Contestability and the Indeterminacy of Free-Entry Equilibria *

Claude d'Aspremont[†], Rodolphe Dos Santos Ferreira[‡] and Louis-André Gérard-Varet[§]

Abstract

A general notion of market perfect contestability is introduced. It coincides with the definition given by Baumol *et al.* under Bertrand competition, but is compatible with Cournot competition as well as monopolistic competition. Using this notion, we largely indeterminate and different levels of positive profits may in many cases be either Cournot competition or product differentiation. Examples are given for both cases. Appropriate conditions of increasing returns are required.

JEL Classification Numbers: D5, D43.

1 Introduction

The role of potential entry in increasing competitive pressure in an industry seems to be an undisputable fact. The classical picture is that of a perfectly competitive industry where, as long as potential entrants have not all entered, the equilibrium price should tend to the minimum average cost and profits should be driven down to zero. With imperfect competition and the introduction of strategic considerations, the pressure is still there, but the conclusions are more ambiguous. Even in the first limit-price models (Bain, 1956; Sylos-Labini, 1956; Modigliani, 1958), using the so-called Sylos postulate, a fundamental asymmetry is introduced between

*Reprinted from *The Japanese Economic Review*, 51(1), 72–83, 2000 – Reprint 1452.

†CORE, Université catholique de Louvain

‡BETA, Université Louis Pasteur, Strasbourg

§GREQAM, Ecole des Hautes Etudes en Sciences Sociales, Marseille

two types of firm, the incumbents and the potential entrants. In those models the incumbents are given the possibility to choose an output (the “limit output”, determining the limit price) to deter entry, under the common postulate that entry would not change this chosen output. From a game-theoretic point of view, two main interpretations should be distinguished. In the first, the incumbents, as established firms, have the option of taking, before the entry decision, some actions that may be costly, and create sunk costs, in order to give more credibility to the threat of maintaining the chosen output post-entry. This first interpretation has given birth to an important literature on the role of commitment or reputation in entry deterrence, and on dealing with equilibrium selection in sequential games (for references, see Gilbert, 1989).

In the second interpretation, the incumbents and the potential entrants all choose their output simultaneously, each potential entrant taking as given the positive outputs of the incumbents (and of other potential entrants), and each incumbent taking as given the non-entry, or zero-output, decision of the potential entrants (and the positive outputs of other incumbents). In this second interpretation there is no possibility for the incumbents strategically to build up any kind of entry barrier, and the Sylos postulate is interpreted as a Cournot conjecture.

It is clearly for this second interpretation that one can make sense of the question of getting (at least approximately) a competitive outcome and the zero-profit conclusion under free entry. However, more symmetry conditions should be introduced to get what Bain calls “easy entry”:

> For easy entry, three conditions must in general be simultaneously fulfilled: ... (a) established firms have no absolute cost advantages over potential entrant firms; (b) established firms have no product differentiation advantages over potential entrant firms; (c) economies of large-scale firm are negligible, in the sense that the output of a firm of optimal (lowest-cost) scale is an insignificant fraction of total industry output. (Bain, 1956, pp. 11–12).

There are various ways, though, to formalize such conditions. As well noted by Novshek (1980), early attempts, taking the number of producing firms as exogenously fixed and then increasing the number for convergence to the competitive outcome, have the serious drawback of ruling out free entry (see Frank, 1965; Ruffin, 1971; Okugushi, 1973). One should, on the contrary, allow the number of producing firms to be determined endogenously and consider situations where the market size is large relative to the firm size. The notion of Cournot equilibrium with free entry was introduced for that purpose (Novshek and Sonnenschein, 1978; Novshek, 1980).

Still keeping the simultaneity of entrants – incumbents' decisions of the second interpretation, there is another concept – the "perfect contestability" concept[1] based on the notion of "sustainable industry configuration" – that can be seen as an alternative to the Sylos postulate and the "easy entry" notion of Bain. The two main differences consist, first, in postulating that potential entrants take as given the price (and no the total quantity) determined by the incumbents' decisions and, second, in allowing or sustainability increasing returns to scale. In other words, Bertrand competition is substituted for Cournot competition and Bain's condition (c) is abandoned. This is enough to get efficient outputs: either one firm is producing, charging the lowest price consistent with non-negative profits (the Ramsey optimum), or more firms are active, each producing the output for which both marginal cost and average cost are equal to price (first-best). Here, we present a generalized notion of perfect contestability (based on a generalized notion of sustainability[2], which will be applicable not only to Bertrand and Cournot competition, but also to monopolistic competition with product differentiation, allowing for the definition of an aggregate "sectoral" demand and respecting Bain's conditions (a) and (b). We will then contradict the common idea that, with increasing returns, free entry is uniquely associated with the maximum admissible number of active firms, and that positive profits may subsist only because the number of firms has to be an integer. Both in the case where firms have Cournot conjectures (Section 2) and in the case where they supply differentiated products (Section 3), perfect contestability is compatible with a multiplicity of free entry equilibria, with different numbers of active firms making positive profits.

[1] See Panzar and Willig (1977) and Baumol *et al.* (1977; 1982).

[2] This is in the line of Knieps and Vogelsang (1982) and Brock and Scheinkman (1983), who specifically use the notions of "price sustainability" and "quantity sustainability".

2 Generalized sustainability and contestability: Bertrand v. Cournot

We start by constructing a unified framework allowing inclusion of both the Baumol *et al.* (1982) notion of a "perfectly contestable market" and the Novshek (1980) concept of "free-entry Cournot equilibrium". In particular, we present an enlarged notion of perfect contestability allowing for product differentiation.

Consider a set of firms $J = \{1, \cdots, j, \cdots, N\}$. Each firm j produces a single product, called product j, in quantity $q_j \in [0, \infty)$, and announces a price $p_j \in [0, \infty]$. Prices and quantities are strategic variables. For each vector of announced prices $p \in [0, \infty]^J$, and for each quantity vector $q_{-j} \in [0, \infty)^{J\setminus\{j\}}$ produced by firms other than j, there is a demand, denoted $D_j(p_j, p_{-j}, q_{-j})$, which is the maximal amount firm j can sell. We assume that $\lim_{p_j \to \infty} p_j D_j(p, q_{-j} = 0$. These demand functions, which for the moment we take as given, will be seen to depend upon the aggregate customers' (competitive) behaviour on the one hand, and on the particular form of competition between firms on the other. There is also, for any firm j, a total cost function $C_j(q_j$, non-negative and defined for every $q_j \in [0, \infty)$.

Definition 1 *A feasible configuration is a triplet* (I, p, q) *where* $I \subset J, p \in [0, \infty]^J$*,* $q \in [0, \infty)^J$*, such that* $\forall\, i \in I$*,* $q_i = D_i(p, q_{-i} > 0$*, and* $\forall\, j \in J \setminus I, q_j = D_j(p, q_{-j}) = 0$*.*

In a feasible configuration, there are two groups of firms: the *active firms* (those in I), which are producing and serve demand, and the *inactive firms* (in $J \setminus I$). To be sustainable, a feasible configuration should be such that every active firm at least breaks even and no inactive firm would find it profitable to produce.

Definition 2 *A feasible configuration* (I, p, q) *is sustainable if,* $\forall\, i \in I, p_i q_i \geq C_i(q_i)$*, and* $\forall\, j \in J \setminus I$*,* $\forall\, (p'_j, q'_j) \in [0, \infty] \times [0, \infty), q'_j \leq D_j(p'_j, p_{-j}$ *sustainable* $q_{-j})$ *implies that* $p'_j q'_j - C_j(q'_j) \leq -C_j(0)$*.*

A sustainable configuration is a weaker concept than that of an equilibrium configuration where, in addition, each active firm is supposed to maximize profit.

Definition 3 *A feasible configuration* (I, p, q) *is an equilibrium if* $\forall\, j \in J, \forall\, (p'_j, q'_j) \in [0, \infty] \times [0, \infty), q'_j \leq D_j(p'_j, p_{-j},$ *equilibrium* $q_{-j})$ *implies that* $p'_j q'_j - C_j(q'_j) \leq p_j q_j - C_j(q_j)$.

Up to now, no assumption has established that the firms (as incumbents or potential entrants) belong to the same industrial sector. This can be formulated by introducing a generalized concept of perfect contestability, which combines three things: a condition of symmetry, the assumed existence of an aggregate "sectoral" demand and an infinite number of potential entrants. Existence of an aggregate sectoral demand reflects the opportunity for firms actually to compete inside the same industry, rather than simply to interact across segmented markets. On the other hand, symmetry ensures that active firms have no *a priori* advantage (to use Bain's words) over inactive firms, i.e. over potential entrants, and eliminates any product differentiation advantage.

The *aggregate sectoral demand*, describing aggregate customers' behaviour in the sector, will here be characterized taking as given the *price* and *quantity aggregates*, $\overline{P}$ and $\overline{Q}$, as symmetric and non-decreasing functions from $[0, \infty]^J$ to $[0, \infty]$. It is a continuous function $\overline{D}$ from $[0, \infty]$ into itself, decreasing (when positive) and such that, at any feasible configuration, the market clears.

Definition 4 *The set of firms* J *is said to form a perfectly contestable sector if:*

(i) $N = \infty$;

(ii) $\forall\, i, j \in J, D_i = D_j = D$ *and* $C_i = C_j = C$;

(iii) there are price and quantity aggregates $\overline{P}$ *and* $\overline{Q}$ *and an aggregate demand function* $\overline{D}$, *such that, for any feasible configuration* (I, p, q), *where* I *is a finite subset of active firms,* $\overline{Q}(q) = \overline{D}(\overline{P}(p))$.

Various types of competition can now be considered by specifying further the firms' demand functions as well as their price and quantity aggregates. The first one, Bertrand competition, is the only type considered in the Baumol *et al.* (1982) approach, where it is integrated to their definitions of "sustainability" and "perfect contestability".[3] It requires perfect substitutability in the sector (all producers produce the same) and the possibility of price undercutting.

Definition 5 *In a perfectly contestable sector J, with price and quantity aggregates $\overline{P}$ and $\overline{Q}$ and aggregate demand function $\overline{D}$, there is Bertrand competition if $\overline{Q}(q) = \sum_{j\in J} q_j$, $\overline{P}(p) = \min_{j\in J} p_j$, and $D_i(p_i, p_{-i}, q_{-i}) = (1/n)\overline{D}(p_i)$, for $i \in I = \arg\min_{j\in J} p_j$ $(n = \#I)$, nil otherwise.*

An equilibrium may then be called a *Bertrand equilibrium with free entry*, generalizing the notion of "long-run competitive equilibrium" under free entry.[4] Baumol (1982/1988 edn) have shown that, under perfect contestability and Bertrand competition, any sustainable configuration implies "zero profits" and either Pareto optimality (with marginal and average costs both equal to price), or Ramsey optimality (with a single producing firm).[5]

The second type of competition – Cournot competition – can be defined by specifying the same quantity aggregate (the sum of all quantities) and any price aggregate that can be expressed as a mean price, and by assuming a different relation between the individual firm demand and aggregate demand (as a residue instead of a constant share[6]). In fact, a larger class of price aggregates is admissible: those that are onto and satisfy $\overline{P}(x, x, \cdots) = x$, for any $x \in [0, \infty]$.

Definition 6 *In a perfectly contestable sector J, with price and quantity aggregates $\overline{P}$ and $\overline{Q}$ and aggregate demand function $\overline{D}$, there is Cournot competition if $\overline{Q}(p) = \sum_{j\in J} q_j$, $\overline{P}$ is a mean price and $D_j(p, q_{-j}) = \max\{\overline{D}(p_j) - \sum_{i\neq j} q_i, 0\}$, for every $j \in J$.*

[3] This integration was not immediately stressed: "No doubt because of lack of clarity in our earlier writings on this subject, the intended role of the Bertrand-Nash assumption in sustainability analysis has been widely misunderstood" (Baumol *et al.*, 1982/1988 edn, p. 11).

[4] In along-run competitive equilibrium under free entry, the price aggregate $\overline{P}$ is ai constant function such that, for all $x \in [0, \infty), \overline{P}x \leq C(x)$.

[5] Notice, however, that under Bertrand competition existence of a sustainable configuration is generally difficult to ensure, owing to the "integer problem". An example of such non-existence is when marginal cost is equal to average cost for a unique quantity of output (the efficient scale), but the total output demanded at the competitive price (the market size) is not an integer multiple of that scale.

[6] As is well known, in Cournot competition each firm behaves as a monopoly facing a "residual demand" (see d'Aspremont *et al.*, 1991). Of course, all firms choose the same price at equilibrium. This is already implied here by our definition of feasibility.

An equilibrium configuration should now be called a *Cournot equilibrium with free entry*.

It may also approximate the pure competitive output, and hence may give approximately zero profits. This property requires that firms be "small relative to the market". This is achieved by Novshek (1980), either on the supply side, by reduction of the *minimum efficient scale* of production $\theta = \inf\{\arg\inf_{x\in[0,\infty)} C(x)/x\}$ (the minimal output leading to minimum average cost), or on the demand side, by increasing the *market size* $\mu = \overline{D}(C(\theta)/\theta)$ (the aggregate demand at the competitive price, assumed positive). He has shown that, under perfect contestability and Cournot competition, any sustainable feasible configuration is such that the aggregate output necessarily belongs to the interval $[\mu - \theta, \mu]$, and hence is relatively close to the competitive output (equal to μ) for θ small or μ large, with the profit of each active firm close to 0. With a large efficient scale of production relative to market size (violating Bain's third condition), perfect contestability and sustainability are not, however, enough to get, even approximately, zero profits at the Cournot equilibrium (in contrast to the Bertrand case). Examples are U-shaped average cost functions (for a large efficient scale) or always-decreasing average cot functions[7] (leading to an infinite efficient size). Then, even with free entry, there might be a great multiplicity of equilibria. The following particular example illustrates this fact rather dramatically. It modifies Novshek's (1980) example B (generalized in B') but leads to a completely different conclusion. Novshek's example allows for two different conclusions: either the non-existence of a o*free-entry* equilibrium whatever the finite number of active firms, but the possible convergence of the *no-entry* equilibrium (i.e. with an exogenous number of active firms) to some non-competitive output; or the existence of a unique free-entry equilibrium[8] approximating the competitive outcome. In our example we obtain a third conclusion: existence of a *free-entry* equilibrium with *any* number of active firms and aggregate output bounded away from the

[7]The question of the convergence to the competitive equilibrium has been studied for this case by several authors: see Dasgupta and Ushio (1981), Frayssé and Moreaux (1981), Ushio (1983) or Frayssé (1986).

[8]In this case, though, there may be two equilibria, owing to the "integer problem". When the ratio of market size to firm size $\mu/\theta \in (n, n+1)$, where n is some integer, there is only one free-entry equilibrium with n active firms making positive (but small) profits. But when $\mu/\theta = n+1$, there is in addition a free-entry equilibrium with $n+1$ firms and zero profits.

competitive one.[9]

Example 1. The demand side is described by a linear inverse demand function $\Delta(\sum_{j\in J} q_j) = \max\{0, \alpha - \beta \sum_{j\in J} q_j\}$, $\alpha > 0$, $\beta > 0$, and the supply side by a quadratic cost function C exhibiting increasing returns to scale, at least up to some point $\overline{\theta} > 0$. This cost function is such that $C(0) = 0$ and

$$\begin{aligned} C(q_j) &= \kappa + \gamma q_j - \tfrac{\delta}{2} q_j^2, \quad && \text{if } q_j \in (0, \overline{\theta}] \\ &= c(q_j), && \text{if } q_j \in [\overline{\theta}, \infty), \end{aligned}$$

with $\kappa \geq 0, \gamma > 0, \delta > 0$, and c any non-decreasing function such that $c(\overline{\theta}) = \kappa + \gamma\overline{\theta} - (\delta/2)\overline{\theta}^2$. We assume: $\alpha > \gamma, 2\beta > \delta > \beta$, and $\gamma/\delta \geq \overline{\theta} \geq \alpha/2\delta$, with $\overline{\theta}$ smaller than or equal to θ, the minimum efficient scale as defined above, which may in fact be infinite. Correspondingly, the market size μ (which is also the competitive output) satisfies:

$$\mu = \frac{\alpha - c(\theta)/\theta}{\beta} \geq \frac{\alpha - \gamma}{\beta} + \frac{\delta}{2\beta}\overline{\theta} - \frac{\kappa}{\beta\overline{\theta}},$$

with equality when $\overline{\theta} = \theta$.

It is easy to verify that the profit of firm j is decreasing in q_j, for $q_j > \overline{\theta}$. Thus, given $\overline{q} = \sum_{i\neq j} q_i$, the profit function of firm j, $\pi(q_j, \overline{q})$ equal to $[(\alpha - \gamma - \beta(\overline{q} + q_j) + \delta q_j/2)q_j - \kappa]$, is maximized in the interval $0 \leq q_j \leq \min\{\overline{\theta}, \alpha/\beta - \overline{q}\}$ and it is strictly concave in q_j in this interval. Given the first-order condition of an interior maximum, $\alpha - \gamma - \beta\sum_{i\in J} q_i - (\beta - \delta)q_j = 0$, the solution is necessarily symmetric for all n active firms and equal to $q(n) = (\alpha-\gamma)/[(n+1)\beta-\delta] > 0$. For this solution to be an equilibrium configuration, the corresponding profits of the active firms should be non-negative; that is, $q(n)^2(\beta - \delta/2) \geq \kappa$. If $\kappa = 0$, this

[9] This is in contrast to the indeterminacy obtained in other examples. For instance, in the cases studied by Ushio (1983), the number of Cournot equilibria with free entry increases as the size of the market is increased, but all such equilibria are approximately competitive. The example analysed by Vickers (1989), with linear demand, constant marginal costs and fixed costs, is a two-stage model where part of the fixed costs may be sunk. The range for the Cournot equilibrium number of firms decreases when sunk costs are increased. (Since total fixed costs remain constant, few-firm equilibria are eliminated first). But even when the proportion of sunk costs is nil – the case comparable to ours, and already treated by others (see the references in fn. 7) – the indeterminacy could never be of the extreme kind described in our example.

inequality is strictly verified for any number of active firms, leading to positive equilibrium profits. Otherwise, n should not be larger than a number $\overline{n} = [(\alpha - \gamma)/\sqrt{(\beta\kappa)}]\sqrt{(1 - \delta/2\beta)} + \delta/\beta - 1$, and indeterminacy $\overline{n} \geq 2$.

Finally, sustainability is ensured whatever the number n of active firms. Indeed, no inactive firms j would increase its profit (which is zero) by producing some q_j (in the interval $0 < q_j \leq \min\{\overline{\theta}, \alpha/\beta - nq(n)\}$), (because its marginal profit is negative:

$$\alpha - \gamma - \beta nq(n) - (2\beta - \delta)q_j < \alpha - \gamma - \beta nq(n) = q(n)(\beta - \delta) < 0.$$

Here, the crucial assumption is $\delta > \beta$. If δ were positive but smaller than β, the number of active firms would have to be larger than some positive $\underline{n}$ (and κ to be positive) to ensure sustainability.[10] In the linear case ($\delta = 0 < \beta$), decreasing the slope β of the inverse demand function (as a consequence of market replication) or decreasing the fixed cost κ (and hence diminishing the degree of increasing returns) indefinitely increases the number of free-entry equilibria, by raising the difference $\overline{n} - \underline{n}$. However, both $\overline{n}$ and $\underline{n}$ tend to ∞ as β or κ tend to 0, and the ratio $\underline{n}q(\underline{q})/\overline{n}q(\overline{n})$ tends to 1, so that the indeterminacy of outcomes decreases (at least in relative terms).

In the present case, with $\delta > \beta$, we have $\underline{n} = 1$ and $\overline{n}$ tending alone to ∞ as κ tends to 0. We then obtain $\underline{n}q(\underline{n})/\overline{n}q(\overline{n})$ tending to $\beta/(2\beta - \delta)$ as κ tends to 0 (which is compatible with sustainability), so that indeterminacy of total output can be quite large in relative terms. Also, observe that total output $nq(n)$ is decreasing in n, so that the set of free-entry Cournot equilibrium outcomes for aggregate output is bounded away from the competitive outcome, since $\mu > q(1)$, as it is easy to verify (for $\kappa = 0$).[11] Finally, as $\pi(x, (n-1)x)$ is positive for $x = q(n)$ and decreasing for $x > (n)$, for an $n < \overline{n}$, equilibrium output is always less than the Ramsey optimal output in the case of zero fixed cost.[12]

[10] An easy calculation gives: $\underline{n} = (1 - \delta/\beta)[(\alpha - \gamma)/\sqrt{(2(2\beta - \delta)\kappa)}]$.

[11] Indeed, $\mu \geq (\alpha-\gamma)/\beta+(\delta/2\beta)\overline{\theta} > (\alpha-\gamma)/(2\beta-\delta) = q(1)$, if $(\alpha-\gamma)(\delta-\beta) <' \delta/2)(\alpha-\alpha\delta/2\beta) \leq (\delta/2)(2\beta-\delta)\overline{\theta}$, which corresponds to the parameter specifications.

[12] This example is robust. Neither has the demand function to be linear nor the cost function to be quadratic. A larger class of examples can be constructed, by imposing appropriate conditions on the inverse demand function

3 Contestability with product differentiation

In this section we introduce a third type of competition: monopolistic competition. This we do in a model of the Dixit and Stiglitz (1977) kind.[13]

Definition 7 *In a perfectly contestable sector J, with pricie and quantity aggregates $\overline{P}$ and $\overline{Q}$ and aggregate demand function $\overline{D}$ there is monopolistic competition if, for every $j \in J$, $p \in [0,\infty]^J$ and $q_{-j} \in [0,\infty)^{J\backslash\{j\}}$, we have $D_j(p_j, p_{-j}, q_{-j}) = D(p_j, \overline{P}(p))$, for some function $D(\cdot,\cdot) : [0,\infty]^2 \to [0,\infty]$, which is decreasing in p_j and satisfies $D(\infty, \overline{P}(\infty, p_{-j}) = 0$. By (iii) for perfect contestability, $\overline{Q}\{[D(p_j, \overline{P}(p))]_{j\in J}\} = \overline{D}(\overline{P}(p))$ should also hold.*

To illustrate, let us recall that, in the Dixit-Stiglitz (1977) example,[14]

$$\overline{S}((q_j)_{j\in J} = \left[\sum_{j\in J} q_j^{(s-1)/s}\right]^{s/(s-1)}, \quad \text{with } s > 1, \overline{P}(p) = \left[\sum_{j\in J} p_j^{1-s}\right]^{1/(1-s)}$$
$$\text{and } D(p_j, P) = [P/p_j]^s \overline{D}(P).$$

However, Dixit and Stiglitz (1977) use a simplified equilibrium concept,[15] introducing the convention that each firm j neglects its influence on the price aggregate $\overline{P}$ when selecting its own

(in order to obtain quasi-concavity of the profit function) and, while maintaining strict concavity of the cost function, by ensuring that this concavity is neither too large to preserve profitability of active firms, nor too small to entail sustainability.

[13]Perfect contestability, implying that any active firm is perfectly interchangeable with any inactive firm, is a way to respect Bain's condition (b). This would not be the case in a spatial (or address) model, where all established firms are symmetrically located but a potential entrant can only consider locating between two established firms, breaking the symmetry. See Eaton and Lipsey (1978) and Eaton (1989).

[14]As is well known, this demand can be derived from utility maximization by a representative consumer. Take the utility function $U(x_0, u(x))$, where $x \in [0,\infty)^J$ is the basket of goods produced in the monopolistic sector and $x_0 \in [0,\infty)$ is a composite numeraire good representing the rest of the economy. The function U is assumed homothetic and u is a symmetric CES function with elasticity of substitution s, hence coinciding with the quantity index $\overline{Q}$. In the case where U is a Cobb-Douglas function and the consumer's income is independent from prices in the monopolistic sector, one gets: $\overline{D}(P) = a/P, a > 0$.

[15]For a discussion of a variant of the Dixit-Stiglitz monopolistic competition model, see d'Aspremont *et al.* (1996).

price p_j. This is a good approximation in cases where the number of active firms is large enough. In the differentiated product case and under monopolistic competition, this convention plays a role analogous to the Bertrand conjecture in the homogeneous-good case. As a consequence, it is clear that any sustainable configuration is characterized by zero profits.[16] On the contrary, once the non-rational conjecture of taking $\overline{P}$ as given is abandoned, the zero-profit conclusion can no longer be generally ensured, as the following example demonstrates.

Example 2. Take the Dixit-Stiglitz (1977) specification of $\overline{D}, \overline{P}, \overline{Q}$ and $D(\cdot, P)$ in the Cobb-Douglas case, i.e. $D(p_j, \overline{P}(p)) = ap_j^{-s}\overline{P}(p)^{s-1}$, with $\overline{P}(p) = [\sum_{j\in J} p_j^{1-s}]^{1/(1-s)}$. Assume the cost function to be $C(q_j) = cq_j^{\gamma_j}$, $c > 0$ and $0 < \gamma < 1 - 1/s$.

Under perfect contestability, a symmetric feasible configuration (I, p, q), with $\#I = n < \infty$ and $\forall\, i \in I$, $p_i = \overline{p}$, $q_i = \overline{q}$, implies positive profits for the active firms if $\overline{pq} - c\overline{q}^{\gamma} > 0$ and $\overline{q} = a/n\overline{p}$, or $c^{-1}a^{1-\gamma}n^{\gamma-1}\overline{p}^{\gamma} > 1$. It is sustainable if, for $(\tilde{p}, \tilde{q}) \geq 0$ such that $\tilde{q} \leq a\tilde{p}^{-s}(\tilde{p}^{1_s} + n\overline{p}^{1_s})^{-1}$, we have $\tilde{p}\tilde{q} - c\tilde{q}^{\gamma} \leq 0$; that is, $c\alpha^{\gamma-1}\tilde{p}^{s(1-\gamma)-1}(\tilde{p}^{1-s} + n\overline{p}^{1_s})^{1-\gamma} \geq 1$, for any $\tilde{p} \geq 0$, or

$$\forall\, \tilde{p} \geq 0, \quad c^{-1}a1 - \gamma n^{\gamma-1}\overline{p}^{\gamma} \leq F(\tilde{p}, \overline{p}, n) \equiv n^{\gamma-1}\overline{p}^{\gamma}\tilde{p}^{s(1-\gamma)-1}(\tilde{p}^{1-s} + n\overline{p}^{1-s})^{1-\gamma}.$$

Taking the two conditions (for sustainability and profit positivity) together, we get: $1 < c^{-1}a^{1-\gamma}n^{\gamma-1}\overline{p}^{\gamma} \leq \inf_{\tilde{p}\geq 0} F(\tilde{p}, \overline{p}, n)$; so we need $\inf_{\tilde{p}\geq 0} F(\tilde{p}, \overline{p}, n) > 1$. This requires that $s(1-\gamma) > 1$, implying $\lim_{\tilde{p}\to 0} F(\tilde{p}, \overline{p}, n) = \lim_{\tilde{p}\to\infty} F(\tilde{p}, \overline{p}, n) = \infty$. Also,

$$\frac{\partial F(\tilde{p}, \overline{p}, n)}{\partial \tilde{p}} \frac{\tilde{p}}{F(\tilde{p}, \overline{p}, n)} = s(1-\gamma) - 1 - (1-\gamma)(s-1)\frac{\tilde{p}^{1-s}}{\tilde{p}^{1-s} + n\overline{p}^{1-s}}.$$

This elasticity is increasing in $\overline{p}$ and takes the value 0 for $\tilde{p} = (bn)^{1/(1_s)}\overline{p}$, where $b = [s(1-\gamma) - 1]/\gamma$. So,

$$\begin{aligned} \inf_{\tilde{p}\geq 0} F(\tilde{p}, \overline{p}, n) &= [(1+b)^{-(1+1/b)}bn]^{b\gamma/(1-s)} > 1 \\ &\Leftrightarrow nw(1+1/b)(1+b)^{1/b}. \end{aligned}$$

[16]Suppose that, at a sustainable configuration with $I \neq J$, firm $i \in I$ has positive profit $p_j D(p_j, \overline{P}(p)) - C[D(p_j, \overline{P}(p))] > 0$. By the symmetry assumed through perfect contestability, any inactive firm $j \in J \setminus I$, neglecting its influence on $\overline{P}$, anticipates the same positive profit by deviating to $p_j = p_i$. This contradicts sustainability.

Since the right-hand side of this last inequality is decreasing in b, going from ∞ to 1 as b goes from 0 to ∞, positive profits are unsustainable in the limit case $b = \infty$, that is if either the monopolistic goods are perfectly substitutable ($s = \infty$, the Bertrand case) or the costs are reduced to a fixed amount ($\gamma = 0$). On the contrary, if $b < \infty$ ($s < \infty$ and $\gamma > 0$), positive profits are compatible with sustainability, at least for $n = 1$. As $b \to 0$ ($\gamma \to 1 - 1/s$), positive profits are compatible with sustainability for almost any n.

Consider now the symmetric equilibrium conditions (more precisely, the profit non-negativity and the first-order conditions)[17]

$$\overline{p}\gamma \geq \overline{p}\left[\frac{1}{(1/n) + (1 - 1/n)s}\right] = c\gamma\left[\frac{a}{n\overline{p}}\right]^{\gamma-1}.$$

The inequality (profit non-negativity) is equivalent to $n \leq 1 + 1/b \equiv \overline{n}$, itself equivalent, when strict, to the first condition $c^{-1}a^{1-\gamma}n^{\gamma-1}\overline{p}^{\gamma} > 1$ when $\overline{p}$ is the equilibrium price (satisfying the first-order equality):

$$\overline{p} = \left(c\gamma\frac{(n-1)s+1}{(n-1)(s-1)}\right)^{1/\gamma}\left(\frac{a}{n}\right)^{1-1/\gamma}.$$

Moreover, plugging this same equilibrium price in the other condition,

$$c^{-1}a^{1-\gamma}n^{\gamma-1}\overline{p}^{\gamma} \leq \inf_{\tilde{p}\geq 0} F(\tilde{p}, \overline{p}, n) = [(1+b)^{-(1+1/b)}bn]^{b\gamma/(1-s)},$$

we finally get:

$$(b+s)(1+b)^{1+b/b+s}b^{-b/b+s}n^{-b/b+s} \geq s + 1/(n-1),$$

an inequality satisfied for $n \geq \underline{1}$. (Notice that the elasticity with respect to n of the left-hand side, $-b/(b+s)$, is larger than the one of the right-hand side for $n \leq \overline{n}$; that is, $b \leq 1/(n-1)$.) A sufficient condition for multiplicity of equilibria with n active firms ($\underline{n} \leq\leq \overline{n}$) is that $\overline{n} - \underline{n} \geq 2$, implying $\overline{n} > 3$ (hence $b < 1/2$, so that $1/(1-\gamma) < s < (1+\gamma/2)/(1-\gamma)$). In order to evaluate the extent of indeterminacy in the outcomes, we may calculate the ratio of quantity aggregates for the extreme cases $n = \overline{n}$ and $= \underline{n}$ (notice, however that $\overline{n}$ and $\underline{n}$ as above defined are not

[17] These conditions are necessary and sufficient for a symmetric equilibrium: see d'Aspremont *et al.* (1995, Proposition 2).

necessarily integers, so that we are in fact calculating an upper bound of the real ratio):

$$\begin{aligned}\frac{Q_{\overline{n}}}{Q_{\underline{n}}} &= \frac{\overline{D}(P_{\overline{n}})}{\overline{D}(P_{\underline{n}})} = \frac{P_{\underline{n}}}{P_{\overline{n}}} = \begin{pmatrix} \overline{n} \\ \\ \underline{n} \end{pmatrix}^{1/(s-1)+1-1/\gamma} \left(\frac{s+1/(\underline{n}-1)}{s+1/\overline{n}-1)}\right)^{1/\gamma} \\ &= \begin{pmatrix} \overline{n} \\ \\ \underline{n} \end{pmatrix}^{-b/s-1} \left(\frac{(b+1)(1+b)^{1+b/b+s}b^{-b/b+s}\underline{n}^{-b/b+s}}{b+s}\right)^{b+s/s-1} = (1+b)^{1/s-1}.\end{aligned}$$

To give a numerical illustration, we may put $\gamma = 0.8$ and $s = 5.3$ (hence $b = 0.05$), and then we have a multiplicity of monopolistic competition equilibria with free entry and positive profits for n active firms, one equilibrium for every n in the interval $8 \leq n \leq 20$; for $n = 21$, the equilibrium exists but profits are zero. The ratio of quantity aggregates for the extreme cases $n = 8$ and $n = 21$ is approximately equal to 1.01. Now, decreasing both the elasticity of the cost function and the elasticity of substitution, and putting $\gamma = 0.14$ and $s = 1.2$ (hence $b \approx 0.23$), we get only three equilibria for n in the interval $3 \leq n \leq 5$, all with positive profits. However, the ratio of quantity aggregates for the extreme cases $n = 3$ and $n = 5$ is now approximately equal to 1.74.

In their example, Dixit and Stiglitz (1977) use another cost function, a linear one with a fixed component. We show that this is enough to get a unique equilibrium with approximately zero profits.

Example 3. Consider (under perfect contestability and monopolistic competition) the same demand specification as in Example 2. Suppose that the cost function is linear: $C(q_j) = c + \gamma q_j$. The first-order condition for an equilibrium configuraiton, with $\#I = n < \infty$, is given by $\overline{p} = \gamma[1 + (n-1)s]/(n-1)(s-1)$. Then imposing positive profits for the active firms, i.e. $a/n - c - \gamma a/n\overline{p} > 0$, reduces to $n < a/sc + 1 - 1/s$. Sustainability, requiring

$$a\frac{\tilde{p}^{1-s}}{\tilde{p}^{1-s} + n\overline{p}^{1-s}} - c - \gamma a\frac{\tilde{p}^{-s}}{\tilde{p}^{1-s} + n\overline{p}^{1-s}} \leq 0$$

for every $\tilde{p} \geq 0$, amounts to $0 \leq cn(\tilde{p}/\overline{p})^{s-1} + \gamma a\tilde{p}^{-1} + c - a \equiv G(\tilde{p}, \overline{p}, n)$. But $\lim_{\tilde{p}\to\infty} G(\tilde{p}, \overline{p}, n) = \infty = \lim_{\tilde{p}\to 0} G(\tilde{p}, \overline{p}, n)$, and $G(\cdot, \overline{p}, n)$ has a minimum at a critical point, namely at $\tilde{p} =$

$[\gamma a\overline{p}^{s-1}/(cn(s-1))]^{1/s}$, leading to the sustainability condition:

$$\inf_{\tilde{p}\geq 0} G(\tilde{p},\overline{p},n) = (cn)^{1/s}a^{1-1/s}s\left(s+\frac{1}{n-1}\right)^{1/s-1} + c - a \geq 0,$$

or

$$n \geq [1+1/(n-1)s]^{s-1}(1-c/a)^s a/sc.$$

Letting $A \equiv (1/s)[(a/c)-1]$, this weak inequality and positive profit give:

$$A\left(\frac{1+1/[(n-1)s]}{1+1/As}\right)^{s-1} \leq n < A+1.$$

Positive profits ($A > n-1$) imply that the term in large parentheses is strictly larger than 1, so that $n > A$. Therefore the combination of the two conditions implies that $A < n < A+1$. There is in general a determinate equilibrium with free entry entailing small profits. An exception is the case where A is an integer, leading to two equilibria, one with $n = A+1$ and zero profits, the other with $n = A$ and, again, small positive profits.

4 Conclusion

It seems clear enough, now, now wrong it is to believe that free entry is uniquely associated with the maximum number of active firms that can be accommodated under increasing returns to scale, and that profits may remain positive only because of the so-called “integer problem”. As we have seen, the number of active firms in free-entry equilibrium may instead be indeterminate, and different levels of positive profits in many cases may be sustained.

This is true in spite of perfect market contestability in our generalized sense, either in the case where firms have Cournot conjectures about their competitors’ behaviour or in the case where they supply differentiated products. Examples have been given for both cases. Appropriate conditions of increasing returns are required, in particular strict concavity of the cost functions in the second case.

This basic intederminacy of the equilibrium number of active firms under free entry, together with the corresponding sustainability of positive profits, has usually been concealed by the restriction to Bertrand conjectures with product homogeneity, or else by the neglect of the impact of individual price deviations on the aggregate price, as advocated by Dixit and Stiglitz.

Final version accepted 18 January 1988.

References

Bain, J.S. (1956). *Barriers to New Competition*. Cambridge, Mass.: Harvard University Press.

Baumol, W.J., Bailey, E. and R.D. Willig (1977). Weak invisible hand theorems on the sustainability of prices in a multiproduct monopoly. *American Economic Review*, 67, 350–365.

Baumol, W.J., Panzar J.C. and R.D. Willig (1982; rev. edn 1988). *Contestable Markets and the Theory of Industrial Structure*. San Diego: Harcourt Brace Jovanovich.

Baumol, W.J.,

Blanchard, Olivier Jean and Kiyotaki, Nobuhiro. Monopolistic competition and the effects of aggregate demand. *American Economic Review*, September 1987, 77(4), 646–666.

Dixit, Avinash K. and Stiglitz, Joseph E. "Monopolistic competition and optimum product diversity." *American Economic Review*, June 1977, 67(3), 297–308.

Dixit, Avinash K. and Stiglitz, Joseph E. "Monopolistic competition and optimum product diversity: reply." *American Economic Review*, March 1993, 83(1), 302-304.

Negishi, Takashi. "Monopolistic competition and general equilibrium". *Review of Economic Studies*, June 1961, 28, 196–201.

Nikaido, Hukukane. *Monopolistic Competition and Effective Demand*. Princeton: Princeton University Press, 1975.

Weitzman, Martin J. "The simple macroeconomics of profit sharing." *American Economic Review*, December 1985, 75(5), 937–953.

Yang, Xiaokai and Heijdra, Ben J. “Monopolistic competition and optimum product diversity: comment.” *American Economic Review*, March 1993, 83(1), 295–301.