Intergenerational Equity in a Model of Endogenous Growth*

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1 Introduction

The main difficulties in trying to derive and apply social welfare criteria, such as utilitarianism or maximin, to the problem of intergenerational justice come from time-irreversibility. Future generations cannot compensate for a sacrifice made by earlier ones, or ask for just sharing of past resources that were wasted. There is a possible altruistic approach to intergenerational equity. However, it requires that the altruism of future generations be secured. In any case, intergenerational equity is not only a matter of gifts from present to future generations. It is also an issue of 'retro-gifts' which have to be socially contracted. In economic terms, this is translated into the problem of finding a just rate of savings, taking into account both the well-being of the present generation and the productivity of capital for future generations and considering the impossibility of *ex post* transfers from future generations to present ones.

This is recognized, for example, by Rawls (1971) in declaring the maximin criterion unsuitable for determining the just rate of savings: 'How the burden of capital accumulation and of raising the standard of civilization is to be shared between generations seems to admit of no definite answer' (1971, p. 286). As well analyzed by Arrow (1973) and Solow (1974) in simple

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growth models,[1] the criterion is inapplicable because it would imply no saving at all (with constant population and no technical progress). With a growing population there is some saving, but just enough to compensate for the increase in population and maintain constant consumption. Technical progress may be used to save less, or even to consume capital. The introduction of exhaustible resources does not change Solow's main conclusion: the maximin criterion is too much at the mercy of the initial conditions. If this means poverty, then poverty is perpetuated. Dasgupta (1974) reaches a similar negative conclusion about the maximin in a model where any generation shows some concern towards the next, and examines the question of what should be an alternative Rawlsian principle of just savings. He proposes to regard this principle as being an intergenerational Nash equilibrium, and shows that it fails to be Pareto-efficient. This lack of efficiency is certainly not in accordance with Rawls' view.

Rawls himself (1971) makes another suggestion based on his own conception of the 'original position'. The original position is a hypothetical negotiation involving 'parties' who are contemporaries and should forget, behind a 'veil of ignorance', the specific features of their own situation and, in particular, to which generation they belong. All possible people belonging to all future generations are only 'virtually" represented in the negotiation. Since they do not know to which generation they belong, 'the persons in the original position are to ask themselves how much they would be willing to save at each stage of advance on the assumption that all other generations are to save at the same rates' (Rawls, 1971, p. 287). This is Rawls's principle of just savings, which, according to him, should be added to the two principles that already have in his theory precedence over the maximin principle (namely, the principles of equal liberty and fair opportunity, regulating respectively the allocation of the first two categories of primary goods). This savings principle is to be understood as a 'principle of recursivity'. The original negotiation can take place at any point in time and should end up with a similar decision.

It is this notion of recursivity that we should like to develop here in looking for a principle of just savings, first in general social welfare terms and then, for illustration, in a simple overlapping

[1]It is well-known that Rawls' maximin is in terms of primary goods and not in utility terms. The maximin criterion in utilitarian terms has been discussed in Kolm (1971).

generations model. One of our main concerns will be to check the compatibility of a savings principle based on recursivity and Pareto efficiency.

In the overlapping generations model, as developed by Allais (1947), Samuelson (1958) and Diamond (1965), savings are generated by individuals living over two periods. With production (as in Diamond, 1965), the capital stock is generated by individuals savings during their working lives to finance their consumption when they retire. An advantage of this approach is to root the concern of people living in one period for those in the next into the concern of the young generation for their own old age. In Rawls, the concern of the people in one generation for their immediate successors is obtained by regarding the parties in the original position as representing family lines: 'a generation cares for its immediate descendants, as fathers, say, care for their sons' (1971, p. 288). In the overlapping generations model no altruism, even so limited, needs to be introduced in that respect. Each generation cares for itself, not about future generations, and the preferences of the generations yet unborn are not registered in the current transactions. Of course, this raises the problem of intergenerational equity. To solve this problem, Samuelson (1958) postulates stationary individual consumption, including both young- and old-age consumption. We argue below that this postulate of stationary consumption, as a way to introduce intergenerational equity, is a particular instance of the principle of recursivity. Samuelson shows, in a model of exponential population growth without capital, that the 'consumption golden rule' among different generations maximizing a representative lifetime utility is that in which each generation faces an interest rate given by the biological rate of population growth. Diamond (1965), adding a productive (neoclassical) capital to the model, has shown that along with the consumption golden rule, maximization of the representative generation utility requires the Phelps (1961) 'production golden rule', in which the marginal productivity of capital is equal to the rate of population growth. This is the proposed way to combine intergenerational equity (stationary consumption) with intertemporal efficiency.

Lerner (1959) objected to the Samuelson-Diamond representative generation biological optimum on ethical grounds. The stationarity of consumption can still be postulated. But, maximizing a representative lifetime utility should be replaced by maximizing the sum of utilities

of all individuals living in the same period, leading to the conclusion that this optimum would be achieved for a capital-less model by a zero interest rate. However, even with capital in the model, a zero interest rate is required for the optimal Lerner consumption allocation when, in terms of production, marginal productivity of capital must, in a golden rule, equal population growth. A consequence is that Lerner's proposal is not compatible with intertemporal efficiency. The prevailing point of view is, finally, that the Samuelson-Diamond approach is providing the relevant criterion, namely the steady-state level of capital satisfying the 'golden rule'. However, this point of view has been put in question by the recent developments on endogenous growth (Romer, 1986; Lucas, 1988). Long-run growth should no longer be considered in terms of a steady-state level of capital (*per capita*, if population is growing). It should be seen in terms of *per capita* growth rates. Could an alternative criterion of intergenerational equity be found justifying a positive *per capita* growth rate? Could this still be compatible, both with a principle of recursivity and with Pareto efficiency?

These are the questions we want to address. In Section 2, we examine, in general welfare terms, various definitions of efficiency criteria and their logical relations, as well as two ways to formulate the principle of recursivity as a principle of intergenerational equity, introducing the idea of individuals living for two periods. In Section 3, we recall the Samuelson-Lerner debate as it can be formulated in the overlapping generations model with production, and show in this model the compatibility of efficiency and intergenerational equity. Finally, in Section 4, a unique positive *per capita* growth rate, satisfying both efficiency and a proportional equity criterion, is derived in an example with endogenous growth.

2 Some intergenerational welfare principles

In order to state some welfare principles in general terms, we introduce an abstract framework. Time is discrete and takes on non-negative integer values. Each generation lives for two periods. All agents have the same tastes. At each date t, the set of all individuals I_t alive in period t is partitioned into two subsets, N_t the set of young individuals born at date t; and N_{t-1} the set

of old individuals born at date $t-1$. Every individual i has a *compound* utility function over their life-cycle $U_i(c_{it}, d_{it+1})$, with c_{it} denoting their utility when young (in period t), and d_{it+1} their utility when old (in period $t+1$). U_i is strictly increasing in both its arguments. For each period t, we let u_t denote the profile of all individual utility levels at period t:

$$u_t = (u_{it})_{i \in I_t} = ((c_{it})_{i \in N_t}, (d_{it})_{i \in N_{t-1}}) = (\tilde{c}_t, \tilde{d}_t).$$

A 'utility path' over time is a sequence $u = (u_t)_{t \geq 0}$. Without loss of generality, we suppose $u_t \geq 0$, and the set of *feasible* utility paths is supposed to be given and denoted $\mathcal{U}$. In order to select among feasible utility paths, we want to introduce two types of welfare principles: efficiency principles and equity principles. Let us start by defining principles of the first kind.

A feasible utility path $u \in \mathcal{U}$ is said to be *intertemporally Pareto-efficient* if no other feasible path $u' \in \mathcal{U}$ could satisfy (with at least one inequality strict):

$$U_i(c'_{it}, d'_{it+1}) \geq U_i(c_{it}, d_{it+1}), \text{ for all } i \in N_t,$$
$$d'_{it} \geq d_{it}, \text{ for all } i \in N_{t-1},$$

at *all* periods $t \geq 0$. Such a property might be difficult to verify (both theoretically and practically), since Pareto-domination in this sense may involve positive changes in an infinite number of periods concerning all generations, past, present and future. But this efficiency criterion implies two weaker concepts, easier to verify, and having the conceptual advantage of involving only presently-living individuals.

The first, in the line of Lerner's optimization programme, concentrates on a single period t and focuses on the redistribution among types of individuals, the young and the old, living together in period t. It is a static concept of efficiency. A feasible utility path $u \in \mathcal{U}$ is said to be *period by period Pareto-efficient* if no other feasible path $u' \in \mathcal{U}$ could satisfy (with at least one inequality strict):

$$u'_{it'} \geq u_{it'}, \text{ for all } i \in I_{t'}, \text{ at } \textit{some} \text{ period } t' \geq 0,$$
$$u'_{it} = u_{it}, \text{ for all } i \in I_t, \text{ at all other periods } t \neq t'.$$

The second concept of efficiency is in the line of Samuelson's optimization programme and focuses on the life-cycle utility of each individual. A feasibility utility path $u \in \mathcal{U}$ is said to be

Pareto-efficient over the life-cycle if no other feasible path $u' \in \mathcal{U}$ could satisfy (with at least one inequality strict):

$$U_i(c'_{it}, d'_{it'+1}) \geq U_i(c_{it'}, d_{it'+1}), \text{ for all } i \in N_{t'}, \text{ at } \textit{some} \text{ period } t' \geq 0,$$
$$(c_{it}, d_{it+1}) = (c'_{it}, d'_{it+1}), \text{ for all } i \in N_t \text{ at all other periods } t \neq t'.$$

This second concept is more interesting than the first, since, under some regularity conditions, it implies Pareto efficiency in the stronger (intertemporal) sense. The set $\mathcal{U}$ is said to be *comprehensive* if, whenever $u \in \mathcal{U}$ and $0 \leq u' \leq u$, then $u' \in \mathcal{U}$. It is said to be *convex over the life-cycle* if, whenever u and u' in $\mathcal{U}$ are such that $(c'_{it'}, d'_{it'+1}) \neq (c_{it'}, d_{it'+1})$, for some $i \in N_{t'}$ at some period $t' \geq 0$, and $(c_{it}, d_{it+1}) = (c'_{it}, d'_{it+1})$, for all $i \in N_t$ at all other periods $t \neq t'$, then any convex combination of u and u' is also in $\mathcal{U}$.

Lemma 1 *If the set of feasible utility paths $\mathcal{U}$ is convex over the life-cycle and comprehensive and if the utility functions $\{U_i\}$ are strictly quasi-concave, then $u \in \mathcal{U}$ is Pareto-efficient over the life-cycle if, and only if, it is intertemporally Pareto-efficient.*

Proof: The 'if' part follows by definition. To prove the 'only if' part, suppose that $u \in \mathcal{U}$ is Pareto-efficient over the life-cycle but not intertemporally. Then using the assumption that $\mathcal{U}$ is comprehensive, there is $u' \in \mathcal{U}, t' \geq 0$ and $j \in N_{t'}$ such that $d'_{jt'+1} > d_{jt'+1}$ but $U_j(c_{jt'}, d_{jt'+1}) = U_j(c'_{jt'}, d'_{jt'+1})$, and $(c_{it}, d_{it+1}) = c'_{it}, d'_{it+1})$ for all $i \in N_t$, at all periods t (and $i \neq j$, when $t = t'$). Since $\mathcal{U}$ is convex over the life-cycle, we may take a convex combination of u and u', say $\overline{u} \in U$, and we obtain:

$$U_j(\overline{c}_{jt'}, \overline{d}_{jt'+1}) > U_j(c_{jt'}, d_{jt'+1}),$$

by strict quasi-concavity of U_j, contradicting Pareto efficiency over the life-cycle. ■

In addition to these efficiency conditions, let us now introduce some concern about distributive justice and propose some equity principles. Adopting first a static point of view, and following standard practice in welfare economics, we use a 'social evaluation function' as an instrument to evaluate the aggregate welfare of a given population (or sub-population) of contemporaneous individuals. Let, for any population S of individuals, E_+^S denote the corresponding utility space,

that is, the set of all vectors $u_S = (u_i)_{i \in S}$, with $u_i \geq 0$. A *social evaluation function* for this population S is a real-valued function $W(S, u_S)$ defined on E_+^S. We assume this function to be strictly increasing (to ensure Pareto efficiency) and symmetric (as a basic equity condition) in each argument. This last condition ensures 'anonymity' in the sense that the value of $W(S, u_S)$ should not be affected by a permutation of the components of u_s. One may add, as a stronger equity condition, that the function $W(S, \cdot)$ be S-concave: for any $|S| \times |S|$-matrix Q, which is doubly stochastic (that is, having all elements between 0 and 1, and each line or column summing to 1), for any u_S in E_+^S,

$$W(S, Qu_S) \geq W(S, u_S).$$

It is well known that this property ensures that any transfer from a relatively better-off individual to a relatively worse-off individual, without changing their rank, is not welfare-deteriorating.[2] We also require that $W(S, \cdot)$ be continuous[3] and homogeneous[4] in u_s, allowing us to define, for every $u_S \in E_+^S$, the notion of *equally distributed equivalent utility level* – that is, the utility level $w(S, u_S)$ satisfying

$$W(S, w(S, u_S), \cdots, w(S, u_S)) = W(S, u_S).$$

When this utility level is given to every individual in S, the resulting egalitarian utility vector is equivalent (in terms of social welfare) to the vector, u_S. It can be used as the welfare level that can be attributed to a *representative* member of group S. By homogeneity,

$$w(S, u_S) = \frac{W(S, u_S)}{W(S, 1_S)},$$

with $1_S = (1, 1, \cdots, 1) \in E_+^S$. In the application to specific growth models developed in the next section, we shall stick to the simplest rule, the average utilitarian rule: for any population S,

[2]This is the weak version of the Pigou-Dalton Principle. In particular, it is weak since it is compatible with the utilitarian rule (which we shall use in this average form).

[3]Continuity excludes the Rawlsian criterion of leximin (the lexicographic completion of maximin). But, by introducing sufficient concavity, the Rawlsian criterion can be approximated (see Moulin, 1988).

[4]Or, to be more general, we could require only that it be homothetic (a monotone increasing transformation of some homogeneous W), by requiring ratio-scale invariance of the underlying social welfare ordering (invariance with respect to linear transformations of the u_S).

and for every $u_S \in E_+^S$,

$$W(S, u_S) = w(S, u_S) = \frac{\sum_{i \in S} u_i}{|S|}.$$

However, the two alternative equity conditions we shall use to make intergenerational welfare comparisons (and so restrict utility paths), may be stated now in more general terms, for any well-defined representative individual utility function $w(S, u_S)$.

The first principle requires to treat all generations equally by imposing stationarity of the welfare level of the representative agent of each group, the young and the old.

Principle of intergenerational equal treatment

A utility path $u \in \mathcal{U}$, with $u_t = ((c_{it})_{i \in N_t}, (d_{it})_{i \in N_{t-1}}) = (\tilde{c}_t, \tilde{d}_t)$, satisfies *intergenerational equal treatment* with respect to the representative individual utility function w, if, for every $t \geq 0$,

$$w(N_t, \tilde{c}_t) = w(N_{t+1}, \tilde{c}_{t+1}),$$
$$w(N_{t-1}, \tilde{d}_t) = w(N_t, \tilde{d}_{t+1}).$$

This principle will be associated with the Samuelson-Diamond theory of growth and its 'golden rule' stead-state level of capital.

The second principle requires that all generations are treated equally only in relative terms. It will be associated with the Romer-Lucas endogenous growth model, and used to justify a positive *per capita* growth rate.

Principle of intergenerational equiproportionality

A utility path $u \in \mathcal{U}$, with $u_t = ((c_{it})_{i \in N_t} (d_{it})_{i \in N_{t-1}}) = (\tilde{c}_t, \tilde{d}_t)$, satisfies *intergenerational equiproportionality* with respect to the representative individual utility function w, if, for every $t \geq 0$,

$$\frac{w(N_{t-1}, \tilde{d}_t)}{w(I_t, \tilde{c}_t, \tilde{d}_t)} = \frac{w(N_t, \tilde{d}_{t+1})}{w(I_{t+1}, \tilde{c}_{t+1}, \tilde{d}_{t+1})}.$$

Under this principle, it is the *relative* welfare of the representative of the old (relative to the welfare of the representative individual of society as a whole) that should be maintained across generations.

We shall now proceed to apply these conditions to specific growth models.

3 Intergenerational equal treatment

Let us first recall the Samuelson-Diamond analysis in the simplest model.

Consider an economy that contains a single commodity produced out of capital and labour. The set (and number) N_t of agents born at time t are all assumed to be identical and to achieve, in the first stage of their lives, the same utility level normalized to the *per capita* consumption c_t of the single produced good (there is no disutility of work). Thus, in our terminology, c_t might be taken as the welfare level of the representative individual in the young population, with a total consumption in period t equal to $N_t c_t$. At time t, the number of identical agents in the second stage of life is N_{t-1}, the set born one period earlier, and their *per capita* consumption during period t is d_t, also identified to the utility of the representative individual in the old population, having a total consumption of $N_{t-1}d_t$. Population grows exponentially at a constant rate $n : N_T = (1+n)N_{t-1}$. Each agent, when young, supplies inelastically one unit of labour. The utility of an individual born at time t, as seen at the beginning of their life-cycle, is given by the function $U(c_t, d_{t+1})$, which we assume to be twice-continuously differentiable, strictly increasing and strictly quasi-concave. Marginal utilities are denoted respectively U'_c and U'_d with respect to present and future consumption. We now add to the model productive capital. The stock of capital available for production at date t is K_t. The productive sector is described by an aggregate production function F, which is homogenous of degree one with respect to labour and capital. The total amount of commodity available for period t is:

$$Y_t = F(K_t, N_t) = N_t F(K_t N_t, 1) = N_t f(k_t),$$

where f gives the output per worker as a function of the capital-labour ratio $k_t = K_t/N_t$, and f is assumed to be increasing, concave and continuously differentiable. The capital fully depreciates over one period. Thus, the resource feasibility constraint for every period t is:

$$N_t c_t + N_{t-1} d_t = F(K_t, N_t) - K_{t+1},$$

or, in *per capita* terms:

$$c_t + \frac{1}{1+n} d_t = f(k_t) - (1+n)k_{t+1}.$$

A feasible path is a sequence $(k_t, c_t, d_t)_{t\geq 0}$ satisfying these constraints at every t, starting with k_0 given. A feasible path $(k_t, c_t, d_t)_{t\geq 0}$ determines correspondingly a feasible utility path $u = (u_t)_{t\geq 0}$. It is intertemporally Pareto-efficient (resp. Pareto-efficient over the life-cycle), if the corresponding utility path is intertemporally Pareto-efficient (resp. Pareto-efficient over the life-cycle). In fact, the two efficiency criteria are trivially equivalent here.

Ensuring intergenerational equal treatment, the Samuelson-Diamond solution sticks to a steady growth path with constant capital *per capita*, maximizing the life-cycle utility of each individual $U(c,d)$, under the above resource constraint. More precisely, we have the following proposition:

Proposition 1 *The set of solutions to the problem:*

$$\begin{aligned} &\max U(c,d) \\ &\textit{such that } c + \tfrac{1}{1+n} d = f(k) - (1+n)k, \\ &c \geq 0, d \geq 0 \textit{ and } k = k_0, \end{aligned}$$

for k_0 such that $f'(k_0) \geq 1+n$, is the set of intertemporally Pareto-efficient paths satisfying intergenerational equal treatment with respect to the average utilitarian rule. The highest utility is obtained for the golden-rule requirement on consumption and production:

$$U'_c(c^*, d^*) = (1+n)U'_d(c^*, d^*) \textit{ and } f'(k^*) = 1+n.$$

For reference, this steady-state path can be compared to the solution of the steady-state version of the Lerner-Benthamite problem, assuming separability of the utility function:

$$U(c_t, d_{t+1}) = U_1(c_t) + U_2(d_{t+1}).$$

This is to maximize the utility sum $N_t U_1(c) + N_{t-1} U_2(d)$, under the same resource constraint, in $c \geq 0, d \geq 0$ and $k \geq 0$. The solution is now characterized by the condition:

$$U'_c(c,d) = U'_1(c) = U'_2(d) = U'_d(c,d) \text{ and } f'(k) = 1+n.$$

It appears clearly that, if $n > 0$, Lerner's solution (which satisfies both the principle of intergenerational equal treatment and period-by-period Pareto optimality) does not verify the condition that $U'_c(c,d) = f'(k)U'_d(c,d)$, which is satisfied by Samuelson's. This last condition, we will show, is quite generally a necessary condition for intertemporal Pareto optimality. In this simple model it is also sufficient.

Let us now consider a more general growth model, with a general production function described by a differentiable increasing function of capital and labour: $Y_t = F(K_t, L_t)$. This stands for a 'social' production function incorporating all external effects. The feasibility constraints are given by:

$$N_t c_t + N_{t-1} d_t = F(K_t, N_t) - K_{t+1},$$
$$N_{t+1} c_{t+1} + N_t d_{t+1} = F(K_{t+1}, N_{t+1}) - K_{t+2}.$$

If we assume that the production function F is concave and that the utility function U is strictly quasi-concave, Lemma 1 can be applied and the two concepts of efficiency are equivalent. The following lemma characterizes the weaker concept of Pareto efficiency over the life-cycle.

Lemma 2 *Let $(K_t, c_t, d_t)_{t\geq 0}$ be a feasible (and interior) path; it is Pareto efficient over the life-cycle if, and only if:*

$$U'_c(c_t, d_{t+1}) = F'_K(K_{t+1}, N_{t+1}) U'_d(c_t, d_{t+1}),$$

for every t.

Proof: Take a feasible path $(K_t, c_t, d_t)_{t\geq 0}$ and freeze all variables but c_t, d_{t+1} and K_{t+1}. A Pareto-optimal path must maximize $U(c_t, d_{t+1})$ under the feasibility constraints with the utility of all generations $s \neq t$ fixed at $U(c_s, d_{s+1})$. For an interior solution $c_t > 0, d_{t+1} > 0$, this implies:

$$U'_c \frac{\partial c_t}{\partial K_{t+1}} + U'_d \frac{\partial d_{t+1}}{\partial K_{t+1}} = 0,$$

which gives the result. ■

In the proof of Proposition 1, the intergenerational equal treatment principle plays a crucial role. The following analogous proposition uses instead the principle of intergenerational equiproportionality.

Proposition 2 *Assume that the production function F is concave and the utility function U strictly quasi-concave. An intertemporal Pareto-efficient path, satisfying intergenerational equiproportionality with respect to the average utilitarian rule, is the solution to the following sequence of problems, taking as given at each period $t = 1, 2, \cdots$, the capital $K_t \geq 0$ and the utility of the old $d_t \geq 0$ (K_0 being initially fixed):*

$$\max U(c_t, d_{t+1})$$

subject to:

$$N_t c_t + N_{t-1} d_t = F(K_t, N_t) - K_{t+1},$$

$$U'_c(c_t, d_{t+1}) = F'_K(K_{t+1}, N_{t+1}) U'_d(c_t, d_{t+1}),$$

$$\frac{d_t}{d_{t+1}} = \frac{F(K_t, N_t)/I_t}{F(K_{t+1}, N_{t+1})/I_{t+1}},$$

$$c_t \geq 0, d_{t+1} \geq 0, K_{t+1} \geq 0.$$

Proof: The first constraint is for feasibility and the second for intertemporal Pareto efficiency, using Lemmata 1 and 2. The third constraint is intergenerational equiproportionality with respect to the average utilitarian rule. ■

4 The study of a particular case

We now construct an example, introducing endogenous growth, where the conditions of intertemporal efficiency and equiproportionality determine a unique positive *per capita* growth rate. This is a one-sector growth model with a labour-saving technical progress:

$$Y_t = F(K_t, A_t N_t),$$

where F is a linear homogenous production function. The assumption of learning-by-doing proposed by Arrow (1962) implies that technical progress is embodied in new equipment, and

measured on the basis of accumulated past investments. Romer (1986, 1989) has introduced a simplified approach, where accumulated knowledge is measured on the basis of the current stock of capital. To obtain an endogenous steady growth, a simple way is to assume that the population is constant and that A_t is a linear function of K_t given by:

$$A_t = AK_t.$$

The output is proportional to the stock of capital and the proportion coefficient is bounded:

$$Y_t = F(K_t, AK_tN) = K_tF(1, AN).$$

Letting $m = F(1, AN) - 1$, the resource constraint in *per capita* terms becomes:

$$c_t + d_t = (1 + m)k_t - k_{t+1}.$$

In this case, the sequence of problems stated in Proposition 2 as an extended version of the Samuelson-Diamond problem, reduces to (with g_t denoting the *per capita* growth rate):

$$\max U(c_t, d_{t+1}),$$

subject to:

$$\begin{aligned} c_t + \frac{d_{t+1}}{1 + g_t} &= (1 + m)k_t - (1 + g_t)k_t \\ U'_c(c_t, d_{t+1}) &= (1 + m)U'_d(c_t, d_{t+1}), \\ c_t &\geq 0, d_{t+1} \geq 0, g_t \geq 0. \end{aligned}$$

Assuming that the utility function U is strictly quasi-concave, intertemporal efficiency is ensured by Lemmata 1 and 2. Assuming, in addition, that U is homothetic, we get a unique solution with c_t and d_{t+1} proportional to k_t, given g_t. Moreover, efficiency leads to a positive and constant *per capita* growth rate $g_t = g^*$. Indeed, under homotheticity, the second constraint is equivalent to $d_{t+1} = \theta c_t$, for some constant $\theta > 0$ (depending on m). So, by the first:

$$c_t = h(g_k)k_t, \text{ with } h(g_t) \equiv \frac{(1 + g_t)(m - g_t)}{(1 + g_t + \theta)}.$$

Hence, the problem is reduced to the maximization of h with respect to $g_t \geq 0$.

Since

$$\frac{h'(g_t)}{h(g_t)} = -\frac{1}{m - g_t} + \frac{1}{1 + g_t} - \frac{1}{1 + g_t + \theta}$$

is decreasing from $-\infty$ to $+\infty$, as g_t goes from -1 to m, there exists a unique solution $g_t = g^*$, which verifies $h'(g_t) = 0$. Computing the positive root x^* of the equation $[x^2+2\theta x-\theta(1+m)] = 0$ determines that $g^* = x^* - 1$, and $g^* > 0$ if $m - \frac{1}{\theta} > 1$.

Moreover, this solution satisfies intergenerational equiproportionality, since

$$\frac{d_{t+1}}{d_t} = 1 + g^*.$$

In this example, we get a unique positive *per capita* growth rate compatible with intertemporal efficiency and equiproportionality.

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