Formal welfarism and intergenerational equity*

Claude d'Aspremont

1 Introduction

Intergenerational justice is a matter that should primarily concern the present generation, since the individuals living now are those to take immediate decisions affecting generations that will be living in the future, and even in the far future, as we know, for example, from the exhaustibility of some resources or from the long-term effects of pollution such as global warming. Of course, each future generation will become 'present' at some point in time, and the reasoning followed for the present 'present generation' about intergenerational justice could be repeated at that point in time. But, to develop this reasoning, each present generation should have a representation of future generations' interests. In that respect, a simple formulation of the problem that has been extensively analyzed consists in trying to find, under equity and efficiency conditions, an ordering of the set of possible 'infinite utility streams', that is, of the set of possible infinite sequences of utility levels attached to the successive generations starting with the present generation. In such a formulation, the welfare of each generation is represented by a single utility level, as if a generation were composed of a single individual or of a cohort of identical individuals with identical allocation. Even though this formulation owes so much to Ramsey (1928), the

*I thank the participants to the IEA Roundtable Meeting on Intergenerational Equity, and in particular Geir Asheim and Walter Bossert, for their fruitful comments and suggestions. Financial support from the Belgian Science Policy Office (CLIMNEG 2 and IAP programs) is gratefully acknowledged.

In J. Roemer and K. Suzumura (eds.), *Intergenerational Equity and Sustainability*. Houndmills, Basingstoke, Palgrave Macmillan, 113–130, 2007. Reprint 2047

particular ranking criterion he proposed – that of maximizing the sum of undercounted utility – was to be rejected for its limitations (see Chakravarty, 1962), and the possibility of representing an ordering of utility streams by a collective utility function (or social welfare function), treating all generations equally, was put into serious question. Impossibility results by Koopmans (1960), diamond (1965) have been followed by others (*e.g.* Basu and Mitra, 2003a). However, if instead of looking for a social welfare function defined on infinite utility streams, one looks for a 'social welfare ordering' of these streams, then, as shown non-constructively by Svensson (1980), satisfying a strong Pareto condition and treating generations equally, in some limited sense, become possible.

Our purpose in this chapter is neither to pursue the investigation of the general possibility or impossibility issue in this formulation of the problem of justice among generations, nor to re-examine the necessity of discounting to obtain a social welfare function. Other chapters of this volume treat these questions. We want to look at the foundations of this formulation within the 'social welfare functional' approach to social choice (as introduced by Sen, 1970). In that respect it can be seen as a supplement to the overview given in d'Aspremont and Gevers[1] (2002), where the intergenerational problem is not treated.

To think about intergenerational justice in terms of infinite generational utility streams, and to look for a social welfare ordering defined on the set of such streams, presume a double reduction. The first is the classical 'welfarist' reduction, as usually defined in ethics and social choice theory, namely that 'utility' provides all the information required to construct a social evaluation rule. Of course the strength of this reduction depends on the precise interpretation given to the concept of utility (for a discussion, see Mongin and d'Aspremont, 1998). We shall not discuss various possible interpretations here and limit our analysis to the formal consequences of welfarism, an attitude that may be called 'formal welfarism'. But the welfarist reduction is not the only one to be subsumed in the infinite utility stream approach. There is also, for each generation, the aggregation of the individual utility levels at each generation into a single

[1] My dear friend and co-author Louis Gevers died in September 2004. His collaboration would have greatly improved the present work.

utility level. We shall argue that this second type of reduction is not innocuous either. Not only does it require to impose additional assumptions on the social welfare functionals tarn as the primitive concept for evaluating social states. It also obliterates the relationship between the value judgments made in the social evaluation of the welfare of the set of individuals forming the present generation with that of future generations. To defend this argument, we rely on standard results in social choice theory showing the capacity for some social evaluation criteria to proliferate (Sen, 1977, and Hammond, 1979), in the sense that adopting such a criterion for a subgroup of individuals (*e.g.* the present generation) forces an ethical observer to use the same criterion for any large group (*e.g.* any larger set of generations). The consequences of adopting some proliferating cirterion in evaluating infinite utility streams, are better examined if these streams remain disaggregated at the individual level, allowing to apply the criterion to a subgroup of individuals, and, most importantly, to the present generation, and also to exploit the bulk of social choice theory as developed for the finite case. In particular, our results concerning the orderings generated respectively by the pure utilitarian rule and the Leximin rule, both having the proliferating property, are compared to the characterisations given by Basu and Mitra (2003b), Asheim and Tungodden (2004) and Bossert, Sprumont and Suzumura (2004) on different infinite-horizon extensions of these rules. To keep with the idea of an overview though, we try to be more general and derive a characterization result (the general overtaking theorem), as well as a simplified criterion, that can be associated to any rule having the proliferating property.

2 Welfarism for successive generations

We consider a countably infinite set of time periods, starting from the present one and denoted $\mathbb{T} = \{0, 1, \cdots, t, \cdots\}$. Associated to each time period t there is a 'generation' made up of a finite set N_t of n_t individuals and there is a set X_t of possible social states. The set of all individuals is represented as a partition of the set of positive integers into successive generations:

$$\mathbb{N} = \{N_0, N_1, \cdots, N_t, \cdots\}.$$

We assume that $n_t \geq 2$ and $|X_t| \geq 3$, for every t. Our objective is to evaluate the respective merits of each program me of social states $x = \{x^t\} \in \mathbb{X} \equiv \chi_{t=1}^{\infty} X_t$ for an infinite future, taking into account individual evaluations. The final evaluation, of course, will have to take into account feasibility constraints, and in particular that the set of social states at some period may depend on the social states realized in previous periods. Here, however, we shall focus on the definition of general evaluation criteria applicable to any set of feasible programmes. Individuals are supposed to live a finite number of periods. For the present analysis we keep in mind two standard cases, the case where each individual lives only one period (N_t is the set of individuals living at period t), and the case where individuals live for two periods (N_t is the set of individuals born at period t) and generations overlap. In both cases, N_t is the set of individuals belonging to generation t.

To introduce intergenerational evaluation a simple and usual approach is to suppose that, for each possible program me of social states, a 'utility level' u^t can be attached to each generation t, allowing to define an infinite 'utility stream' $u = (u^0, u^1, \cdots, u^t, \cdots) \in \mathbb{R}^{\mathbb{T}}$ (with $\mathbb{R}$ the set of real numbers), and then to look for an ordering of all infinite utility streams satisfying some efficiency or equity properties. However, this approach requires us to proceed in two stages. The first stage is to construct an ordering of all infinite *individual* welfare evaluation streams (or 'utility streams'). The second is to determine by aggregation, for every infinite individual welfare evaluation stream, the welfare level attached to each generation, and then to reduce the previous ordering to an ordering defined on the set of all infinite *generational* welfare evaluation streams.

To examine these problems we start from the concept introduced by Sen (1970) of a 'Social Welfare Functional', using it both in the case of non-overlapping and in the case of overlapping generations, and then go on to the associated concept of 'Social Welfare Ordering' (the terminology fixed by Gevers, 1979).

This, formally, consists in assuming that we have an individual evaluation function (or profile for short) given by a real-valued function U defined on $\mathbb{X} \times \mathbb{N}$. That is, if $i \in N_t$ for any $t \in \mathbb{T}$, the function $U(\cdot, i)$, or U_i for short, is a real-valued function, defined on $X_t \times X_{t+1}$ in the overlapping generation case and on X_t in the non-overlapping case, and is called individual i's evaluation function. Also, for every $x \in \mathbb{X}$, the vector $U(x, \cdot)$, or U_x for short, is a point in the infinite *individual welfare evaluation space* $\mathbb{R}^{\mathbb{N}}$ and called an infinite individual welfare evaluation stream associated to x. Given any individual evaluation function U in some admissible subset $\mathcal{D} \subset \{U \mid U : \mathbb{X} \times \mathbb{N} \to \mathbb{R}\}$, we are to recommend a social ranking of $\mathbb{X}$, that is an element in the set $\mathcal{R}$ of all complete and transitive binary relations on $\mathbb{X}$. A *social welfare functional* (SWFL) is a map $F : \mathcal{D} \to \mathcal{R}$ with generic image $R_U = F(U)$ (I_U and P_U denoting respectively the associated indifference and strict preference relations). If x is ranked socially at least as high as y whenever the relevant profile is U, we write xR_Uy (resp. xI_Uy or xP_Uy in case of indifference or strict preference).

To reduce the SWFL approach to the comparisons of generational welfare evaluation streams, we need to introduce conditions ensuring that the relative welfare (in a formal sense) of two social states can be entirely judged by comparing their respective individual evaluation vectors, independently from the other aspects of the individual profile at hand. With individuals partitioned in a sequence of successive generations, formal welfarist social evaluation may be defined and applied at different levels, according to the domain of evaluation vectors which is considered (individual or generational). Standard conditions are the following:

Domain Attainability (AD). $\forall\, u, v, w \in \mathbb{R}^{\mathbb{N}}, \exists\, x, y, z \in \mathbb{X}, \exists\, U \in \mathcal{D}$ *such that* $U_x = u$, $U_y = v$ *and* $U_z = w$.

This condition ensures that the set $\{r \in \mathbb{R}^{\mathbb{N}} \mid \exists\, x \in \mathbb{X}, \exists\, U \in \mathcal{D}$ such that $U_x = r\}$ fills the whole individual evaluation space $\mathbb{R}^{\mathbb{N}}$. The next condition is a Paretian principle:

Pareto Indifference (PI). $\forall\, U \in \mathcal{D}, \forall\, x, y \in X$, *if* $U_x = U_y$ *then* xI_Uy.

The third condition is an Arrowian inter-profile consistency requirement imposing that the ranking of two alternatives depends only on the evaluation of these two alternatives.

Binary Independence (BI). $\forall\, U, V \in \mathcal{D}, \forall\, x, y \in \mathbb{X}$ *such that* $V_x = U_x, V_y = U_y, xR_Uy \Leftrightarrow xR_Vy$.

As it will be recalled in the next theorem, these three conditions characterize formal welfarism, that is, the possibility to define, on the individual evaluation space $\mathbb{R}^{\mathbb{N}}$, an ordering R^*, called a *Social Welfare Ordering* (SWO), which is derived from the SWFL F. Now, since we also need to introduce the possibility of aggregating the welfare of each generation, we have to add three conditions which refer explicitly to generations. The first is a separability condition on F which allows to isolate the evaluation of the welfare of each generation.

Generational Separability (GS). $\forall t \in \mathbb{T}, \forall U, V \in \mathcal{D}$, *if* $\forall i \in N_t, U_i = V_i$ *whereas,* $j \in \mathbb{N} \setminus N_t, \forall x, y \in \mathbb{X}, U(x,j) = U(y,j)$ *and* $V(x,j) = V(y,j)$, then $R_U = R_V$.

This condition allows to derive from F, for each generation t, a SWFL F_t defined on a domain $\mathcal{D}_t$ contained in$\{U^t \mid U^t : \mathbb{X} \times N_T \to \mathbb{R}\}$ with range in $\mathcal{R}$. For each $i \in N_t$, we define $U^t(\cdot, i)$ on the whole set $\mathbb{X}$ for simplicity of notation, but it is constant for variables outside $X_t \times X_{t+1}$ in the overlapping generation case and outside X_t in the non-overlapping case. If the relevant profile is U^t and $x \in \mathbb{X}$ is ranked socially at least as high as $y \in \mathbb{X}$, we then write $xR^t_{U^t}y$ ($xI^t_{U^t}y$ or $xP^t_{U^t}y$ in case of indifference or strict preference). Moreover, our first three conditions can be straightforwardly reformulated for each t to be applied to each derived SWFL F_t: replacing $\mathbb{R}^{\mathbb{N}}, \mathcal{D}, U, V, R_U, R_V$ and I_U by, respectively $\mathbb{R}^{\mathbb{N}_t}, \mathcal{D}_t, U^t, V^t, R^t_{U^t}, R^t_{V^t}$ and $I^t_{U^t}$ in the conditions AD, PI and BI, we get the conditions AD$_t$, PI$_t$ and BI$_t$. They will ensure the existence of an associated SWO R^*_t defined on $\mathbb{R}^{\mathbb{N}_t}$.

Lemma 1 *Assume the SWFL F satisfies conditions AD, PI, BI and GS. Then, for every generation $t \in \mathbb{T}$, there is a SWFL $F_t : \mathcal{D}_t \to \mathcal{R}, \mathcal{D}_t \subset \{U^t \mid U^t : \mathbb{X} \times N_t \to \mathbb{R}\}$, which can be identified to the restriction of the SWFL F to some subset $\overline{\mathcal{D}}_t \subset \mathcal{D}$. The SWFL F_t satisfies the conditions AD$_t$, PI$_t$ and BI$_t$.*

Proof: Under GS, F_t can be identified to the restriction of F to the set $\overline{\mathcal{D}}_t = \{U \in \mathcal{D} : \forall j \in \mathbb{N} \setminus N_t, \forall x \in \mathbb{X}, U(x,j) = \overline{U}(x,j)\}$, for some arbitrary $\overline{U} \in \mathcal{D}$ such that $\forall x \in \mathbb{X}, \overline{U}_x = \overline{u} \in \mathbb{R}^{\mathbb{N}}$, so that $\mathcal{D}_t = \{U^t \mid U^t : \mathbb{X} \times N_t \to \mathbb{R}$ and $(U^t, \overline{U}^{-t}) \in \overline{\mathcal{D}}_t\}$, with $\overline{U}^{-t} = (\overline{U}_i)_{i \in \mathbb{N} \setminus N_t}$. Since F satisfies AD, PI and BI, F_t satisfies AD$_t$, PI$_t$ and BI$_t$. Indeed PI and BI should hold on $\overline{\mathcal{D}}_t$, meaning that SP_t and BI_t hold on $\mathcal{D}_t$. Similarly, AD_t is simply the condition AD applied to all $u, v, w \in \mathbb{R}^{N_t} \times \mathbb{R}^{\mathbb{N}\setminus N_t}$ such that $u^{-t} = v^{-t} = w^{-t} = \overline{u}^{-t} \in \mathbb{R}^{\mathbb{N}\setminus N_t}$. ■

Two additional conditions, both based on the derived SWFLs $\{F_t\}$, are needed in the following theorem. One is continuity. We say that a sequence $(U^{tk})_{k=1}^{\infty} \subset \mathcal{D}_t$ converges point wise to $U^{t0} \in \mathcal{D}_t$, if $\lim_{k\to\infty} U^{tk}(x,i) = U^{t0}(x,i)$, $\forall\,(x,i) \in \mathbb{X} \times N_t$. The condition is then stated as

Generational Continuity (GC). $\forall\, t \in N, \forall\, x, y \in \mathbb{X}, \forall\, U^{t0} \in \mathcal{D}_t$ *and for any sequence* $(U^{tk})_{k=1}^{\infty} \subset \mathcal{D}_t$ *converging pointwise* to U^{t0}, *if* $xR^t_{U^{tk}}y, \forall\, k \geq 1$, *then* $xR^t_{U^{t0}}y$.

This property will allow to represent each SWO R^*_t by a continuous function w_t, called a *Social Welfare Function* (SWF): $\forall\, u^t, v^t \in \mathbb{R}^{N_t}, w_t(u^t) \geq w_t(v^t) \Leftrightarrow u^t R^*_t v^t$.

The last condition is an 'extended Pareto' condition (Dhillon, 1998). It is a Pareto indifference condition but applied to a partition of all individuals into groups (here the generations): if all the groups are indifferent between two alternatives, then society should also be indifferent.

Generational Pareto Indifference (GPI). $\forall\,(U^t)_{t\in\mathbb{T}} \in \times_{t\in\mathbb{T}}\mathcal{D}_t$, $\forall\, x, y \in \mathbb{X}$, *if,* $xI^t_{U^t}y$, $\forall\, t \in \mathbb{T}$, *then* $xI_U y$ *with* $U \in \mathcal{D}$ *such that,* $\forall\, t \in \mathbb{T}, \forall\, i \in N_t, U_i = U^t_i$.

We can now prove the following result.

Theorem 1 *Intergenerational welfarism. Assume the SWFL F satisfies conditions AD, SP and BI. Then, (i) there exists a SWO R^* on $\mathbb{R}^N$ such that, for all $x, y \in \mathbb{X}$ and for all $U \in \mathcal{D}$,*

$$U_x R^* U_y \Leftrightarrow x R_U y; \tag{1}$$

also, (ii) under GS and for every generation $t \in \mathbb{T}$, there exists a SWO R^*_t on $\mathbb{R}^{N_t}$ such that, for all $x, y \in \mathbb{X}$ and for all $U^t \in \mathcal{D}_t$,

$$U^t_x R^*_t U^t_y \Leftrightarrow x R^t_{U^t} y; \tag{2}$$

and, (iii), with GC and GPI in addition, there is, for every generation $t \in \mathbb{T}$, a social welfare function $w_t : \mathbb{R}^{N_t} \to \mathbb{R}$ such that, for all $u^t \in \mathbb{R}^{N_t}$, $\mathbf{1}_{N_t} w_t(u^t) I^*_t u^t$ (with $\mathbf{1}_{N_t} = (1, \cdots, 1) \in \mathbb{R}^{N_t}$)

and there exists a SWO $R^{\#}$ on $\mathbb{R}^{\mathbb{T}}$ such that, for all $u, v \in \mathbb{R}^{\mathbb{N}}$,

$$(w_0(u^0), w_1(u^1), \cdots, w_t(u^t), \cdots) R^{\#} (w_0(v^0), w_1(v^1), \cdots, w_t(v^t), \cdots) \Leftrightarrow uR^*v.$$

Proof: As is well-known from the case of a finite number of individuals, under AD, the conditions PI and BI are equivalent to the condition of *Strong Neutrality* (SN): $\forall\, U, V \in \mathcal{D}, \forall\, x, y \in \mathbb{X}$, if there are $x', y' \in \mathbb{X}$ such that $V_{x'} = U_x$, $V_{y'} = U_y$ then $xR_Uy \Leftrightarrow x'R_Vy'$. The argument consists in choosing $z \in \mathbb{X} \setminus \{y, y'\}$, with z a third alternative if the two pairs coincide, and, thanks to AD, in constructing profiles U^1, U^2 and U^3 such that $U_x^1 = U_z^1 = u$, $U_y^1 = v$, $U_z^2 = u$, $U_y^2 = U_{y'}^2 = v$, $U_{x'}^3 = U_z^3 = u$ and $U_{y'}^3 = v$. Applying alternately BI and PI, we get $xR_Uy \Leftrightarrow xR_{U^1}y \Leftrightarrow zR_{U^1}y \Leftrightarrow zR_{U^2}y \Leftrightarrow zR_{U^2}y' \Leftrightarrow zR_{U3}y' \Leftrightarrow x'R_{U^3}y' \Leftrightarrow x'R_Vy'$, and SN follows. Then, defining R^* by (1), for some $x, y \in \mathbb{X}$ and for some $U \in \mathcal{D}$, we get by SN the same relation for any $x', y' \in \mathbb{X}$ such that $V_{x'} = U_x$, $V_{y'} = U_y$, so that R^* is well-defined. Completeness and transitivity of R^* follow from AD and from the completeness and transitivity of each R_U. This proves (i). To prove (ii), we know from Lemma 1 that each F_t satisfies AD_t, PI_t and BI_t. Then, repeating the same argument as in (i), we get the required SWO R_t^* defined on $\mathbb{R}^{N_t}$ (see (2)). To prove (iii), we need in addition R_t^* to be continuous, *i.e.* $\forall\, v^t \in \mathbb{R}^{N_t}$, that the sets $\{u^t \in R^{N_t} \mid u^tR_t^*v^t\}$ and $\{u^t \in R^N \mid v^tR_t^*u^t\}$ be closed in R^{N_t}. This property is implied by GC. Indeed, if for some $v_t \in \mathbb{R}^{N_t}$ the set $\{u^t \in R^{N_t} \mid u^tR_t^*v^t\}$, say, was not closed, it would be possible to find a sequence $(u^{tk})_{k=1}^{\infty}$ in $\mathbb{R}^{N_t}$, converging to some u^{t0}, with $u^{tk}R_t^*v^t$ for all $k \geq 1$ and $v^tP^*u^{t0}$; but it would then be possible to construct a sequence $(U^{tk})_{k=1}^{\infty}$ converging point wise to U^{t0} such that, $\forall\, k \geq 1, (U^{tk}(x,i))_{i \in N_t} = u^{tk}$ and $(U^{tk}(y,i))_{i \in N_t} = v^t$ for some $x, y \in \mathbb{X}$, implying $xR_{U^{tk}}y$, $\forall\, k \geq 1$, but $yP_{U^{t0}}x$, in contradiction with GC. Thus, we obtain (see *e.g.* Theorem 3 in Blackorby, Bossert and Donaldson, 2002) that, for every $t \in \mathbb{T}$, there exists a SWF $w_t : \mathbb{R}^{N_t} \to \mathbb{R}$ such that for every $u^t \in \mathbb{R}^{N_t}$, $\mathbf{1}_{N_t}w_t(u^t)I_t^*u^t$. Now, by AD, for any $u \in \mathbb{R}^{\mathbb{N}}$, there are $x, y \in \mathbb{X}$ and $U \in \mathcal{D}$ such that $U_x = u$ and $U_y = (\mathbf{1}_{N_0}w_0(u^0), \mathbf{1}_{N_1}w_1(u^1), \cdots, \mathbf{1}_{N_t}w_t(u^t), \cdots)$ so that, by GPI, if $xI_{U^t}^ty$, $\forall\, t \in \mathbb{T}$, then

$$uI^*(\mathbf{1}_{N_0}w_0(u^0), \mathbf{1}_{N_1}w_1(u^1), \cdots, \mathbf{1}_{N_t}w_t(u^t), \cdots).$$

So, whenever uR^*v we may write as well,

$$(\mathbf{1}_{N_t} w_t(u^t))_{t\in\mathbb{T}} R^* (\mathbf{1}_{N_t} w_t(v^t))_{t\in\mathbb{T}},$$

or taking only one representative component per generation we can write equivalently

$$(w_0(u^0), w_1(u^1), \cdots, w_t(u^t), \cdots) R^\# (w_0(v^0), w_1(v^1), \cdots, w_t(v^t), \cdots),$$

thereby defining a SWO $R^\#$ on $\mathbb{R}^{\mathbb{T}}$. ■

This 'welfarism theorem', as any other welfarism theorem[2], opens the possibility to work directly in terms of SWOs and to add conditions formulated in those terms only. Of course these additional properties will, almost always, be easily translated back in SWFL terms.

3 Intergenerational social welfare orderings

In the previous section we have shown that, even with an infinite number of individuals partitioned into a sequence of generations, it is possible to prove a welfarism theorem, transforming the problem of finding an acceptable Social Welfare Functional into the problem of finding an appropriate Social Welfare Ordering. The theorem above even left us with two possible SWOs: R^* or $R^\#$. Formally they are completely similar. Both are orderings of all infinite evaluation streams and, if we want to get a SWO satisfying both collective efficiency and intergenerational equity conditions, the problem of constructing the ordering remains as difficult whether in the case of R^* or in the case of $R^\#$. Existence of such an ordering is difficult to establish. We shall rely on the result by Svensson (1980). Other existence results are provided in Fleurbaey and Michel (2003). In that respect, the literature studies principally the SWO $R^\#$. We want to argue that it is preferable to work with the more basic SWO R^*. A first advantage is that R^* can be derived from a SWFL under a weaker set of assumptions (using neither GS nor GC nor GPI). However, this is a formal argument and, for some results, we will have to add separability or continuity assumptions anyway. The main argument in favour of R^* is that it forces

[2]See *e.g.* d'Aspremont and Gevers (1977), Sen (1977), d'Aspremont (1985) and Blackorby, Bossert and Donaldson (2002).

an explicit consideration of the relationship between the problem of justice among generations and the problem of justice among individuals within each generation and, primarily, within the present generation. Should not the solutions proposed for the intragenerational problem, which has up to now been the main domain of investigation in ethics and social choice, have some bearing on the solutions that should be considered for the intergenerational problem? Using standard welfarist arguments, this section answers this question positively.

To represent collective efficiency and intergenerational equity requirements, the two basic conditions that we use are the following. The first is the welfarist translation of the strong Paretian condition, Pareto indifference being trivially satisfied by construction of R^*:

Strong Pareto (SP).* $\forall\, u, v \in R^{\mathbb{N}}$, *if* $u \geq v$ *and* $u \neq v$, *then* uP^*v.

By $u \geq v$ we mean $u_i \geq v_i$, $\forall\, i \in \mathbb{N}$. This is a strong but standard efficiency condition.

As for equity, the basic condition is to keep social indifference for finite permutations of individual evaluations both within and across generations. Although this condition seems to be introducing a minimal condition of impartiality, it already excludes the use of a discount factor. A much more demanding condition would be to allow for all permutations, but then it becomes incompatible with Pareto conditions (see Lauwers and Van Liedekerke, 1995; Lauwers, 1998). Other, intermediate, impartiality conditions are studied in Fleurbaey and Michel (2003).

Finite Anonymity (FA).* *If* σ *is a permutation of* $M \subset \mathbb{N}$, $|M| < \infty$, *and* $u, v \in \mathbb{R}^{\mathbb{N}}$ *are such that* $u_i = v_i$, $\forall\, i \in \mathbb{N} \setminus M$, *and* $u_i = v_{\sigma(i)}$, $\forall\, i \in M$, *then* uI^*v.

The two conditions SP^* and FA^* are defined as properties of a SWO R^* defined in $\mathbb{R}^{\mathbb{N}}$. But they can also be defined as properties of a *quasi-ordering* R, that is, a reflexive and transitive binary relation defined in $\mathbb{R}^{\mathbb{N}}$ (resp. in a subspace $\mathbb{R}^M$, $M \subset \mathbb{N}$, or just in $\mathbb{R}^m$, $m < \infty$, with SP* and FA* then restricted to such domains), which we call a Social Welfare Quasi-ordering (SWQ) on $\mathbb{R}^{\mathbb{N}}$ (resp. on $\mathbb{R}^M$ or on $\mathbb{R}^m$). A SWO is a complete SWQ. A SWQ R is a *sub-relation* to another SWQ R' (or, equivalently, R' is an *extension* of R) if they have the same domain and for any u and v in this domain, $uPv \Rightarrow Up'v$ and $uIv \Rightarrow uI'v$ (P and I denoting respectively the strict preference and indifference relations associated to R). If R' is a SWO, then R' is called an *ordering extension* of R (see Bossert, Sprumont and Suzumura, 2004).

Combining the two basic conditions we obtain a well-known SWQ, first proposed by Suppes (1966) and further analyses by Sen (1970), Kolm (1972) and Hammond (1976, 1979). This version is adapted to the infinite case (see Svensson, 1980).

Definition 1 (The m-Grading Principle) The m-Grading Principle is the SWQ R^S such that: for any permutation $\sigma : M \to M, M \subset \mathbb{N}$ with $|M| = m$, and for any $u, v \in R^{\mathbb{N}}$, if $u_i \geq v_{\sigma(i)}, \forall\, i \in M$ and $u_i \geq v_i$, $\forall\, i \in \mathbb{N} \setminus M$, then $uR^S v$; if in addition $u_j > v_{\sigma(j)}$ for some $j \in M$, then $uP^S v$.

The m-Grading Principle is a quasi-ordering on $\mathbb{R}^{\mathbb{N}}$ that satisfies both SP* and FA*. The following result, easily adapted from Lemma 3.1.1 in d'Aspremont (1985), demonstrates the capacity (at least known[3] since Sen, 1977) of the Grading Principle to proliferate through and across generations:

Lemma 2 *If a SWQ R is an extension of the 2-Grading Principle then it satisfies SP* and FA*. Moreover, if R satisfies these two conditions, then R is an extension of the m-Grading Principle for every $m < \infty$.*

Proof: The only new argument (with respect to the finite case) is to show that the 2-Grading Principle implies SP*. But, for $u, v \in R^{\mathbb{N}}$, supposing $u \geq v$ and $u_j > v_j$ for some j, and applying the 2-Grading Principle to the pair $\{j, j+1\}$, we immediately get uP^*v. ■

There are, of course, many quasi-orderings satisfying SP* and FA*, but all have the m-Grading Principle as sub-relation. The interest in the Grading Principle in comparing infinite utility streams comes from the following existence theorem for SWOs given by Svensson (1980, Theorem 2), and based on a result due to Szpilrajn (1930) and adapted by Arrow (1951, section 3 of chapter VI).

[3] The property is mentioned by Sen (1977, n 26) as suggested by Hammond as a step to derive the same property for Leximin. For a proof, see Hammond (1979).

Theorem 2 (Svensson, 1980) *If a SWQ R is an extension of the m-Grading Principle for every $m < \infty$, then there exists a SWO R^* which is an ordering extension of R (and hence R^* satisfies SP* and FA*).*

An important observation is that the 'proliferating' property of a SWQ, as illustrated by the Grading Principle, can be defined in general terms.

Definition 2 (proliferating sequence) For any $M \subset \mathbb{N}$, $|M| = m, 2 \leq m < \infty$, let R_m denote a SWQ defined on $\mathbb{R}^m$ (with P_m and I_m denoting respectively the associated strict preference and indifference relations) and, for any $u \in \mathbb{R}^{\mathbb{N}}$, let $u_M \in real^m$ be such that $u_M = (u_i)_{i \in M}$. Then:

- (i) A SWQ R is said to *extend* R_m if, $\forall\, M \subset \mathbb{N}$, $|M| = m, \forall\, u, v \in \mathbb{R}^{\mathbb{N}}$, if $u_M P_m v_M$ and $u_j \geq v_j$ (resp. $u_M I_m v_M$ and $u_j = v_j$), $\forall\, j \in \mathbb{N} \setminus M$, then uPv (resp. uIv).
- (ii) A sequence of SWQs $(R_m)_{m:2}^{\infty}$, with each R_m defined on $\mathbb{R}^m$, is said to be *proliferating* if every SWQ R defined on $\mathbb{R}^{\mathbb{N}}$ and extending R_2, also extends R_m for every $m < \infty$.

A useful property of a proliferating sequence is the following:

Lemma 3 *Consider a proliferating sequence of SWQs $(R_m)_{m=2}^{\infty}$, with each R_m defined on $\mathbb{R}^m$. If a SWQ R^* defined on $\mathbb{R}^{\mathbb{N}}$ extends R_2 and R_2 satisfies SP* and FA* restricted to $\mathbb{R}^2$, then R satisfies SP* and FA*.*

Proof: Since R_2 satisfies SP* and FA* restricted to $\mathbb{R}^2$, it is an extension of the 2-Grading Principle on $\mathbb{R}^2$ (see, e.g. Lemma 3.1.1 in d'Aspremont (1985)). So R is an extension of the 2-Grading Principle on $\mathbb{R}^{\mathbb{N}}$ and the result follows from Lemma 2. ■

Clearly, by Lemma 2, the sequence of SWQs corresponding to the m-Grading Principle (as defined on $\mathbb{R}^m$) is proliferating. But there are other well-known proliferating sequences. One example is based on the pure utilitarian rule.

Definition 3 (Pure m-Utilitarianism) The pure utilitarian SWO on $\mathbb{R}^m$, denoted R_m^{pu}, is such that for any $u, v \in \mathbb{R}^m$,$uR_m^{pu}v$ if only if $\sum_{i=1}^m u_i \geq \sum_{i=1}^m v_i$.

We call $(R_m^{pu})_{m=2}^{\infty}$ the *pure utilitarian sequence*. We then have:

Lemma 4 *The pure utilitarian sequence $(R_m^{pu})_{m=2}^{\infty}$ is proliferating.*

Proof: The proof (adapted from Lemma 3.3.1 in d'Aspremont, 1985) goes by induction. Suppose a SWQ R extends $R_2^{pu}, R_3^{pu}, \cdots$, and R_m^{pu}, we want to show that it extends R_{m+1}^{pu}. Take any $M \subset \mathbb{N}$, $|M| = m \geq 2$, and $j \in \mathbb{N} \setminus M$, and any $u, v \in \mathbb{R}^{\mathbb{N}}$ sic that $u_i \geq v_i$, $\forall\, i \in \mathbb{N} \setminus M'$, with $M' = M \cup \{j\}$. For simplicity of notation, suppose $M' = \{1, 2, \cdots, m\} \cup \{m+1\}$. Then, we can find $w \in \mathbb{R}^{\mathbb{N}}$ such that $w_i = u_i$ for $1 \neq i \neq m+1$

$$w_1 + w_{m+1} = u_1 + u_{m+1},$$

and $w_{m+1} = v_{m+1}$. Since R extends R_2^{pu}, we have wIu. Also, since R extends R_m^{pu}, we get

$$\sum_{i=1}^{m+1} u_i = \sum_{i=1}^{m+1} w_i > \sum_{i=1}^{m+1} v_i \Rightarrow \sum_{i=1}^{m} w_i > \sum_{i=1}^{m} v_i \Rightarrow wPv \Rightarrow uPv,$$

and uIv if the inequalities wire replaced by equalities. ■

Hence, if the pure utilitarian rule is used to evaluate the welfare of a set of individuals belonging to some generation, say the present generation, then it has to be used to evaluate the welfare of any finite set of subsequent generations.

A second example is the Leximin (the lexicographic completion of the maximin), the proliferating property of which is known since Sen (1976, 1977) and Hammond (1979). We start by defining m-Leximin using, for any $u \in \mathbb{R}^m$, the notation $u_{i(\cdot)} \in \mathbb{R}^m$ to denote the vector with the same set of components as u but increasingly ranked.

Definition 4 (m-Leximin) For $2 \leq m < \infty$, the m-Leximin SWO on $\mathbb{R}^m$, denoted R_m^{lx}, is such that: for any $u, v \in \mathbb{R}^m$, $uP_m^{lx}v$ if and only if $\exists\, k \in \{1, 2, \cdots, m\}$ such that $u_{i(k)} > v_{i(k)}$ and $u_{i(h)} = v_{i(h)}$, for $h = 1, 2, \cdots, k-1$.

We call $(R^{lx}_m)^\infty_{m=2}$ the *Leximin sequence*. The proliferating property of m-Leximin can be shown by a simple adaptation of the argument in d'Aspremont (1985, Lemma 3.4.1).

Lemma 5 *The Leximin sequence $(R^{lx}_m)^\infty_{m=2}$ is proliferating.*

Proof: The proof goes by induction. But, first, it can be verified that if a SWQ R extends R^{lx}_2, then R is an extension of the 2-Grading Principle, hence, by Lemma 2, R satisfies SP* and FA*. Suppose, now that R extends $R^{lx}_2, R^{lx}_3, \cdots,$ and R^{lx}_m. We want to show that it extends R^{lx}_{m+1}. With FA* we can take $M = \{1, 2, \cdots, m\}$, and $m+1 \in \mathbb{N} \setminus M$, and consider any $u, v \in \mathbb{R}^{\mathbb{N}}$ such that $u_i \geq v_i$, $\forall\, i > m+1, u_1 \leq u_2 \leq \cdots \leq u_{m+1}$ and $v_1 \leq v_2 \leq \cdots \leq v_{m+1}$. Clearly uIv if and only if $u = v$. If $u_1 > v_i$, for some $i > 1$, and $u_j \geq v_j$, for $j = 1, \cdots, i-1$, then uPv by applying R^{lx}_k, $2 \leq k \leq m$, or simply by SP*. It remains to be shown that uPv, whenever $u_1 > v_1$ and $u_j w v_j$, for $j = 2, \cdots, m+1$, so that $v_2 > v_1$ (otherwise we would have $u_2 > v_2$). But, then, we can find $w \in \mathbb{R}^{\mathbb{N}}$ such that $v_1 < w_1 < \min v_2, u_1\}$, $w_2 = u_2$ and $w_h = v_h$, $1 \neq h \neq 2$, implying uPw by application of R^{lx}_m, and uPv by application of R^{lx}_2. ■

Our purpose in defining proliferating sequences is to apply Theorem 2 and to look for SWOs that are extensions of rules that are well-known and well characterized in the intragenerational case, such as Pure Utilitarianism and Leximin, in order to rank infinite utility streams. Indeed, the choice of such a rule for any generation constrains the choice of a similar rule for intergenerational comparisons. For that purpose we define a notion of 'generalized overtaking criterion', inspired by von Weizsäcker (1965) and Atsumu (1965), but applied to other rules than the pure utilitarian rule and adapted to the case where the individual evaluation of each generation is not aggregated. Such criteria consist in 'transforming the comparison of any two infinite utility paths to an infinite number of comparisons of utility paths each containing a finite number of generations' (Asheim and Tungodden, 2004). However, the present formulation of the criterion differs in two different important respects. The first is that it aims at an ordering of all infinite individual welfare evaluation streams without aggregating them into streams of each generation utility. The second is that the criterion is not only applied to any finite number of successive generations, from some point on, but to any finite set of individuals (belonging to any generation), from some size on. This leads to a more demanding criterion (implying less completeness), but which does not privilege in a specific way the present generation and the generations in the near future. The criterion treats all generations symmetrically.

Criterion 1 (general overtaking). *A SWQ R^0 defined in $\mathbb{R}^{\mathbb{N}}$ is a generalized overtaking criterion generated by a profile rating sequence of SWQs $(R_m)_{m=2}^{\infty}$ if it extends R_2 and is such that: $\forall\, uv \in \mathbb{R}^{\mathbb{N}}, uP^0v$ (resp. uI^0v) whenever $\exists\, \overline{m} \geq 2$ such that, $\forall\, M \subset \mathbb{N}$ with $|M| = m \geq \overline{m}$, $u_M P_m v_M$ (resp. $u_M I_m v_M$).*

The proliferating feature of an overtaking sequence of SWOs implies that, accepting only the first element of the sequence, because we take it as an acceptable condition, we are forced to accept any subsequent element of the sequence. Hence the properties of R_2, the first accepted SWQ, are crucial. Coming back to Svensson's theorem this remark has the following important application:

Theorem 3 (general overtaking) *Suppose R^0 is a generalized overtaking criterion generated by a proliferating sequence of SWQs $(R_m)_{m=2}^{\infty}$, with R_2 satisfying SP* and FA* restricted to $\mathbb{R}^2$. Then there exists a SWO R^* defined on $\mathbb{R}^{\mathbb{N}}$, an ordering extension of R^0, satisfying SP* and FA*. Moreover, a SWQ R defined on $\mathbb{R}^N$ extends R_2 (and hence satisfies SP* and FA*) if and only if R^0 is a subrelation of R.*

Proof: Consider the generalized overtaking criterion R^0 generated by the sequence $(R_m)_{m=2}^{\infty}$. Because the sequence is proliferating, R^0 extends R_m, $\forall\, m \geq 2$. Since R_2 satisfies SP* and FA* restricted to $\mathbb{R}^2$, R^0 satisfies SP* and FA* and is an extension of the m-Grading Principle for every $m < \infty$ (by Lemma 2 and 3). Then, by Theorem 2, there exists a SWO R^* defined on $\mathbb{R}^{\mathbb{N}}$ which is an ordering extension of R^0 and satisfying SP* and FA*.

Now, consider a SWQ R defined on $\mathbb{R}^{\mathbb{N}}$ extending R_2, and so satisfying SP* and FA*. Since the sequence $(R_m)_{m=2}^{\infty}$ is proliferating, R extends all subsequent R_m, $2 \leq m < \infty$. Suppose R^0, the overtaking criterion generated by this sequence, is not a sub relation of R. Then $\exists\, u, v \in \mathbb{R}^{\mathbb{N}}$ such that uPv (resp. uIv) does *not* hold although uP^0v (resp. uI^0v) holds, meaning that for some $\overline{m} \geq 2$, $\forall\, M \subset \mathbb{N}$, $|M| = m \geq \overline{m}$, $u_M P_m v_M$ (resp. $u_M I_m v_M$). Thus,

$\forall M \subset \mathbb{N}, |M| \geq \overline{m}$, $\exists i \in \mathbb{N} \setminus M$ such that $v_i > u_i$ (resp. $v_i \neq u_i$). Otherwise there would be some $M \subset \mathbb{N}$, $|M| = m \geq \overline{m}$, such that $u_M P_m v_M$ (resp. $u_M I_m v_M$) and, $\forall i \in \mathbb{N} \setminus M$, $u_i \geq v_i$ (resp. $u_i = v_i$) implying uPv (resp. uIv), since R extends R_m. So we may select sets $M_1, \cdots, M_{\overline{m}}$ and $M' = \{i_1, \cdots, i_{\overline{m}}\}$ such that $|M_k| \geq \overline{m}$, $i_k \in \mathbb{N} \setminus M_k$, and $v_{i_k} > u_{i_k}$ for $k = 1, \cdots, \overline{m}$ (resp. either $v_{i_k} > u_{i_k}$ for all k, or $u_{i_k} > v_{i_k}$ for all $k, 1 \leq k \leq \overline{m}$) and $\{i_1, \cdots, i_{k-1}\} \subset M_k$, for $k = 2, \cdots, \overline{m}$. Then by SP*, $(v_{M'}, v_{\mathbb{N}\setminus M'}) P (u_{M'}, v_{\mathbb{N}\setminus M'})$ (resp. $(v_{M'}, v_{\mathbb{N}\setminus M'}) P (u_{M'}, v_{\mathbb{N}\setminus M'})$ or $(u_{M'}, v_{\mathbb{N}\setminus M'}) P (v_{M'}, v_{\mathbb{N}\setminus M'})$) which contradicts $u_{M'} P_{\overline{M}} v_{M'}$ (resp. $u_{M'} I_{\overline{m}} v_{M'}$), since R satisfies $R_{\overline{m}}$. ■

This theorem can be applied to any generalized overtaking criterion R^0 generated by a proliferating sequence of SWQs, whenever the first element in the sequence, R_2, satisfies SP* and FA* restricted to $\mathbb{R}^2$. It can in particular be applied to the pure utilitarian generalized overtaking criterion, say R^{pu}, generated by the pure utilitarian sequence $(R_m^{pu})_{m=2}^{\infty}$.

Basu and Mitra (2003b) and Asheim and Tungodden (2004) propose alternative pure utilitarian criteria. They are formulated to compare streams of generational aggregated utility streams and give precedence to the present and near futures generations. Reformulated in our framework, Basu and Mitra (2003b) pure utilitarian SWQ, say R^{PU}, is defined by:

$$\begin{aligned}&\forall u, v \in \mathbb{R}^{\mathbb{N}}, uR^{PU}v \text{ if and only if, for some } M \subset \mathbb{N},\\ &\sum_{i \in M} u_i \geq \sum_{i \in M} v_i \text{ and } u_j \geq v_j, \forall j \in \mathbb{N} \setminus M.\end{aligned}$$

By the above theorem, since R^{PU} satisfies SP*, FA* and extends R_2^{pu}, it is an extension of R^{pu}. Conversely, since R^{pu} satisfies SP*, FA* and is translatable (*e.g.* for any $u, v, w \in \mathbb{R}^{\mathbb{N}}, uR^{pu}v \Leftrightarrow (u+w)R^{pu}(v+w)$), the argument[4] of Theorem 1 on Basu and Mitra (2003b) can be used to get that R^{pu} is an extension of R_U. Asheim and Tungodden (2004) propose two alternative pure utilitarian criteria (a Catching Up and an Overtaking criterion) defined on infinite utility streams. These extensions are respectively characterized by SP*, FA*, a translation invariance condition and two alternative 'Preference Continuity' conditions. Reformulated in our framework, both

[4]Basu and Mitra (2003b) assume that utility streams belong to $[0,1]^{\mathbb{N}}$, but the argument in their Theorem 1 can be readily adapted.

criteria would satisfy R_2^{pu} so that they are extensions of R^{pu} (but less partial).

These results suggest to use the following 'simplified criterion':

Criterion 2 (simplified) *Given a sequence of SWQs* $(R_m)_{m=1}^{\infty}$*, a SWQ* R^0 *is a simplified criterion if* $\forall\, u, v \in \mathbb{R}^{\mathbb{N}}$*,* uP^0v *(resp.* uI^0v*) if and only if, for some* $M \subset \mathbb{N}$ *with* $|M| = m, u_M P_m v_M$ *and* $u_j \geq v_j$ *(resp.* $u_M I_m v_M$ *and* $u_j = v_j$*),* $\forall\, j \in \mathbb{N} \setminus M$*.*

The following result shows that, when the sequence $(R_m)_{m=2}^{\infty}$ is proliferating, then a simplified criterion can be used equivalently to the generalized criterion generated by this sequence.

Theorem 4 *Suppose* R^0 *is a generalized overtaking criterion generated by a proliferating sequence of SWQs* $(R_m)_{m=2}^{\infty}$*, with* R_2 *satisfying SP* and FA* restricted to* $\mathbb{R}^2$*. Then* R^0 *is a simplified criterion.*

Proof: Let $\hat{R}$ be a simplified criterion. That is: $\forall\, u, v \in \mathbb{R}^{\mathbb{N}}$, $u\hat{P}v$ (resp. $u\hat{I}v$) if and only if $u_M P_m v_M$ and $u_j \geq v_j$ (resp. $u_M I_m v_M$ and $u_j = v_j$), $\forall\, j \in \mathbb{N} \setminus M$, for some $M \subset \mathbb{N}$ with $|M| = m$. We want to show that, if $\hat{R}$ extends R_2, then $\forall\, u, v \in \mathbb{R}^{\mathbb{N}}$, $u\hat{R}v \Leftrightarrow uR^0v$. Since $\hat{R}$ extends R_2 and, by Lemma 3, satisfies SP* and FA*, we can apply Theorem 3 and so get that $\hat{R}$ is an extension of R^0. Now, suppose $u, v \in \mathbb{R}^{\mathbb{N}}$ are such that for some $M \subset \mathbb{N}$ with $|M| = m, u_M P_m v_M$ and $u_j \geq v_j$ resp. $u_M I_m v_M$ and $u_j = v_j$), $\forall\, j \in \mathbb{N} \setminus M$. Since R^0 extends R_m, we have uP^0v (resp. uI^0v). ■

This theorem provides a much simpler characterization of the generalized overtaking criterion associated to a proliferating sequence. We have seen how it can be used in the pure utilitarian case. As another example it can be applied to the Leximin overtaking criterion, say R^{LX} generated by the Leximin sequence $(R_m^{\ell x})_{m=2}^{\infty}$. We have the following simplified criterion:

$$\begin{aligned}&\forall\, u, v \in \mathbb{R}^{\mathbb{N}}, uR^{LX}v \text{ if and only if, for some } M \subset \mathbb{N}, |M| = m,\\ &u_M R_m^{\ell x} v_M \text{ and } u_j \geq v_j, \forall\, j \in \mathbb{N} \setminus M.\end{aligned}$$

By Theorem 3, a SWQ R defined on $\mathbb{R}^{\mathbb{N}}$ is an extension of R^{LX} if and only if it satisfies $R_2^{\ell x}$ (and hence satisfies SP* and FA*). As a result, R^{LX} is equivalent to the Leximin criterion defined by Bossert, Sprumong and Suzumura (2004), any extension of which is characterized by SP*, FA* and a two-person equity axiom, called Strict Equity Preference. This axiom is implied by Hammond Equity and SP* and, together with FA* implies the satisfaction of $R_2^{\ell x}$ (d'Aspremont, 1985, Theorem 3.4.2). To recall, Hammond Equity, as a condition on a SWQ R, is:

Hammond Equity (HE). For any pair $\{i,j\} \subset N$, if u and v in $R^{\mathbb{N}}$ are such that $u_j = v_j$ for all $j \in \mathbb{N} \setminus \{i,j\}$, and $v_i < u_i < v_j$, then uRv.*

Asheim and Tungodden (2004) propose two alternative Leximin criteria the extensions of which are characterized by SP*, FA*, Hammond Equity and, again, two alternative 'Preference Continuity' conditions. In our framework, both criteria extend $R_2^{\ell x}$ so that they are extensions of R^{LX} (but less partial).

There are many other examples of generalized overtaking criteria generated by proliferating sequences, since the proliferation property is a very common phenomenon among SWOs. To illustrate, we can show that pure utilitarianism can be generalized to a very large class of rules, the generalized pure utilitarian rules. The class is defined by:

Definition 5 (Generalized Pure m-Utilitarianism) The generalized pure utilitarian SWO on $\mathbb{R}^m$, denoted R_m^{gu}, is such that: for any $u, v \in \mathbb{R}^m$, $uR_m^{gu}v$ if and only if $\sum_{i=1}^m g(u_i) \geq \sum_{i=1}^m g(v_i)$, where the transformation g is a continuous and increasing real-valued function defined on $\mathbb{R}$.

There are as many rules as there are transformations g. All these rules have the proliferation property and can therefore be used to define overtaking sequences in order to generate generalized overtaking criteria, to which the overtaking theorem can be applied. For any given transformation g, we call $(R_m^{gu})_{m=2}^{\infty}$ the *generalized pure utilitarian sequence*. We then have:

Lemma 6 *The generalized pure utilitarian sequence $(R_m^{gu})_{m=2}^{\infty}$ is proliferating.*

Proof: The proof is similar to the one for pure utilitarianism. Suppose a SWQ R extentds $R_2^{gu}, R_3^{gu}, \cdots,$ and R_m^{gu}, we want to show that it extends R_{m+1}^{gu}. Again for simplicity, take

$M' = \{1, 2, \cdots, m\} \cup \{m+1\}$, and any $u, v \in \mathbb{R}^{\mathbb{N}}$ such that $u_i \geq v_i$, $\forall\, i \in \mathbb{N} \setminus M'$. Choose $w \in \mathbb{R}^{\mathbb{N}}$ such that $w_i = u_i$ for $1 \neq i \neq m+1$,

$$g(w_1) + g(w_{m+1}) = g(u_1) + g(u_{m+1}),$$

and $g(w_{m+1}) = g(v_{m+1})$. Since R extends R_2^{gu}, we have wIu. Also, since R extends R_m^{gu}, we get

$$\sum_{i=1}^{m+1} g(u_i) = \sum_{i=1}^{m+1} g(w_i) > \sum_{i=1}^{m+1} g(v_i) \Rightarrow \sum_{i=1}^{m} g(w_i) > \sum_{i=1}^{m} g(v_i) \Rightarrow wPv \Rightarrow uPv,$$

and uIv if the inequalities are replaced by equalities. ■

This class is characterized in Blackorby, Bossert and Donaldson (2002) in the finite, and in the variable, population case (for variants see Fleming, 1952, and Debreu, 1960).

The proliferating sequence $(R_m^{gu})_{m=2}^{\infty}$ can be used to characterize a generalized pure utilitarian criterion R^g for any transformation g, defined as a simplified criterion:

$$\forall u, v \in \mathbb{R}^{\mathbb{N}}, uR^g v \text{ if and only if, for some } M \subset \mathbb{N},$$
$$\sum_{i \in M} g(u_i) \geq \sum_{i \in M} g(v_i), \text{ and } u_j \geq v_j, \forall\, j \in \mathbb{N} \setminus M.$$

There are as many criteria as there are specifications of the transformation g, and these specifications depend on the additional conditions on wishes to impose. Various axiomatized specifications have been derived in the literature (see Blackorby, Bossert and Donaldson, 2002). We shall not review them here.

4 Conclusion

The characterizations we have derived for various SWOs over the infinite individual welfare evaluation space, all rely on a general overtaking criterion, and its simplified version, generated by some anonymous and strongly efficient SWQ that has the proliferating property. This property leads to the definition of an associated proliferating sequence the first element of which (satisfying strong Pareto and anonymity) plays the decisive role in the characterization. This first element can sometimes be interpreted as an equity axiom. This is notably the case of the

2-leximin (the first element in the lexicon proliferating sequence). But, this first element in the sequence can also be replaced by a set of other axioms that are known to be equivalent (from characterizations derived in the finite case). These characterizations, though, very often involve an 'invariance' condition, restricting the measurability and interpersonal comparability properties of individual evaluation profiles. Not all invariance conditions can be admissible. For example, already in the finite case, FA* and noncomparability, in the sense of invariance with respect to individual increasing, or simply positive affine, transformations, imply universal social indifference (excluding SP*). Such results are reviewed in d'Aspremont and Gevers (2002). The extension of an invariance condition to the infinite-horizon case is even more delicate. However, some characterizations do work as the results of Asheim and Tungodden (2004) and of Basu and Mitra (2003b), mentioned above, do show. This line of investigation should be pursued.

Here, the overtaking theorem and its applications have been presented in order to stress an important consequence of sticking to welfarism in choosing criteria for intergenerational justice: adopting a criterion in evaluating the welfare of the present generation (or a subgroup) forces use of the same criterion for any subset of subsequent generations. This can be viewed as a very restrictive consequence, since it means that the moral value judgements of the first generation have to be imposed to all the subsequent ones. But it can also be seen, more positively, as a 'time consistency' property. If the present generation solves, in some specific way, the social welfare evaluation problem, taking into consideration the social welfare evaluation of all future generations, this solution will be consistent with the solutions that future 'present generations' should advocate. This is analogous to Rawls' conception (1971, p. 287) that looks at the problem 'from the standpoint of the original position', where 'the parties do not know to which generation they belong'. Each generation solving the same problem (behind the veil of ignorance), and expecting the same solution being adopted by the other generations, has in fact correct expectations.

References

Arrow, K.J. (1951, 2nd ed. 1963), *Social Choice and Individual Values* (New York: Wiley).

Asheim, G.B. and Tungodden, B. (2004) 'Resolving distributional conflicts between generations', *Economic Theory*, 24, 221–230.

d'Aspremont, C. (1985) 'Axioms for social welfare orderings', in Hurwicz, L., Schmeidler, D. and Sonnenschein, H. (eds), *Social Goals an Social Organizations: Essays* (Cambridge: Cambridge University Press), 19–76.

d'Aspremont, C. and Gevers, L. (1977) 'Equity and the informational basis of collective choice', *Review of Economic Studies* 44, 199–209.

d'Aspremont, C. and Gevers, L. (2002) 'Social welfare functionals and interpersonal comparability', in Arrow, K., Sen, A. and Suzumura, K. (eds), *Handbook of Social Choice and Welfare*, vol. I (Amsterdam: Elsevier), 459–541.

Atsumi, H. (1965) 'Neoclassical growth and the efficient program of capital accumulation', *Review of Economic Studies*, 32, 127–136.

Basu, K. and Mitra, T. (2003a) 'Aggregating infinite utility streams with intergenerational equity: the impossibility of being Paretian', *Econometrica*, 71, 1557–1563.

Basu, K. and Mitra, T. (2003b) 'Utilitarianism for infinite utility streams: a new welfare criterion and its axiomatic characterization', Working Paper no. 03-05, Department of Economics, Cornell University.

Blackorby, C., Bossert, W. and Donaldson, D. (2002) 'Utilitarianism and the theory of justice', in Arrow, K., Sen, A. and Suzumura, K. (eds), *Handbook of Social Choice and Welfare*, vol. I (Amsterdam: Elsevier), 543–596.

Bossert, W., Sprumont, Y. and Suzumura, K. (2004) 'The possibility of ordering infinite utility streams', Cahier 12-2004, CIREQ.

Chakravarty, S. (1962) 'The existence of an optimum savings program', *Econometrica*, 30, 178–187.

Debreu, G. (1960) 'Topological methods in cardinal utility theory', in Arrow, K.J., Karlin, S. and Suppes, P. (eds), *Mathematical Methods in the Social Sciences* (Stanford: Stanford University Press), 16–26.

Diamond, P. (1965) 'The evaluation of infinite utility streams', *Econometrica*, 33, 170–177.

Dhillon, A. (1998) 'Extended Pareto rules and relative utilitarianism', *Social Choice and Welfare*, 15, 521–542.

Fleming, M. (1952) 'A cardinal concept of welfare', *Quarterly Journal of Economics*, 66, 366–384.

Fleurbaey, M. and Michel, P. (2003) 'Intertemporal equity and the extension of the Ramsey criterion', *Journal of Mathematical Economics*, 39, 777–802.

Gevers, L. (1979) 'On interpersonal comparability and social welfare orderings', *Econometrica*, 47, 75–89.

Hammond, P.J. (1976) 'Equity, Arrow's conditions, and Rawls' difference principle', *Econometrica*, 44, 793–804.

Hammond, P.J. (1979) 'Equity in two person situations: some consequences', *Econometrica*, 47, 1127–1135.

Kolm, S.C. (1972) *Justice et Equité* (Paris: CNRS).

Koopmans, T.C. (1960) 'Stationary ordinal utility and impatience', *Econometrica*, 28, 287–309.

Lauwers, L. (1998) 'Intertemporal objective functions. Strong Pareto versus anonymity', *Mathematical Social Sciences*, 35, 37–55.

Lauwers, L. and Van Liedekerke, L. (1995) 'Ultraproducts and aggregation', *Journal of Mathematical Economics*, 24, 217–237.

Mongin, P. and d'Aspremont, C. (1998) 'Utility theory and ethics', in Barbera, S., Hammond, P. and Seidl, C. (eds), *Handbook of Utility Theory*, vol. 1: Principles (Dordrecht: Kluwer), 371–481.

Ramsey, F. (1928) 'A mathematical theory of savings', *Economic Journal*, 38, 543–559.

Rawls, J. (1971) *A Theory of Justice* (Cambridge, MA: Harvard University Press).

Sen, A.K. (1970) *Collective Choice and Social Welfare* (San Francisco: Holden-Day).

Sen, A.K. (1976) 'Welfare inequalities and Rawlsian axiomatics', *Theory and Decision*, 7, 243–262.

Suppes, P. (1966) 'Some formal models of grading principles', *Synthèse*, 6, 284–306.

Svensson, L.-G. (1980) 'Equity among generations', *Econometrica*, 48, 1251–1256.

Szpilrajn, E. (1930) 'Sur l'extension de l'ordre partiel', *Fundamenta Mathematicae*, 16, 386–389.

Weymark, J.A. (1981) 'Generalized Gini inequality indices', *Mathematical Social Sciences*, 1, 409–430.

von Weizsäcker, C.C. (1965) 'Existence of optimal programs of accumulation for an infinite time horizon', *Review of Economic Studies*, 32, 85–104.