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SYMMETRIES AND SYMMETRY BREAKING, FROM GEOMETRY TO PHYSICS, VIA PAINTING

GIUSEPPE LONGO
Centre Cavaillès, République des Savoirs
CNRS & Ecole Normale Supérieure, Paris
https://www.di.ens.fr/~longo/

Name: Giuseppe LONGO
Profession / Speciality: Directeur de Recherche Emeritus, former Professor of Logic and Computability
Fields of interest: Foundation of Mathematics and of Biology, Epistemology
Affiliation: CNRS and Ecole Normale Supérieure, Paris
E-mail: giuseppe.longo@ens.psl.eu
Homepage: *https://www.di.ens.fr/~longo/*
Awards: Unione Matematica Italiana, National Award for young mathematicians 1974; Member of the **ACADEMIA EUROPAEA**, the European Academy of Sciences**.**
Publications and/or Exhibitions: for papers, see https://www.di.ens.fr/users/longo/download.html; books:
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"Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty and perfection" (Weyl 1952))

"... substituting to the concept of law that of symmetry... I consider this concept as being the principal means of access to the world we create in theories." (van Fraassen 1994)

ABSTRACT

Symmetries, invariances, and their breaking not only manifest in physical phenomena but also appear to underpin its objectivity at a deeper level of abstract determination—and perhaps even of cognitive regulation. It is within this framework that we will explore the relationships between symmetry principles, which help construct and shape scientific knowledge, and their cognitive origins. Specifically, we will start by examining foundational issues in mathematics, beginning with Euclid's geometry, then move on to consider causal structures in physics through the lens of symmetry. An explicit reference to the symmetries of space and their metaphysical origin will allow us to mention the theological invention of Renaissance pictorial perspective that preceded the construction of the geometry of modern science. We will conclude with some reflections on human cognition as an approach to a renewed epistemology of mathematics.

SYMMETRIES IN EUCLID

Let's take as a starting point Euclid's Aithemata (Requests/Postulates), the minimal constructions required to do geometry.

1. To draw a straight line from any point to any point.
2. To extend a finite straight line continuously in a straight line.
3. To draw a circle with any center and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles. (Heath 1908; pp. 190-200).

These "Requests" are constructions performed by ruler and compass: an abstract ruler and compass, of course, not the carpenter's tools but tools for a dialogue with the Gods. More precisely, they provide the minimal "construction's principles" the geometer should be able to apply. We will try now to understand them in Husserlian terms: "The primary evidence should not be interchanged with the evidence of the "axioms"; since the axioms are mostly the result already of an original formation of meaning (*Sinnbildung*) and they already have this formation itself always *behind them*" (Husserl 1933, p. 193).

Note that these requests are all active gestures, they ask to draw, to extend, to "produce"… The primary evidence which is behind and grounds them on meaningful action in space is a "*maximal symmetry principle*". A symmetry in space is full of immediate "meaning" for us, humans (animals), with a fundamental bilateral symmetry; it is a fortiori so for a Greek man whose art and science is full of symmetries (and sometimes, slight symmetry breaking).

Now, by drawing a straight line between two points (Request/postulate 1), one obtains the most symmetric possible structure: any other line, different from this one, would introduce a-symmetries by breaking the axial symmetry of the straight line. The same can be said for the second Request, where any other extension of a finite line, differently from a straight continuous line, would yield fewer symmetries. Similarly, the third, a complete rotation symmetry, generates the most symmetric figure for a line enclosing a point. In the fourth, equality is defined by congruence; that is, by a translation symmetry of plane figures. Finally, the fifth construction again is a matter of drawing, intersecting and then extending. The most symmetric construction occurs when the two given lines do not intersect: then the two inner angles given by the falling one are right angles on both sides of this line intersecting the two given lines. In this case, the two lines are parallel. The other two cases, as negations of this one, would reduce the number of symmetries. The equivalent formulations of the other two cases, as shown by theorem in book I n. 29 in Euclid's Elements (more than one parallel in one point to a line, no parallel at all) both yield fewer symmetries, on a Euclidian plane, than having exactly one parallel line, for a point to a given line (we are developing here some remarks in (Longo 2011)).

In short, Euclid's requests found geometry by actions on plane figures, implicitly governed by symmetries, with their precise mathematical meaning, such as reflections, rotations and translations - all plane symmetries. Now, "symmetries" are at the core of Greek culture. They refer to "balanced" situations or, more generally, "measurable/comparable" entities or forms (Chiurazzi 2018). They acquire a precise mathematical sense in Greek geometry, where symmetries of translation, rotation, reflection… are commonly used in constructions and proofs. Similarly, Archimedes' principle, "the upward force on an object submerged in a fluid is equal to the weight of the fluid the object displaces" or his remark on "why a balance with equal weights is at equilibrium" are both based on symmetry reasons. The meaning we give to symmetries here underlies also Greek "aesthetics" (in the Greek sense of the word), knowledge and art, from sculpture to myth and tragedy. As a matter of fact, loss of symmetries (symmetry-breaking) originate the world as well as human tragedy; as breaking of equilibria among the Gods, they underlie the sense of human life. As tools for mathematical construction, they are part of the "original formation of sense", as Husserl writes.

Concerning the postulates/axioms of geometry, the modern formalist "universal-existential" version ("For any two points on a plane, there exists one and only one segment between these points") misses the constructive sense and misleads the foundational analysis into the anguishing questions: does this line really exists? Is it unique? In Euclid, existence is by construction, unicity by symmetry: they pose no problem. Hilbert's proposal of 1900 to bypass these questions by proving the consistency (no contradiction) of the formalized theory was a vane attempt to avoid both ontological commitment and active construction by a proof of formal consistency. We know how this quest for consistency of theories and, in particular, of existential and unicity statements ended: by Gödel's theorem, 1931, there is no such proof for any theory containing the paradigm of finitism in mathematics, formal number theory (arithmetic).

From axioms to theorems, up to infinity

"Theorem" derives from the Greek *theoria*, meaning "vision," as in "theater": a theorem shows by constructing. The first theorem of Euclid's *Elements* shows how to construct an equilateral triangle on a given finite straight line (a segment). It is posed as a problem and suggests to solve it by tracing (semi-)circles centered on each endpoint, using the segment as the radius. These upper semi-circles intersect at a single point. Drawing straight lines from the endpoints of the segment to this point yields an equilateral triangle. For over a century, we've been told this is not a proof (in Hilbert's axiomatic sense): the existence of the intersection point must first be formally proven. We claim that these detractors might better refer to the Greeks' dialogue with their gods and their reference to ideal objects as a key way to organize reality; they miss the "diagrammatic" sense of the proof in Euclid (Panza 2012).

As Schrödinger notes, a fundamental feature of Greek philosophy is the absence of *"the unbearable division which affected us for centuries…: the division between science and religion"* (quoted in (Fraisopi 2009)). Lines, in Greek geometry, are ideal objects—cohesive continua with no thickness. Both points and continuous lines are foundational notions, yet the conceptual path that links them reverses the point-wise constructions that have dominated mathematics since Cantor, 19th century. For Euclid, points are *signs* (*σημεῖα*). It was Boethius, six centuries later, who introduced the Latin word "point" along with its definition (*definition alpha:* a point has no parts). Points-signs are at the boundary of a line or segment (*definition gamma*). They arise, and this is the issue here, from the intersection of lines: two one-dimensional, thickness-less lines intersecting produce a point and, under the good conditions of the theorem, only one, with no parts—that is, of no dimension. This proves theorem I. Moreover, we claim here that this first theorem implicitly defines continuity: a line with no thickness is continuous if, when intersecting any another such line, it always produces one point and, *under good conditions,* only one. Note that before the rigor of Hilbert (1862 - 1943) and the Bourbaki school (1930), often definitions where implicitly given in theorems – this actually continued in the excellent Russian school of mathematics till the fall of the Berlin wall (this author's experience in reading Kolmogorov and Ershov, 1970s). As a matter of fact, the definitions *alpha*, *beta* and *gamma* in the *Elements* are consistent with Euclid's constructions, but were introduced six centuries later (Toth 2002).

In summary, the immense leap toward abstraction in Greek geometry lies in the invention of the continuous, *thickness-less* line—an abstraction as divine as it is conceptual. How else could one propose a general measure theory of surfaces, the very aim of *geo-metry*? If a plane figure has thick borders, what exactly would define its surface? Greek geometry is thus built upon the invention of the notion of *border*, this is the sense of the line with no thickness—a concept that remains deep and challenging in modern topology. This astonishing conceptual (and metaphysical) invention—the continuous, border-defining line with no thickness—emerged from philosophy (Aristotle, in particular) and was later formalized as *definition beta* (a line is breadthless length). Points—dimensionless but nameable, signs as Euclid defines them—are produced either by the intersection of lines, as we have said, or they sit at the endpoints of a segment (*definition gamma*). But lines are not composed of sign-points, an idea made rigorous by Cantor in the 19th century. A continuous line is a *gestalt*—not a set of points, but a unified whole, a trace to be drawn ("draw/produce a line", says Euclid). It is thus generated by a "gesture" on the plane, a trajectory in space. Greek geometric figures and their theatrical properties arise from constructions based on these fundamental gestalts—sign-points (as names for locations, *σημεῖα*) and lines—in a game of rotations and translations, of constructing and breaking symmetries. These gestalts underlie proofs then and still today (Longo 2011a).

In spite of Schrödinger's remark above, concerning the loss of the Greek relation to ideal constructions of divine inspiration, the creation of the spatial frameworks of the Scientific Revolution was likewise inspired by a renewed dialogue with divinity. In the 14th century, painter-priests—Ambrogio Lorenzetti among the first, see (Panofsky 1925)—invented modern "linear" perspective, thereby constructing a new geometry for three-dimensional space.

Ambrogio Lorenzetti, *Annunciation* 1344, tempera on wood, Siena, Pinacoteca Nazionale.

This pictorial construction proved that actual infinity could have a *symbolic form*—it could be represented. That form is the convergence point, at "infinity", or *vanishing point*, of Italian perspective. According to (Arasse 1999), that vanishing point evokes the presence of God in the unique moment of His encounter with the mother of His Son. He observes that for about fifty years, it is present mostly and much more emphasized in Annunciations. Even later, up to Leonardo da Vinci (see below), the geometric perspective is particularly highlighted in Annunciations (Leonardo, in his other paintings, describes spatial depth by the "nuances" of colors, with no explicit geometry). Thus, this symbolic invention represents, through a pictorial-theological gesture, as Arasse writes, the infinity of God in scenes of the *Annunciation,* see also (Longo S. 2022).

Leonardo da Vinci, *Annunciation* 1472-75, tempera on wood, Galleria degli Uffizi, Florence

Between the two—Lorenzetti and Leonardo—architects such as Brunelleschi and Alberti, in the 15th century, made extensive and secular use of perspective in the preparatory drawings for their constructions. Meanwhile, another towering painter, Piero della Francesca, authored a mathematical treatise, *De prospectiva pingendi* (1450), an introduction to the practice of "projective geometry" for painters. Piero was a master also at breaking the new global symmetry in the painting established by the central vanishing point. By moving it down to the right, he imposes here a different "view point" (perspective) on the viewer – and this further highlights the "mirror" symmetry between the point at infinity and the position of the viewer:

Piero della Francesca, *The Flagellation* (detail), c. 1455, Oil and tempera on panel, Galleria Nazionale delle Marche, Urbino

Note that the Euclidean conception of the line as a cohesive continuum—traced, drawn, or produced in practice—is embraced by Leonardo, who asserts that a line is not composed of points, but rather constitutes the path or trace of a moving point (Arasse 1997). More closely, as discussed in (Longo, Longo 2021), "for Leonardo... the action of the pencil is an extension of the visual act, capable of capturing not only the form, but its becoming, its movement. The drawing and the swirling action on the sheet reproduce the momentum of nature and the world, to realize its "energies..."... an essential aspect of Leonardo's art consists in wanting to make one feel, in accordance with what he perceives of the world, the "*formation beneath the form*" (Chastel 1959, p.19)". Leon Battista Alberti instead claims, in *De Pictura* (1435), that the line is made up of an infinity of points. In view of Cantor's very rigorous and effective Set Theoretic re-construction of the line by uncountably many points, in late 19^{th} century, most of contemporary mathematics followed Alberti's view. Yet, alternative proposals are possible in Categories (*Toposes*) "without enough points", constructed following the contemporary ideas by Grothendieck, see (Zalamea 2012) and (Longo 2015).

In conclusion, it is from the early work of the painters-theologians of the 14^{th} century that emerged the perfect three-dimensional geometry of the paintings by Piero della Francesca (1412 – 1492), a painter-mathematician, and then Descartes and Desargues' spaces, in the 16^{th} century. Within these three-dimensional spaces—by using and extending Greek geometry and its symmetries—Galileo, Kepler, and Newton constructed modern physics (Longo, Longo 2020).

BREAKING SYMMETRIES

The breaking of symmetry in Euclid's fifth postulate lies at the heart of non-Euclidean geometries. Riemann's main aim (Riemann 1854) was to account for Newton's unexplained concept of "action at a distance." He sought to understand gravitation—along with electromagnetism and heat propagation—through the *structure of space* itself.

It is important to note that Euclid's fifth postulate is equivalent to the invariance, at any scale, of the sum of the internal angles of a triangle: in Euclid's geometry, this sum is always 180°. Homotheties—that is, enlarging or shrinking the triangle—do not alter this value; whether at human scale or between three distant stars, the sum remains the same. But this is precisely what fails in Einstein's theory of relativity, using Riemannian geometry. When measured on a triangle formed by three stars, the sum of the angles differs—an empirical fact observed since 1919 and considered a "confirmation" of Einstein's relativity. Homotheties are symmetries on a plane—the plane defined by the triangle. Therefore, a key aspect of the unity in Euclid's framework is lost: physical space—whether at the microscopic level of quantum physics or the vast scale of astrophysics—can have properties that defy sensory experience, particularly with regard to the fundamental symmetry expressed by the fifth postulate.

In Riemann's approach, the relationship between the local and the global is governed by a complex and novel mathematics: local maps are "glued" together using differential methods. Homotheties no longer allow for the transfer of measurements and knowledge at a "medium scale" to other scales. And this is profoundly modern: from relativity and quantum physics, we have learned that access, measurement, and operations at both the very large and the very small cannot rely on the naïve, sensory-based analyses of classical intuition. Departing from symmetry transformed physics through a new geometry of its spaces.

As a matter of fact, yet another fundamental symmetry breaking reshaped physics at the microphysical level. In classical and relativistic physics, measurements *commute*—that is, the order of operations does not affect the outcome, a symmetry property. This is not the case in quantum mechanics. The measurements of a particle's position and momentum, for example, do *not* commute: performing one before the other may yield a different result than doing them in reverse order—yet another form of symmetry breaking. This is crucial, because in quantum physics, position and momentum—or energy and time—are key observables. Measuring them with instruments is our only access to *physical reality*. More precisely, knowledge in microphysics can be constructed only through the experimental setup of measurements: there is no other way to build evidence and understanding. This is our starting point.

From this empirical fact and its theoretical consequences in quantum physics (Heisenberg's indetermination of measurement), Alain Connes (Connes 1994) developed a unifying geometry for quantum spaces. His work consists of reconstructing geometry from the non-commutative algebra of measurement (based on the non-symmetric algebraic structure of Heisenberg's calculus of matrices). In this respect, there is here a conceptual continuity with the approaches by Euclid and Riemann. But the instruments of measurement in quantum physics are far more complex than the ruler and compass, or even Riemann's more general notion of the "rigid body." Measurement in microphysics depends on elaborate physical and conceptual instruments; and the foundational certainty of quantum mechanics lies in a handful of observable phenomena that reveal the non-commutativity of measurement. Both of these approaches—Riemann–Einstein relativistic spaces and Connes' non-commutative geometry—represent a radical *geometrization* of physics, with far-reaching consequences. Symmetries, and their breaking, lie at the heart of it all.

THE SYMMETRIES BEHIND PHYSICAL CAUSALITY

> "All a priori statements in physics have their origin in symmetry"
> (Weyl 1952, p. 65)

Noether's theorem, 1918, (Kosman-Schwarback 2010) established a fundamental correlation between *continuous symmetries* in space and time and the conservation of certain physical quantities—such as energy, momentum, and electric charge—that characterize a system and govern its dynamics. The theorem describes these physical invariants in terms of space-time symmetry groups. For instance, energy conservation corresponds to invariance under temporal translations (a time symmetry). More generally, a continuous translation in time leads to energy conservation, while a continuous translation in space conserves momentum. Similarly, angular momentum conservation is associated with rotational symmetry. As a consequence, geodesic (optimal) curves define the trajectories along which quantities are conserved—as they are stable, minimal paths

Noether's theorem applies to continuous symmetry transformations which preserve the equations of motion derived from Hamilton's principle (i.e., the Euler–Lagrange equations). Thus, each such transformation corresponds to a conserved physical quantity. For example, invariance under spatial translation—reflecting the impossibility of defining an absolute spatial origin—implies conservation of linear momentum. Similarly, invariance under time translation—reflecting the impossibility of defining an absolute temporal origin—implies conservation of energy. From a less classical viewpoint, invariance under phase shifts of a wave function implies conservation of electric charge.

These deep links between invariance and conservation underlie the gauge theories of quantum physics—another major contribution of Hermann Weyl to mathematical physics. For nearly a century now, conservation laws (of energy, momentum, etc.) have been understood as resulting from spatio-temporal continuous symmetries.

Einstein's theory of relativity made extensive use of these principles. By establishing the equivalence of gravitation and inertia (gravitation being interpreted as inertial motion in curved spaces), Einstein could claim that "an apple falls for symmetry reasons." Newton's apple, just like the planets' movement around the Sun, follow an inertial trajectory within the gravitational fields—a manifestation of space-time curvature. Now, inertia, as the conservation of momentum, is itself a symmetry property, by Noether. This is why, as Weyl noted, relativistic physics introduced a profound shift in perspective: from causal laws to the structural organization of space and time. As we put it in (Bailly, Longo 2006, §1.3.5): "Causes become interactions, and these interactions themselves constitute the fabric of the universe, its geometry: deform this fabric, and the interactions change; intervene in the interactions, and the fabric is deformed."

As mentioned above, this normative role of geometry is now being extended to microphysics, notably through Alain Connes' work on non-commutative geometry. In a similar vein, foundational analysis in mathematics and its applications may benefit from this broader shift in the paradigm of scientific explanation—from laws to geometric intelligibility. An even more radical role for symmetry breaking is proposed in the recent mathematics of "heterogenesis" (Sarti et al., 2022), where the "laws"—understood as differential constraints governing a system's dynamics—evolve during the dynamics itself. Actually, the "phase space" itself (pertinent observables and parameters) is not preserved – this may be viewed as *the* fundamental symmetry breaking. This framework could offer a productive approach in the historical sciences—most notably in Darwinian evolution, where organisms actively modify forms and functions, thus "norms", of their internal structures and their interactions within ecosystems (Kauffman 2000; Soto et al. 2016).

Curie's Principle

Curie's principle states that "the symmetries of causes are to be found in those of the effects." Spontaneous symmetry breaking might appear to violate this principle. However, in classical frameworks, such symmetry breaking typically involves a random event which—unlike intrinsic quantum fluctuations—does not challenge causality itself, but only concerns observability. That is, it is *epistemic*: we may be unable to observe the fluctuation, below classical measurement, that produces the effect. Yet the symmetry, or its breaking, must still be posited in order to explain the resulting change in symmetry. It is thus the symmetry breaking in the invisible cause that breaks the symmetry in the measurable effect – and Curie's principle remains valid, in principle.

A critical transition—such as the sudden formation of a snowflake or a crystal—often involves a symmetry breaking and a corresponding shift in some physical property: for example, a change in density during a liquid–solid transition, or in total magnetic moment during a paramagnetic–ferromagnetic transition. The cause lies in the change of an *order parameter*, typically arising from a non-observable fluctuation. Note that phase transitions, in one way or another, represent a shift between relative disorder and relative order, in classical physics.

Quantum physics radically transforms the theoretical framework. The "spin up" or "spin down" of an electron, for example, has *no cause,* in Bohr's interpretation. Einstein famously rejected this view, proposing instead to search for hidden variables in continuous fields—"hidden parameters" whose fluctuations would determine whether the electron's spin is up or down – thus symmetries (and their breaking) would be preserved.

In conclusion, *symmetry reasoning*—whether rooted in physical theory, or in the language of social interaction, logical coherence, or aesthetic harmony—often leads to the generation of new mathematical concepts. Also the invention of the limit, vanishing point of the Italian Perspective, changes the symmetries in the painting and is at the origin of a major mathematical invention, the spaces of the scientific revolution. The new mathematics may correspond to richer mathematical representations, in space for instance, or even to new physical entities. Notable examples are the positron, a physical object predicted purely through a mathematical symmetry in Dirac's equation for microphysics, which describe the properties of the electron, and the symmetry breaking that lead to Higgs boson (Wikipedia).

ON THE FOUNDATIONS OF MATHEMATICS

> "The powerful dogma of the fundamental break between *epistemological clarification* and *historical explanation*, as well as the psychological explicitation in the frame of sciences of mind, the dogma of the break between *epitemological origin* and *genetic origin*, this dogma, as long as the concept of "history", of "historical analysis" and of "genesis" are not limited in an inadmissible way, as they usually are, this dogma must be reversed from top to bottom"
> (Husserl 1933, p. 201).

Mathematical intuition is the result of historical praxis; it is a constructed framework for active reasoning, grounded in action within space and stabilized through language and writing within intersubjective experience. Pure intuition refers to *what can be done*, rather than to what is. It is the perception of a mental construction—an appreciation of active experience, of an active (re-)construction of the world.

As Poincaré observed:

"A motionless being could never have acquired the notion of space, since, being unable to correct the effects of changes in external objects through its own movements, it would have had no reason to distinguish those changes from changes in its own state."

"…for each posture of my body, my index finger determines a point—and this, and only this, defines a point in space."

"To locate an object at any given point means to imagine the movement (that is, the muscular sensations accompanying it...) that one must perform to reach it."

(Poincaré 1902, pp. 78 and 92; Poincaré 1905, p. 67)

Euclid's principles of construction (his Postulates) may serve as a paradigm for this: the "seeing" of symmetries in gestures (drawing a line, a circle…) lies at its core. In our view, these constructions are *founded* in active praxis and their symmetries. Their analysis, as developed here, provides a historical and "genetic" foundation—an epistemological grounding of geometry on these symmetries-and their breaking.

We are capable of intuition because we actively construct (mathematical) knowledge on the phenomenal interface between ourselves and the world. Take, for instance, the early and fundamental gestalt of the continuous line: our evolutionary and historical brain constructs contours that are not actually in the world, beginning with the activity of the primary cortex.

Neurons in the primary visual cortex activate by contiguity and connectivity along non-existent lines, effectively *projecting* continuous contours onto objects (where, in reality, there are only singularities in the change of light frequencies). More precisely, recent studies of the primary cortex (see Petitot 2017) highlight the role of intra-cortical synaptic linkages in the perceptual construction of edges and trajectories. In the primary cortex, neurons are directionally sensitive: they respond when aligned with the tangent of a detected direction or contour. That is, neurons that respond to nearly parallel directions— along approximately the same straight line—are more strongly connected than others. In other words, neurons whose receptive fields are approximately aligned along a straight line (or along parallel directions) are more interconnected. Thus, the activation of one neuron facilitates or primes the activation of other, similarly aligned neurons—akin to tangents in space. This "gluing" of neural activations results in the perception of a continuous line—a trajectory in space. Its extension is made possible by the activation in advance of neurons that will be along the expected trajectory. This expectation (*protension*) is memorized thanks to previous practices of seeing, chasing, pursuing… . That is, their *retension* allows the protensive actions (preceding a prey or a predator) that are at the core of our movements as "geometrical" animals acting in space.

Of course, there is a significant gap—indeed, an abyss—between the biological-evolutionary pathway and the historical-conceptual construction; and yet, this is what we tried to bridge in these few pages. Symmetries and continua root mathematics in our deepest and oldest forms of cognition and action in the world. Mathematics is "a proud tree which freely raises its crown of branches into the thin air, but which at the same time sucks its strength through thousands of roots from the earth of intuitions and real representations" (Weyl 1910, quoted in the editor's commentary, 1985 reprint).

CONCLUSION

After a short detour in physics and art, we came back to the foundation of mathematics. The rooting of knowledge in active practices in an ecosystem, with our bodies integrated to and by an always active brain, is at the core of the perspective hinted here. Symmetries guide our action in space and our organization of it, they justify fundamental principles in physics. The relevance also of their breaking in science as well as their presence in art prove the key role of symmetries in knowledge construction and in our relation to the world.

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