_i,$$
$$\int_{A_{-i}} [v(\alpha_i, \alpha_{-i}) - g_{-i}(\alpha_{-i}) + h_i(\alpha_i, \alpha_{-i})]\, \pi_i(\alpha_{-i}) d\alpha_{-i} \geq U_i(x^0; \alpha_i).$$

4.2 A sufficient condition for strong individual rationality

Let us consider a two-person collective decision problem, where $N = \{1, 2\}$, $A_1 = \{\alpha_1^i; = 1, 2\}$, $A_2 = \{\alpha_2^j; j = 1, 2\}$ and where the (independent) beliefs are described by the two probability distributions of Table 1.

Table 1

α_2^j	$\pi_1^j = \pi_1(\alpha_2^j)$	α_1^i	$\pi_2^i = \pi_2(\alpha_1^i)$
α_2^1	3/5	α_1^1	1/10
α_2^2	2/5	α_1^2	9/10

The individual payoffs associated to the outcome efficient decision rule are given by Table 2, where

$${}^1u_i^{ij} = U_1(d(\alpha_1^i, \alpha_2^j); \alpha_1^i); \qquad {}^2u_i^{ij} = U_2(d(\alpha_1^i, \alpha_2^j); \alpha_2^j)$$

and

$$v^{ij} = U_1\big(d(\alpha_1^i, \alpha_2^j); \alpha_1^i\big) + U_2\big(d(\alpha_1^i, \alpha_2^j); \alpha_2^j\big)$$
$$= \max_x U_1(x; \alpha_1^i) + U_2(x; \alpha_2^j).$$

We assume that the payoffs are zero-normalized, i.e.

$$U_1(x^0; \alpha_1^1) = U_1(x^0; \alpha_1^2) = 0; \qquad U_2(x^0; \alpha_2^1) = U_2(x^0; \alpha_2^2) = 0.$$

Everybody reporting the truth, the payoff space is pictured in Figure 1.

Table 2

	${}^1u_i^{ij}$	${}^2u_j^{ii}$	v^{ij}
(α_1^1, α_2^1)	${}^1u_1^{11} = 30$	${}^2u_1^{11} = 10$	40
	${}^1u_1^{12} = 40$	${}^2u_1^{12} = 0$	
	${}^1u_1^{21} = 5$	${}^2u_1^{21} = 30$	
	${}^1u_1^{22} = 10$	${}^2u_1^{22} = 4$	
(α_1^2, α_2^1)	${}^1u_2^{21} = 60$	${}^2u_1^{21} = 30$	90
	${}^1u_2^{11} = 30$	${}^2u_1^{11} = 10$	
	${}^1u_2^{12} = 20$	${}^2u_1^{12} = 0$	
	${}^1u_2^{22} = 10$	${}^2u_1^{22} = 4$	
(α_1^1, α_2^2)	${}^1u_1^{12} = 40$	${}^2u_2^{12} = 20$	60
	${}^1u_1^{11} = 30$	${}^2u_1^{11} = 0$	
	${}^1u_1^{21} = 5$	${}^2u_1^{21} = 10$	
	${}^1u_1^{22} = 10$	${}^2u_1^{22} = 50$	
(α_1^2, α_2^2)	${}^1u_2^{22} = 10$	${}^2u_2^{22} = 50$	60
	${}^1u_2^{12} = 20$	${}^2u_2^{12} = 20$	
	${}^1u_2^{11} = 30$	${}^2u_2^{11} = 10$	
	${}^1u_2^{21} = 60$	${}^2u_2^{21} = 0$	

Let us now consider the following transfer scheme:

$$\forall\,(\alpha_1^i, \alpha_2^j), t_1^{ij} = t_1(\alpha_1^i, \alpha_1^j) = g_1(\alpha_1^i) - g_2(\alpha_2^j),$$
$$t_2^{ij} = t_2(\alpha_1^i, \alpha_2^j) = g_2(\alpha_2^t) - g_1(\alpha_1^i),$$

where

$$g_1(\alpha_1^i) = \sum_{j=1}^{2} {}^2u_j^{ij}\pi_1^j \text{ and } g_2(\alpha_2^j) = \sum_{i=1}^{2} {}^1u_i^{ij}\pi_2^i,$$

and

$$t_1^{11} = -t_2^{11} = -43; \quad t_1^{21} = -t_2^{21} = -14;$$
$$t_1^{12} = -t_2^{12} = 1; \qquad t_1^{22} = -t_2^{22} = 25.$$

Finally we get, for the mechanism $m = (d, t)$:

$$V_1\big(m(\alpha_1^1, \alpha_2^1); \alpha_1^1\big) = {}^1u_1^{11} + t_1^{11} = 30 - 43 = -13,$$
$$V_2\big(m(\alpha_1^1, \alpha_2^1); \alpha_2^1\big) = {}^2u_2^{11} + t_2^{11} = 10 + 43 = 53.$$

Since, for one pair (α_1^i, α_2^j), one player receives less than his status-quo payoff, the mechanism is not strongly individually rational.

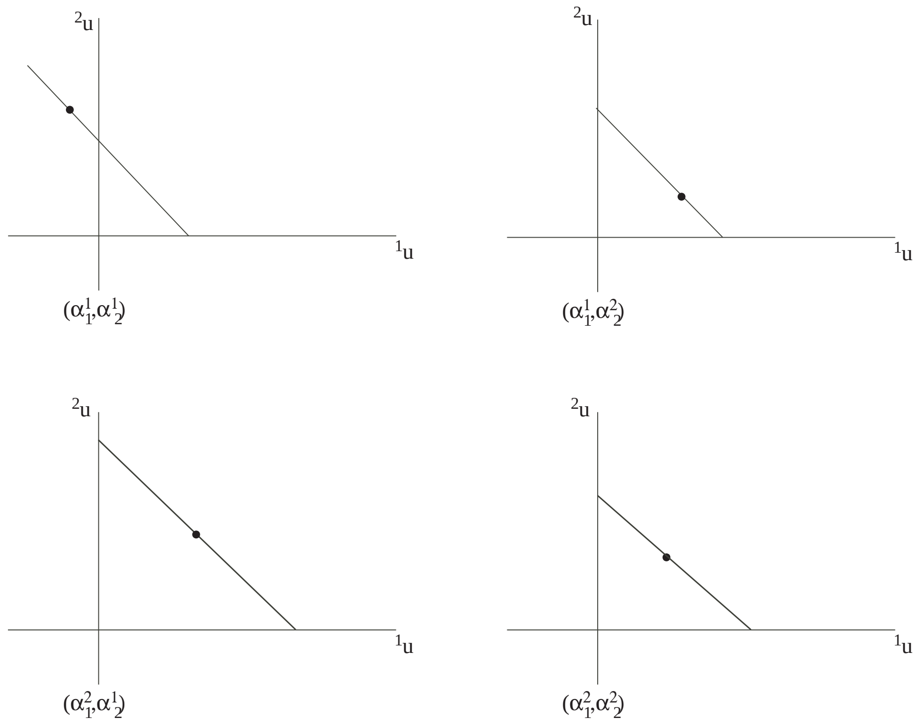

Figure 1

Notice that there exist, for the same problem, other mechanisms that are Bayesian incentive compatible and collectively rational and which may have the strong individual rationality property.

Along that line, one may find situations for which there exist mechanisms which simultaneously are Bayesian incentive compatible, collectively rational and strongly individually rational. Let us consider again two-persons decision problems. We have:

Theorem 5 *Assume H_1 and that the beliefs are independent. Given any efficient outcome decision rule d, consider the mechanism $m = (d, t)$ such that*

$$\forall\, a \in A,\; t_1(a) = g_1(a_1) - g_2(a_2) + h_1(a),$$
$$t_2(a) = g_2(a_2) - g_1(a_1) + h_2(a),$$

with

$$g_i(a_i) = \int_{A_j} U_j\big(d(a), a_j\big)\, \Pi_i(da_j),\ \text{with}\, j \neq i;$$

and

$$\begin{aligned} h_1(a) = U_2\big(d(a), a_2\big) - \frac{1}{2}v(a) - \frac{1}{2}\int_{A_2} U_2\big(d(a); a_2\big)\, \Pi_1(da_2) \\ + \frac{1}{2}\int_{A_1} U_1\big(d(a); a_2\big)\, \Pi_2(da_1) + \frac{1}{2}\int_{A_2} U_1\big(d(a); a_1\big)\, \Pi_1(da_2) \\ - \frac{1}{2}\int_{A_1} U_2\big(d(a); a_2\big)\, \Pi_2(da_1), \end{aligned}$$

$$h_2(a) = U_1\big(d(a), a_1\big) - \frac{1}{2}v(a) - \frac{1}{2}\int_{A_1} U_1\big(d(a); a_1\big)\,\Pi_2(da_1)$$
$$+ \frac{1}{2}\int_{A_2} U_2\big(d(a); a_2\big)\,\Pi_1(da_2) + \frac{1}{2}\int_{A_1} U_2\big(d(a); a_2\big)\,\Pi_2(da_1)$$
$$- \frac{1}{2}\int_{A_2} U_1\big(d(a); a_1\big)\,\Pi_1(da_2).$$

The mechanism $m = (d, t)$ is Bayesian incentive compatible, collectively and strongly individually rational whenever

$$\forall\, a \in A,$$
$$\frac{1}{2}\left|\int_{A_2} v(a_1, a_2)\,\Pi_1(da_2) - \int_{A_1} v(a_1, a_2)\,\Pi_2(da_1)\right| \leq \frac{1}{2}v(a) - \max_{i\in\{1,2\}} U_i(x^0; a_i).$$

Proof
(i) First we have

$$\forall\, a \in A, h_1(a) + h_2(a) = 0,$$

and

$$\int_{A_2} h_1(a_1, a_2)\,\Pi_1(da_2) = \frac{1}{2}\Big[\int_{A_1}\int_{A_2} U_1\big(d(a_1; a_2); a_1\big)\,\Pi_1(da_2)\,\Pi_2(da_1)$$
$$- \int_{A_1}\int_{A_2} U_2\big(d(a_1, a_2); a_1\big)\,\Pi_1(da_2)\,\Pi_2(da_1)\Big],$$

and

$$\int_{A_1} h_2(a_1, a_2)\,\Pi_2(da_1) = \frac{1}{2}\Big[\int_{A_1}\int_{A_2} U_2\big(d(a_1; a_2); a_2\big)\,\Pi_1(da_2)\,\Pi_2(da_1)$$
$$- \int_{A_1}\int_{A_2} U_1\big(d(a_1, a_2); a_2\big)\,\Pi_1(da_2)\,\Pi_2(da_1)\Big].$$

Thus, by Theorem 3, the mechanism $m = (d, t)$ is Bayesian incentive compatible and collectively rational.
(ii) Secondly, we have for any $\alpha \in A$:

$$t_1(\alpha) = U_2\big(d(\alpha); \alpha_2\big) - \frac{1}{2}\left[v(\alpha) - \int_{A_2} v(\alpha_1, \alpha_2)\,\Pi_1(d\alpha_2) + \int_{A_1} v(\alpha_1, \alpha_2)\,\Pi_2(d\alpha_1)\right]$$

and similarly,

$$t_2(\alpha) = U_1\big(d(\alpha); \alpha_1\big) - \frac{1}{2}\left[v(\alpha) - \int_{A_2} v(\alpha_1, \alpha_2)\,\Pi_1(d\alpha_2) - \int_{A_1} v(\alpha_1, \alpha_2)\,\Pi_2(d\alpha_1)\right].$$

Thus, for any $\alpha \in A$,

$$\begin{aligned} V_1\big(m(\alpha);\alpha_1\big) &= U_1\big(d(\alpha);\alpha_1\big) + t_1(\alpha) \\ &= \frac{1}{2}v(\alpha) + \frac{1}{2}\left[\int_{A_2} v(\alpha)\,\Pi_1(d\alpha_2) - \int_{A_1} v(\alpha)\,\Pi_2(d\alpha_1)\right], \\ V_2\big(m(\alpha);\alpha_2\big) &= U_2\big(d(\alpha);\alpha_2\big) + t_2(\alpha) \\ &= \frac{1}{2}v(\alpha) - \frac{1}{2}\left[\int_{A_2} v(\alpha)\,\Pi_1(d\alpha_2) - \int_{A_1} v(\alpha)\,\Pi_2(d\alpha_1)\right]. \end{aligned}$$

By assumption we have

$$\forall\,\alpha \in A, \frac{1}{2}v(\alpha) - \frac{1}{2}\left|\int_{A_2} v(\alpha)\,\Pi_1(d\alpha_2) - \int_{A_1} v(\alpha)\,\Pi_2(d\alpha_1)\right| \geq \max_{i\in\{1,2\}} U_i(x^0;\alpha_i),$$

which clearly implies

$$\forall\, i \in \{1,2\}, \forall\,\alpha \in A, V\big(m(\alpha),\alpha_i\big) \geq U_i(x^0;\alpha_i).$$

■

It is easy to construct examples where the sufficient condition given in this theorem is violated. For instance if we change v^{21} to 110 and v^{22} to 70 in the example above and modify the ${}^1u_i^{ij}$'s and the ${}^2u_j^{ij}$'s accordingly (in a consistent way) then the condition is violated. In this modified example, even weak individual rationality is not satisfied.

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