Oligopolistic competition as a common agency game*

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Received 23 February 2008 – Available online 20 May 2009

Abstract

Oligopolistic competition is analyzed in a complete information multi-principal common agency framework, where principals are firms supplying differentiated goods and the agent is a representative consumer. We first propose a canonical formulation of common agency games, and a parameterization of the set of equilibria based on the Lagrange multipliers associated with the participation and the incentive compatibility constraints of each principal. This is used to characterize the set of equilibria in the intrinsic and non-intrinsic games. The former includes the latter, as well as the standard price and quantity equilibrium outcomes. It may also include the collusive solution.

Keywords: Common agency; oligopolistic competition; competitive toughness; tacit collusion

JEL Classification: D43, L13

* *Games and Economic Behavior*, 70, 21–33, 2010. Reprint 2238

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1 Introduction

We propose to analyze oligopolistic competition in the common agent multi-principal framework, as introduced by Bernheim and Whinston (1985, 1986) and further developed by Martimort, Stole and others.[1] This framework is known to be flexible enough for studying several economic situations such as the regulation of a firm by several government authorities, the control of a common distributor by several competing producers or, on the contrary, the relationship between several retailers distributing the output of a common manufacturer. Here, we shall study competition among several firms producing differentiated goods for a common (representative) consumer, whose preferences are separable with respect to those goods. Our approach is in some sense close to the seminal contribution of Bernheim and Whinston (1985), who consider competing firms delegating control over their marketing activity to a common agent. They ignore, as we do, some relevant factors such as economies of scale or asymmetric information to concentrate on the use of common agency as a device to facilitate collusion. We will examine this issue in an even more straightforward way, by directly treating a representative consumer as the common agent, to which firms offer price-quantity contracts. In our approach, the essential feature of Bernheim and Whinston (1985) will be preserved, namely the coordinating role played by the common participation constraint, characterizing the "intrinsic" common agency and allowing to attain full cooperation. For our part, we want to stress the emulating role played by the incentive compatibility constraint, fostering competition. We will show that the combination of these two forces creates a large equilibrium indeterminacy in the intrinsic common agency game, allowing to obtain the perfectly competitive outcome at one extreme and, for some values of the fundamentals, the collusive solution at the other. For that purpose, we will introduce a canonical formulation of common agency games, leading to a natural parameterization of the set of equilibria. This canonical formulation may also be applied to the "non-intrinsic" common agency game, where participation constraints are individualized, in order to compare the set of equilibria in the two games.

[1]See Stole (1991), Martimort (1992, 1996), Mezzetti (1997), Martimort and Stole (2002, 2003, 2008), Peters (2001), Laussel and Le Breton (2001), and references therein.

Our analysis of the intrinsic common agency is related to previous work on oligopolistic equilibrium indeterminacy.[2] This previous work exploits another coordination device, given by the so-called "facilitating practices" (Salop, 1986; Kalai and Satterthwaite, 1994),[3] which include various meeting competition policies characterized by different clauses in the sales contract. The present approach, resorting to the common agency framework, is more comprehensive and more appropriate to deal with differentiated goods. Besides, the model developed in the present paper is more general, not requiring nomothetic preferences.

In Section 2 we introduce a canonical formulation of the common agency game and the associated equilibrium parameterization. Section 3 is devoted to the incentive compatibility constraint in relation to the representative consumer's behavior. Section 4 presents two types of participation constraints, and analyzes and compares the two corresponding (intrinsic and non-intrinsic common agency) games. It shows that the set of intrinsic common agency equilibria includes the standard price and quantity equilibrium outcomes, as well as the set of non-intrinsic common agency equilibria. Section 5 shows that collusion is not always enforceable. A symmetric duopoly example is fully developed as an illustration. The conclusion follows.

2 Common agency game

Adopting the common agency framework, we shall analyze the situation where several firms compete to sell differentiated goods to a representative consumer. We assume complete information, but the representative consumer is supposed to decide voluntarily and to have some outside option, introducing a dimension of moral hazard.

2.1 Canonical formulation

A typical formulation of common agency is to suppose that the principals offer singleton contracts in order to maximize their objective functions under two constraints, an incentive compatibility

[2] See d'Aspremont et al. (1991) and d'Aspremont and Dos Santos Ferreira (2009) assuming a single homogeneous good, as well as d'Aspremont et al. (2007) assuming nomothetic preferences over differentiated goods.

[3] See also Grether and Plott (1984), Holt and Scheffman (1987), Doyle (1988).

constraint and a participation constraint, which take into account the common agent's behavior. Incentive compatibility requires that each principal's offer, given those of the other principals, maximize the agent's objective function, provided the agent accepts to contract. Indeed, voluntary participation by the agent is only ensured if no better outside opportunities are available. The two constraints refer to two different kinds of rivalry. The incentive constraint integrates the moral hazard created by the agent and faced by all the principals, and therefore refers to the rivalry among them. The participation constraint integrates the outside options available to the agent, and therefore refers also to the rivalry of each principal with respect to outsiders to the game.

Formally, each principal $i \in \{1, \cdots, n\}$ $(n \geq 2)$ makes a price-quantity offer $(p_i, q_i) \in \mathbb{R}^2_+$, chosen in the set of incentive compatible contracts, which we shall assume to be represented by the constraint $f_i(p_i, p_{-i}, q_i, q_{-i}) \leq 1$. The function f_i is assumed differentiable and will be explicitly derived in Section 3. Similarly, we shall assume that the participation constraint takes the form $g_i(p_i, p_{-i}, q_i, q_{-i}) \leq 1$, where g_i is again a differentiable function, explicitly derived in Section 4, for two different specifications of the common agency game. The offer (p_i, q_i) is expected to generate a profit $p_i q_i - C_i(q_i)$, where C_i is an increasing differentiable cost function defined on $\mathbb{R}_+$ and such that $C_i(0) = 0$.

A common *agency game* between the n principals is defined by the set of offers $(p_i, q_i) \in \mathbb{R}^2_+$ as principal i's strategies and payoffs $p_i q_i - C_i(q_i)$ if both constraints are satisfied, zero otherwise. In order to eliminate irrelevant equilibria (as exemplified in the following sections), we shall restrict our attention to Nash equilibria $(p^*, q^*) \in \mathbb{R}^{2n}_+$ of this game such that all $2n$ constraints are satisfied as equalities. This restriction allows a complete parameterization of the selected set of equilibria.

2.2 Parameterization of equilibria

Take a common agency equilibrium (p^*, q^*) such that, for any i, $f_i(p^*, q^*) = g_i(p^*, q^*) = 1$. The equilibrium strategy of each firm i must solve the program:

$$\max_{(p_i,q_i)\in \mathbb{R}^2_+} \{p_i q_i - C_i(q_i) : f_i(p_i, p^*_{-i}, q_i, q^*_{-i}) \leq 1 \text{ and } g_i(p_i, p^*_{-i}, q_i, q^*_{-i}) \leq 1\}. \tag{1}$$

Associating with these constraints the Kuhn-Tucker multipliers ϕ_i and γ_i respectively, and using the normalization $\theta_i \equiv \phi_i(\phi_i + \gamma_i) \in [0, 1]$, the *first order conditions* can be written in terms of the Lerner index of degree of monopoly μ^*_i as measured at equilibrium:

$$\mu^*_i \equiv \frac{p^*_i - C'_i(q^*_i)}{p^*_i} = \frac{\theta_i \epsilon_{q_i} f_i(p^*, q^*) + (1 - \theta_i)\epsilon_{q_i} g_i(p^*, q^*)}{\theta_i \epsilon_{p_i} f_i(p^*, q^*) + 1(1 - \theta_i)\epsilon_{p_i} g_i(p^*, q^*)}, \tag{2}$$

where ϵ denotes the elasticity operator. Thus, the normalized multipliers may serve to parameterize the set of equilibria. The multiplier θ_i expresses the relative implicit price principal i would be ready to pay for relaxing the incentive constraint, and can hence be interpreted as the *weight of moral hazard* for principal i.

In order to facilitate the interpretation and the applications of the preceding formula for the degree of monopoly μ^*_i, we can transform it into the equivalent expression:

$$\mu^*_i = \frac{\theta_i \epsilon_{q_i} f_i(p^*, q^*) + (1 - \theta_i)\epsilon_{q_i} g_i(p^*, q^*)}{\theta_i \epsilon_{q_i} f_i(p^*, q^*) s^*_i + (1 - \theta_i)\epsilon_{q_i} g_i(p^*, q^*) \sigma^*_i}, \tag{3}$$

where μ^*_i appears as a weighted harmonic mean of the reciprocals of the elasticities

$$s^*_i \equiv \frac{\epsilon_{p_i} f_i(p^*, q^*)}{\epsilon_{q_i} f_i(p^*, q^*)} = -\frac{dq_i}{dp_i}\frac{p_i}{q_i}\bigg|_{f_i(p_i, p^*_{-i}, q_i, q^*_{-i})=1} \text{ and} \tag{4}$$

$$\sigma^*_i \equiv \frac{\epsilon_{p_i} g_i(p^*, q^*)}{\epsilon_{q_i} g_i(p^*, q^*)} = -\frac{dq_i}{dp_i}\frac{p_i}{q_i}\bigg|_{g_i(p_i, p^*_{-i}, q_i, q^*_{-i})=1}, \tag{5}$$

that is, the elasticities of q_i with respect to p_i for derivations from equilibrium (p^*, q^*) along the curves representing each one of the two constraints.

3 Incentive compatibility and competition among the principals

Since incentive compatibility requires principals' offers to maximize the agent's objective function, we must specify his behavior. In the interpretation of the common agency game we want to focus on, the agent is a representative consumer considering the purchase of a composite good, and the principals are firms selling specific components of this good.

3.1 Representative consumer

This consumer is assumed to have a *separable* utility function $U(u(x), z)$, where $x \in \mathbb{R}^n_+$ is the basket of goods sold by the n firms, and $z \in \mathbb{R}_+$ is a good taken as numeraire.[4] The consumer is thus assumed to be able to isolate a set of linked goods (either substitutes or complements) and to evaluate them globally through a sub-utility index u, providing a 'quantity' of a composite good. To make things concrete, we may imagine a household organizing trips involving several connected flights (complementarity) and several possible airlines for each flight (substitutability). Another example is to consider a consumer ordering a stock of wine bottles of different types and different wineries.

We assume the utility function U to be increasing and strongly quasi-concave over $\mathbb{R}^2_{++}$ (*i.e.* strictly quasi-concave and twice-differentiable, with a regular bordered Hessian). We also assume the sub-utility function u to have the same properties over some admissible subset of $\mathbb{R}^n_+$, except in the two limit cases of perfect substitutability(the homogeneous product case), where $u(x) = \sum_i x_i$, and perfect complementarity, where $u(x) = \min_i\{x_i\}$.

Because of separability, maximization of U under the budget constraint $px + z \leq w$, where wealth w does not depend upon the vector of given prices p, can be performed in two stages. At the first stage, the consumer minimizes the expenditure ensuring a given consumption level $\overline{u}$ of the composite good:

$$\min_{x \in \mathbb{R}^n_+} \{px : u(x) \geq \underline{u}\} \equiv e(p, \underline{u}), \tag{6}$$

with solution $H(p, \underline{u}) = \bigtimes_i H_i(p, \underline{u}) \in \mathbb{R}^n_+$, defining the *Hicksian demand function*, and corresponding first order condition:

$$p = \partial_u e(p, \underline{u}) \partial u(x). \tag{7}$$

[4]This amounts to suppose that the prices of all goods outside the n-firm sector are fixed throughout, so that z can be treated as the quantity of a Hicksian composite good.

At the second stage, the consumer chooses the optimal consumption level of the composite good:

$$\max_{(\underline{u},z)\in\mathbb{R}^2_+} \{U(\underline{u}, z) : e(p, \underline{u}) + z \leq w\}, \tag{8}$$

with solution in $\underline{u}$ denoted $\overline{D}(p)$. For simplicity of notation, we omit reference to the variable w, assumed fixed throughout.

These two stages correspond to the two constraints in the principals' programs. The incentive compatibility constraint is meant to ensure that the contract is compatible with the consumer expenditure minimizing behavior for the composite good. It requires that the expenditure $p_i q_i$ does not exceed the amount that the agent would want to spend for good i, at prices (p_i, p_{-i}), in order to obtain the quantity $u(q_i, q_{-i})$ of the composite good entailed by the set of all contracts, that is, by the definition of the Hicksian demand, the amount $p_i H_i(p_i, p_{-i}, u(q_i, q_{-i}))$. Hence,

$$f_i(p_i, p_{-i}, q_i, q_i = \frac{q_i}{H_i(p_i, p_{-i}, u(q_i, q_{-i}))} \leq 1. \tag{9}$$

3.2 Intra-second substitutability

In order to apply formulae (3) and (4), we have to compute, using (9),

$$\begin{aligned} \epsilon_{q_i} f_i(p, q) &= 1 - \epsilon_u H_i(p, u(q))\epsilon_i u(q) \equiv 1 - \varepsilon_i(p, q), & (10) \\ \epsilon_{p_i} f_i(p, q) &= -\epsilon_{p_i} H_i(p, u(q)), & (11) \end{aligned}$$

so that, by (4),

$$s_i(p, q) = -\frac{\epsilon_{p_i} H_i(p, u(q))}{1 - \varepsilon_i(p, q)}. \tag{12}$$

By consumer's first order condition (7) and Shephard's lemma, we have $\varepsilon_i(p, q) = \epsilon_{p_i} \partial_u e(p, u(q))$, which thus appears as the *elasticity of the marginal purchasing cost of the composite good* with respect to the price of good i. Furthermore, $s_i(p, q)$ can be seen as the elasticity (in absolute value) of the share $H_i(p, u(q))/u(q)$ of good i in the composite good with respect to the corresponding marginal rate of substitution $p_i/\partial_u e(p, u(q))$, that is, the *elasticity of substitution* of good i for the composite good. This elasticity is a measure of the degree of intra-sectoral substitutability.

Example 1 *Quadratic sub-utility).* An example repeatedly used in the following involves quasi-linearity in the numeraire good and quadratic sub-utility, a specification which is frequently chosen since it leads to linear demand. More specifically, we assume in this case that u is symmetric, given by

$$u(x) = \beta \sum_{i=1}^{n} x_i - \frac{1}{2} \sum_{i=1}^{n} x_i^2 - \frac{\gamma}{2} \left(\sum_{i=1}^{n} x_i \right)^2, \tag{13}$$

with $\beta > 0$ and $\gamma > -1/n$ for increasingness and (strict) concavity of u, x being constrained to satisfy for any i:

$$\partial_i u(x) = \beta - x_i - \gamma \sum_j x_j \geq 0. \tag{14}$$

The parameters β and γ can be seen as indices of market size (the consumer's reservation price) and intra-sector substitutability, respectively. The n goods are substitutes (respectively complements) if $\gamma > 0$ (respectively $\gamma < 0$). Solving in x Eq. (7), which gives the first order condition for expenditure minimization, and denoting $S = 1 + n\gamma \in (0, \infty)$, we obtain the Hicksian demand for good i

$$H_i(p, \underline{u}) = \frac{1}{S} \left(\beta - \frac{Sp_i + (1 - S)(1/n) \sum_j p_j}{\partial_u e(p, \underline{u})} \right) \tag{15}$$

for any $\underline{u} \in (0, n\beta^2/2S)$ and any admissible price vector p (*i.e.* entailing a non-negative vector x). Using the definitional property $u(H(p, \underline{u})) = \underline{u}$, it is then straightforward to compute

$$\partial_u e(p, \underline{u}) = \sqrt{\frac{S(1/n) \sum_j p_j^2 + (1 - S)((1/n) \sum_j p_j)^2}{\beta^2 - 2S(1/n)\underline{u}}}. \tag{16}$$

Given quasi-linearity in the numeraire good and under the normalization $U = u(x) + z$, a necessary condition for expenditure minimization at $\underline{u} = \overline{D}((p)$ is $\partial_u e(p, \overline{D}\,(p)) = 1$. Also, for any symmetric profile $(p, q) = ((\overline{p}, \overline{p}), (\overline{q}, \overline{q})), \varepsilon_i(p, q) = 1/n$. Then, the elasticity $s_i(p, q)$ of substitution of any good i for the composite good (as given by (12)) simplifies to

$$s_i(p, q) = \frac{S}{\beta/\overline{p} - 1} = \frac{1 + n\gamma}{\beta/\overline{p} - 1}. \tag{17}$$

4 Agent's participation and principals' mutual interests

Suppose the consumer, considering the purchase of the composite good, has the option of buying it either in the market place, at the market prices, or on the web, where the producers have posted their price-quantity offers. The advantage for him of contracting through the web over searching in the market place is to reduce transaction costs and/or to benefit from price discounts (although, for simplicity, both the former and the latter will be kept implicit in our analysis). Now, if the consumer can mix the two buying possibilities in acquiring the composite good, the participation constraint that each producer will have to take into account will express the fact that the consumer is certainly not ready to spend on the web, *for that producer's good*, more than he would have optimally spent in the market place, at the same prices. Whether on the web or in the market place, search is however time consuming. For that reason, the two options may well be mutually exclusive, so that the participation constraint will then apply *for the composite good as a whole*. The latter participation constraint characterizes the so-called *intrinsic* common agency game, whereas the former characterizes the *non-intrinsic* game (see Bernheim and Whinston, 1985, 1986, and Martimort and Stole, 2008).

4.1 Intrinsic common agency

The participation constraint is introduced to ensure that the consumer would accept the set of contracts offered by the firms rather than go to the market place. So, participation requires that the minimal expenditure $e(p, u(q))$ associated with those contracts does not exceed the budget optimally allocated to the composite good[5] $B(p) = e(p, \overline{D}(p))$;

$$g_i^I(p_i, p_{-i}, q_i, q_{-i}) = \frac{e(p, u(q))}{B(p)} \leq 1. \tag{18}$$

Notice that the participation constraint in the intrinsic common agency is in fact common to all principals. Notice also that any set of contracts (p, q) satisfying the common participation constraint and all the incentive compatibility constraints also satisfies:

$$pq \leq pH(p, u(q)) = e(p, u(q)) \leq B(p). \tag{19}$$

Requiring that a set of contracts (p, q) saturate all the constraints is then equivalent to imposing the no-rationing condition

$$q = D(p), \tag{20}$$

where $D(p) = H(p, \overline{D}(p))$ is the *Walrasian demand function.* We call (p^*, q^*) an *intrinsic common agency equilibrium* if it is a Nash equilibrium of the corresponding game that satisfies condition (20). This condition indeed implies, as $pq = Pd(p) = B(p)$, that both inequalities in (19) are satisfied as equalities, and finally that all participation and incentive compatibility constraints hold as equalities. Conversely, if all constraints hold as equalities, $q = H(p, \overline{D}(p))$, so that the no-rationing condition is satisfied. The use of this condition as a definitional property of the equilibrium is justified: it eliminates unsatisfactory Nash equilibria and allows us to fully exploit duality.

In order to apply formulae (3) and (5), we compute:

$$\epsilon_{q_i} g_i^I(p, q) \quad = \quad \epsilon_u e(p, u(q)) \epsilon_i u(q) \equiv \alpha_i(p, q), \tag{21}$$

[5] The budget $B(p)$ can also be determined according to the following dual procedure. First consider the program

$$\max_{x \in \mathbb{R}^n_+} \{u(x) : px \leq b\} \equiv v(p, b),$$

defining the indirect sub-utility of budget b at prices p, as well as the Marshallian demand function $X(p, b)$ as its solution. Then $B?$ is simply the solution to the problem:

$$\max_{(b,z) \in \mathbb{R}^2_+} \{U(v(p, b), z) : b + z \leq w\}.$$

We omit reference to wealth w, supposed fixed throughout.

$$\epsilon_{q_i} g_i^I(p,q) \quad = \quad \epsilon_{p_i} e(p,u(q)) - \epsilon_i B\,(p)\,. \tag{22}$$

By consumer's first order condition (7), we have $\alpha_i(p,q) = p_i q_i / e(p,u(q))$, the *budget share* of good i. Also, by Shephard's lemma, $\alpha_i(p,q) = \epsilon_{p_i} e(p,u(q))$. We thus obtain, using (5):

$$\sigma_i(p,q) = 1 - \frac{\epsilon_i B\,(p)}{\alpha_i(p,q)}. \tag{23}$$

The elasticity $\sigma_i(p,q)$ may be interpreted[6] as the *price elasticity of demand for the composite good* via good i.

Example 2 (*Quadratic sub-utility, continued).* Using $\partial_u e(p,\underline{u}) = 1$ at $\underline{u} = \overline{D}\,(p)$, we obtain from the expression for the Hicksian demand (15) the following expression for the Walrasian demand:

$$D_i\,(p) = \beta/S - p_i - (1/S-1)(1/n)\sum_j p_j. \tag{24}$$

The price elasticity σ_i of demand for the composite good via good i (as computed from (23) using $B\,(p) = pD\,(p)$) simplifies, in the case of a symmetric profile (with $(p_i,q_i) = (\overline{p},\overline{q})$ for all i) to

$$\sigma_i(p,q) = \frac{1}{\beta/\overline{p}-1}. \tag{25}$$

Thus, since $s_i(p,q) = S/(\beta/\overline{p}-1)$, the parameter $S = 1 + n\gamma$ can be interpreted as the ratio between the elasticities of intra- and intersect oral substitution. It is this ratio, rather than s_i and σ_i which is independent of $(\overline{p},\overline{q})$.

4.2 Potential equilibria

The preceding discussion allows us to characterize what we may call the set of potential equilibria of the intrinsic common agency game, meaning that they must satisfy necessary first order

[6] The function $e(\cdot,u(q))$ represents the minimal amount the consumer has to spend, as the prices vary, to obtain the *fixed* quantity of $u(q)$ of the composite good (as opposed to the function $B(\cdot)$ which takes quantity adjustments into account). It can consequently be assimilated to a price index for this good. Thus, by subtracting $\epsilon_{p_i} e(p,u(q))$ from $\epsilon_{p_i} B\,(p)$, we are left with the impact on the quantity of the composite good of a variation in price p_i. By then dividing by $\epsilon_{p_i} e(p,u(q))$ (and taking the absolute value), we obtain $\sigma_i(p,q)$, justifying the interpretation given in the text.

conditions, expressed by Eq. (3), and make all constraints hold as equalities (so that the consumer is not rationed). This is summarized by the following proposition.

Proposition 1 *An intrinsic common agency equilibrium (p^*, q^*) of the corresponding game, with incentive compatibility constraints given by (9) and a common participation constraint given by (18), must satisfy the no-rationing condition $q^* = D(p^*)$ and exhibit, for each firm i, a degree of monopoly*

$$\mu_i^* = \frac{\theta_i(1-\varepsilon_i^*) + (1-\theta_i)\alpha_i^*}{\theta_i(1-\varepsilon_i^*)s_i^* + (1-\theta_i)\alpha_i^*\sigma_i^*}, \tag{26}$$

for some $\theta_i \in [0,1]$ and with $\varepsilon_i^*, s_i^*, \alpha_i^*$ and σ_i^* respectively given by the functions defined by (10), (12), (21) and (23), evaluated at (p^*, q^*).

The degree of monopoly of firm i at equilibrium (p^*, q^*) is a weighted harmonic mean of the reciprocals of the elasticity s_i^* of substitution of good i for the composite good and of the elasticity σ_i^* of demand for the composite good via good i. These elasticities reflect the degrees of intra- and intersect oral substitutability, and their ratio indicates the degree of conflict of principals' interests. The weight of moral hazard θ_i, measuring the relative weight of the incentive compatibility constraint, expresses the *competitive toughness* of firm i at equilibrium (p^*, q^*). When maximal ($\theta_i = 1$), the degree of monopoly μ_i^* takes the extreme value $1/s_i^*$, which also obtained when the budget share α_i^* attains its minimum ($\alpha_i^* = 0$), that is, when firm i becomes negligible in the sector. Thus, when it exists, the equilibrium associated with $\theta_i = 1$ for all i is equivalent to a *monopolistic competition* equilibrium. By contrast, when competitive toughness is minimum ($\theta_i = 0$), the degree of monopoly μ_i^* takes the extreme value $1/\sigma_i^*$. This value might also result from a situation where a deviation in p_i has a full repercussion on the composite good price ($\varepsilon_i^* = 1$). When it exists, the equilibrium associated with $\theta_i = 0$ for all i coincides with the *collusive solution:*

$$\arg \max_{(p,q)\in\mathbb{R}_+^{2n}} \left\{ e(p, u(q)) - \sum_i C_i(q_i) : e(p, u(q)) \leq B\,(p) \right\}.$$

Example 3 *(Quadratic sub-utility, continued).* Let us consider a symmetric equilibrium $(\overline{p}^*, \cdots, \overline{p}^*, \overline{q}^*, \cdots, \overline{q}^*)$ associated with the uniform competitive toughness $\overline{\theta}$. By symmetry,

$\varepsilon_i^* = \alpha_i^* = 1/n$, the market share of each individual firm. Using Eq. (26), we obtain for the degree of monopoly:

$$\mu^* = \frac{\overline{\theta}(1-1/n) + (1-\overline{\theta})(1/n)}{\overline{\theta}(1-1/n)S + (1-\overline{\theta})(1/n)}(\beta/\overline{p}^* - 1). \tag{27}$$

By assuming a constant marginal cost, normalized to one, the equilibrium price becomes a function of the sole degree of monopoly, $\overline{p}^* = 1/(1-\mu^*)$, and can consequently be eliminated in the preceding equation, to obtain:

$$\mu^* = \frac{\beta - 1}{\beta + \dfrac{\overline{\theta}(1-1/n)S + (1-\overline{\theta})(1/n)}{\overline{\theta}(1-1/n) + (1-\overline{\theta})(1/n)}} \equiv \mu(\overline{\theta}, S, n, \beta). \tag{28}$$

Thus, the degree of monopoly increases with market size (the consumer's reservation price) $\beta \in (1, \infty)$, decreases with the ratio $S \in (0, \infty)$ between intra- and intersect oral substitutability's, and decreases (respectively increases) with the number n of firms when S is larger (respectively smaller) than 1. In addition to these structural parameters, the competitive toughness $\overline{\theta} \in [0, 1]$ also influences the degree of monopoly, in the same way as the number of firms. The degree of monopoly takes two extreme values, the collusive one

$$\mu(\overline{\theta}, S, 1, \beta)|_{\overline{\theta}<1} = \frac{\beta - 1}{\beta + 1} = \mu(0, S, n, \beta) \tag{29}$$

and the monopolistic competition one

$$\mu(\overline{\theta}, S, \infty, \beta)|_{\overline{\theta}>0} = \frac{\beta - 1}{\beta + S} = \mu(1, S, n, \beta). \tag{30}$$

The latter value of μ^* is lower (respectively higher) than the collusive value when $S > 1$ (respectively $S < 1$).

4.3 Price and quantity equilibria

We now show that the outcomes of two current equilibrium concepts, the price and quantity equilibria, are both included in the set of intrinsic common agency equilibria, hence ensuring non-vacuity of this set under standard assumptions. The *price equilibrium* is the solution to the following program for each firm i

$$\max_{p_i \in \mathbb{R}_+} \{p_i D_i(p_i, p^*_{-i}) - C_i(D_i(p_i, p^*_{-i}))\}. \tag{31}$$

As this definition implies that firm i serves the whole demand for any deviation from equilibrium, a standard assumption for that obligation to make sense is to assume linear costs ($C_i(q_i) = c_i q_i$ for all i). Under this assumption we can prove the following proposition.

Proposition 2 *Assume linear costs and suppose that a price equilibrium p^* exists. Then $(p^*, D(p^*))$ is an intrinsic common agency equilibrium.*

Proof: Suppose that, for some i and some contract (p_i, q_i) satisfying the two constraints (9) and (18), we have: $(p_i - c_i)q_i > (p^*_i - c_i)D_i(p^*)$. By (9), $q_i \leq H_i(p_i, p^*_{-i}, u(q_i, q^*_{-i}))$. By (18) and since by definition $B(p) = e(p, overline(p))$, we also have: $e(p_i, p^*_{-i}, u(q_i, q^*_{-i})) \leq e(p_i, p^*_{-i}, \overline{D}(p_i, p^*_{-i}))$. Hence $u(q_i, q^*_{-i}) \leq \overline{D}(p_i, p^*_{-i})$, and $H_i(p_i, p^*_{-i}, u(q_i, q^*_{-i})) \leq H_i(p_i, p^*_{-i}, \overline{D}(p_i, p^*_{-i})) = D_i(p_i, p^*_{-i})$. Thus

$$(p_i - c_i)D_i(p_i, p^*_{-i} \geq (p_i - c_i)q_i > (p^*_i - c_i)D_i(p^*)$$

and the result follows by contradiction. ■

First order conditions for the price equilibrium lead, for each firm i, to a degree of monopoly[7]

$$\mu_i^* = -\frac{1}{\epsilon_i D_i(p^*)} = \frac{1}{(1-\varepsilon_i^*)s_i^* + \varepsilon_i^* \sigma_i^*}, \tag{32}$$

equal to the harmonic mean of $1/s_i^*$ and $1/\sigma_i^*$, with weights respectively equal to $1-\varepsilon_i^*$ and ε_i^*. The same expression for μ_i^* can be directly obtained from the general formula (26) by taking the intermediate competitive toughness

$$\theta_i = \frac{\alpha_i^*}{\alpha_i^* + \varepsilon_i^*}. \tag{33}$$

Consider now a *quantity equilibrium*, namely a solution q^* to the following program for each firm i

$$\max_{q_i \in \mathbb{R}_+} \{(D^{-1})_i(q_i, q_{-i}^*)q_i - C_i(q_i)\}, \tag{34}$$

where D^{-1} is the inverse Walrasian demand function, such that

$$D^{-1}(q) = \partial_u e(D^{-1}(q), u(q))\partial u(q). \tag{35}$$

Proposition 3 *Assume a decreasing demand $\overline{D}$ for the composite good (assumed not to be a Giffen good) and suppose that a quantity equilibrium q^* exists. Then $(D^{-1}(q^*), q^*)$ is an intrinsic common agency equilibrium.*

[7]Recall from footnote (6) the notations $v(p,b)$ and $X_i(p,b)$ of the indirect utility at prices p of budget b and of the Marshallian demand function for good i, respectively; Using the identity $D_i(p) \equiv X_i(p, B(p))$ and the Slutsky equation, we get

$$\begin{aligned} \epsilon_i D_i(p^*) &= \epsilon_{p_i} X_i(p^*, B(p^*)) + \epsilon_b X_i(p^*, B(p^*))\epsilon_i B(p^*) \\ &= \epsilon_{p_i} H_i(p^*, v(p^*, B(p^*))) - \epsilon_b X_i(p^*, B(p^*)) \left(\frac{p_i^* D_i(p^*)}{B(p^*)} - \epsilon_i B(p^*) \right). \end{aligned}$$

By (12), $\epsilon_{p_i} H_i(p^*, v(p^*, B(p^*))) = -(1-\varepsilon_i^*)s_i^*$. Using the identity $X_i(p, B(p)) \equiv H_i(p, v(p, B(p)))$, the equality $\varepsilon_b v(p,b) = 1/\partial_u e(p, v(p,b))$ and Eqs. (10) and (21), we have $\epsilon_b X_i(p^*, B(p^*)) = \varepsilon_i^*/\alpha_i^*$. By (21) and (23), $p_i^* D_i(p^*) B(p^*) - \epsilon_i B(p^*) = \alpha_i^* \sigma_i^*$. Hence, we finally get:

$$-\epsilon_i D_i(p^*) = (1-\varepsilon_i^*)s_i^* + \varepsilon_i^* \sigma_i^*.$$

Proof: Suppose that, for some i and some contract (p_i, q_i) satisfying the two constraints, we have: $p_i q_i - C_i(q_i) > (D^{-1})_i(q^*)q_i^* - C_i(q_i^*)$. Using consumer's first order condition (7) to reformulate the incentive compatibility constraint (9) in dual terms, we obtain $p_i \leq \partial_u e(p_i, p_{-i}^*, u(q_i, q_{-i}^*))$ $\partial_i u(q_i, q_{-i}^*)$, as well as $D^{-1}(q_i, q_{-i}^*) = \partial_u e(D^{-1}(q_i, q_{-i}^*), u(q_i, q_{-i}^*)) \times \partial u(q_i, q_{-i}^*)$. Thus, if

$$\delta \equiv \frac{\partial_u e(p_i, p_{-i}^*, u(q_i, q_{-i}^*))}{\partial_u e(D^{-1}(q_i, q_{-i}^*), u(q_i, q_{-i}^*))} \leq 1,$$

implying $p_i \leq (D^{-1})_i(q_i, q_{-i}^*)$, q_i is a profitable deviation from the quantity equilibrium q^*, and we get a contradiction. If $\delta > 1$, from the equalities

$$\begin{aligned}
&\delta e(D^{-1}(q_i, q_{-i}^*), u(q_i, q_{-i}^*)) \\
&\quad = \delta \partial_e(D^{-1}(q_i, q_{-i}^*), u(q_i, q_{-i}^*))\partial u(q_i, q_{-i}^*) \cdot (q_i, q_{-i}^*) \\
&\quad = \partial_u e(p_i, p_{-i}^*, u(q_i, q_{-i}^*))\partial u(q_i, q_{-i}^*) \cdot (q_i, q_{-i}^*) = e(p_i, p_{-i}^*, u(q_i, q_{-i}^*))
\end{aligned}$$

we see that, to obtain the same quantity $u(q_i, q_{-i}^*)$ of the composite good the consumer has to spend more at prices (p_i, p_{-i}^*) than at the same relative prices $D^{-1}(q_i, q_{-i}^*)$ (given by the vector $\partial u(q_i, q_{-i}^*)$). This means that the price level is higher (by a factor δ) at prices (p_i, p_{-i}^*). As $\overline{D}$ is a decreasing function,

$$u(D(p_i, p_{-i}^*)) = \overline{D}(p_i, p_{-i}^*) < \overline{D}(D^{-1}(q_i, q_{-i}^*)) = u(q_i, q_{-i}^*),$$

and hence, since $e(p, \cdot)$ is increasing,

$$B(p_i, p_{-i}^*) = e(p_i, p_{-i}^*, u(D(p_i, p_{-i}^*))) < e(p_i, p_{-i}^*, u(q_i, q_{-i}^*)),$$

violating the participation constraint (18) and leading again to a contradiction. ∎

First order condition for the quantity equilibrium lead, for each firm i, to a degree of monopoly

$$\mu_i^* = -\epsilon_i(D^{-1})_i(q^*) = \sum_j \varepsilon_j^*[-\epsilon_i(D^{-1})_j(q^*)] - [\epsilon_u \partial_u e(D^{-1}(q^*), u(q^*))\epsilon_i u(q^*) + \epsilon_i \partial_i u(q^*)], \quad (36)$$

using $\varepsilon_j^* = \epsilon_{p_j} \partial_u e(D^{-1}(q^*), u(q^*))$. This expression cannot be simplified as easily as in the price equilibrium case, because the first term on the right-hand side cannot be transformed in general into a simple function of the elasticity σ_i^* of the demand for the composite good via good i. However, when sub-utility u is nomothetic, the expenditure function is linear in $u(e(p, \underline{u}) = P(p)\,\underline{u}$ for some function P), and hence $\varepsilon_i^* = \epsilon_i P(p^*) = \alpha_i^*$ (by Shephard's lemma). The same coincidence prevails in symmetric equilibria, where $\varepsilon_i^* = \alpha_i^* = 1/n$. After some computations[8] we then obtain in this case:

$$\mu_i^* = \frac{1-\varepsilon_i^*}{s_i^*} + \frac{\varepsilon_i^*}{\sigma_i^*}, \tag{37}$$

the *arithmetic* mean of $1/s_i^*$ and $1/\sigma_i^*$, with weights respectively equal to $1 - \varepsilon_i^*$ and $\varepsilon *_i$, as for price equilibrium. The degree of monopoly (37) can then be directly obtained from the general formula (26) with

$$\theta_i = \frac{1}{1 + s_i^*/\sigma_i^*}, \tag{38}$$

where s_i^*/σ_i^* can be seen as the ratio between *elasticities of intra-* and *intersect oral substitution* for good i. Comparing (33) and (38), we see that the quantity equilibrium is less competitive (in the sense of a lower θ_i) than the price equilibrium (where θ_i is then equal to 1/2) only if goods are more substitutable within the sector than relative to its environment. In spite of their similarity, comparing the two equilibrium conditions (32) and (37) is however not in general an easy task even when $\varepsilon_i^* = \alpha_i^*$, because the values s_i^* and σ_i^* are not necessarily the same at the price and the quantity equilibria. Only in more specific cases, as when these values are fixed as structural parameters of the game (*e.g.*, when U and u are both CES), can the comparison be pushed further. Our example also illustrates this possibility.

[8] Using the dual of definition (12), hence letting $f_i(p,q) = p_i/[\partial_u e(p, u(q))\partial_i u(q)]$ in (4), we can obtain the following equivalent expression for the elasticity of substitution of good i for the composite good:

$$s_i^* = \frac{1 - \epsilon_{p_i}\partial_u e(p^*, u(q^*))}{\epsilon_u \partial_u e(p^*, u(q^*))\epsilon_i u(q^*) + \epsilon_i \partial_i u(q^*)}.$$

Also, we obtain for the price elasticity of the demand for the composite good via good i the equivalent expression:

$$\sigma_i^* = -\frac{dq_i}{dp_i}\frac{p_i}{q_i}\bigg|_{e(p_i, p_{-i}^*, u(q_i, q_{-i}^*)) = e(D^{-1}(q_i, q_{-i}^*), u(q_i, q_{-i}^*))} = \frac{\epsilon_{p_i} e(p^*, u(q^*))}{\sum_j \epsilon_{p_j} e(p^*, u(q^*))[-\epsilon_i(D^{-1})j(q^*)]} = \frac{\alpha_i^*}{\sum_j \alpha_j^*[-\epsilon_i(D^{-1})_j(q^*)]}.$$

Notice that the denominator on the right-hand side is an arithmetic mean of the same elasticities as in the first term of the right-hand side of (36) but with different weights in general. We are exploiting in the text the coincidence $\alpha_i^* = \varepsilon_i^*$.

Example 4 *(Quadratic sub-utility, continued).* By referring to Eqs. (28), (33) and (38), it is easy to determine the values of μ^* corresponding to price equilibrium

$$\mu\left(\frac{1}{2}, S, n, \beta\right) = \frac{\beta - 1}{\beta + (1 - 1/n)S + (1/n)} \tag{39}$$

and quantity equilibrium

$$\mu\left(\frac{1}{1+S}, S, n, \beta\right) = \frac{\beta - 1}{\beta + \frac{1}{(1-1/n)/S + (1/n)}}. \tag{40}$$

Since the arithmetic mean of two different numbers is always larger than their harmonic mean (with the same weights) the degree of monopoly (and hence the price) corresponding to price equilibrium is lower than the one corresponding to quantity equilibrium.

4.4 Non-intrinsic common agency

When access to the market is cheap and easy, the common agency may become *non-intrinsic*, principal i's participation constraint then reflecting the possibility for the agent to purchase good i in the market place, whatever his decision about other principals' offers. Thus, in order to warrant the consumers' participation, firm i cannot offer a quantity q_i of its good exceeding consumer's demand at price (p_i, p_{-i}). In our canonical formulation we thus have:

$$g_i^{NI}(p_i, p_{-i}, q_i, q_{-i}) = \frac{q_i}{D_i(p_i, p_{-i})} \leq 1. \tag{41}$$

The elasticity of q_i with respect to p_i for deviations from an equilibrium (p^*, q^*) along the curve representing this constraint is now

$$\frac{\epsilon_{p_i} g_i^{NI}(p^*, q^*)}{\epsilon_{q_i} g_i^{NI}(p^*, q^*)} = -\epsilon_i D_i(p^*). \tag{42}$$

Hence, by (3), (5), (10) and (41), the equilibrium degree of monopoly of firm i is

$$\mu_i^* = \frac{\theta_i(1 - \varepsilon_i^*) + (1 - \theta_i)}{\theta_i(1 - \varepsilon_i^*)s_i^* + (1 - \theta_i)[-\epsilon_i D_i(p^*)]}. \tag{43}$$

If we consider the lowest possible competitive toughness ($\theta_i = 0$), we obtain $\mu_i^* = 1/[-\epsilon_i D_i(p^*)]$, which is also, by (32), the degree of monopoly corresponding to the price equilibrium. Thus, what is by (33) an intermediate value of the equilibrium degree of monopoly in the intrinsic common agency game appears now as an extreme value. This suggests that going from the intrinsic to the non-intrinsic common agency game reduces the set of equilibria. The following proposition confirms this claim.

Proposition 4 *The set of non-intrinsic common agency equilibria is included in the set of intrinsic common agency equilibria.*

Proof: Suppose $(p^*, q^*) \in \mathbb{R}^{2n}_+$ is an equilibrium of the non-intrinsic common agency game, but not of the intrinsic common agency game. This means that there exists, for some firm i, a pair $(p_i, q_i) \in \mathbb{R}^2_+$ such that $p_i q_i - C_i(q_i) > p_i^* q_i^* - C_i(q_i^*)$, $D_i(p_i, p_i^*) < q_i \leq H_i(p_i, p^*_{-i}, u(q_i, q^*_{-i}))$ and $e(p_i, p^*_{-i}, u(q_i, q^*_{-i})) \leq B(p_i, p^*_{-i})$. Thus

$$H_i(p_i, p^*_{-i}, \overline{D}(p_i, p^*_{-i})) = D_i(p_i, p^*_{-i}) < H_i(p_i, p^*_{-i}, u(q_i, q^*_{-i})),$$

so that $\overline{D}(p_i, p^*_{-i} < u(q_i, q^*_{-i})$. Since $B(p_i, p^*_{-i} = e(p_i, p^*_{-i}, \overline{D}(p_i, p^*_{-i})) < e(p_i, p^*_{-i}, u(q_i, q^*_{-i}))$, a contradiction follows. ■

In fact, the reduction of the set of equilibria in the non-intrinsic case can be substantial, as illustrated by our example.

Example 5 *(Quadratic sub-utility, continued).* The price elasticity of the Walrasian demand can be easily computed for a symmetric profile from (24), giving

$$-\epsilon_i D_i(p^*) = \frac{(1 - 1/n)S + (1/n)}{\beta/\overline{p}^* - 1}, \tag{44}$$

an expression equal, by (25), to the one of the price elasticity σ_i of the demand for the composite good, up to the factor $(1 - 1/n)S + (1/n)$. Hence, the difference between the set of the intrinsic and non-intrinsic common agency equilibria is larger the larger the difference between S and 1 (or the difference between intra- and intersect oral substitutability's). With a constant marginal cost normalized to one, the equilibrium degree of monopoly is indeed

$$\mu^* = \frac{\beta - 1}{\beta + \frac{(1-1/n)S+(1-\overline{\theta})(1/n)}{1-1/n+(1-\overline{\theta})(1/n)}}, \tag{45}$$

taking values between $(\beta - 1)/(\beta + S)$ and $(\beta - 1)/(\beta + (1 - 1/n)S + (1/n))$ instead of between $(\beta - 1)/(\beta + S)$ and $(\beta - 1)/(\beta + 1)$, as in the intrinsic case (by (28)).

5 Collusion enforceability

The common agency framework was introduced by Bernheim and Whinston (1985), who analyzed a similar model where two firms sell differentiated products through a common marketing agent. The main issue they examine is whether the presence of this common marketing agent may facilitate collusion. They show that, under intrinsic common agency, there always exists a collusive equilibrium, *i.e.* an equilibrium where strategic variables are set at their fully cooperative levels. Our analysis allows to enlarge this result, first by showing that in the intrinsic common agency framework we get in fact a whole spectrum of equilibria that can be parameterized, and second, as we will now show, that this spectrum of equilibria may or may not include the collusive solution, according to the values of the fundamentals.

Another possible way of facilitating collusion is through "meeting competition" clauses, as introduced among other facilitating practices by Salop (1986) and formally analyzed in a pricing game by Kalai and Satterthwaite (1994). In a previous paper (d'Aspremont and Dos Santos Ferreira, 2009), we have shown that, if price-quantity competition to sell a homogeneous good is substituted to price competition, the set of equilibria never includes the collusive solution (the set of equilibria is a continuum between Cournot and the perfect competitive outcome). However, once goods are differentiated, the collusive solution may or may not be enforceable, according to the degree of substitutability among those goods, as shown in d'Apremont et al. (2007) under the assumption of homothetic preferences allowing for an extended meeting competition clause. The present intrinsic common agency leads to the same general conclusions without requiring homothetic preferences and without calling forth an extended meeting competition clause.

The reason why the collusive solution is not always enforceable, or more generally why equilibria close to the collusive solution do not always exist, is not difficult to grasp. When goods are highly substitutable within the sector, there is a strong incentive for firms to deviate from strategy profiles close to the collusive one, since they can benefit through a price decrease from a significantly higher market share. At the other extreme, when the goods in the sector are much more substitutable for outside goods than among themselves, firms have a strong incentive to debate from such profiles by increasing their prices, taking advantage of an unresponsive market share. These two opposite threats to collusion or quasi-collusion enforceability are well illustrated by our example in the symmetric duopoly case.

Example 6 *(Symmetric duopoly with quadratic sub-utility).* From (28) we get, in the intrinsic common agency game, the symmetric equilibrium degree of monopoly

$$\mu^* = \mu(\overline{\theta}, S, 2, \beta) = \frac{\beta - 1}{\beta + \overline{\theta} S + (1 - \overline{\theta})},$$

with the corresponding equilibrium price $\overline{p}^* = 1/(1 - \mu^*)$ and quantity $\overline{q}^* = (\beta - \overline{p}^*)/S$.

- If deviating from potential equilibrium $(\overline{p}^*, \overline{p}^*, \overline{q}^*, \overline{q}^*)$, firm i must first take into account the incentive compatibility constraint. Formulated in dual terms, by referring to the first order condition of consumer's expenditure minimizing problem, this constraint requires the relative price $p_i/\overline{p}^*$ to be at most equal to the marginal rate of substitution $\partial_i u(q_i, \overline{q}^*)/\partial_j u(q_i, \overline{q}^*)$ (where both the numerator and the denominator must be non-negative). Using (14) and recalling that $S = 1 + 2\gamma$, we get:

$$\frac{p_i}{\overline{p}^*} \leq \frac{2\beta/\overline{q}^* - (S-1) - (S+1)q_i/\overline{q}^*}{2\beta/\overline{q}^* - (S+1) - (S-1)q_i/\overline{q}^*}, \text{ with} \tag{46}$$

$$\begin{aligned} \partial_i u(q_i, \overline{q}^*) &= (2\beta/\overline{q}^* - (S-1) - (S+1)q_i/\overline{q}^*)\overline{q}^*/2 \geq 0 \text{ and} \\ \partial_j u(q_i, \overline{q}^*) &= (2\beta/\overline{q}^* - (S+1) - (S-1)q_i/\overline{q}^*)\overline{q}^*/2 \geq 0. \end{aligned} \tag{47}$$

  With utility $U = u(x) + z$, the participation constraint for firm i can be expressed as $\partial_u e(p_i, \overline{p}^*, u(q_i, \overline{q}^*)) \leq 1$, so that the incentive compatibility constraint is active only if $\overline{p}^*/\partial_j u(q_i, \overline{q}^*) \leq 1$, that is, only if $(S-1)q_i/\overline{q}^* \leq S - 1$, or $q_i/\overline{q}^* \leq 1$ for $S > 1$ and $q_i/\overline{q}^* \geq 1$ for $S < 1$.

- As regards the participation constraint

$$u(q_i, \overline{q}^*) \leq u(D(p_i, \overline{p}^*)), \tag{48}$$

with u as given by(13), we must take into account the case where, for $S > 1$, the price p_i is small enough to entail $D_j(p_i, \overline{p}^*) = 0$, leading, by (14) and (24), to

$$u(q_i, \overline{q}^*) \leq u\left(2\frac{\beta - p_i}{S+1}, 0\right) \text{ for } \frac{p_i}{\overline{p}^*} \leq \frac{S + 1 - 2\beta/\overline{p}^*}{S - 1} \text{ and } S > 1. \tag{49}$$

We must also consider the case where, for $S < 1$, the quantity q_i is small enough to entail $\partial_j u(q_i, \overline{q}^*) = 0$, leading, again by (14), to

$$u\left(q_i, \frac{2\beta + (1 - S)q_i}{1 + S}\right) \leq u(D_i(p_i, \overline{p}^*), D_j(p_i, \overline{p}^*)) \quad \text{for } \frac{q_i}{\overline{q}^*} \leq \frac{S + 1 - 2\beta/\overline{q}^*}{1 - S} \text{ and } S < 1. \tag{50}$$

- We can now construct two cases of unenforceable collusive (or close to collusive) degree of monopoly, each firm having an incentive to deviate to a low (respectively high) price because of excessive relative substitutability (respectively complementarity) inside the sector. In Figs. 1 and 2, the relative quantity $q_i/\overline{q}^*$ is measured along the horizontal axis, the relative price $p_i/\overline{p}^*$ along the vertical axis, and the consumer's reservation price is assumed to be $\beta = 4$, which leads to a collusive value of the degree of monopoly equal to 0.6. In Fig. 1, depicting a case of high substitutability inside the sector, the admissible strategy set is upper-bounded by the thick broken curve, where the segment corresponding to $q_i/\overline{q}^* < 1$ and $p_i/\overline{p}^* > 1$ represent the incentive compatibility constraint (46), and the segments corresponding to $q_i/\overline{q}^* > 1$ and $p_i/\overline{p}^* < 1$ represent the two pieces of the participation constraint (48) and (49). The thin curve is the isoprofit curve through the potential symmetric equilibrium point $(1, 1)$ at the intersection of the curves representing the two constraints. This is clearly not an equilibrium, as firm i can increase its profit by decreasing p_i and increasing q_i. In Fig. 2, depicting a case of low relative substitutability inside the sector, the admissible strategy set is again upper-bounded by the thick broken curve, where the segments corresponding to $q_i/\overline{q}^* < 1$ and $p_i/\overline{p}^* > 1$ represent the two pieces of the participation constraint (50) and (48), the segment corresponding to $q_i/\overline{q}^* > 1$ and $p_i/\overline{p}^* < 1$ representing the incentive compatibility constraint (46). The thin curve is again the isoprofit curve through $(1, 1)$, which is clearly not an equilibrium since firm i can increase its profit by increasing its price and decreasing its quantity.

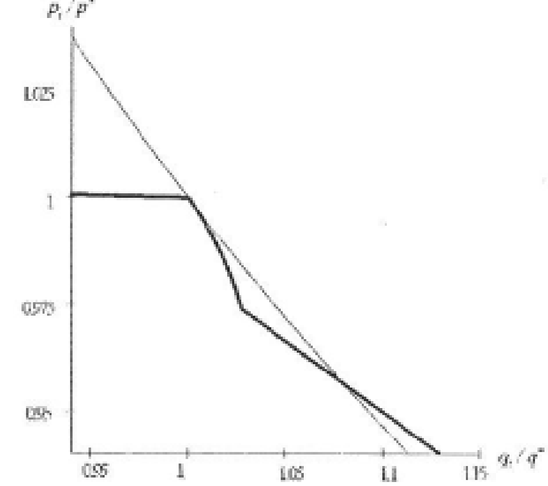

Figure 1: $S = 60, \mu = 0.59$

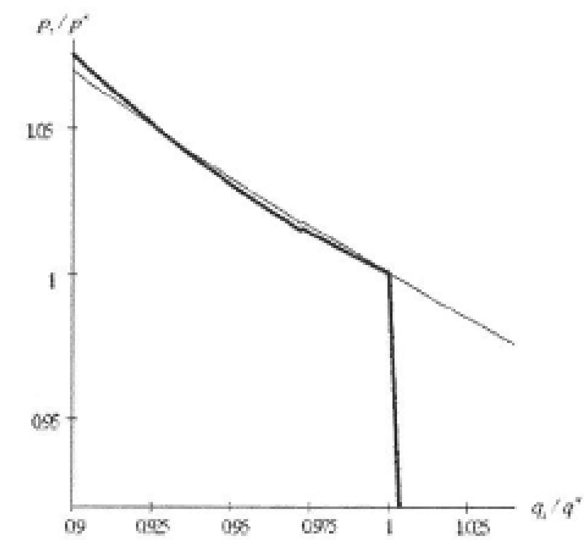

Figure 2: $S = 0.02, \mu = 0.63$

- Finally, we represent in Fig. 3, for each degree of relative intra-sectoral complementarity $\kappa = 1/(1+S) \in [0,1]$ measured along the horizontal axis, the set of degrees of monopoly $\mu^* \in [0,1]$ that are enforceable at a symmetric duopolistic equilibrium (with $\beta = 4$). This set is bounded by two thick curves. The first one, with a horizontal segment, results from minimum admissible competitive toughness, thus coinciding with the collusive level of degree of monopoly $\mu(0, S, 2, 4) = 0.6$ for intermediate degrees of relative complementarity. The second boundary results from maximum competitive toughness, and corresponds to the values of the degree of monopoly $\mu(1, S, 2, 4)$ associated with monopolistic competition (or perfect competition when $\kappa = 0$). The two thin, concave and convex, curves represent the degrees of monopoly in price and quantity equilibrium, respectively. They both link the two opposite extremes of the boundaries of the equilibrium set.

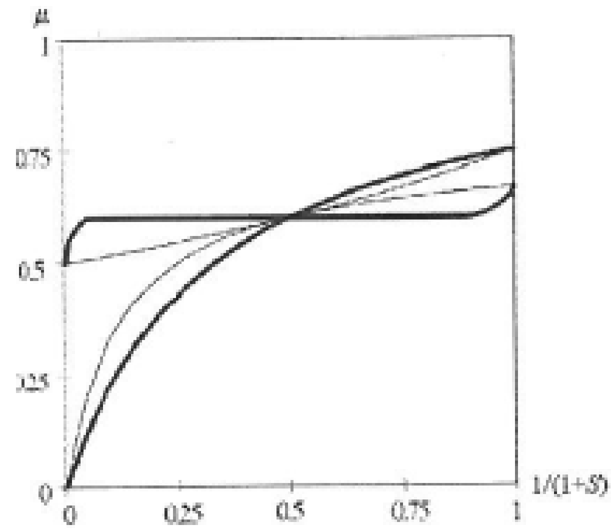

Figure 3:

6 Conclusion

In the analysis developed in this paper, we have tried to shed more light on the oligopoly problem through its reformulation in the multi-principal common agency framework. Two types of common agency games, intrinsic and non-intrinsic, have been presented, according to the common or individualized nature of the participation constraint. We have shown that there is a large equilibrium indeterminacy, in particular in the intrinsic case, where the set of equilibria may include the collusive solution. Obviously, the set of contracts that are fully (and not only potentially) enforceable depends upon the fundamentals, principally the ratio of intra- to intersectoral substitutability. We have seen that collusion is certainly not enforceable for extreme values of this ratio.

Our analysis is based on a canonical formulation of the common agency game and an associated natural parameterization of the set of equilibria. Hopefully, such a formulation might be used in more general environments, including asymmetric information. This is a topic for further research.

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