Nonholonomic motion planning

Jean-Paul Laumond
CNRS – INRIA – PSL
DI-ENS
Tribute to the wheel

Marcel Duchamp
© The Museum of Modern Art, New York
Tribute to the wheel
From the rolling car to the rolling man

\[ \dot{x} \sin \theta - \dot{y} \cos \theta = 0 \]
\dot{x} \sin \theta - \dot{y} \cos \theta = 0
Motion planning
Motion planning

Any admissible motion for the 3D mechanical system appears a collision-free path for a point in the CSpace
Motion planning

Any admissible motion for the 3D mechanical system appears a collision-free path for a point in the CSpace

Converse:
- true for holonomic systems
- - not true for nonholonomic systems
Holonomy *versus* nonholonomy: an integrability question

\[ \dot{x} = \frac{1}{12} \dot{y} \quad \dot{x} \sin \theta - \dot{y} \cos \theta = 0 \]
Holonomy *versus* nonholonomy: an integrability question

\[ \dot{x} = \frac{1}{12} \dot{y} \]

Dimension of the manifold: 2
Dimension of the reachable set: 1
Holonomy *versus* nonholonomy: an integrability question

\[ \dot{x} = \frac{1}{12} \dot{y} \]

The time that will never happen!!!
Holonomy *versus* nonholonomy: an integrability question

\[ \dot{x} \sin \theta - \dot{y} \cos \theta = 0 \]

Dimension of the manifold: 3
Dimension of the reachable set: 3
Nonholonomic systems

Motion planning and small-space controllability
Motion planning and small-space controllability
Motion planning and small-space controllability
Holonomy *versus* nonholonomy: two central questions

- Is my system nonholonomic?
- Is my nonholonomic system small-space controllable?
Holonomy *versus* nonholonomy: two central questions

- Is my system nonholonomic?  
  Frobenius theorem

- Is my nonholonomic system small-space controllable?  
  Lie Algebra Rank Condition
Car-like optimal control
Car-like optimal control
Car-like optimal control
Hilare-like optimal control
Hilare-like optimal control

Mobile robot with trailers
Mobile robot with trailers
Mobile robot with trailers

F. Jean
Open problem

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Mobile robot with trailers
Humanoid walking
Optimality in Robot Motion: Optimal versus Optimized Motion

The first book dedicated to robot motion was published in 1982 with the subtitle “Planning and Control.” The distinction between motion planning and motion control has a mainly historical root. Sometimes motion planning refers to geometric path planning, sometimes it refers to open loop control; sometimes motion control refers to open loop control, sometimes it refers to closed loop control and stabilization; sometimes planning is considered as an offline process whereas control is real time. From a historical perspective, robot motion planning arose from the ambition to provide robots with motion autonomy; the domain was born in the computer science and artificial intelligence communities.2

Exploring the distinction between an optimal robot motion and a robot motion resulting from the application of optimization techniques.

BY JEAN-PAUL LAUMOND, NICOLAS MANSARD, AND JEAN-BERNARD LASSERT

Motion planning is about deciding on the existence of an optimal motion to reach a given goal and computing one if this one exists. Robot motion control arose from manufacturing and the control of manipulators with rapid effective applications in the automotive industry. Motion control aims at transforming a task defined in the robot workspace into a set of control functions defined in the robot motor space: a typical instance of the problem is to find a way for the end effector of a welding robot to follow a predefined welding line.

What kind of optimality is about in robot motion? Many facets of the question are treated independently in different communities ranging from control and computer science; to numerical analysis and differential geometry, with a large and diverse corpus of methods including, for example, the minimum principle, the applications of Hamilton-Jacobi-Bellman equation, quadratic programming, neural networks, simulated annealing, genetic algorithms, or Lagrange invariants. The ultimate goal of these methods is to compute a so-called optimal solution whatever the problem is. The objective of this article is not to overview this entire corpus but to focus on independently from robotics, but...

key insights
- Computing an optimal robot motion is a fascinating idea illustrated by more than 30 years of research on mobile robots.
- Geometric control theory and numerical analysis highlight two complementary perspectives on optimal robot motion.
- Most of the time, robot algorithms almost always computing an optimal motion provide an optimal motion, which is not optimal at all, but to the output of a given optimization method.
- When optimal motions exist, numerical algorithms mostly fail in accounting for their non-continuous nature. Moreover, optimization algorithms bypass just as an exercise the question of the existence of optimal motions.
Robots move to act. While actions operate in a physical space, motions begin in a motor control space. So how do robots express actions in terms of motions?

BY JEAN-PAUL LAUMOND, NICOLAS MANSARD, AND JEAN-BERNARD LASSERRE

Optimization as Motion Selection Principle in Robot Action

Movement is a fundamental characteristic of living systems (see Figure 1). Plants and animals must move to survive. Animals are distinguished from plants in that they have to explore the world to feed. The carnivorous plant remains at a fixed position to catch the intruder insect. Plants must make use of self-centered motions. At the same time the cheetah goes out looking for food.

Feeding is a paragon of action. Any action in the physical world requires self-centered movements, exploration movements, or a combination of both. By analogy, a manipulator robot makes use of self-centered motions, a mobile robot moves to explore the world, and a humanoid robot combines both types of motions.

Key Insights

1. For robots and living beings, the link between actions as present in the physical space and motions established in the motor space, home to geometry in general and, in particular, to linear systems. In this context, the application of optimization principles to sensorimotor control appears natural and elegant.

2. Among all possible motions performing a given action, the optimality operator identifies the best motion according to a given performance criterion. More than that, they minimize the realization of secondary actions.

3. Optimal motions are action signatures. How to reveal what optimality criteria underlie a given output? The question arises challenging issues ininverse optimal control.