Strategic R& D investment, competitive toughness and growth*

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Abstract

We show, within a single industry, the possibility that R&-investment is non-monotonically related to competitive toughness: increasing when competition is soft and decreasing when competition is tough. This possibility results from the combination of a Schumpeterian markup squeezing effect discouraging innovation, and a concentration effect spurring innovators. It is obtained in a sectoral model where the number of innovators is random and where non-successful investors may remain productive. The result is extended to a multisectoral stochastic endogenous growth model with overlapping generations of consumers and firms, the number of which is endogenously determined in the capital market.

Key words: competitive toughness, R& incentives, strategic investment, endogenous growth

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1 Introduction

The relationship between product market competition and innovation is not simple to assess, either empirically or theoretically. It varies according to the market, the industry and the innovation characteristics. Following the Schumpeterian view (Schumpeter 1942), monopoly rent is required to support innovative activity and tougher competition on the product market has a negative impact on innovation. This conclusion is contrary to the view that "the incentive to invent is less under monopolistic than under competitive conditions" (Arrow 1962)[1] or to the "Darwinian" view for which competition is needed to force firms to innovate to survive.

Empirical evidence is not conclusive either. For example, Link and Lunn (1984) exhibit a positive effect of concentration on the returns to R& D for process innovation, whereas Geroski (1995), Nickell (1996) and Blundell, Griffith, and Van Reenen (1999) find a negative effect of concentration on innovation.[2] Another way of tackling the problem consists in looking for a non monotone relationship between competition and innovation. It can be traced back to Scherer (1965, 1967) who shows that the effect of firm size on patented inventions is diminishing for large sizes.[3] An inverted-U relationship between competition and innovation is further explored by Levin, Cohen, and Mowery (1985),[4] and is reexamined in a panel study by Aghion it et al. (2005), who obtain a clear inverted-U shape when plotting patents against the Lerner index.

[1]Tirole (1997) calls this the "replacement effect": the profits that are replaced by those resulting from innovation are larger for a monopoly.

[2]For more references and a discussion of these results, see Gilbert (2006).

[3]Dasgupta and Stiglitz (1980), referring to the earlier empirical literature surveyed in Scherer (1970) and Kamien and Schwartz (1975), stress that innovative activity might become negatively correlated to concentration when an industry is too concentrated.

[4]In a preliminary statistical investigation, the authors find a significant inverted-U relationship between industry concentration and R&D intensity or the innovation rate. However, in a further data analysis including more variables designed to capture the influence of technological opportunity and appropriability, the statistical significance of the concentration variable (C4 index) is much lower. "These econometric studies suggest that whatever relationship exists at a general economy-wide level between industry structure and R& D is masked by differences across industries in technological opportunities, demand, and the appropriability of inventions" (Gilbert 2005, p. 191).

Moreover, building upon previous work,[5] Aghion *et al.* (2005) provide a theoretical explanation for their observations. They suppose a fixed number of firms (a duopoly) in each industry, competing both at the research and the production levels. The set of industries can be partitioned into two groups: those where the two firms are at the same technological level (neck-and-neck) and those where one firm leads and the other lags. More intense competition enhances R& D investment in the former group, but discourages it in the latter (according to the Schumpeterian view).[6] It is by averaging R&D intensities across all industrys that an inverted$U$-relationship between the (average) innovation rate and product market competition is obtained (through a "composition effect").[7]

Our goal here is to propose an alternative theoretical model of product market competition and innovation explaining a non monotone relationship. This is done in a framework combining features of tournament models (Reingaum 1985, 1989) and non tournament models (Dasgupta and Stiglitz 1980). As in tournament models, the expected incremental gain of innovating creates the incentive for R& D investment by firms, and each industry cane partitioned into successful and unsuccessful firms. However, a special feature of our model is that we allow for multiple simultaneous innovators.[8] As in non tournament models, the concentration effect

[5]See Aghion, Harris, and Vickers (1997) and Aghion, Harris, Howitt, and Vickers (2001). The latter has been extended by Encaoua and Ulph (2000), allowing for the possibility that the lagging firm leapfrogs the leader without driving it out of the market, and also obtains a non-monotone relationship between competition and innovation. Aghion, Dewatripont, and Rey (1999) introduce agency considerations with nonprofit maximizing firms, leading to non-Schumpeterian conclusions. A synthesis of this stream of the literature is provided by Aghion and Griffith (2005).

[6]In a nec-and-neck industry, R& D intensity increases with product market competition because firms invest in R&D to escape competition (the "escape competition effect"). Only in an unleveled industry can the traditional "Schumpeterian effect" dominate (and will dominate when product market competition is sufficiently tough): as there is no incentive for the leader to invest in R& D (because of an assumed automatic catching up by the follower), only the laggard firm innovates, its chosen R&D intensity decreasing a competition becomes tougher in the product market, dissipating the rents that can be captured after innovation.

[7]A similar composition effect is exploited by Mukoyama (2003) to obtain an inverted-U relationship between competition and growth in a tournament model introducing the possibility of imitation.

[8]This feature significantly differentiates our model from the one in Denicolò and Zanchettin (2004), where

plays an essential role, not only through the variation of the endogenous number of (identical) firms (can de Klundert and Smulders 1997; Peretto 1999), but also by taking into account the distribution of market shares between successful and unsuccessful (incremental) innovators.[9]

In our model, identical firms compete first at the research and then at the production levels. At the first stage, they have equal access to technological knowledge and their R& D investments determine their respective probabilities of innovating. At the second stage, firms compete in the product market both in prices and in quantities (as in d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1991] and d'Aspremont and Dos Santos Ferreira [2009]). There is a continuum of oligopolistic equilibria, corresponding to intermediate regimes between Cournot and Bertrand and parameterized by an index of "competitive toughness."[10] As in Arrow (1962), the key notion is the incentive to invent, represented here by the incremental gain of innovating; that is, the difference for an investing firm between the profit it earns when successful and that which it earns when unsuccessful. As a function of competitive toughness, the incremental gain of innovating is affected by two opposite effects: a negative "markup squeezing effect," clearly Schumpeterian, and a positive concentration effect. Under soft competition, and assuming that innovators obtain only a small cost advantage,[11] unsuccessful firms remain active at equilibrium. Then the gap between market shares of the two groups of firms increases with competitive toughness, implying that concentration as measured by an index such as the Herfindahl index increases (other things equal) and that the incentive to invest in R& D, evaluated by the expected incremental gain of

there is a single technological leader in each period, although several past innovators might remain active.

[9]Thompson and Waldo (1994) discuss the two kinds of innovative capitalism described by Schumpeter (1928); "competitive capitalism," under which only the innovative firm remains active in the market and "trustified capitalism," where losing firms may remain active. They argue that, empirically, trustified capitalism is more important than competitive capitalism.

[10]Various continuous measures of the intensity of competition have been used to examine the relationship between R& D investment and competition: the degree of substitutability and the mass of differentiated products (as in Aghion, Dewatripont, and Rey 1999); the conjectural variations parameter (as in Encaoua and Ulph 2000); the inverse of the market price of a homogeneous product (as in Denicolò and Zanchettin [2004], in a reduced form model inspired by Cabral [1995] and Maggi [1996]); and the complement to one of the degree of collusion (as in Aghion *et al.* 2005).

[11]This supposes a small innovation step or imperfect patent protection (large spillovers).

innovating, also increases. At some level of competitive toughness, however, unsuccessful firms are eliminated and competition becomes symmetric, thus eliminating any further gain in market shares, so that the incentive to invest eventually decreases with higher competitive toughness. On this basis we show the possibility of obtaining a non monotone relationship between R& D investment and competitive toughness (increasing when competition is soft and decreasing when it is tough) in a single sector partial equilibrium model with a fixed number of firms. A corresponding non monotone relationship is then obtained, independently of any composition effect, in a multisector endogenous growth model where the number of firms in each sector is endogenously determined.

The paper is organized as follows. In Section 2, we introduce a one-sector model and give the definition of the oligopolistic two-stage game under a continuum of possible regimes of competition. We then present the basic nonmonotonicity results. In Section 3, we extend the model to a continuum of sectors and analyze the consequences of the basic nonmonotonicity results in this general setting where the number of firms is endogenously determined. We conclude in Section 4.

2 A representative oligopolistic sector

Let us start by considering a typical industry involving a set $\mathcal{N}$ of N firms. We shall successively consider the second and the first stages of the oligopolistic game. First suppose that each firm j has already chosen its level of investment in R&D, and that uncertainty in relation to innovation is resolved, resulting in a partition of the industry into successful and unsuccessful firms. Each firm j has to choose a price-output pair (p_j, y_j). Output y_j can be produced at unit cost c_j. This unit cost takes three values: the lowest one for successful firms, and intermediate value for non successful firms benefitting from innovators' spillovers,[12] and the highest value when no firms are successful. The demand, D, for the good is a function of market price, P, with a finite

[12]There are more sophisticated ways of introducing spillovers. For instance Amir and Wooders (1999) consider stochastic apillovers. In a paper that is more directly related to the present work, Reis and Traca (2008) analyze the impact of the intensity of spillovers on R&D productivity (rather than on the diffusion of innovations).

negative continuous derivative over all the domain where it is positive. A particular specification of the demand function will be introduced below. We let Y denote the total output, $\sum_k y_k$, and $Y_{-j} = \sum_{k \neq j} y_k$ the total output produced by firms other than j.

2.1 The oligopolistic equilibrium

Because our goal is to study the relationship between the degree of competition and R&D investment, we want to compare different competition regimes, including perfect competition, Bertrand and Cournot competition, but also other intermediate regimes. This is common practice in the new empirical industrial organization literature, where "behavioral equations" are used to estimate firm behavior, and in which the degree of competitiveness of a firm is represented by a conduct parameter.[13] From a theoretical point of view, there are various ways to derive these behavioral equations.[14] We shall not review all these various theories here but shall rely on the canonical, and Cournot-like, representation of oligopolistic competition introduced in d'Aspremont and Dos Santos Ferreira (2009), which is readily usable in the present framework. In this canonical representation beach firm j is assumed to choose a price-output pair (p_j, y_j) to maximize its profit under two constraints representing the competitive pressure coming, respectively, from inside and from outside the industry. In the first constraint, firm j preserves its market share by matching its competitors' prices. In the second, firm j adjusts for the market size. At equilibrium, the consumers should not be rationed. Formally, a 2 N-tuple (p^*, y^*) is an *oligopolistic equilibrium* if, for each firm j, (p_j^*, y_j^*) is a solution to the program

$$\max_{(p_j, y_j) \in \mathbb{R}_+^2} \{(p_j - c_j) y_j : p_j \leq \min_{k \neq j}\{p_k^*\} \text{ and } p_j \leq D^{-1}(y_j + Y_{-j}^*)\}, \tag{1}$$

and satisfies the no-rationing condition

$$Y^* = D(P^*), \text{ with } P^* = \min_j\{p_j^*\}. \tag{2}$$

Introducing Kuhn and Tucker multipliers $(\lambda_j, \nu_j) \in \mathbb{R}_+^2 \setminus \{0\}$ associated with the first and second constraints in (1), respectively, general first-order conditions require, by the positivity

[13] For a synthesis, see Bresnahan (1989) and Martin (2002).

[14] See Martin (2002).

of p_j^* and the non negativity of y_j^*, that $y_j^* - \lambda_j - \nu_j = 0$, and $p_j^* - c_j + \nu_j/D'(P^*) \leq 0$ with $(p_j^* - c_j + \nu_j/D'(P^*))y_j^* = 0$. The multiplier λ_j, associated with the market share constraint, and the multiplier ν_j, associated with the market size constraint, can be interpreted as the shadow costs for firm j of relaxing the pressure coming from its competitors, respectively, inside and outside the industry. Defining the normalized parameter $\theta_j \equiv \lambda_j/(\lambda_j + \nu_j) \in [0, 1]$, the first-order conditions for the N^* producing firms (with $p_j^* = P^*$ and $y_j^* > 0$) can then be expressed as a function of the θ_js:

$$\frac{P^* - c_j}{P^*} = (1 - \theta_j)\frac{y_j^*/Y^*}{-\epsilon\, D(P^*)}, \tag{3}$$

where the left-hand side is the firm j Lerner degree of monopoly and ϵ is the elasticity operator.[15] The behavioral equations of the empirical literature coincide with these first-order conditions where the θ_js are called conduct parameters. However, here the θ_j s are determined endogenously and parameterize the set of equilibria. Each parameter θ_j can be viewed as an index of the competitiveness (or the competitive toughness) of firm j at some equilibrium. Alternatively, the θ_js could be taken as exogenous parameters and, by varying their values, a whole set of games can be defined thus describing the set of competitive regimes between Cournot competition and perfect competition. In such an intermediate game, each firm maximizes a convex combination of its Cournot and its price-taking profits. An equilibrium consists of an equilibrium price P^* and equilibrium quantities $y^* \in \mathbb{R}_+^N$ such that, for each firm j,

$$y_j^* \in \arg\max_{y_j \in \mathbb{R}_+} \{(1 - \theta_j)[D^{-1}(y_j + Y_{-j}^*) - c_j]y_j + \theta_j(P^* - c_j)y_j\}$$

and

$$P^* = D^{-1}(Y^*).$$

An equilibrium is characterized by the same first-order conditions (3), given the fixed value of the θ_js.

[15] For a differentiable function $f(x)$,

$$\epsilon\, f(x) \equiv \frac{df(x)}{dx}\frac{x}{f(x)}.$$

At one extreme, whenever $\theta_j = 0$ for all j, this equilibrium coincides with the Cournot solution (P^C, y^C) satisfying

$$y_j^C \in \arg\max_{y_j \in [0,\infty)} [D^{-1}(y_j + Y_{-j}^C) - c_j] y_j, \quad j = 1, \cdots, N, \tag{4}$$

and $P^C = D^{-1}(Y^C)$. At the other extreme, whenever $\theta_j = 1$ for all j, the equilibrium coincides with the perfectly competitive (Walrasian) outcome (P^W, y^W) satisfying

$$y_j^W \in \arg\max_{y_j \in [0,\infty)} (P^W - c_j) y_j, \quad j = 1, \cdots, N, \tag{5}$$

with P^W such that $\sum_i y_i^W = D(P^W)$. This also corresponds to the Bertrand outcome in the case where all costs are identical. The Bertrand outcome (P^B, y^B) with $P^B = \min_j\{p_j^B\}$ is defined by

$$p_j^B \in \arg\max_{p_j \in [0,\infty)} (p_j - c_j) d_j(p_j, p_{-j}^B) \tag{6}$$

$$\begin{aligned} \text{with } d_j(p_j, p_{-j}^B) &= \frac{D(\min\{p_j, p_{-j}^B\})}{\#\arg\min\{p_j, p_{-j}^B\}}, \text{ if } p_j = \min\{p_j, p_{-j}^B\}, \\ &= 0 \text{ otherwise} \end{aligned} \tag{7}$$

It corresponds to the case where at least one θ_j is equal to 1. When costs are not identical, the standard way to ensure the existence of a Bertrand outcome is to assume that profits are measured in (small ε) discrete units of account (e.g. in cents), then to compute Bertrand prices accordingly and take their limit as $\varepsilon \to 0$.[16] Only the most efficient firms produce and have a positive profit. All prices converge to some price between the first and the second lowest cost. Such an outcome also corresponds to an oligopolistic equilibrium.

In the following, we shall treat the θ_js as parameters indicating the relative degree of competition and use the first-order conditions (3) to investigate their impact on R&D investment. However, to simplify the analysis, we introduce additional assumptions.

2.2 The gain of innovating under varying toughness

A first simplifying assumption is to suppose a unit-elastic demand to the industry,

$$D(P) = \frac{A}{P}, \tag{8}$$

[16]See, for example, Mas-Colell, WHinston, and Green (1995).

with A and P positive and denoting, respectively, the sectoral expenditure and the market price. A second simplification is to limit our analysis to the case in which competitive toughness is the same for all producing firms; that is, $\theta_j = \theta \in [0,1]$ for any j.[17] A third assumption is to let the unit cost of each firm depend on its type (successful or unsuccessful) and to take into account the possibility of incomplete appropriability by the innovators through a spillover coefficient $\sigma, 0 \leq \sigma < 1$. Formally, for $c(\cdot)$, a strictly decreasing function, we let

$$c_j = c(\delta_j), \tag{9}$$

with $\delta_j = 1$, if j is successful, $\delta_j = 0$ if no firm succeeds and $\delta_j = \sigma$, if at least one firm succeeds but not firm j. We note by $\kappa \equiv (c(\sigma) - c(1))/c(\sigma)$ the *relative cost advantage* of the innovators. Clearly, more spillovers, or less appropriability of invention, decreases the relate cost advantage. Under these assumptions, the first-order condition (3) becomes:

$$1 - \frac{c(\delta_j)}{P^*} = (1-\theta)\frac{y_j^*}{y^*}, \tag{10}$$

with $Y^* = A/P^*$.

Consider first the symmetric case where firms are all unsuccessful. The equilibrium price, as a function of the number n of successful firms, the number N of competing firms and competitive toughness, θ, is easily computed to be

$$P(0,N,\theta) = \frac{Nc(0)}{N-(1-\theta)}. \tag{11}$$

A second case is when n firms succeed $(1 \leq n \leq N)$ and all N firms are active at equilibrium. We then obtain the equilibrium price

$$P(n,N,\theta) = \frac{nc(1) + (N-n)c(\sigma)}{N-(1-\theta)}, \tag{12}$$

provided $P(n,N,\theta) \geq c(\sigma)$ or $\kappa \leq (1-\theta)/n$. For $\kappa > (1-\theta)/n$, the innovation is drastic, unsuccessful firms are eliminated and we obtain the third case, where only successful firms are

[17] Competitivie toughness, θ, as now defined, satisfies the axioms characterizing a measure of the intensity of competition according to Boone's (2001) definition (except for the normalization axiom associating a zero θ with local monopoly).

active (getting equal market shares), the price becoming:

$$P(n,n,\theta) = \frac{nc(1)}{n-(1-\theta)}. \tag{13}$$

Observe that there is a borderline case where the competitive toughness is just sufficient to eliminate unsuccessful firms; that is,

$$\theta = \theta^L(n) \equiv \max\{1-n\kappa, 0\}. \tag{14}$$

We call this borderline case the *limit-pricing regime*. The corresponding θ varies with n. It can be extended to the cases in which there are no successful firms and in which all firms succeed, by taking the limit values of $\theta^L(n)$, denoted, respectively, $\theta^L(0)$ (equal to 1 and corresponding to the Bertrand outcome) and $\theta^L(N)$.

For the following analysis, it is convenient to introduce a specific notation for market shares. For $0 < n \leq N$, we let $m(1,n,N,\theta)$ and $m(\sigma,n,N,\theta)$ denote, respectively, the market share of a successful and an unsuccessful firm. From (10) and the price equations, it is easy to verify that the equilibrium market share is:

$$\frac{Y_j^*}{Y^*} = m(\delta_j,n,N,\theta) = \frac{1}{1-\theta}\left(1 - \frac{N-(1-\theta)}{nc(1)+(N-n)c(\sigma)}c(\delta_j)\right) \tag{15}$$

or, using the notation κ and taking into account the case of drastic innovations (with $\kappa > (1-\theta)/n$),

$$m(1,n,N,\theta) = \min\left\{\frac{1}{1-\theta}\frac{(N-n)\kappa+(1-\theta)(1-\kappa)}{N-n\kappa}, \frac{1}{n}\right\}, \tag{16}$$

$$m(\sigma,n,N,\theta) = \max\left\{\frac{1}{1-\theta}\frac{1-\theta-n\kappa}{N-n\kappa}, 0\right\}. \tag{17}$$

When $n = 0$, we let $m(\sigma,0,N,\theta) = m(0,0,N,\theta) = 1/N$.

Clearly, $m(1,n,N,\theta) > m(\sigma,n,N,\theta)$: the market share of a successful firm is bigger than the market share of an unsuccessful one. Notice also that, for $0 < n < N$ and $\theta < \theta^L(n)$; that is, when, given the competitive toughness, innovations are non-drastic, the market share $m(1,n,N,\theta)$ is increasing, whereas $m(\sigma,n,N,\theta)$ is decreasing in θ, so that the gap in market shares between successful and unsuccessful firms increases with competitive toughness. In particular, the Herfindahl concentration index is increasing in θ.[18] It is through this effect on market shares, hence by enhancing concentration (as measured by the Herfindahl index), that tougher competition can stimulate R&D. Note that the concentration effect exhibited here is not measured by the reduction in the number of firms (at zero profit equilibrium under free entry) when competition becomes tougher, as in the case of non-tournament models when all firms are identical (e.g. van de Klundert and Smulders, 1997). We have to use a more sophisticated notion of concentration because, although we assume symmetry *ex ante*, we lose it *ex post* when there are successful and unsuccessful firms.

Using the first-order condition (10), the equilibrium profit Π per unit of expenditure of firm j is then equal to

$$\Pi(\delta_j, n, N, \theta) = (1-\theta) m(\delta_j, n, N, \theta)^2. \tag{18}$$

We see that an increase in competitive toughness has two effects on a firm equilibrium profit, first a negative markup squeezing effect through $(1-\theta)$, and second an effect through the variation in market share $m(\delta_j, n, N, \theta)$. When all active firms have the same cost, so that they all get the same market share at equilibrium, profits decrease as competition becomes tougher. This is also true as regards the profit of an unsuccessful firm ($\delta_j = \sigma$, with $n \geq 1$) because its market share is a decreasing function of competitive toughness. However, as a positive effect, which might more than compensate for the negative markup squeezing effect, provided its relative cost advantage, κ, is strong enough, but not so as to make the innovation drastic (see Equation 16).

However, as we shall see in the following, it is not so much the equilibrium profit that determines a firm incentive to innovate, but rather the incremental gain G of innovating

$$G(n, N, \theta) \equiv \pi(1, n+1, N, \theta) - \Pi(\sigma, n, N, \theta), \tag{19}$$

[18] The Herfindahl index is defined as

$$\mathcal{H} = \sum_{j=1}^{N} m_j^2 = \frac{1}{N} + NV,$$

with m_j the market share of firm j and the variance

$$V = \frac{1}{N} \sum_{j=1}^{N} \left(m_j - \frac{1}{N} \right)^2.$$

for $0 \leq n < N$ (using the equality $\Pi(\sigma, 0, N, \theta) = \Pi(0, 0, N, \theta)$). Here an increase in competitive toughness has clearly two opposite effects. There is still the negative markup squeezing effect, together with a positive concentration effect through the difference in the squares of market shares. We may thus expect to obtain a non-monotonic relationship between the incremental gain of innovating G and the competitive toughness, θ. This is formalized in the following lemma showing that G either has an inverted-U shape or is decreasing for all θ.

Lemma 1 *If the relative cost advantage κ (or the number n) of successful firms is small enough, then the incremental gain of innovating G is increasing in the competitive toughness θ for sufficiently small, and decreasing for sufficiently large, values of θ. Otherwise, G is decreasing in θ in the whole interval $[0, 1]$. In any case, the function G is strictly quasi-concave in θ in the whole interval $[0, 1]$. Moreover, G is decreasing in n for small enough values of κ.*

Proof: See appendix ■

In the case of drastic innovations, that is, when $\theta^L(n) = 0$ due to a combination of many innovators and a large relative cost advantage, laggards are eliminated even under Cournot competition, so that the concentration effect vanishes and tougher competition can only have a negative effect on the incentive to innovate through the markup squeeze. In the case of non-drastic innovations, the concentration effect is positive but weak when innovation is sufficiently appropriable (with a small spillover coefficient, σ, resulting in a large relative cost advantage, κ) and when the number of innovators, n, is large. The incentive to innovate is then stronger even if competition is soft; in conformity with Schumpeter's view. By contrast, with small values of κ and n, the incentive to innovate is first increasing and eventually decreasing with competitive toughness, so that we may obtain a non-monotone curve relating the incremental gain of innovating to competitive toughness. This non-monotonicity derives from a strong concentration effect when competition is soft; that is, from the incentive created by a high prospective increase of the innovator's market share, an effect that entirely depends upon the probabilistic nature of the model, and disappears as soon as innovation is approached as a deterministic process. The last statement of the lemma asserts that the incentive to innovate decreases with the number of innovators, a property that will play a crucial role in the following (see the proof of Proposition 1), and that relies on the possibility of multiple winners, usually excluded in tournament models. Hence, uncertainty and multiplicity of simultaneous innovations are crucial to obtain a non-monotone relation between competitive toughness and R&D-investment.

2.3 Strategic R&D investment

In the preceding analysis, we supposed that each firm had already chosen its level of investment in R&D and that uncertainty of innovation was resolved, implying that the sets of successful and of unsuccessful firms were fixed. We now introduce a first stage during which each firm, j, chooses a level of R&D investment leading to discovery with some probability of success, s_j. Innovation is assumed to be a Bernoullian random process that depends upon these investment levels. The two-stage game will depend on the expected expenditure, A, on the number of firms, N, and, of course, on the selected competitive toughness, θ, and will be denoted $\Gamma(A, N, \theta)$.

For simplicity, firm j investment is directly represented by the independent probability s_j of success in the next period. We specify the investment cost to be quadratic: $C(s_j) = \phi + (\gamma/2)s_j^2$ for $s_j \geq 0$ (with $\phi > 0$ and $\gamma > 0$). The sunk cost, ϕ, corresponds to the investment that is required for having access to the technology and for benefitting from technological spillovers. The probability that a subset S of firms innovate simultaneously, while firms in the complementary subset $\overline{S}$ do not succeed, is given by $\prod_{j\in S} s_j \prod_{j\in\overline{S}}(1-s_j)$. We denote $\Pr\{\nu \mid \mathbf{s}_{-j}\}$ the probability of having ν innovators among the $N-1$ competitors of firm j with investment strategies $\mathbf{s}_{-j} \equiv (s_1, \cdots, s_{j-1}, s_{j+1}, \cdots, s_N)$.

Investment s_j is decided by maximizing the profit expectation $\tilde{\Pi}(s_j, \mathbf{s}_{-j})A$, for given values of A, N and θ, with

$$\begin{aligned}\tilde{\Pi}(s_j, \mathbf{s_{-j}}) \quad &= \quad s_j \sum_{\nu=0}^{N-1} \Pr\{\nu \mid \mathbf{s}_{-j}\}\Pi(1, \nu+1, N, \theta) \\ &+ \quad (1-s_j)\sum_{\nu=0}^{N-1} \Pr\{\nu \mid \mathbf{s}_{-j}\}\Pi(\sigma, \nu, N, \theta) - \tfrac{C(s_j)}{A}.\end{aligned} \tag{20}$$

This expectation is strictly concave in the strategy variable s_j, by the specification of the cost function. Therefore, we obtain for each j the necessary and sufficient first-order condition for an interior maximum (at $s_j \in (0,1)$):

$$\begin{aligned} \tfrac{C'(s_j)}{A} &= \textstyle\sum_{\nu=0}^{N-1} \Pr\{\nu \mid \mathbf{s}_{-j}\}[\Pi(1,\nu+1,N,\theta) - \Pi(\sigma,\nu,N,\theta)] \\ &= \textstyle\sum_{\nu=0}^{N-1} \Pr\{\nu \mid \mathbf{s}_{-j}\} G(\nu,N,\theta), \end{aligned}$$

the equality of the marginal R&D investment cost and of the expected value of the incremental gain of innovation (both by unit of expenditure).

At a symmetric equilibrium (with $s_j = s$, for any j), this first-order condition becomes:

$$\frac{\gamma^s}{A} = \sum_{\nu=0}^{N-1} \Pr\{\nu \mid (s,\cdots,s)\} G(\nu,N,\theta) \equiv \overline{G}(s,N,\theta), \tag{21}$$

where

$$\Pr\{\nu \mid (s,\cdots,s)\} = \frac{(N-1)!}{(N-1-\nu)!\nu!} s^{\nu}(1-s)^{N-1-\nu}. \tag{22}$$

By continuity of $\overline{G}$ as a function of s, and because $\overline{G}(0,N,\theta) = G(0,N,\theta) > 0$, either there is a value $s \in (0,1)$ satisfying (21), or $\overline{G}(s,N,\theta) > \gamma/A$ for any $s \in (0,1)$ and the corner solution $s = 1$ applies.

Using Lemma 1 and the probabilities given by (22), we may easily derive the following conclusions:

Lemma 2 *If the relative cost advantage, κ, of successful firms (or the probability of success, s) is small enough, then the expected incremental gain of innovating $\overline{G}(s,N,\theta)$ is increasing with the competitive toughness θ for sufficiently small, and decreasing for sufficiently large, values of θ. Otherwise, $\overline{G}$ is decreasing in 0 in the whole interval* $[0,1]$.

Proof: Given $N, \overline{G}(s,N,\theta)$, as a function of θ, is an expectation (determined by s) computed from the set of functions $G(\nu,N,\theta)$ for $\nu = 0,1,\cdots,N-1$. By Lemma 1, whenever κ is small enough, every such $G(\nu,N,\theta)$ is increasing in θ for any θ close to 0. In addition, for s small enough, we see by (22) that most of the weight is put on small values of ν entailing (again by Lemma 1) that $G(\nu,N,\theta)$ is increasing in θ for any θ close to 0. Because, for any $\nu, G, (\nu,N,\theta)$

is decreasing in θ for any θ close to 1 (and for all θ in $[0, 1]$ when neither κ nor s are too low), $\overline{G}(s, N, \theta)$ inherits the same property. ■

Except for quasi-concavity, the properties exhibited in Lemma 1 for the function G are preserved in expected terms for the function $\overline{G}$, with a condition imposing a small probability of success replacing the condition of a small number of successful firms. An implication is that when the relative cost advantage of innovation is significant, the possibility of deviating from a strict Schumpeterian view arises from the probabilistic nature of the model.

Figure 1 illustrates Lemma 2 by showing how the expected incremental gain varies with competitive toughness for three values of the probability of success, $s (s = 0.25, 0.5, 0.75)$.[19] For the lowest value of s, we obtain a hump-shaped curve dominating the other two. For the largest value, the curve is decreasing overall.

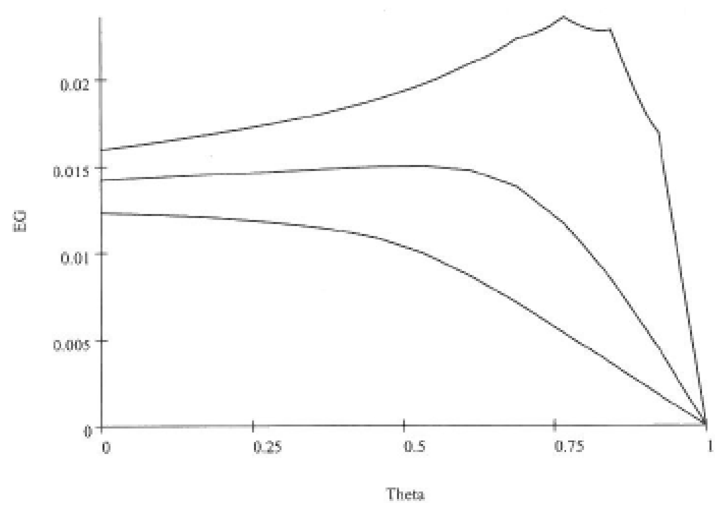

Figure 1: The relationship between competitive toughness and the expected incremental gain of innovating

The next proposition describes properties of the (symmetric) equilibrium where all firms choose the (same) level of R&D-investment, a probability of success, s, for various values of

[19]Figure 1 is based on the parameter values $N = 9$ and $\kappa = 0.078$.

competitive toughness.

Proposition 1 *If the relative cost advantage, κ, of successful firms is small enough, the symmetric equilibrium level, s of R&D investment is uniquely determined for every value of θ. If, in addition, the slope γ/A of the marginal investment cost per unit of aggregate expenditure is large enough (so as to exclude a corner solution), then s is increasing with competitive toughness θ for sufficiently small, and decreasing for sufficiently large, values of θ. Moreover, s is decreasing in γ/A.*

Proof: By Lemma 1, the incremental gain, G, is decreasing in n for small enough values of κ. An implication of this property of G is that the expected incremental gain $\overline{G}$ is decreasing in s. Indeed, by (22), the elasticity of the weight $\Pr\{\nu \mid (s, \cdots, s)\}$ with respect to s is equal to $(\nu - (N-1)s)/(1-s)$, which has the sign of the excess of ν over its mean. Therefore, an increase in s displaces the mass towards the terms corresponding to a larger number of successful firms, those for which the incremental gain is lower. It results that the first-order condition for equilibrium investment (21) uniquely determines the symmetric equilibrium value of s at the intersection of the decreasing curve on the right-hand side with the increasing line on the left-hand side, provided this line has a large enough slope γ/A (otherwise, we obtain the corner solution $s = 1$). Then, from Lemma 2, for a sufficiently low competitive toughness, the expected incremental gain is increasing, implying by (21) an increasing equilibrium level s of R&D-investment. The reverse holds for sufficiently soft competition. Finally, notice that the equilibrium value s (a solution to Equation 21) is smaller for a higher slope γ/A. ■

The proof of Proposition 1 is illustrated in Figure 2, where the increasing line corresponds to the left-hand side of (21), and the decreasing curves to its right-hand side, for values of the competitive toughness $\theta = 0$ and $\theta = 0.12$ (the upper thick and thin curves, respectively) and $\theta = 0.8$ and $\theta = 0.9$ (the lower thick and thin curves, respectively).[20] In the situation depicted in Figure 2, the equilibrium level s of R&D investment increases (respectively, decreases) as one switches from the thick to the thin curve; that is, as competition becomes tougher, starting from

[20]Figure 2 is based on the parameter values $N = 3$, $\kappa = 0.1$ and $\gamma/A = 1.5$.

soft (respectively, tough) competition. The reverse result would be attained, in the case of soft competition, for lower values of the slope γ/A of the increasing line (because the two decreasing curves intersect).

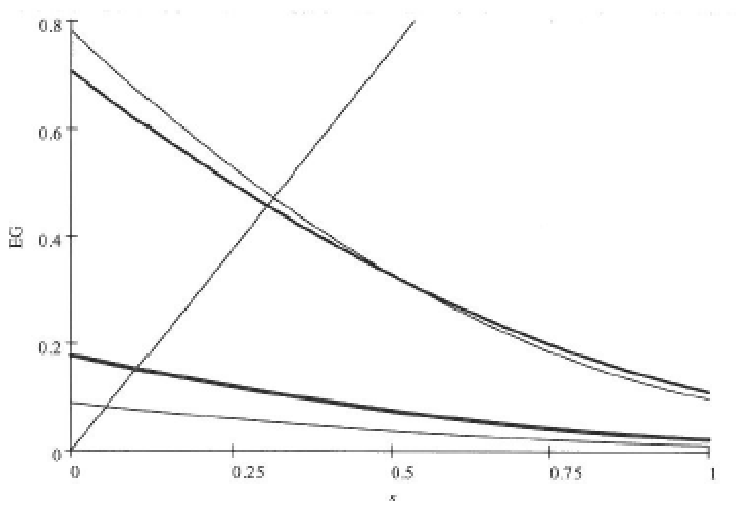

Figure 2: Variation of the equilibrium R&D investment as competition becomes tougher

Notice that Proposition 1 uses the main properties of the incremental gain function that were exhibited in Lemma 1, in particular the fact that it decreases with the number of innovators, which has the consequence that the expected incremental gain is decreasing in s. This means that assuming the number of innovators to be a random variable, following a Bernoullian process, is essential to establish the proposition.

3 Competition and innovation in endogenous growth: A non-monotone relationship

The previous proposition shows the possibility of obtaining a non-monotone relationship between R&D investment and competitive toughness (increasing at low values and decreasing at high values), by using a single sector partial equilibrium model. As mentioned in the Introduction, an inverted-U relationship has been theoretically derived by Aghion *et al.* (2005) in a multisectoral endogenous growth model, through a "composition effect of competition on the steady-state distribution of technology gaps across sectors." When competition increases from its minimal to its maximal level, the aggregate innovation rate first increases (the "escape competition effect" dominates because of a larger percentage of neck-and-neck industries) and then decreases (the "Schumpeterian effect" dominates because of a larger percentage of industries exhibiting a technology gap).

We shall show that the one-sector model constructed above and the associated two-stage game can serve as a building block in an endogenous growth general equilibrium model with a continuum of uniformly distributed oligopolistic industries and that the non-monotone pattern can also be predicted for cross-section investigations. In our model, however, we do not assume that every industry has a fixed number of firms (two in Aghion *et al.* 2005). It is composed of a finite but endogenously determined number of firms, each having a two-period life, competing first at the research and then at the production levels. The number of successful firms is a random variable, the realization of which might differ across industries.

3.1 A model with overlapping generations of firms and consumers

We use an overlapping generations model with a continuum of produced goods and an infinite number of periods ($t = 0, 1, \cdots$). Both firms and consumers live for two periods, corresponding to R&D investment and production stages for firms, and young and old ages for consumers.

On the firms' side, there is a continuum of identical oligopolistic industries of mass 1. At date $t+1$, there is in each industry i a number $N_{it} (N_{it} \geq 2)$ of firms created at date t, which produce good i for immediate consumption, on a one-to-one basis with respect to effective labor, supplied by young consumers:

$$y_{ijt+1} = \ell_{ijt+1} H_{it} \eta^{\delta_{ijt}},$$

where y_{ijt+1} and ℓ_{ijt+1} are the output and labor input of firm j, respectively, H_{it} is the inherited stock of knowledge (available to the whole industry at the beginning of period t), and $\eta > 1$ is the innovation step if the firm has succeeded to innovate (with probability s_{ijt} and cost $C(s_{ijt})$) at the end of period t. As indicated in Subsection 2.2, δ_{ijt} is equal to 1 for an innovator, to σ

in $[0,1)$ for an unsuccessful firm benefitting from spillovers from successful competitors, and to 0 if there were no innovators in period t. Accordingly, and taking labor as the numeraire, the unit production cost is $c_{it}(\delta_{ijt} = 1/H_{it}\eta^{\delta_{ijt}}$, so that the innovators' relative cost advantage is a constant $\kappa = 1 - \eta^{\sigma-1}$. We also assume that public knowledge accumulates in proportion to the percentage of successful firms:

$$\frac{H_{it+1} - H_{it}}{H_{it}} = (\eta - 1)\frac{n_{it}}{N_{it}}. \tag{23}$$

On the consumers' side, there is a continuum of identical consumers of constant unit mass at each generation. One unit of labor is inelastically supplied at wage 1 by each young consumer, who has to choose present consumption $x_t \in \mathbb{R}_+^{[0,1]}$ and saving $z_t \in \mathbb{R}_+$, under the budget constraint $\langle P_t, x_t\rangle + z_t = 1$, where $P_t \in \mathbb{R}_+^{[0,1]}$ is the vector of market prices.[21] Anticipated future consumption $\tilde{x}_{t+1}$ is a random variable induced by $\tilde{r}_{t+1}$ and $\tilde{P}_{t+1}$ according to the budget constraint $\langle \tilde{P}_{t+1}, \tilde{x}_{t+1}\rangle = \tilde{r}_{t+1} z_t$, where $\tilde{P}_{t+1}$ is the vector of anticipated market prices and $\tilde{r}_{t+1}$ is the expected return factor on capital. Saving is supposed to be invested in funds that allow canceling out of idiosyncratic risks, but not aggregate risk, so that $\tilde{r}_{t+1}$ is a random variable depending upon the success of the innovative efforts by all investing firms. For simplicity, we assume symmetric log-linear sub utility functions:

$$u(x_t, \tilde{x}_{t+1}) = \alpha \int_0^1 \ln x_{it} di + (1-\alpha) \int_0^1 \ln \tilde{x}_{it+1} di, \text{ with } \alpha \in]0,1[,$$

so that $x_{it} = (1 - z_t)/P_{it}$ and $\tilde{x}_{it+1} = \tilde{r}_{t+1} z_t / \tilde{P}_{it+1}$. Maximizing expected utility reduces to maximizing $\alpha \ln(1 - z_t) + (1 - \alpha) \ln z_t$, leading to the solution $z_t = 1 - \alpha$.

Given r_t (the actual return factor) and P_t, old consumers at period t optimally choose consumptions $x'_{it} = r_t z_{t-1}/P_{it} = (1-\alpha) r_t / P_{it}$. Therefore, adding consumptions by young and old, we obtain the aggregate demand A_t/P_{it} for good i, with aggregate expenditure

$$A_t = \alpha + (1 - \alpha) r_t. \tag{24}$$

Observe that, as we have assumed a continuum of sectors, A_t is unaffected by sectoral idiosyncratic variations.

[21] Given P and x belonging to $\mathbb{R}^{[0,1]}$, we let $\langle P, x\rangle$ denote the inner product $\int_0^1 P_i x_i di$.

3.2 Intertemporal stochastic equilibrium

The N_{it} firms in industry i, created at date t, can be seen as involved in a two-stage game $\Gamma(A_{t+1}, N_{it}, \theta_{it+1})$ of the kind analyzed in Section 2. In the first stage of this game, corresponding to the investment period t, each firm j, producing good i, chooses strategically a probability s_{ijt} of success and accordingly invests $C(s_{ijt})$ in R&D. Uncertainty on innovation is resolved at the end of period t, resulting for each industry i in a number of successful firms, n_{it}. In the second stage, corresponding to the production period $t+1$, each firm j chooses a price-output pair $(p_{ijt+1}, y_{ijt+1}))_{it}$ of solutions to the sequence $\Gamma(A_{t+1}, N_{it}, \theta_{it+1}))_{it}$ of these two-stage games, for all industries. We assume such solutions to be symmetric within each sector relative to investing firms ($\mathbf{s}_{it} = (s_{it}, \cdots, s_{it})$) and within each category of producing firms (successful and unsuccessful).

This sequence of solutions is determined by the sequence of vectors $(\theta_{it+1})_{it}$ of competitive toughness that can be treated as exogenously given (except in the limit pricing regime, where $\theta_{it+1} = \theta^L(n_{it})$). For this sequence of solutions to deliver an inter temporal stochastic equilibrium, the sequence of values of the variables $(n_{it}, N_{it}, A_{t+1})$ must satisfy three sequences of conditions. The first sequence of conditions corresponds to the first-order conditions for equilibrium investment (see Equation 21):

$$\frac{\gamma}{A_{t+1}} s_{it} = \overline{G}(s_{it}, n_{it}, \theta_{it+1}). \tag{25}$$

The second sequence of conditions corresponds to capital market clearing:

$$\int_0^1 N_{it} C(s_{it}) di = 1 - \alpha, \tag{26}$$

expressing the equality of aggregate R&d investment and aggregate saving. Finally, the third sequence of conditions corresponds to labor market clearing. Total labor supply is 1, but, by the capital market clearing conditions, a proportion $1-\alpha$ of labor is employed by investing firms, leaving α to producing firms, so that we get

$$A_{t+1} \int_0^1 L(n_{it}, N_{it}, \theta_{it+1}) di = \alpha, \tag{27}$$

where $L(n_{it}, N_{it}, \theta_{it+1})$ is labor demand per unit of expenditure in industry i. As the wage is normalized to 1, labor demand must be equal at equilibrium to expenditure minus the total profits of successful and unsuccessful firms, so that, by (18), we obtain the following expression:[22]

$$\begin{aligned} L(n_{it}, N_{it}, \theta_{it+1}) &= 1 - n_{it}(1 - \theta_{it+1})m(1, n_{it}, N_{it}, \theta_{it+1})^2 \\ &- (N_{it} - n_{it})(1 - \theta_{it+1})m(\sigma, n_{it}, N_{it}, \theta_{it+1})^2. \end{aligned} \tag{28}$$

To compare the implications of our model with the cross-sectional observations of Aghion *et al.* (2005), we refer to the first-order conditions for equilibrium investment (Equation 25) in two industries of the same size, where the coefficient (γ/A_{t+1}) on the left-hand side of (25) and the relative cost advantage, κ, are the same. As a direct corollary of Proposition 1, we may derive the following cross-section result.

Proposition 2 *Consider an inter temporal stochastic equilibrium and any period t. Suppose that two industries i and i' have the same size ($N_{i't} = N_{it}$), the same sufficiently low relative cost advantage, κ, and the same sufficiently steep marginal investment cost per unit of expenditure (high γ/A_{t+1}). In addition, assume that both industries have a sufficiently low (respectively, high) competitive toughness, but that i' is more competitive than $i : \theta_{i'} > \theta_i$. Then the R&D investment of the more competitive sector is larger (respectively, smaller): $s_{i'} > s_i$ (respectively, $s_{i'} < s_i$).*

In other words, a statistical cross-section of otherwise identical industries (with low relative cost advantage and steep marginal investment cost) should reveal that if competition is soft (respectively, tough) for two of them, the more competitive one invests more (respectively, less).

3.3 Implications of an endogenous number of firms

Aghion *et al.* (2005) limit their analysis to an economy where every product market is a duopoly. By contrast, van de Klundert and Smulders (1997) introduce a non-tournament model, where

[22] Observe that labor demand can be expressed as

$$L(n_{it}, N_{it}, \theta_{it+1}) = 1 - (1 - \theta_{it+1})\mathcal{H}(n_{it}, N_{it}, \theta_{it+1}),$$

where $\mathcal{H}(n_{it}, N_{it}, \theta_{it+1})$ is the Herfindahl concentration index for industry i at date $t + 1$.

the number of firms is endogenously determined under free entry by the zero profit condition. This model allows them to show that the (differentiated) Bertrand equilibrium always implies a higher rate of innovation than the Cournot equilibrium. This is so because tougher competition, meaning lower markups and prices, enlarges the market for high-technology goods and weakens the relative weight of R&D costs, increasing the attractiveness of R&D investment. However, at the same time, tougher competition also reduces the equilibrium number of firms, implying larger firm size and more means devoted to R&D activity. Although our model is a tournament model, we shall see that a comparison with the approach of van de Klundert and Smulders (1997) is straightforward.

To achieve this comparison, we assume identical regimes of competition across time and sectors ($\theta_{it} = \theta$ for any i and t), the symmetry of the model leading to stochastic equilibria that are symmetric and quasi-stationary; that is, with random variables $(\tilde{n}_{it})_{it}$ following the same binomial law of constant parameters (N, s).[23] The capital market clearing condition then simplifies to

$$NC(s) = N\left(\phi + \frac{\gamma}{2}s^2\right) = 1 - \alpha. \tag{29}$$

In addition, the labor market clearing condition can be rewritten (using the weak law of large numbers) as

$$A\sum_{\nu=0}^{N}\frac{N!}{}(N-\nu)!\nu!s^{\nu}(1-s)^{N-\nu}L(\nu, N, \theta) \equiv A\overline{L}(s, N, \theta) = \alpha. \tag{30}$$

Combining this condition with the first-order condition (25) for equilibrium investment, we obtain the equilibrium investment condition:

$$\frac{\gamma}{\alpha}\overline{L}(s, N, \theta)s = \overline{G}(s, N, \theta). \tag{31}$$

The equilibrium level of the probability s and the equilibrium (average) number of firms, N, can

[23] The number of investing firms in each industry depends upon the way savings are allocated to firms in the capital market. Here we suppose that this allocation results in a common number N of firms. However, to be precise, the value of N resulting from the equilibrium conditions is not necessarily an integer, so that it should be seen as a weighted average of the (integer) numbers of firms in the different industries, for instance of the two integers that are closest to N.

be determined by solving equations (29) and (31),[24] for given competitive toughness, θ (or by taking $\theta^L(\nu)$ as the third argument of $L(\nu, N, \theta)$ in (30) if we consider the limit pricing regime). Once s and N are determined, all other variables can be readily computed, in particular the expected growth rate $(\eta - 1)s$.

To further facilitate the comparison between the two approaches, we take an example, solving our model numerically for specific values of the parameters: $\eta = 1.1$ and $\sigma = 0.15$ for the innovation step and the spillover coefficient, respectively (leading to the relative cost advantage $\kappa = 1 - \eta^{\sigma-1} = 0.078$), $\alpha = 0.75$ for the propensity to consume, $\gamma = 0.03$ for the variable investment cost and the two values $\phi_1 = 0.025$ and $\phi_2 = 0.055$ for the fixed cost. Figure 3 gives a geometrical representation of this example.

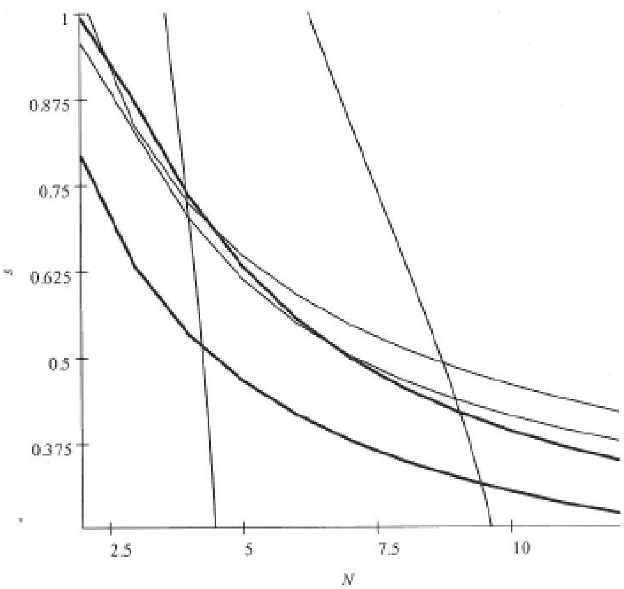

Figure 3: Individual R&D investment and the number of investing firms in equilibrium

[24]The zero profit curve in van de Klundert and Smulders (1997) plays a role analogous to the curve expressing the capital market clearing condition (Equation 29) in our model. In addition, their capital market equilibrium curve, resulting in particular from conditions for profit maximizing relative to (non-strategic) investment decisions, plays in their model a role equivalent to the curve expressing the equilibrium investment condition (Equation 31) in our model.

The equilibrium levels of s and N are given by the intersection of one of the two steep capital market clearing curves (the one to the right for the low value ϕ_1 and the one to the left for the high value ϕ_2) with one of the four flatter equilibrium investment curves (for different values of competitive toughness, θ). The four equilibrium investment curves correspond to $\theta = 0$ (Cournot regime, upper thick curve), $\theta = 0.5$ (lower thin curve), $\theta = 0.9$ (close to Bertrand regime, lower thick curve) and limit pricing (upper thin curve). The relationship between R&D effort, as represented by the probability of success, s, and competitive toughness is monotone decreasing in the case of high fixed cost, ϕ_2 (left steep curve). The equilibrium value of s decreases indeed with competitive toughness from $s \simeq 0.75$ (for $\theta = 0$) through $s \simeq 0.69$ (for $\theta = 0.5$) to $s \simeq 0.52$ (for $\theta = 0.9$) (limit pricing giving an intermediate value $s \simeq 0.73$) with an increasing average number of investing firms close to 4. The sense of this relationship, resulting from a concentration effect too small to dominate the markup squeezing effect, conforms with the Schumpeterian prediction. It contradicts van de Klundert and Smulders' conclusions.[25]

Besides, according to Proposition 1, the relationship between R&D effort and competitive toughness is non-monotone in our model for parameter values entailing lower equilibrium probabilities of success. Indeed, in the case of a low fixed cost, ϕ_1 (right steep curve), leading to smaller equilibrium probabilities (and to higher numbers of investing firms, fluctuating between 8 and 10), s first increases with competitive toughness from $s \simeq 0.42$ (for $\theta = 0$) to $s \simeq 0.44$ (for $\theta = 0.5$) and then falls back to $s \simeq 0.32$ (for $\theta = 0.9$): with limit pricing entailing the highest value $s \simeq 0.49$. This non-monotonicity is possible because the equilibrium investment curves for $\theta = 0$ and $\theta = 0.5$ intersect in this example. As already emphasized, this is in accordance with the inverted-U pattern empirically found by Aghion *et al.* (2005).

A final observation is in order. When the concentration effect of an increase in competitive toughness, θ, dominates the markup squeezing effect, so that the equilibrium investment curve

[25] In their model, they have two sectors, one producing high-technology differentiated goods, where innovation takes place, and another perfectly competitive-sector. The positive concentration effect of tougher competition is reinforced by an increase in the high-technology market size (because the relative price of high-technology goods falls). This feature is absent in our model.

is locally shifting upwards, the average number of investing firms, N, must decrease, because the capital market clearing curve is decreasing. This means that the concentration effect prevalent at the sectoral partial equilibrium level is in this case reinforced at the general equilibrium level.

4 Conclusion

Incentives to innovate depend upon multiple and conflicting effects, and it is only natural that there is no clear-cut answer to the question of determining whether tough competition tends to spur or deter potential R&D investors. By combining features of tournament and non-tournament models, more specifically by admitting the possibility for investing firms either to innovate along with some of their rivals, or to fail in their R&D effort and yet to remain productive, we have obtained a relationship between innovation and product market competition, which may well be non-monotone for an individual industry. This result does not rely on a composition effect, as is the case for the inverted-U pattern of Aghion *et al.* (2005). Non-monotonicity follows straightforwardly from the interplay of two conflicting effects: the negative Schumpeterian effect of tougher competition through markup squeeze and the positive concentration effect expanding innovators' market shares. For the latter to dominate the former, one must assume non-drastic innovations (a small relative cost advantage of the innovators), so that unsuccessful firms maintain a positive market share, and conditions for a low equilibrium probability of R&D success, so that the incremental gain of those that do succeed is high enough to encourage R&D effort (and the more so the tougher the competition). It should be noted that the concentration effect at work in our model is not primarily related to the reduction of the number of firms[26] (as in van de Klundert and Smulders 1997, and other symmetric non-tournament models) but rather to the increase in innovators' market shares.[27] However, higher competitive toughness

[26]The concentration effect of competition resulting from the endogenous reduction of the number of firms is a known paradox for antitrust policy (see d'Aspremont and Motta 2000).

[27]In the (tournament) model of sequential and cumulative innovation of Denicolò and Zanchettin (2004), both the number of active firms and the laggards' market shares decrease as competition becomes tougher. The resulting concentration effete is dominant in the vicinity of Bertrand competition: "competition is good for growth [. . .] if

is favorable to innovation only when competition is initially soft, otherwise one obtains the traditional Schumpeterian deterring effect of competition on innovative activity.

Appendix: Proof of Lemma 1

We start by studying the function $G(n, N, \theta)$, the expression of which differs in three intervals: $[0, \theta^L(n+1)]$, $[\theta^L(n+1), \theta^L(n)]$ and $[\theta^L(n), 1]$, with $\theta^L(n) = \max\{1 - n\kappa, 0\}$. Using (16), (17), (18) and (19), we obtain for $\theta \in [0, \theta^L(n+1)]$:

$$\begin{aligned} G(n,N,\theta) &= (1-\theta)[m(1,n+1,N,\theta) - m(\sigma,n,N,\theta)] \\ &\times [m(1,n+1,N,\theta) + m(\sigma,n,N,\theta)] \\ &= a(n,N)\left(\tfrac{N}{1-\theta} - 1\right)[2 - b(n,N)(N - (1-\theta))], \end{aligned} \tag{32}$$

with $a(n,N) \equiv \frac{\kappa}{N-(n+1)\kappa}\left(1 - \frac{1}{N-n\kappa}\right) > 0$

and $b(n,N) \equiv \frac{1-\kappa}{N-(n+1)\kappa} + \frac{1}{N-n\kappa} > 0$.

We can then obtain, for the sign of the elasticity of G with respect to θ,

$$\text{sign}\{\epsilon_\theta G(n,N,\theta)\} = \text{sign}\left\{(1-\theta)^2 - N^2\left(1 - \frac{2}{Nb(n,N)}\right)\right\}. \tag{33}$$

For $\theta \in [\theta^L(n+1), \theta^L(n)]$, we have:

$$G(n,N,\theta) = (1-\theta)\left(\left(\frac{1}{n+1}\right)^2 - \left(\frac{1-n\kappa/(1-\theta)}{N-n\kappa}\right)^2\right), \tag{34}$$

with sign of its elasticity with respect to θ

$$\text{sign}\{\epsilon_\theta G(n,N,\theta)\} = \text{sign}\left\{1 - \left(\frac{N-n\kappa}{n+1}\right)^2 - \left(\frac{n\kappa}{1-\theta}\right)^2\right\}. \tag{35}$$

Finally, for $\theta \in [\theta^L n), 1]$,

$$G(n,N,\theta) = \frac{1-\theta}{(n+1)^2}. \tag{36}$$

the intensity of competition is high." In fact, to obtain an inverted U-shaped relationship, the authors have to allow for equilibrium prices below the Bertrand price (eliminating in particular the concentration effect).

Looking at (33) and (35), we see that the elasticity of G can change signs at most once, from positive to negative, in each one of the two first intervals. Also, because by (16) and (17) the partial derivative of $m(1, n+1, N, \theta)$ with respect to θ switches from positive to nil at $\theta = \theta^L(n+1)$, and the corresponding parietal derivative of $m(\sigma, n, N, \theta)$ is continuous at the same point, we see from (18) and (19) that the right-hand partial derivative of G with respect to θ at $\theta^L(n+1)$ must be smaller than the corresponding left-hand derivative. Hence, G is strictly quasi-concave in θ when restricted to the interval $[0, \theta^L(n)]$. As G is clearly decreasing in θ in the interval $[\theta^L(n), 1]$, we can conclude that G is, in fact, strictly quasi-concave in θ. We may add that G is never monotonically increasing, because the interval $[\theta^L(n), 1]$ is non-degenerate for $n > 0$ and, by (35), G is otherwise decreasing in the interval $[\theta^L(n+1), \theta^L(n)] = [\theta^L(1), 1]$.

Now let us consider the case where either the number of successful firms, n, or their relative cost advantage, κ, is small, so that $\theta^L(n+1) > 0$. By (32) and (33), the function G is increasing for θ close to zero if and only if

$$Nb(n, N) - 2 = \left(\frac{n}{N - n\kappa} - \frac{N - (n+1)}{N - (n+1)\kappa} \right) \kappa < \frac{2}{N^2 - 1}. \tag{37}$$

A simple inspection shows that this inequality holds either for κ close to 0, or of $n = 0$, and is violated if both κ and n are large enough.

It remains to show that the incremental gain, G, is a decreasing function of n when κ is small enough. Using again (16), (17), (18) and (19), we obtain for $\theta \in [0, \theta^L(n+1)]$:

$$\begin{aligned} G(n, N, \theta) &= \left[\left(\tfrac{1}{N-1+\theta} - \tfrac{1_\kappa}{N-\kappa-n\kappa} \right)^2 - \left(\tfrac{1}{N-1+\theta} - \tfrac{1}{N-n\kappa} \right)^2 \right] \\ &\times \tfrac{(N-1+\theta)^2}{1-\theta}. \end{aligned} \tag{38}$$

We see that the terms $(1_\kappa)/(N - \kappa - n\kappa)$ and $1/(N - n\kappa)$ both have positive elasticities, the elasticity of the former term being the larger one. As a consequence, the first square within the brackets decreases faster than the second as n increases, so that G is indeed decreasing in n. In addition, for $\theta \in [\theta^L(n+1), \theta^L(n)]$, the two squares in (34) are both decreasing in n, but the elasticity of the second one is smaller in absolute value, leading to the same conclusion, for κ small enough.

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