Nonholonomic motion planning

Jean-Paul Laumond
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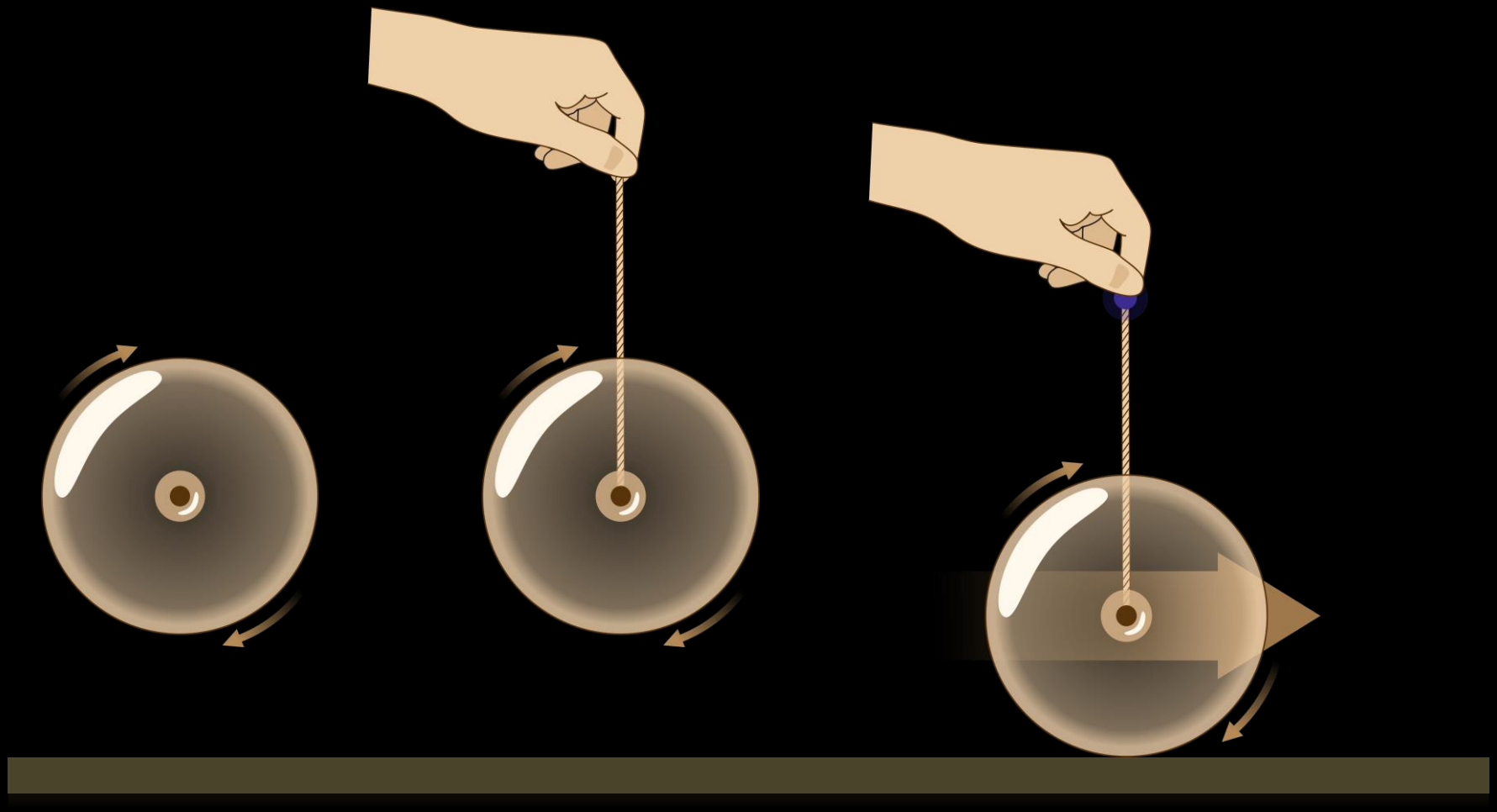
DI-ENS

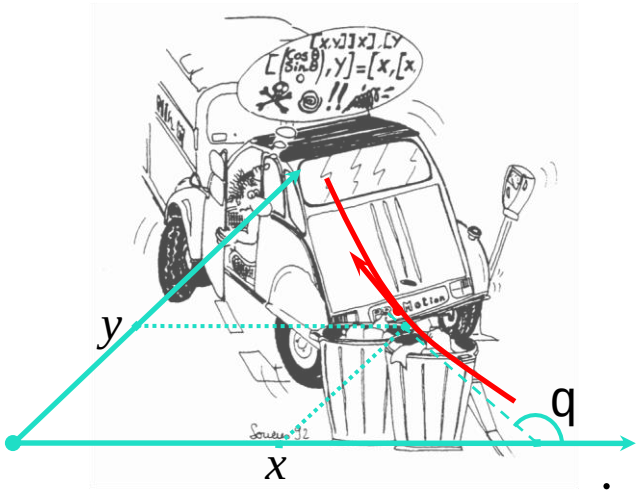
Tribute to the wheel

Marcel Duchamp

Tribute to the wheel



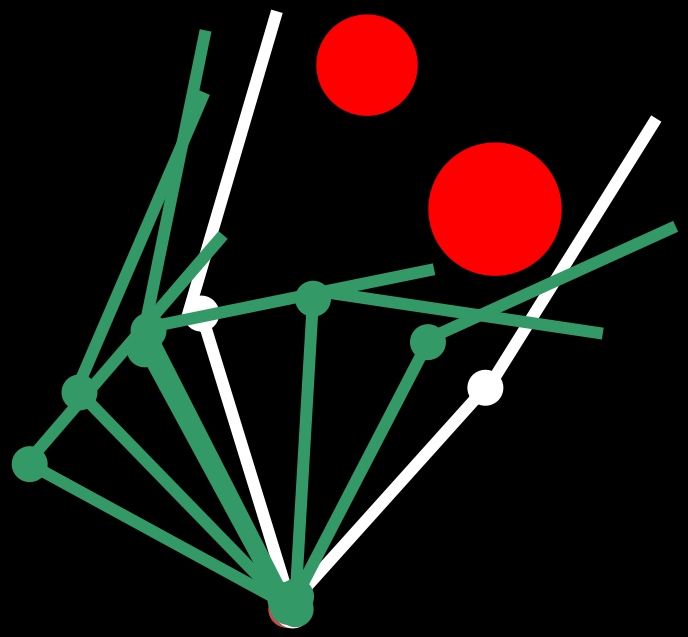
From the rolling car to the rolling man

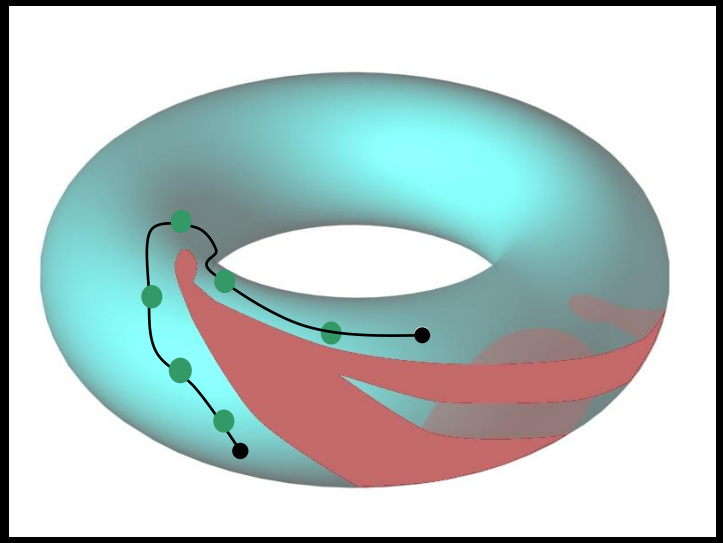


$$\dot{x}\,\sin q \;-\; \dot{y}\,\cos q \;=\; 0$$

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

Motion planning





Motion planning

Any admissible motion for the **3D mechanical system** appears a collision-free path for a **point** in the CSpace

Motion planning

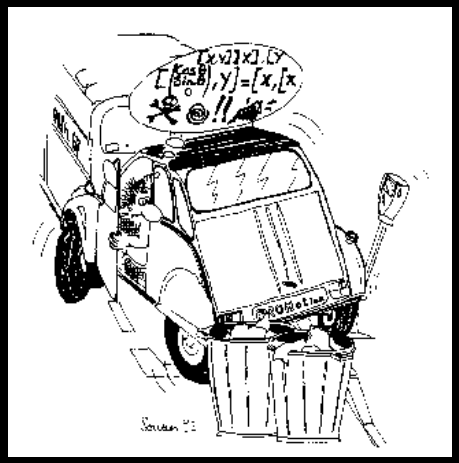
Any admissible motion for the **3D mechanical system** appears a collision-free path for a **point** in the CSpace

Converse:

- true for holonomic systems
- - not true for nonholonomic systems

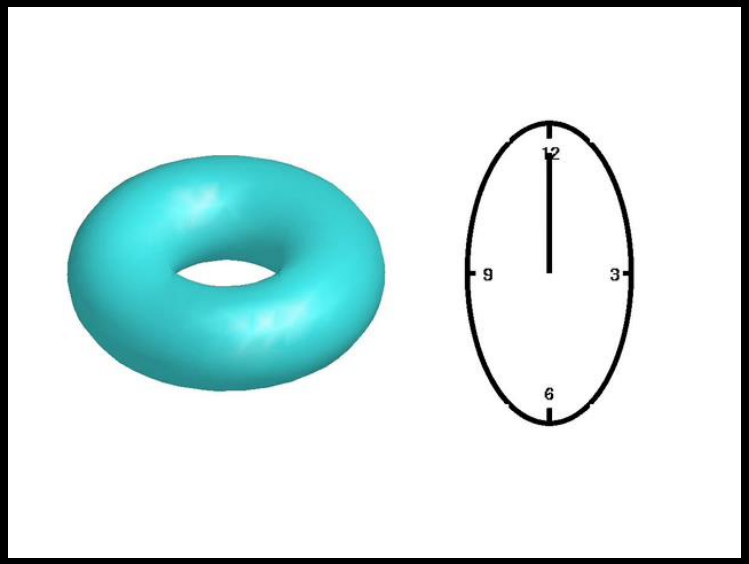
Holonomy *versus* nonholonomy: an integrability question

$$\dot{x} = \frac{1}{12}\,\dot{y}$$



$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

Holonomy *versus* nonholonomy: an integrability question



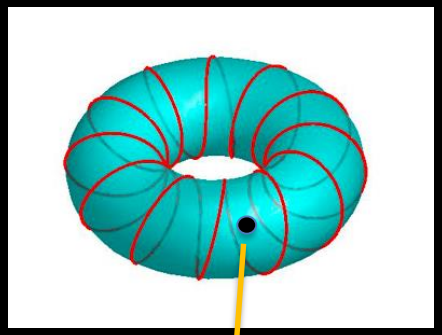
$$\dot{x} = \frac{1}{12}\,\dot{y}$$

Dimension of the manifold: 2

Dimension of the reachable set: 1

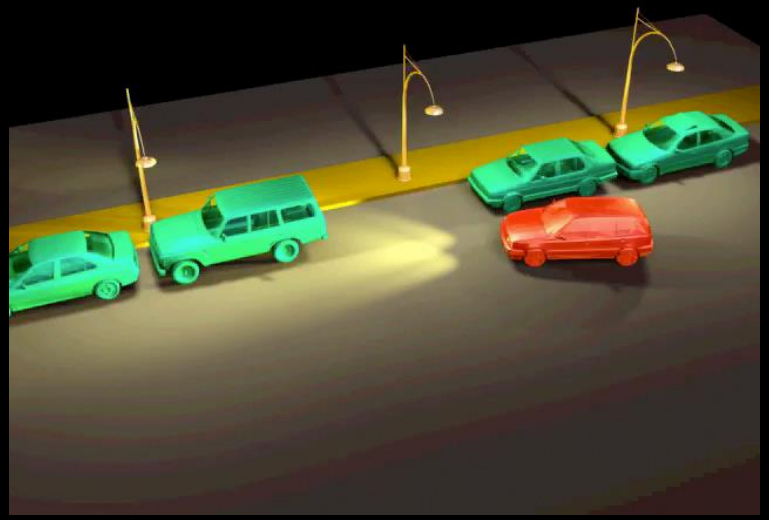
Holonomy *versus* nonholonomy: an integrability question

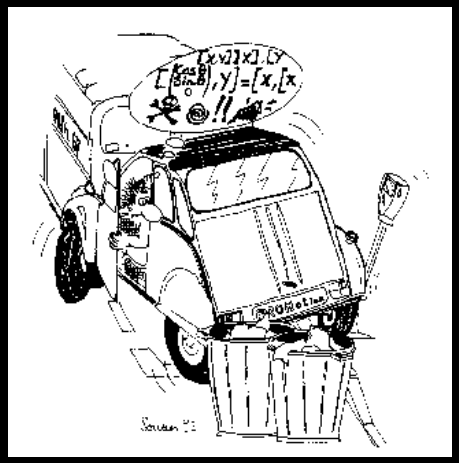
$$\dot{x} = \frac{1}{12}\,\dot{y}$$



The time that will never happen!!!

Holonomy *versus* nonholonomy: an integrability question

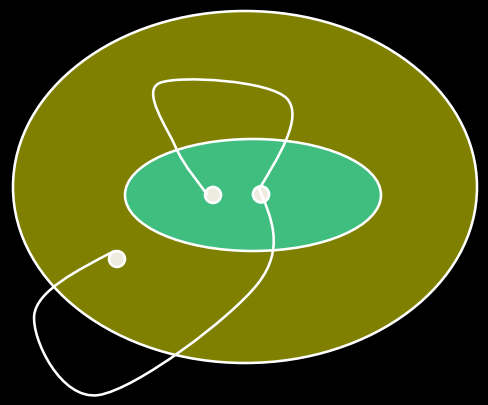




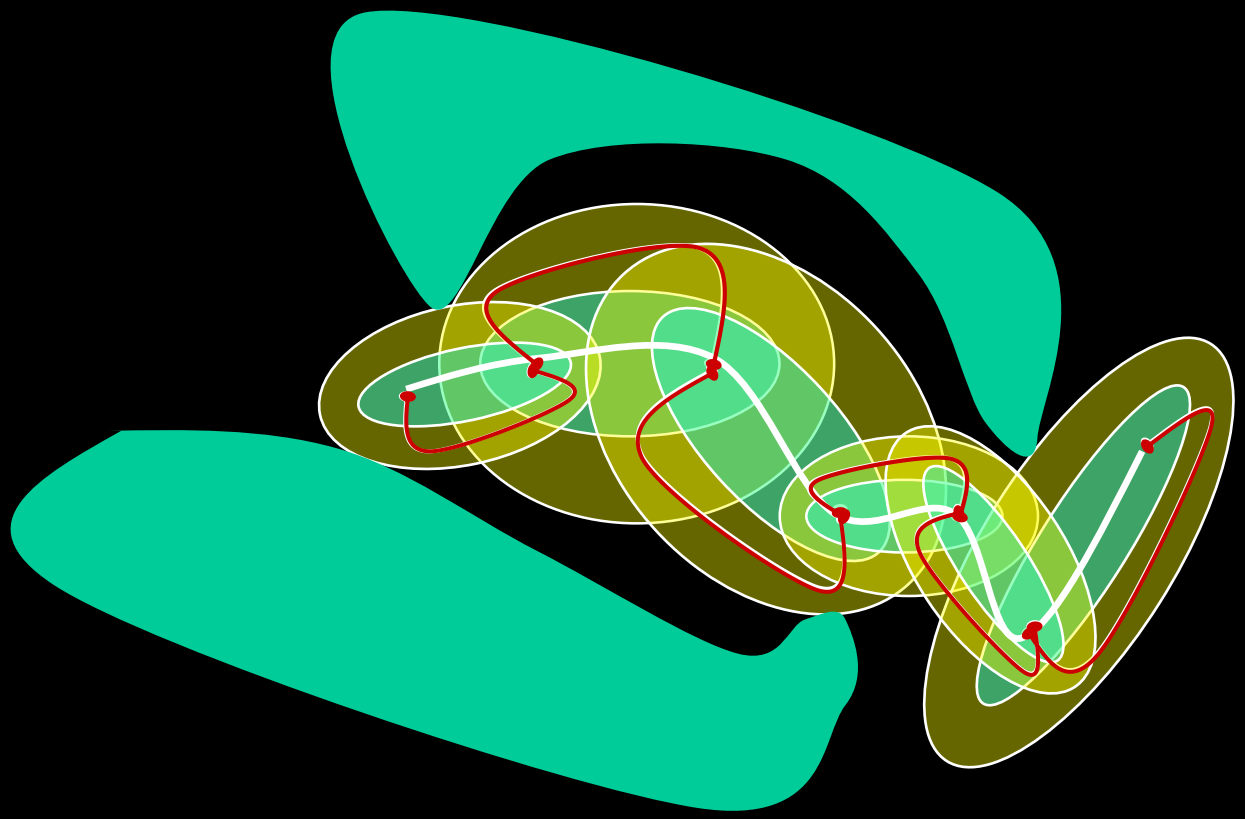
$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

Dimension of the manifold: 3
Dimension of the reachable set: 3

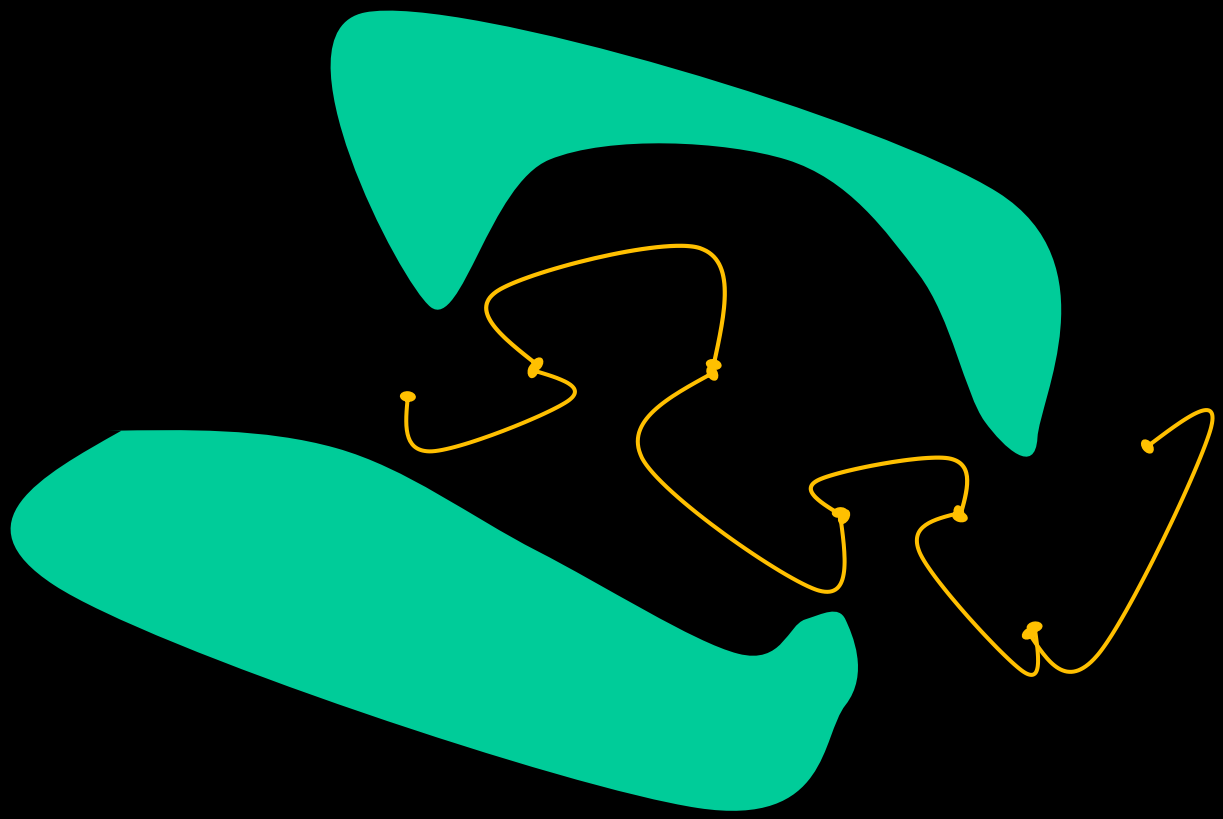
Motion planning and small-space controllability



Motion planning and small-space controllability



Motion planning and small-space controllability



Holonomy *versus* nonholonomy: two central questions

- Is my system nonholonomic?
- Is my nonholonomic system small-space controllable?

Holonomy *versus* nonholonomy: two central questions

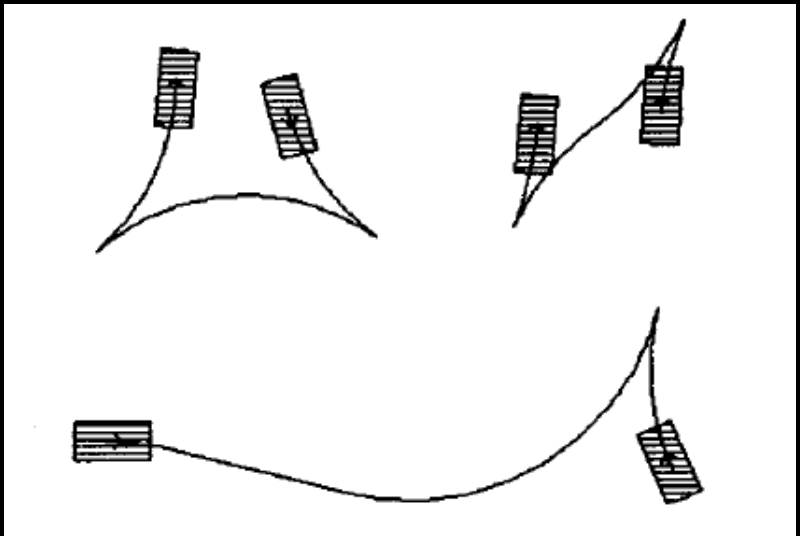
- Is my system nonholonomic?

  Frobenius theorem

- Is my nonholonomic system small-space controllable?

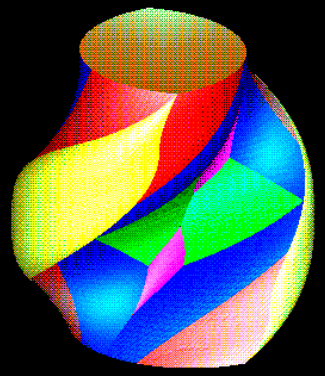
  Lie Algebra Rank Condition

J. A. Reeds and L. A. Shepp
Pacific J. Math. Vol. 145, N. 2, 1990



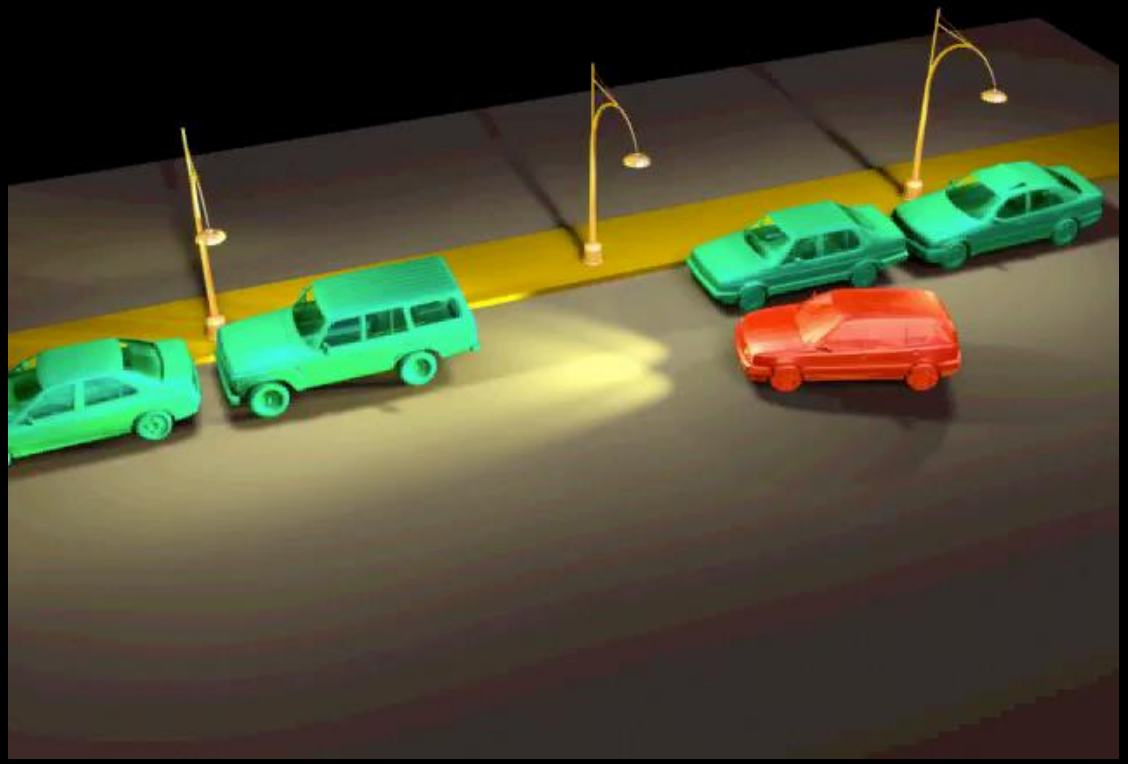
Car-like optimal control

P. Souères, J.P. Laumond,
IEEE Transactions on Automatic Control, Vol.41, N. 05, 1996

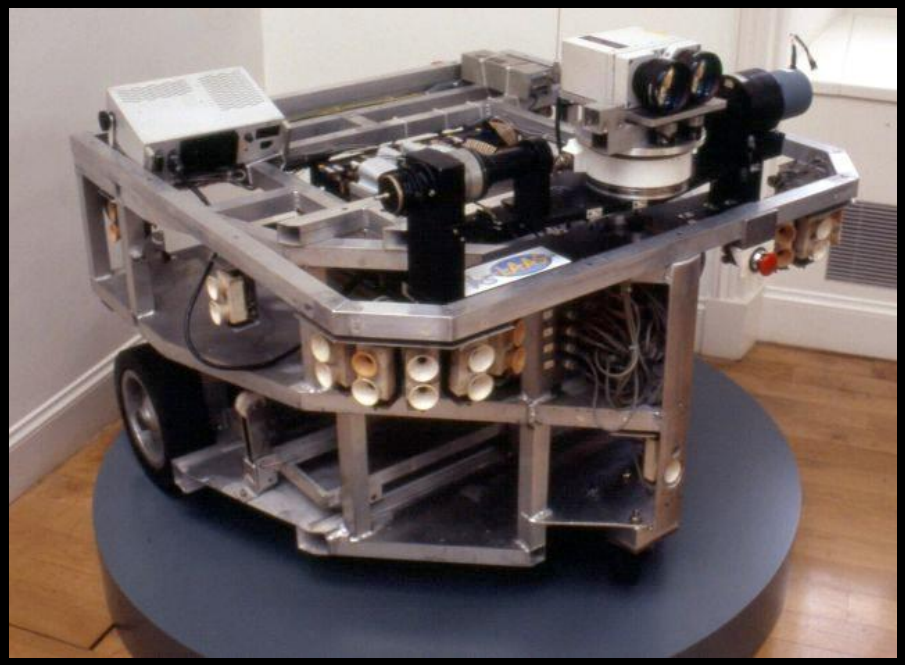


Car-like optimal control

J.P. Laumond, P. Jacobs, M. Taix, R. Murray,
IEEE Transactions on Robotics and Automation, Vol.10, N. 05, 1994

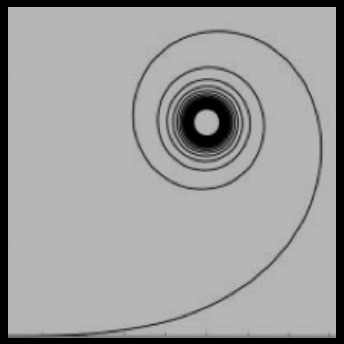


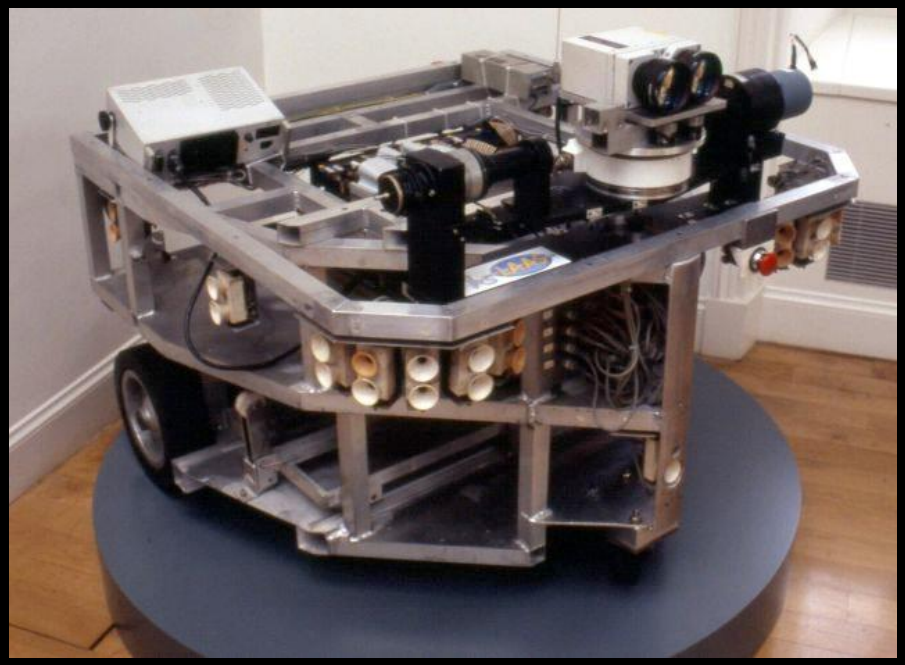
Car-like optimal control

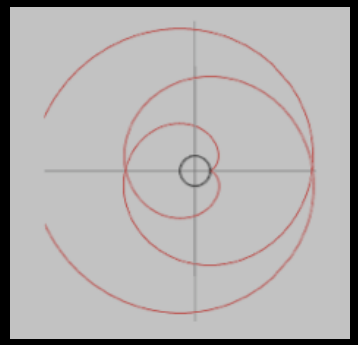


Hilare-like optimal control

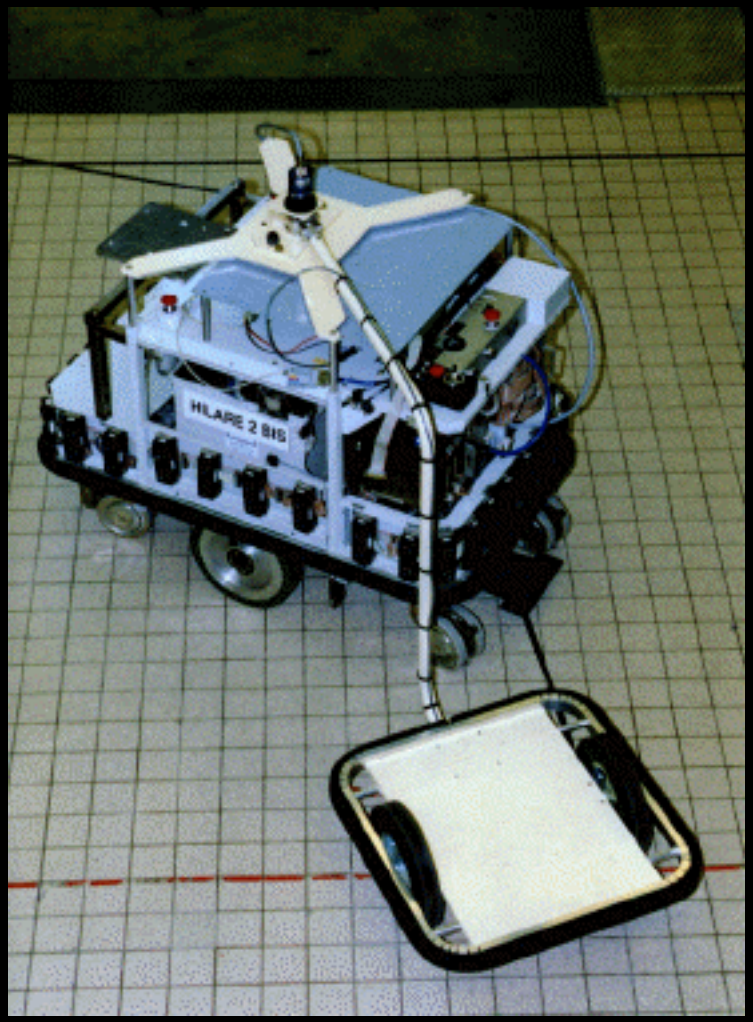
P. Jacobs, J.P. Laumond, A. Rege,
Intelligent Robotics, Vidyasagar & Trivedi Eds, McGraw Hill, 1991

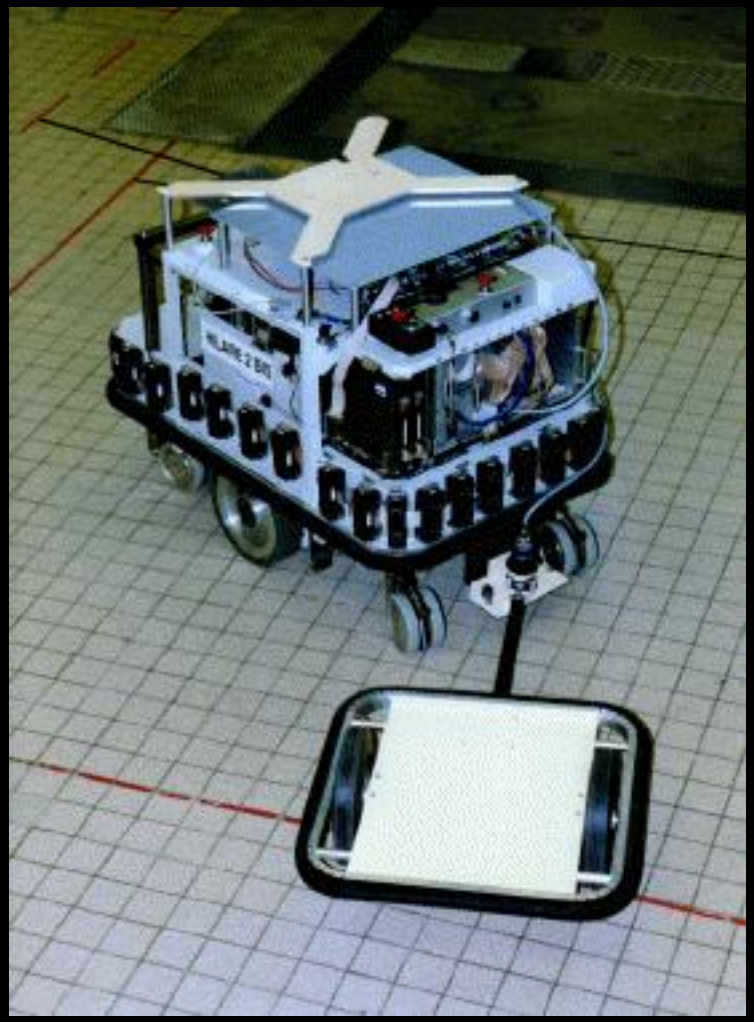




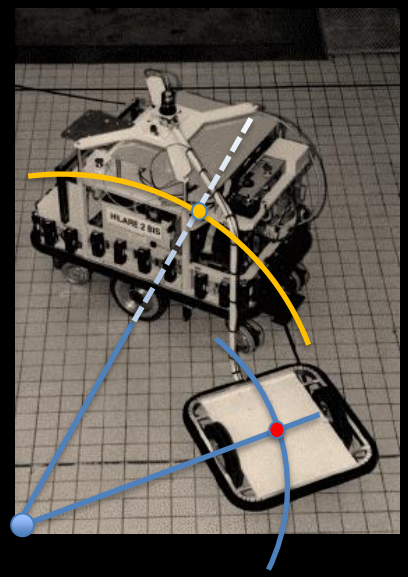


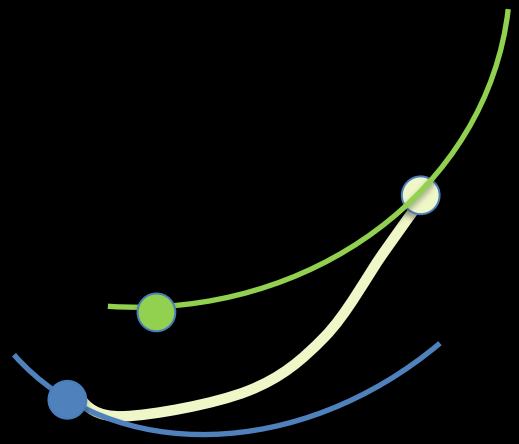
Hilare-like optimal control


HILARE 2 BIS



F. Lamiraux, J.P. Laumond,
IEEE Transactions on Automatic Control, Vol. 45, N. 10, 2000





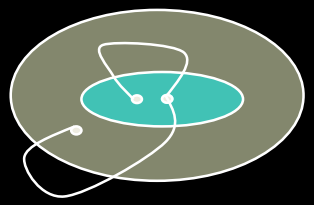
Flatness

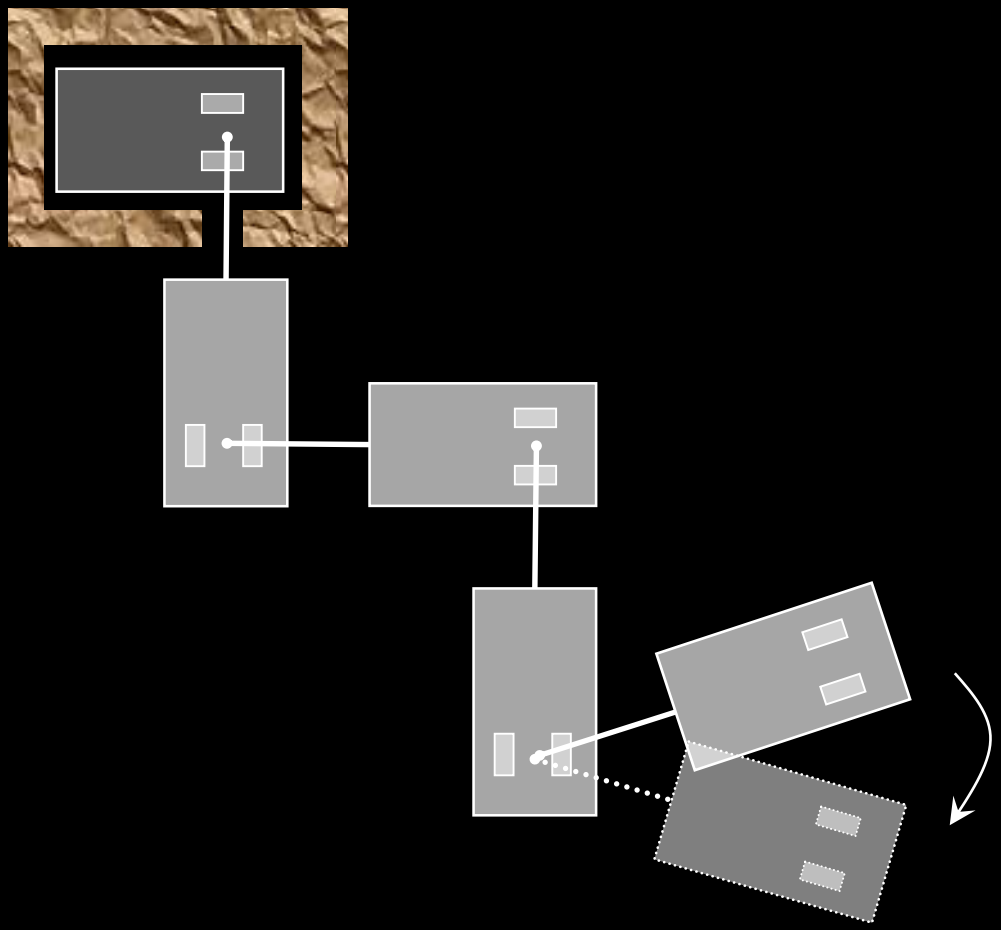
Mobile robot with trailers

F. Lamiraux, J.P. Laumond,
IEEE Transactions on Automatic Control, Vol. 45, N. 10, 2000

Mobile robot with trailers

J.P. Laumond,
IEEE J. on Robotics and and Automation, Dec. 1993

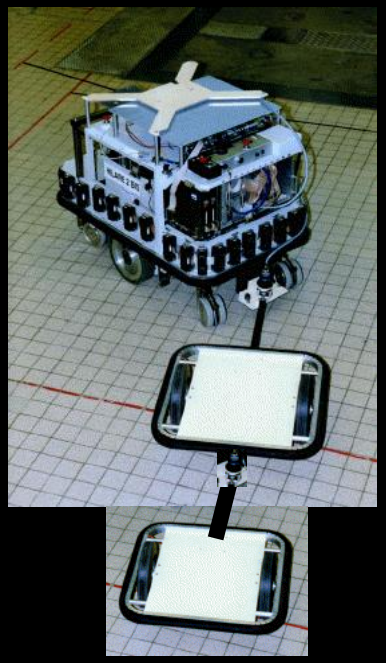




Mobile robot with trailers

F. Jean
ESAIM: Cont. Opt. Calc. Var, vol. 1, 1996

Open problem



Hooking	centered	not
f_1, f_2	2	2
$[f_1, f_2]$	3	3
$[f_1,[f_1, f_2]]$, $[f_2,[f_1, f_2]]$	4	5
$[f_1,[f_1,[f_1, f_2]]]$,....	5	

(2,3,4,5) ***versus*** **(2,3,5)**

Mobile robot with trailers

S. Dalibard, A. El Khoury, F. Lamiraux, A. Nakhaei, M. Taix, J.P. Laumond,
International Journal of Robotics Research, Vol. 32, N. 9-10, 2013

Humanoid walking

DOI:10.1145/2629535

Exploring the distinction between an optimal robot motion and a robot motion resulting from the application of optimization techniques.

BY JEAN-PAUL LAUMOND, NICOLAS MANSARD, AND JEAN-BERNARD LASSERRE

Optimality in Robot Motion: Optimal versus Optimized Motion

THE FIRST BOOK dedicated to robot motion was published in 1982 with the subtitle "Planning and Control."[5] The distinction between motion planning and motion control has mainly historical roots. Sometimes motion planning refers to geometric path planning, sometimes it refers to open loop control; sometimes motion control refers to open loop control, sometimes it refers to close loop control and stabilization; sometimes planning is considered as an offline process whereas control is real time. From a historical perspective, robot motion planning arose from the ambition to provide robots with motion autonomy: the domain was born in the computer science and artificial intelligence communities.[22]

Motion planning is about deciding on the existence of a motion to reach a given goal and computing one if this one exists. Robot motion control arose from manufacturing and the control of manipulators[10] with rapid effective applications in the automotive industry. Motion control aims at transforming a task defined in the robot workspace into a set of control functions defined in the robot motor space: a typical instance of the problem is to find a way for the end-effector of a welding robot to follow a predefined welding line.

What kind of optimality is about in robot motion? Many facets of the question are treated independently in different communities ranging from control and computer science, to numerical analysis and differential geometry, with a large and diverse corpus of methods including, for example, the maximum principle, the applications of Hamilton-Jacobi-Bellman equation, quadratic programming, neural networks, simulated annealing, genetic algorithms, or Bayesian inference. The ultimate goal of these methods is to compute a so-called *optimal* solution whatever the problem is. The objective of this article is not to overview this entire corpus that follows its own routes independently from robotics, but

» key insights

- **Computing an optimal robot motion is a challenging issue illustrated by more than 20 years of research on wheeled mobile robots.**
- **Geometric control theory and numerical analysis highlight two complementary perspectives on optimal robot motion.**
- **Most of the time, robot algorithms aimed at computing an optimal motion provide an *optimized* motion, which is not optimal at all, but is the output of a given optimization method.**
- **When optimal motions exist, numerical algorithms mostly fail in accounting for their combinatorial structure. Moreover, optimization algorithms bypass (not overcome) the question of the existence of optimal motions.**

DOI:10.1145/2743132

Robots move to act. While actions operate in a physical space, motions begin in a motor control space. So how do robots express actions in terms of motions?

BY JEAN-PAUL LAUMOND, NICOLAS MANSARD, AND JEAN BERNARD LASSERRE

Optimization as Motion Selection Principle in Robot Action

MOVEMENT IS A fundamental characteristic of living systems (see Figure 1). Plants and animals must move to survive. Animals are distinguished from plants in that they have to explore the world to feed. The carnivorous plant remains at a fixed position to catch the imprudent insect. Plants must make use of self-centered motions. At the same time the cheetah goes out looking for food.

Feeding is a paragon of action. Any action in the physical world requires self-centered movements, exploration movements, or a combination of both. By analogy, a manipulator robot makes use of self-centered motions, a mobile robot moves to explore the world, and a humanoid robot combines both types of motions.

Actions take place in the physical space. Motions originate in the motor control space. Robots—as any living system—access the physical space only indirectly through sensors and motors. Robot motion planning and control explore the relationship between physical, sensory, and motor spaces; the three spaces that are the foundations of geometry.[36] How to translate actions expressed in the physical space into a motion expressed in motor coordinates? This is the fundamental robotics issue of inversion.

In life sciences, it is recognized that optimality principles in sensorimotor control explain quite well empirical observations, or at least better than other principles.[40] The idea of expressing robot actions as motions to be optimized was first developed in robotics in the 1970s with the seminal work by Whitney.[41] It is now well developed in classical robot control,[27] and also along new paradigms jointly developed in multidisciplinary approaches.[25] Motion optimization appears to be a natural principle for action selection. However, as we explained in the companion article,[24] optimality equations are intractable most of the time and numerical optimization is notoriously slow in practice. The article aims

» key insights

- **For robots and living beings, the link between actions expressed in the physical space and motions originated in the motor space, turns to geometry in general and, in particular, to linear algebra. In life science the application of optimality principles in sensorimotor control unravels empirical observations. The idea to express robot actions as motions to be optimized has been developed in robotics since the 1970s.**
- **Among all possible motions performing a given action, optimization algorithms tend to choose the best motion according to a given performance criterion. More than that, they also allow the realization of secondary actions.**
- **Optimal motions are action signatures. How to reveal what optimality criterion underlies a given action? The question opens challenging issues to inverse optimal control.**