Competition for market share or for market size: oligopolistic equilibria with varying competitive toughness *

Claude d'Aspremont[†], Rodolphe Dos Santos Ferreira [‡], and Louis-André Gérard-Varet[§]

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Abstract

For an industry producing a composite commodity, we propose a comprehensive concept of oligopolistic equilibrium, allowing for a parameterized continuum of regimes varying in competitive toughness. Each firm sets simultaneously its price and its quantity under two constraints, relative to its market share and to market size. The price and the quantity equilibrium outcomes always belong to the set of oligopolistic equilibria. When firms are identical and we let their number increase, any sequence of symmetric oligopolistic equilibria converges to the monopolistic competition outcome. Further results are derived in the symmetric CES case, concerning in particular the collusive solution enforceability.

1 Introduction

The Cournot–Bertrand debate has been central to the static analysis of oligopolistic competition since the origins of oligopoly theory. It is generally viewed as opposing quantities and prices as the relevant strategic variables. But such a view bypasses another opposition, possibly more

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[†]CORE, Louvain University, Louvain-la-Neuve, Belgium

[‡]BETA, Louis-Pasteur University Strasbourg, Institut Universitaire de France

[§]GREQAM, Ecole des Hautes Etudes En Sciences Sociales, Marseille, France. Sadly, during the process of revision of this article, Louis-André Gérard-Varet passed away following a painful disease.

significant, between market size and market share as the relevant targets. An oligopolistic firm faces in any case a price–quantity trade-off, with higher prices inducing lower sales and higher sales requiring lower prices. But in the approach of Cournot (1838, ch. VII), a lower price, which "is necessarily the same" for all firms, entails higher sales, given competitors' supplies, through the resulting increase in *market size*, whereas in the approach of Bertrand (1883), undercutting competitors' prices is meant to secure a higher *market share*, maybe a full appropriation of the market, independently of its size. Of course, market size and market share considerations are not mutually exclusive and may well coexist in various degrees, either orienting producers' strategies mainly against competitors outside the industry or directing them more aggressively against insiders, respectively, and leading to a large spectrum of possible regimes of oligopolistic competition. This variety of regimes should not be reduced to the dichotomous choice between Bertrand and Cournot or between price and quantity competition. Many applications of oligopoly theory in the fields of industrial organization and international trade, and also in macro-economic analysis of business cycles, innovation and growth, limit themselves to this pairwise comparison. Our objective is to offer, for modeling purposes, a unified formulation of the whole spectrum of enforceable noncooperative equilibria with varying degree of competition, allowing for easy intertemporal or intersectoral comparisons within the same model.

The proposed concept remains in the spirit of Cournot, and considers oligopolistic competition as a generalization of monopoly, by adding to the competitive pressure coming from outside the industry the one due to the multiplicity of producers within the industry. Switching from monopoly to oligopoly implies combining the struggle of the whole industry for market size with the struggle of each individual firm for its market share. Accordingly, we suppose that each firm, while maximizing its profit *in both price and quantity*, faces two constraints: one bounding the size of the market as determined by the price level in the industry, the other bounding its share of that market as determined by its relative price. We thus obtain a large set of solutions (i.e., of competition regimes) that can be parameterized using the values of the Lagrange multipliers associated with each one of the two constraints, on market share and on market size. For each firm, this parameter is chosen to be the relative shadow cost of the market share constraint and appears as a measure of the *competitive toughness* of the firm conduct.

An advantage of this parameterization is that it is isomorphic to the parameterization used in the so-called conduct parameter method of the new empirical industrial organization and its well-known econometric modeling of firm and industry behavior.[1] Hence, it allows us to test competitive toughness in our sense using standard empirical techniques. Conversely, our approach proposes a theoretical foundation for this method, which, in the homogeneous good case, is often apologetically associated with the conjectural variations approach.[2] Moreover, this approach is not current practice in the case of product-differentiated industries.

Competitive toughness is not the only dimension of competition intensity. This article will make clear how the set of enforceable oligopolistic equilibria varies along other dimensions, such as the degree of concentration as measured by the number of competitors (or more generally by some concentration index) – less concentration entailing more intense competition – or such as the degree of substitutability between differentiated products – greater substitutability resulting in more intense competition.[3]

From a theoretical viewpoint, the market-share and the market-size constraints define the two branches of a kinked feasibility frontier, expressing the price – quantity trade-off imposed on the profit-maximizing firm. This frontier is of course reminiscent of the kinked demand curve introduced by Hall and Hitch (1939) and Sweezy (1939), but with two major differences. The first difference is that the kink obtained by these authors results from a fundamental asymmetry in producers' conjectures about the reaction of their competitors to any price deviation: price matching if the deviation is downwards, inertia otherwise. By contrast, we resort to straight Nash conjectures on competitors' strategies, but admit that catalogue prices are automatically adjusted before transactions. This may be interpreted according to some kind of "facilitating

[1] Clear and complete overviews are provided by Bresnahan (1989) and Martin (2002). See also Corts (1999), for additional references.

[2] For a critique, see Corts (1999).

[3] Boone (2000, 2001) are attempts to axiomatize a general measure of intensity of competition, merging all dimensions.

practice" (Salop, 1986), such as the "best price guarantee" when the industry product is homogeneous. The second differ- ence is that Hall and Hitch, as well as Sweezy, refer strictly to price competition, assuming that the market share is kept constant whenever a price cut is matched by the competitors. We refer instead to price – quantity competition, so that the constraint on market size generates a Cournotian residual demand.[4]

The concept of oligopolistic equilibrium is defined here for an industry supplying a group of differentiated products aggregated into a composite commodity,[5] in a similar but more general framework than the popular one introduced by Dixit and Stiglitz (1977) and Spence (1976a, 1976b). The homogeneous product case is a limit case, when the degree of substitutability is in finite.[6] Then the set of oligopolistic equilibria includes the Cournot solution as the softest enforceable competition regime at one extreme, as well as the competitive equilibrium at the other extreme, when competitive toughness is maximal. All competition regimes that are enforceable in the homogeneous case are intermediate to these two extremes. When products are differentiated, the set of enforceable oligopolistic equilibria still includes the quantity and price equilibrium outcomes (usually called Cournot and Bertrand equilibria). However, as substitutability decreases, softer competition regimes become also enforceable, since market shares become less and less responsive to price cuts, which makes them less and less attractive for a potential deviator. For a sufficiently low elasticity of substitution, even the collusive regime

[4] The same approach has already been adopted, although in a different setting (where market price is determined by the so-called min-pricing scheme), by d'Aspremont et al. (1991). Kalai and Satterthwaite (1994) also refer to facilitating practices, and obtain a kinked demand curve from a price matching policy. But, contrary to our Cournotian approach, they adhere to strict price competition, in the line of the founders of the kinked demand model.

[5] See Gorman (1959) and Green (1964, ch. 4).

[6] In the industry we consider, products are not necessarily substitutes, perfect complementarity being actually another limit case. This case corresponds to Cournot' s complementary monopoly (Sonnenschein, 1968). Sonnenschein uses the expression "complementary monopoly" for Cournotian (price) competition between firms producing perfectly complementary goods. Cournot himself used the expression *concours des producteurs* (producers' concurrence, ch. IX), by contrast with *concurrence des producteurs* (producers' competition, ch.VII) in the homogeneous oligopoly case.

may become enforceable as an oligopolistic equilibrium. But a reverse argument applies when the degree of substitutability becomes too low, making upward price deviations profitable because of unresponsive market shares. Moreover, for any elasticity of substitution larger than 1, when indefinitely increasing the number of firms, and as market shares become negligible, all oligopolistic equilibria converge to the monopolistic competition equilibrium as long as competitive toughness is bounded away from 0. But, if we allow competitive toughness to become itself negligible along with market shares, then indeterminacy persists and even the collusive solution may remain enforceable. In Section 2, we define the concept of oligopolistic equilibrium for an industry producing a composite commodity, and we characterize the set of oligopolistic equilibria. We show in Section 3 that this set includes quantity and price equilibrium outcomes, and in Section 4 that it may also include monopolistic competition and, at the other extreme, collusive outcomes. In Section 5, we analyze in more detail the symmetric case, with a CES aggregator. Finally, we conclude in Section 6.

2 Oligopolistic equilibrium

We consider an industry producing a composite commodity, and consisting of n firms ($n > 1$), each firm i producing a single component of the composite good with a technology that is described by a cost function C_i, which is continuously differentiable on $(0, \infty)$, and such that $C_i(0) = 0$.

2.1 Demand to the industry and demand to the firm

We suppose that any basket $x \in \Re^n_+$ of industry products, viewed as elements of the composite good, can be aggregated into a quantity $Q(x)$ of this good, which could be interpreted as a subutility function, identical for all consumers.[7] The aggregator Q is assumed to be a twice

[7] We can alternatively suppose that the composite good is demanded by down streams firms, its quantity appearing as an argument of the corresponding production functions.

continuously differentiable, increasing, strongly quasi-concave[8] function, which is homogeneous of degree 1. A well-known example of this class is the CES aggregator, introduced by Dixit and Stiglitz (1977) and Spence (1976a, 1976b).

Any household h minimizes at prices $p = (p_1, \cdots, p_n)$ the cost $\sum_i p_i x_{hi}$ of a basket $x_h = (x_{h1}, \cdots, x_{hn})$ under the constraint $Q(x_h) \geq X_h$, where X_h is an argument of its utility function. The expenditure function, associating with the price vector p and the quantity X_h, the minimal attainable expenditure, can be written as $P(p) X_h$, where P is a price aggregator with the same properties as Q. There are two important dual limit cases for the aggregators, which will not be ignored in spite of being, strictly speaking, outside our framework (because of loss of regularity and differentiability). The first one is the homogeneous commodity case, where the quantity and price aggregators can be, respectively, defined as $\overline{Q}(x_h) = \sum_i x_{hi}$ and $\underline{P}(p) = \min_i\{p_i\}$. The second one is the Leontief case of perfect complements, where the quantity and price aggregators are, respectively, given by $\underline{Q}(x_h) = \min_i\{x_{hi}\}$ and $\overline{P}(p) = \sum_i p_i$.

We shall in the following use the first-order condition for expenditure minimization (denoting by ∂f the gradient of f)

$$p = P(p)\, \partial Q(x_h) \tag{1}$$

and, by Shephard's lemma, the dual condition

$$x_h = Q(x_h)\, \partial P(p)\,. \tag{2}$$

By the latter condition,

$$\sum_h x_h = \partial\, P(p) \sum_h Q(x_h) \text{ and } Q(\partial\, P(p)) = 1,$$

so that

$$\sum_h Q(x_h) = Q\left(\sum_h x_h\right).$$

[8]By strong quasi-concavity of a utility (production) function, we mean strict quasi-concavity, together with regularity of its bordered Hessian. This property entails twice continuous differentiability of the expenditure (cost) function, and continuous differentiability of the demand function (see Barten and Böhm, 1982).

Thus, through aggregation over all h's, we obtain multiplicative separability of the aggregate *demand to firm* i, $d_i P(p) = \partial_i P(p) Q(\sum_h x_h)$, allowing identification of two components: *the market share*,

$$\frac{d_i P(p)}{Q(\sum_h x_h)} = \partial_i P(p), \tag{3}$$

and the *market size*, as determined by the *demand to the industry* D,

$$Q\left(\sum_h x_h\right) = D(P(p)), \tag{4}$$

where D is assumed to be a decreasing, continuously differentiable function.

2.2 Definition and characterization of the equilibrium

We can now define the concept of oligopolistic equilibrium, where each firm strategies are supposed to be price-quantity pairs: each firm i announces a pair $(p_i, q_i) \in \mathbb{R}^2_+$, where p_i is a list price and q_i is a quantity produced in advance. For concreteness, we shall start from a particular interpretation applying to markets (such as retail markets) where "price-making policies" are common practice. But the concept that we will finally derive below (Lemma 1), with each firm facing two constraints, one on market size as determined by the market price level, the other on its market share as determined by its relative price, will appear to be more general and applicable to a larger set of industries.

To define the profit function of each firm, let us start by the limit case where the composite good is actually homogeneous, so that firms produce perfect substitutes. When establishing sales contracts, firm i must be aware that consumers might want to look elsewhere for a lower price. A standard way to trigger consumer decision immediately is for the firm to advertise a clause guaranteeing the best price, that is, ensuring that any competitor's lower price will instantaneously be matched. Alternatively, in the case where consumers are well informed and each firm commits to serve all demand, no consumer would accept paying more than the lowest price. Formally, this means that the revenue of firm i will also depend on the prices listed by the other firms and that the relevant price, or "market price," is given by $\underline{P}(p_i, p_{-i}) = \min\{p_i, \min_{j \neq i} p_j\}$, the profit of firm i being defined as $\Pi_i(p, q) = \underline{P}(p) q_i - C_i(q_i)$. In a more abstract formulation in terms of "pricing schemes" (i.e., mechanisms associating a market price to profiles of list prices[9] the function $\underline{P}$ is simply the so-called min-pricing scheme. Our approach here can be seen as a generalization of this notion to the composite good case.

In the composite good case, with general quantity index $Q(q)$, the equivalent of the best price guarantee should take into account product differentiation. In practice, this could take the form of a detailed specification of the characteristics differentiating each product, together with a list of prices, one for each characteristic. In our model, the formal representation of product differences is given by the partial derivatives $\partial_i Q(q)$, $i = 1, 2, \cdots, n$, measuring the specific contribution of each good to the composite good (all equal to one in the homogeneous case). The price actually paid should be adjusted to take into account these differences. Our formulation follows from the first-order condition (1) leading to the contracted price $P(p)\,\partial_i Q(q)$, where the price aggregator $P(p)$ is weighted by $\partial_i Q(q)$ (leading to $P(p)\,\partial_i Q(q) = \min_j\{p_j\}$ when the goods are perfect substitutes). As the price paid by the consumers should in addition never exceed the listed price p_i, the profit of firm i is consequently defined as

$$\pi_i(p, q) = \min\{p_i, P(p)\,\partial_i Q(q)\}q_i - C_i(q_i).$$

Moreover, as usual in oligopoly theory since Cournot, the firms should integrate in their computations the constraint imposed by total demand. Accordingly, the firm strategies should satisfy the constraint $Q(q) \leq D(P(p))$ (with $Q(q) = \sum_j q_j$ when the goods are perfect substitutes). We do not constrain firm i to choose its price-quantity pair (p_i, q_i) such that $Q(q) = D(P(p))$, which would allow us to reduce each firm strategy to a quantity, with market price given by the inverse demand, as in the common understanding of Cournot. But the equality is still imposed at equilibrium, so that consumers are not rationed.

Accordingly, we define an *oligopolistic equilibrium* to be a $2n$-tuple (p^*, q^*) such that (p_i^*, q_i^*) satisfies, for any i,

$$\begin{aligned}&\pi_i(p^*, q^*) \geq \pi_i(p_i, p_{-i}^*, q_i, q_{-i}^*)\\ &\text{for all } (p_i, q_i) \geq 0 \text{ such that } Q(q_i, q_{-i}^*) \leq D(P(p_i, p_{-i}^*)),\end{aligned}$$

[9] Cf. d'Aspremont et al. (1991).

and such that the additional restriction

$$Q(q^*) = D(P(p^*)) \tag{5}$$

holds.[10]

The following result gives two alternative definitions of the oligopolistic equilibrium concept, thus enlarging the set of possible interpretations and the class of industries for which it makes sense. These two characterizations will generate two equivalent parameterizations of the set of competition regimes and the associated potential oligopolistic equilibria.

Lemma 1 *A $2n$-tuple (p^*, q^*) is an oligopolistic equilibrium if and only if*

$$Q(q^*) = D(P(p^*)) \tag{6}$$

and, for each firm i, (p_i^*, q_i^*) solves the program

$$\max_{(p_i,q_i)\in\mathbb{R}_+^2} \left\{ \begin{array}{c} p_i q_i - C_i(q_i) : p_i \le P(p_i, p_{-i}^*)\partial_i Q(q_i, q_{-i}^*) \\ \text{and } P(p_i, p_{-i}^*) \le D^{-1}(Q(q_i, q_{-i}^*)) \end{array} \right\} \tag{7}$$

or, alternatively, for each firm i, (p_i^*, q_i^*) solves the program

$$\max_{(p_i,q_i)\in\mathbb{R}_+^2} \left\{ \begin{array}{c} p_i q_i - C_i(q_i) : q_i \le Q(q_i, q_{-i}^*)\partial_i P(p_i, p_{-i}^*) \\ \text{and } Q(q_i, q_{-i}^*) \le D(P(p_i, p_{-i}^*)) \end{array} \right\}. \tag{8}$$

Condition (6) ensures, for each alternative program formulation, that both constraints hold as equalities at equilibrium.

Proof: Clearly, hour our assumptions and using duality (see (1) and (2)), each constraint in program (7) is equivalent to the corresponding constraint in program (8), so that the two programs are equivalent.

Suppose now that for some oligopolistic equilibrium (p^*, q^*) and for some firm i, (p_i^*, q_i^*) does not solve (7), so that there is $(p_i, q_i) \in \mathbb{R}_+^2$ satisfying both program constraints and such

[10] In order for an oligopolistic equilibrium to be a Nash equilibrium of a *game*, one should specify the payoff whenever $Q(q) > D(P(p))$. An easy way is to specify that, for each i, $\Pi_i(p, q) < 0$ in such a case.

that $p_i^* q_i^* - C_i(q_i^*) < p_i q_i - C_i(q_i) \leq \min\{p_i, P(p_i, p_i^*)\partial_i Q(q_i, q_{-i}^*)\} q_i - C_i(q_i)$. We thus obtain a contradiction.

Conversely, take (p^*, q^*) satisfying (6) and such that, for each i, (p_i^*, q_i^*) solves (7), and suppose that it is not an oligopolistic equilibrium. Then, there must be $(p_i, q_i) \in \mathbb{R}_+^2$ satisfying the total demand constraint (equivalent to the second constraint of (7)) and such that $\min\{p_i, P(p_i, p_{-i}^*)\partial_i Q(q_i, q_{-i}^*)\} q_i - C_i(q_i) > p_i^* q_i^* - C_i(q_i^*)$. If $p_i \leq P(p_i, p_{-i}^*)\partial_i Q(q_i, q_{-i}^*)$, we immediately obtain a contradiction. If not, the contradiction results from taking (p_i', q_i), with $p_i' = P(p_i', p_{-i}^*) \times \partial_i Q(q_i, q_{-i}^*) < p_i$, satisfying both constraints of (7). ■

In this lemma, it is shown that an oligopolistic equilibrium can be characterized by having each firm maximizing profit under two alternative pairs of constraints. In the first pair, we have one constraint imposing that firm i does not set a relative price $p_i/P(p)$ higher than the (common) marginal utility $\partial_i Q(q)$ of the ith good and another bounding the market price by the inverse demand:

$$p_i/\partial_i Q(q_i, q_{-i}^*) \leq P(p_i, p_{-i}^*) \leq D^{-1}(Q(q_i, q_{-i}^*)). \tag{9}$$

In the second, dual, pair of constraints

$$q_i/\partial_i P(p_i, p_{-i}^*) \leq Q(q_i, q_{-i}^*) \leq D(P(p_i, p_{-i}^*)), \tag{10}$$

the first inequality maybe viewed as a constraint imposed on *market share* $q_i/Q(q)$, and the second one as a constraint imposed on *market size* $Q(q)$.

As illustrated in Figure 1 (drawn in the (q_i, p_i) space, for an aggregator with constant elasticity of substitution s and a demand function with constant Marshallian elasticity σ), the individual firm outcome is generically corner solution at the kink of the feasibility frontier, represented by the thick decreasing curve.[11] This frontier has two branches, corresponding to the two constraints, with elasticities at the kink equal to $-1/s$ for the constraint on market share

[11]The data underlying this figure are those of a symmetric duopoly in which firms produce at zero-fixed cost and constant marginal cost $c = .75$. We take $\sigma = 2$ and $s = 25$. The represented symmetric equilibrium is $(p_i, q_i) = (1, 1)$, for $i = 1, 2$.

and $-1/\sigma$ for the constraint on market size (so that the relative position of the two branches depends on the relative values of s and σ).[12]

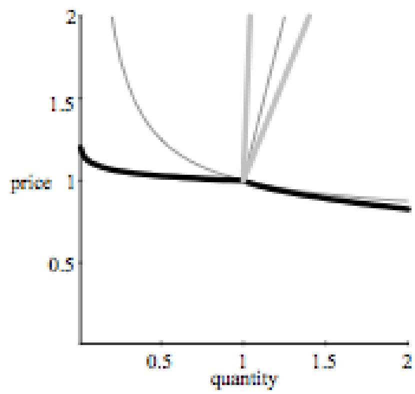

Figure 1: Profit maximization under the constraints on market share and market size

The thin isoprofit curve is tangent to neither of the two branches. But the gradient of the profit function at the kink (the direction of which is represented by the thin line) belongs to the normal cone (bounded by the two thick gray lines) generated at the same point by the two constraints.

2.3 Parameterization of the set of equilibria

The first-order necessary conditions for the maximization of $p_i q_i - C_i(q_i)$ under the two alternative pairs of constraints, (9) and (10), are, respectively,

$$\begin{bmatrix} q_i^* \\ p_{-i}^* - C_i'(q_i^*) \end{bmatrix} = \frac{\lambda_i}{\partial_i Q(q^*)} \begin{bmatrix} 1 - \partial_i Q(q^*) \partial_i P(p^*) \\ - p_i^* \partial_{ii}^2 Q(q^*)/\partial Q(q^*) \end{bmatrix} + \nu_i \begin{bmatrix} \partial_i P(p^*) \\ - \partial_i Q(q^*)/D'(P(p^*)) \end{bmatrix} \quad (11)$$

[12]The two branches of the kinked feasibility fonder are local approximations (at the kink) of Chamberlin's (1933) depend curves dd' (with other prices kept constant when varying p_i) and DD' (with other prices in the industry varying proportionally with p_i). Chamberlin's curves are the branches of the kinked demand curve in Hall and Hitch (1939).

and

$$\begin{bmatrix} q_i^* \\ p_i^* - C_i'(q_i^*) \end{bmatrix} = \frac{\lambda_i'}{\partial_i P(p^*)} \begin{bmatrix} -q_i^* \partial_{ii}^2 P(p^*)/\partial_i P(p^*) \\ 1 - \partial_i P(p^*) \partial_i Q(q^*) \end{bmatrix} + \nu_i' \begin{bmatrix} -D'(P(p^*))\partial_i P(p^*) \\ \partial_i Q(q^*) \end{bmatrix} \tag{12}$$

for some pairs of Kuhn and Tucker multipliers $(\lambda_i, \nu_i), (\lambda_i', \nu_i')$ in $\mathbb{R}_+^2 \setminus \{0\}$. The normalized parameter $\theta_i = \lambda_i/(\lambda_i + \nu_i)$ (resp. $\theta_i' = \lambda_i'/(\lambda_i' + \nu_i')$), representing the relative shadow cost associated with the market share constraint of firm i, can be taken to measure the *competitive toughness* of firm i within the industry, at equilibrium (p^*, q^*).

Denote by $s(p_i/P(p))$ the elasticity of substitution of good i for the composite commodity, which is the absolute value of the elasticity of $q_i/Q(q)$ with respect to $p_i/P(p)$. Using the elasticity operator ϵ,[13] the elasticity of $\partial_I P(p)$ with respect to its ith argument can then be written $\epsilon_i \partial_i P(p) = -s(p_i/P(p))[1 - \epsilon_i P(p)]$. Using the simplifying notations $s_i^* \equiv s(p_i^*/P(p^*))$ for the *elasticity of substitution* relative to good i, $\sigma^* \equiv -\epsilon D(P(p^*))$ for the Marshallian *elasticity of demand*, and $\alpha_i^* \equiv \epsilon_i P(p^*) = \epsilon_i Q(q^*) = p_i^* q_i^* / \sum_j p_j^*/q_j^*$ for the *budget share* of good i, all determined at equilibrium, conditions (11) and (12) can now be expressed in terms of Lerner's index of degree of monopoly $(p_i^* - C_i'(q_i^*))/p_i^* \equiv \mu_i^*$,[14] giving respectively

$$\mu_i^* = \mu_i(\theta, p^*) \equiv \frac{1}{\theta_i(1-\alpha_i^*) + (1-\theta_i)\alpha_i^*} \left(\frac{\theta_i(1-\alpha_i^*)}{s_i^*} + \frac{(1-\theta_i)\alpha_i^*}{\sigma^*} \right) \tag{13}$$

and

$$\mu_i^* = \mu_i'(\theta', p^*) \equiv \frac{\theta_i'(1-\alpha_i^*) + (1-\theta_i')\alpha_i^*}{\theta_i'(1-\alpha_i^*)s_i^* + (1-\theta_i')\alpha_i^*\sigma^*} \tag{14}$$

for every i. We observe that the equilibrium degree of monopoly μ_i^* is a mean of two terms, $1/s_i^*$ and $1/\sigma^*$. It is the arithmetic mean in (13), and the harmonic mean in (14).

From these equations a one-to-one relationship between the two indices of competitive toughness of firm i can be derived for $s_i^* \notin \{0, \sigma^*, \infty\}$:

$$\frac{1/\theta_i - 1}{\sigma^*} = \frac{1/\theta_i' - 1}{s_i^*}. \tag{15}$$

[13] The elasticity operator ϵ, applied to a differentiable function $F(\cdot)$ at $x \in \mathbb{R}^n$, is defined by $\epsilon_i F(x) \equiv \partial_i F(x) x_i / F(x)$. The subscript is omitted when $n = 1$.

[14] Notice that the elasticity of the isoprofit curve at the point of intersection of the three curves in Figure 2 is equal to $-\mu_i^*$.

These two parameterizations can also be put into a one-to-one relationship with the parameters indexing the competitiveness of oligopoly conduct and appearing in the supply relations of econometric models of an industry such as those used by the "new empirical industrial organization." It can be viewed as a way to reconcile the two approaches to estimate the conduct parameters, one restricting estimation to those specific values associated with well-known theories (e.g., the competitive theory vs. the Cournot theory), the other treating the conduct parameters as continuous-valued. Our (static) framework indeed provides a continuum of theories.[15]

The theoretical justification we propose for these parameterizations is better founded than the one based on conjectural variations. Consider, for example, the model presented (and questioned) in Nevo (1998) for a differentiated oligopoly. In the duopoly case (each firm producing one good), the first-order conditions[16] are perturbed by the conjectural derivative parameters $\zeta_{i,j}, i = 1, 2, j = 1, 2$, and cane written (in our notation) as:

$$\begin{bmatrix} p^* - c_1 \\ p_2^* - c_2 \end{bmatrix} = A \begin{bmatrix} -\zeta_{22}\partial_2 d_2(p^*) & \zeta_{21}\partial_2 d_1(p^*) \\ \zeta_{12}\partial_1 d_2(p^*) & -\zeta_{11}\partial_1 d_1(p^*) \end{bmatrix} \begin{bmatrix} d_1(p^*) \\ d_2(p^*) \end{bmatrix},$$

with $A = (\zeta_{11}\zeta_{22}\partial_2 d_2(p^*)\partial_1 d_1(p^*) - \zeta_{12}\zeta_{21}\partial_1 d_2(p^*)\partial_2 d_1(p^*))^{-1}$. Using our specification $d_i\,(p) \equiv \partial_i\,P\,(p)\,D(P\,(p))$, these first-order conditions canbe simplified to

$$\mu_i^* = \frac{1}{\zeta_{11}\zeta 12 - \zeta_{12}\zeta_{21}} \left(\frac{\zeta_{ij}}{(1-\alpha_i^*)s_i^* + \alpha_i^*\sigma} - \frac{(\alpha_j^*/\alpha_i^*)\zeta_{ji}}{(1-\alpha_j^*)s_j^* + \alpha_j^*\sigma} \right) \tag{16}$$

for $i, j = 1, 2,\ i \neq j$, an expression which is easy to compare to our conditions (13) or (14). Clearly our conditions are not equivalent to this new one. First, from an estimation point of view, this conjectural variations approach introduces more parameters to be identified (four instead of two, more generally n^2 instead of n). Second, from a theoretical point of view, our conditions, giving μ_i^* as a mean of $1/s_i^*$ and $1/\sigma^*$, are easier to interpret. Also, in the limit case

[15] Anyway, Bresnahan (1989) is not really worried about the free parameter approach. "It risks the possibility that values of θ which are 'in between' existing theories will be estimated, but that is hardly a disaster… The researcher who has estimated θ from a continuum will test theories by nested methods. The other researcher will use non-nested tests to distinguish among the few theories entertained." This can continue with better foundations.

[16] Corresponding to equation (6) in Nevo (1998).

of perfect substitutability ($s_1^* = s_2^* = \infty$), the $\mu_i^* s$ are equal to zero, so that all solutions (in particular the Cournot equilibrium) are excluded except the competitive one, a fact that is not implied by our model, as we will see now.

3 Quantity and price equilibria

To investigate the set of oligopolistic equilibria, let us first consider the standard, and often used, quantity and price equilibrium concepts, usually referred to as Cournot and Bertrand equilibrium, respectively. We will show that both are included in the set of oligopolistic equilibria. These concepts result from restricting the strategy space of each firm to one decision variable only. This amounts to reducing to a single equality the constraints in (9), for the quantity equilibrium, and in (10), for the price equilibrium.

More precisely, a *quantity equilibrium* is a quantity vector q^* such that, for every firm i, q_i^* belongs to the solution set

$$\arg \max_{q_i \in \mathbb{R}_+} \{\partial_i Q(q_i, q_{-i}^*) D^{-1}(Q(q_i, q_{-i}^*)) q_i - C_i(q_i)\}. \tag{17}$$

A simple calculation yields the first-order condition:

$$\frac{p_i^* - C_i'(q_i^*)}{p_i^*} = \frac{1 - \alpha_i^*}{s_i^*} + \frac{\alpha_i^*}{\sigma^*} \tag{18}$$

with $p_i^* = \partial_i Q(q^*) D^{-1}(Q(q^*))$ and $\alpha_i^* = \epsilon_i Q(q^*) = \epsilon_i P(p^*)$, for every i. Looking at (13), we observe that it coincides[17] with the first-order condition for profit maximization in an oligopolistic equilibrium when $\theta_i = 1/2$. This amounts to giving the same weight to the two constraints as expressed in (9). In particular, whenever the elasticity of intraindustry substitution tends to infinity, the degree of monopoly of any firm tends to its Cournot value.

Similarly, a *price equilibrium* is a price vector p^* such that, for every firm i, p_i^* belongs to

[17] Both in this case and in the case of a price equilibrium, the coincidence of the first-order conditions does not imply that the conditions for existence of an equilibrium coincide. These will generally be weaker for the corresponding oligopolistic equilibrium.

the solution set

$$\arg\max_{p_i \in \mathbb{R}_+} \{p_i \partial_i P(p_i, p^*_{-i}) D(P(p_i, p^*_{-i})) - C_i(\partial_i P(p_i, p^*_{-i}) D(P(p_i, p^*_{-i})))\}. \tag{19}$$

A straightforward calculation leads to the first-order condition

$$\frac{p *_i - C_i'(q_i^*)}{p_i^*} = \frac{1}{(1-\alpha_i^*) s_i^* + \alpha_i^* \sigma^*}, \tag{20}$$

with $q_i^* = \partial_i P(p^*) D(P(p^*))$. By referring to (14), we see that it coincides with the first-order condition for profit maximization in an oligopolistic equilibrium when $\theta_i' = 1/2$, which amounts to giving the same weight to each of the two constraints as expressed in (10). Observe that whenever the elasticity of intraindustry substation tends to infinity the degree of monopoly tends to zero, and thus, the price equilibrium tends to the corresponding Bertrand homogeneous product equilibrium (in the symmetric linear cost case). Also, if the elasticity of intraindustry substitution tends to zero (the Leontief case of perfect complementarity), the degree of monopoly tends to its Cournot value for complementary monopoly.[18]

The following simple proposition confirms that quantity and price equilibrium outcomes belong to the set of oligopolistic equilibria.

Proposition 1 *To every quantity equilibrium q^* corresponds an oligopolistic equilibrium (p^*, q^*) with $p^* = \partial Q(q^*) D^{-1}(Q(q))$. Also, with constant marginal costs $c_i \geq 0$, to every price equilibrium p^* corresponds an oligopolistic equilibrium (p^*, q^*) with $q^* = \partial P(p^*) D(P(p^*))$.*

Proof: Suppose that q^* is a quantity equilibrium, but that (p^*, q^*), with $p_i^* = \partial_i Q(q^*) D^{-1}(Q(q^*))$ for all i, is not an oligopolistic equilibrium. Then $p_i q_i - C_i(q_i) > p_i^* q_i^* - C_i(q_i^*)$, for some i and some (p_i, q_i) such that $p_i \leq \partial_i Q(q_i, q^*_{-i}) P(p_i, p^*_{-i})$ and $P(p_i, p^*_{-i}) \leq D^{-1}(Q(q_i, q^*_{-i}))$. This implies $p_i \leq \partial_i Q(q_i, q^*_{-i}) D^{-1}(Q(q_i, q^*_{-i})$, so that we get

$$\partial_i Q(q_i, q^*_{-i}) D^{-1}(Q(q_i, q^*_{-i})) q_i - C_i(q_i) > \partial_i Q(q^*) D^{-1}(Q(q^*)) q_i^* - C_i(q_i^*),$$

a contradiction.

[18] See Cournot (1838, ch. IX), and Sonnenschein (1968).

With constant marginal costs $c_i \geq 0$, a similar argument shows that (p^*, q^*) with $q_i^* = \partial_i P(p^*) D(P(p^*))$ for all i, is an oligopolistic equilibrium whenever p^* is a price equilibrium. Indeed, $(p_i - c_i) q_i > (p_i^* - c_i) q_i^*$ for some i and some (p_i, q_i) such that $q_i \leq \partial_i P(p_i, p_{-i}^*) Q(q_i, q_{-i}^*) \leq \partial_i P(p_i, p_{-i}^*) D(P(p_i, p_{-i}^*))$ implies

$$(P - i - c_i) \partial_i P(p_i, p_{-i}^*) D(P(p_i, p_{-i}^*)) > (p_i^* - c_i) q_i^*,$$

a contradiction. ■

However, the inclusion of price (or Bertrand) equilibrium outcomes in the set of oligopolistic equilibria is not generally true for any cost structure. Indeed, the Bertrand equilibrium requires that the price-setting producers always satisfy demand (even when deviating),whereas this is only an equilibrium requirement for the oligopolistic equilibrium.

4 Monopolistic competition and tacit collusion

To further explore the set of oligopolistic equilibria, let us come back to Equations (13) and (14). We see that the degree of monopoly μ_i varies with θ_i (or μ_i' with θ_i') between

$$\mu_i(1, p^*) = \frac{1}{s_i^*}, \tag{21}$$

the reciprocal of the elasticity of substitution, when the competitive toughness of firm i is maximal ($\theta_i = \theta_i' = 1$), and

$$\mu_i(0, p^*) = \frac{1}{\sigma^*}, \tag{22}$$

the reciprocal of the elasticity of demand to the industry, when the competitive toughness of firm i is minimal ($\theta_i = \theta_i' = 0$). The former extreme value $1/s_i^*$ is also the degree of monopoly prevailing when firm i budget share α_i^* becomes negligible, that is, in *monopolistic competition* (in Chamberlin's "large group"). The latter extreme value $1/\sigma^*$ is the *collusive* degree of monopoly, corresponding to joint profit maximization.[19]

[19] Joint profit is equal to $\sum_{i=1}^n (p_i q_i - C_i(q_i)) = P\,(p)) Q\,(q) - \sum_{i=1}^n C_i(q_i)$, and can be maximized in two stages, first in terms of P and q, under the constraint $Q\,(q) = D\,(P)$, and then in terms of p, using demander's optimizing

We have seen that the equilibrium degree of monopoly is a mean of these two extreme values. When calculating such a mean, the (monopolistically) competitive value $1/s_i^*$ is weighted by the product of the competitive toughness of firm i and of the budget share of its competitors, so that it can alternatively be approached either by increasing competitive toughness of firm i or by reducing its budget share. In particular, in the limit homogeneous case ($s_i^* = \infty$), a zero degree of monopoly may result either from extreme competitive toughness *à la* Bertrand, or from the vanishing market share of every element of a continuum of perfectly competitive firms. These comments lead to the result for symmetric oligopolistic equilibrate stated in the following proposition.

Proposition 2 *Any sequence of symmetric oligopolistic equilibria $(p^n, q^n)_n$ corresponding to an increasing number of identical firms ($n = 2, 3, \cdots$), each having competitive toughness $\overline{\theta}_n$ (resp $\overline{\theta}^n$) bounded away from 0, converges to the monopolistic competition equilibrium (corresponding $\overline{\theta} = \overline{\theta}' = 1$).*

Proof: The proposition is an immediate consequence of the fact that, along a sequence of symmetric oligopolistic equilibria with an increasing number of firms, the budget share $\alpha_i^n = p_i^n q_i^n / [P(p^n)Q(q^n)] = 1/[P(1, \cdots, 1)Q(1, \cdots, 1)]$ of each firm i is decreasing and eventually becomes negligible. Applying this fact to the first-order conditions (13) or (14), with competitive toughness bounded away from 0, leads to $\mu_i(1, p^*) = 1/s_i^*$, for every i. ∎

5 The CES case with identical linear costs

To illustrate the different kinds of oligopolistic equilibria and make our analysis more precise, we now examine the standard case of the (symmetric) CES aggregators

$$Q\left(q\right) = \left(\sum_j q_j^{(s-1)/s}\right)^{s/(s-1)} \quad \text{and } P\left(p\right) = \left(\sum_j p_j^{1-s}\right)^{1/(1-s)}, \tag{23}$$

conditions. In the first stage, we obtain as first-order conditions that, for any i, $(1 - 1/\sigma)P\partial_i Q\left(q\right) = C_i'(q_i)$. At the second stage, we use $\partial_i Q\left(q\right) = p_i/P\left(p\right)$, and finally get $(p_i - C_i'(q_i))/p_i = 1/\sigma$.

with $s > 0$, and such that $Q(q)$ and $P(p)$ tend to $\overline{Q}(q) = \sum_j q_j$ and $\underline{P}(p) = \min_j\{p_j\}$ (resp. to $\underline{Q}(q) = \min_j\{q_j\}$ and $\overline{P}(p) = \sum_j p_j$) as s tends to infinity (resp. zero). Assuming for simplicity an isoelastic demand $aP^{-\sigma}$ to the industry (with $a > 0$ and $\sigma > 0$), the demand addressed to firm i takes the explicit form:

$$d_i(p) = a\partial_i P(p)(P(p))^{-\sigma}, \text{ with } \partial_i P(p) = (p_i/P(p))^{-s}. \tag{24}$$

We also assume the same positive constant marginal cost c for all firms, and restrict our analysis to symmetric equilibria.

5.1 Bounds on competitive toughness

The set of values of competitive toughness (measured by $\overline{\theta}$ or $\overline{\theta}'$ for all i) that parameterize actual (as opposed to merely potential) symmetric oligopolistic equilibria does not coincide with the whole interval $[0, 1]$ for any elasticity of substitution s. Take for instance the case $\overline{\theta} = \overline{\theta}' = 1$, and notice that it is excluded as soon as $s \leq 1$, the least upper bound of $\overline{\theta}$ (or $\overline{\theta}'$) being then the value,[20] smaller than 1, which implies a degree of monopoly equal to 1. Taking $\overline{\theta} = \overline{\theta}' = 1$ with $s > 1$, we get an equilibrium degree of monopoly equal to $1/s$, which corresponds *exactly*, not just *approximately*, to the one obtained by Dixit and Stiglitz.[21]

Looking at the Dixit-Stiglitz outcome as an approximation, acceptable for large enough n, is of course in the line of Chamberlin's "large group," but one can also take it as a precise result and relate the underlying type of competition to Bertrand. Indeed, in Bertrand's argument (for $s = \infty$), the market size is immaterial: No price higher than marginal cost can be sustained, since any firm would otherwise have the possibility of slightly undercutting such price to catch

[20] The least upper bound of admissible $\overline{\theta}$ tends to 0 as s tends to 0, making it more and more inconvenient (impossible in the limit) to use this measure of competitive toughness to parameterize oligopolistic equilibria. In the case of strong complementarity, it is preferable to use $\overline{\theta}'$, the least upper bound of which tends to $(\sigma - 1)/(\sigma + n - 2)$ as s tends to 0.

[21] This well illustrates that the Dixit-Stiglitz monopolistic competition equilibrium is not a price equilibrium. Both coincide only in the limit situation of an infinite number of firms (see Yang and Heijdra, 1993; d'Aspremont et al., 1996).

the whole demand, whatever its level. In the original Dixit-Stiglitz approach (for $s \in (1, \infty)$), market size also remains immaterial for the price outcome because each firm is assumed to take the aggregate price P (hence market size) as independent of its own decision. Finally, in our approach the same is still true because of a zero relative shadow cost of the constraint on market size ($\overline{\theta} = \overline{\theta}' = 1$).

The other extreme case $\overline{\theta} = \overline{\theta}' = 0$ leads to an equilibrium degree of monopoly equal to the collusive value $1/\sigma$ (if $\sigma > 1$), but necessary second-order conditions for local profit maximization may not be verified, contrary to the former extreme case. As shown in the following proposition, the symmetric collusive solution (in fact any symmetric profile satisfying first-order conditions, whatever the common competitive toughness) is enforceable as an oligopolistic equilibrium if s belongs to some interval $[\underline{s}(\sigma, n), \overline{s}(\sigma, n)] \supset [1, \sigma]$. When σ tends to 1, this interval degenerates into $\{1\}$. When n increases, this sufficient condition becomes stricter, the interval eventually shrinking to $[1, \sigma + \sqrt{\sigma(\sigma - 1)}]$, this limit interval being however non degenerate for $\sigma > 1$.

This means that, if intraindustry substitutability is moderate, the symmetric collusive solution is enforceable as an oligopolistic equilibrium even for an arbitrarily large number of firms. The reason why such enforceability is lost when the elasticity of substitution reaches a high level is that, as market shares strongly respond to the corresponding relative price changes, it becomes profitable for any individual firm to deviate by decreasing its price and accordingly increasing its quantity. The reverse holds when the elasticity of substitution is low, market shares becoming insensitive, which makes price increases appealing.

Formally, as s becomes large or small, one must move from the collusive value $1/\sigma$ of the degree of monopoly toward $1/s$ in order to obtain existence of an oligopolistic equilibrium. Denote by $\mu^C(s)$ the degree of monopoly that, for the elasticity of substitution s, is enforceable and closest to its collusive value, the one that results from the minimum competitive toughness $\overline{\theta}^C(s)$ (or $\overline{\theta}'^C(s)$) compatible with an oligopolistic equilibrium. For intermediate values of s, in any case for $s \in [1, \sigma + \sqrt{\sigma(\sigma - 1)}, \mu^C(s) = 1/\sigma$, but, as s tends to infinity or to zero, $\mu^C(s)$ tends to the values calculated by Cournot, $\mu^C(\infty) = 1/n\sigma$ for the homogeneous oligopoly case, and $\mu^C(0) = n/\sigma$ for the complementary monopoly case.[22]

[22]Sonnenschein (1968) rightly stress the unity of the two Cournot theories, of homogeneous oligopoly and complementary monopoly, through the duality of the relevant strategies, quantities, and prices, respectively. Here, we see that the unity also stems from another source, the closeness to the collusive solution while maintaining enforceability.