

## **Rhythms, Retention and Protention: Philosophical Reflections on Geometrical Schemata for Biological Time**

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### **Abstract**

In this paper, following the technical approach to biological time, rhythms and retention/protention in (Longo, Montévil, 2014), we develop a philosophical frame for the proposed dimensions and mathematical structure of biological time, as a working example of “theory building”. We first introduce what “theory building” means to our perspective, in order to make explicit our theoretical tools and discuss the general epistemological issue. Then, through a conceptual articulation between physics and biology, we introduce protention (anticipation) and retention (memory), as proper biological observables. This theoretical articulation, which we consider at the core of moving from physical to biological theorizing, allows us to use some of the properties of these observables as principles around which it is possible to outline a proper geometrical schema for biological time. We then philosophically motivate the analysis of “time” as an operator that acts in biological dynamics in a constitutive way. In other words, space and time become special concepts of order, actively involved in the theoretical organization of biology, in contrast to existing theories in physics where they appear as parameters. In this approach, we first

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consider the usual dimension of an irreversible physical time. We then add to it a dimension specific to the internal rhythms of organisms. We motivate this dimensional extension by the relative autonomy of biological rhythms with respect to physical time. This second dimension of time is "compactified" in a simple but rigorous mathematical sense. In short, as soon as there are life phenomena, their rhythms scan biological time. We will consider such a statement as a starting point for an original notion of biological inertia. In a conclusion, we will summarize the general meaning of this concrete theory building.

### **Keywords**

Biological inertia

Biological time

Geometrical Schema

Protention

Retention

Rhythms

### **1 Introduction**

In this paper, we provide a philosophical account to an original approach on biological time, developed in (Longo, Montévil, 2014). This approach is a working example of "theory building". Before going into more details, we aim to state some philosophical considerations on the practice of "theory building", that we understand with the meaning of "constituting objectivity" [see (Bitbol, Kerszberg, & Petitot, 2009)].

Our constructivist approach is primarily built on the philosophical believe that science is not the mere description of phenomena, as well as theories are not the plain representation of reality. The objectivity of the *laws of nature* does not belong to nature itself, as an intrinsic truth. Rather, it results from the scientific activity, in interaction with the perception of the world. That means, the scientific object differs from the simple perception. Its configuration is related to the conceptual and practical tools used to access the phenomena and it is part of a *construction of objectivity* that is proper to scientific knowledge.

It should be clear that, for us, theory building is not an arbitrary speculation on "reality",

whatever this word may mean. Instead, it begins with the active access to the world by our sensitive and scientific tools for “measuring”, in a broadest sense. But these tools, starting with our physiological sensitivity, are biased by our actual being in the world: we may be insensitive to a visual perception or even to pain, say, by ongoing forms of conscious or preconscious being. Similarly, the construction of measurement tools, in science, is biased by strong theoretical commitment, such as the construction of the instruments of microphysics. These depend on the choice of the observables to be measured, for example, the physical quantities expressed by the bounded operators in Schrödinger's equations, (Bitbol1996), as well on countless technical details all depending on theoretical and engineering frames. Thus, they depend on both previously theorized experiences (the search for the spin of a particle by these or that tools, say) as well as on their mathematical expression (Schrödinger's approach, typically). The choice of the observables and measurement contexts and tools is at least as important in biology, as our working example will show.

In other words, the process of building theories happens in a halfway place between thinking and reality. The convergence of these two *ways of access* to phenomena is the place where concepts appear and stabilize. According to this perspective, a main difficulty of the scientific activity has been to take the conceptual elements of this intermediate world, as reality themselves. It is typically the case of categories such as causality, of numbers, of symbolics structures as time and space, and so on. In the particular case we account for, we make extensive use of the conceptual construction of time and geometry.

In the approach to biological time presented here, we recognized two different forms of observable time in the same dimension, the irreversible time of thermodynamics and the time of the developmental process in biology (embryo/ontogenesis). Then, we also distinguish between the dimension of these two forms of time and a new one, where it is possible to represent biological rhythms. The choice of this new observable is far from being arbitrary as it is motivated by the relevant empirical role of rhythms from the analyses of minimal forms of metabolism to the clinical relevance (and extensive literature) on the heart and the respiratory rhythms (see below and the references). Our philosophical bias should be clear: we noticed a different practical role, given to rhythms, in concrete actions on living beings, from investigations to therapies. Historically, this role has been made possible by the invention, since long, of a variety of tools for their

measurement. We then integrated these knowledge and practical activities into a basic, schematic representation which may suggest both further theoretical developments and new tools for measurements.

## **2 Philosophy and geometrical schemata for biological time**

Contemporary studies on the temporal orientation of consciousness insist on the importance of anticipation and memory. The role played by these two temporal elements has been largely explored through a long philosophical and phenomenological tradition, among others [see for example (Depraz, 2001; Petitot, Varela, & Pachoud, 1999) for recent approaches and syntheses]. Husserl designated as “protention” the particular extension of the present towards the future by anticipation. Moreover, he proposed to link this aspect to “retention” as the extension of the present towards the past by memory (Husserl, 1964). Recent works, directly inspired by the phenomenological approach, formalize this aspect in an increasingly precise way [see for example (Gallagher & Varela, 2003; Voegeley & Kupke, 2007)]. This philosophical background is an essential component of the theory building introduced here. Too often the scientific work hides philosophical biases, (Perret et al., 2017); by our worked example we will also consider the implicit consequences of an intended philosophy.

In our theoretical approach, we propose to see protention and retention as the starting point to exhibit a specific temporal structure of the *preconscious* living systems, as a compatible extension of the phenomenological view. Through a conceptual articulation between the role of observables and parameters in physics and in biology, we argue that these two elements are specific observable for biology. Accordingly, it is possible to outline a geometry of the temporal structure specific to living organisms (Bailly, Longo, & Montévil, 2011). This is a broader theoretical approach that seeks to propose specific principles for a conceptual organization of the living. This global perspective is grounded on the new theoretical framework of “extended criticality” (Bailly & Longo, 2008; Longo & Montévil, 2011).

### **2.1 Constitutive Space and Time**

In this paper, we invoke a special constitution of time for the living. Inspired by the

philosopher Kant (2000), we consider that reflecting on time and space means reflecting on the deep conceptual conditions of a scientific domain as *a priori* forms. That means, we are not invoking a description of some real temporal properties that would be present within biological objects, and not even a measurement of quantities, but more a mathematical conceptualization of some temporal specificities that we recognize as observables in the living. To do this operation, we have to clarify which is the theoretical role of mathematics in our perspective. According to the Kant of the *Critique*, the *a priori* forms of space and time receive and shape phenomena. Mathematics, because of its *synthetic* and *a priori* character, organizes the content of the *a priori* forms by a conceptualisation. This amounts, therefore, to a constitutive role of mathematics in the construction of scientific concepts (Cassirer, 2004). Space and time, then, are the *conditions of possibility* of constituting objectivity operated by mathematics (Kant, 2000).

Now, through a further abstraction of the *a priori* transcendental principles<sup>3</sup>, it is possible to overcome the Kantian dualism between, on the one hand, the pure forms of the *a priori* intuition of space and time and concepts, on the other hand. In particular, time becomes an *operator*. It participates to the organizing and constitutive activity of mathematics rather than being set frameworks making such activity possible. In other words, space and time become true *concepts of order* actively involved in the conceptual organization of a science. As for our perspective, this a major methodological shift when moving from physical to biological theorizing.

In this context, the project of making mathematically intelligible the geometrical complexity of biological temporality corresponds to the construction of a geometrical schema specific to living phenomena, which is characterized, in a broadened Kantian sense, by a set of principles of order. Indeed, in the process of constituting scientific objects, the determination of sensory perception through concepts provide not only the

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The process of relativisation of the Kantian *a priori* comes from the neo-Kantian School of Marburg and especially from Cassirer. With non-Euclidean geometries, *a priori* forms of intuition of space and time (which, for Kant, had the form of Euclidean geometry) could no longer constitute a scientific foundation for localisation. Moreover, after the formulation of the theory of relativity (restrained and general, both basing themselves on non-Euclidean spaces), the very concept of an object and its relationship to space was no longer immediate in intuition. More specifically, in classical mechanics, the dependency of the notion of “object” upon a complex of universal laws was founded on the laws of geometry. In , relativity theory, instead, the localisation of an object takes place through operations that enable a transition from one reference system to another. It is the invariants of such transformations that may be deemed “objects”. We refer here to (Cassirer, 2004); for broad overviews of the possible modulations of the *a priori* we refer to (Kauark-Leite, 2012; Lassègue, 2015).

scientific object, but also the rule of construction of this object. We call this rule, in a Kantian sense, a *schema*. That means, the process of determination assumes genericity, because together with it, goes the exhibition of a general process of construction. Therefore, providing a geometrical schema for biological time means operating a geometrical determination on the specificity of time shown by the living, and constructing, through this determination, a general theory. Now, a biological object, as a *physical singularity* (Bailly & Longo, 2011), presents specific temporal characteristics, as described in (Longo, Montévil, 2014) and that we will survey and discuss here.

Note that, in general and as in any scientific theorizing, a sound theory of life phenomena must stem from its specificities in order to construct a conceptual organization adapted to the biological. The physicalistic reduction of the biological constitutes an illegitimate theoretical operation, based upon a realist prejudice according to which the laws of physics represent real properties of phenomena. Instead, physical theories are also conceptual organizations of “reality” constructed from transcendental principles (Bitbol, 1998, 2000). Furthermore, they propose various notions of a *causal field*, which are not even reducible to one another, as, for example, in classical and relativistic vs quantum physics. So, to which of these fields should we reduce life phenomena? Note that both classical and quantum phenomena are relevant in biology, as they may both have phenotypic effects, from cells to organisms (Buiatti, Longo, 2013); not to mention the hydrodynamics of fluids, a field far from being understood in terms of quantum (particle) physics (Chibbaro et al, 2014).

It seems then more pertinent to construct a new “causal field” for the biological, which is founded upon its own specific principles, the same as the physical causal fields within their own theoretical domains. Later on, one may better establish a unification project, as it is also considered in physics (like, for example, in the ongoing tentative unifications of relativistic/quantum physics: what would there be to unify if there were not two theories?). Now, for biology we suggest to ground the causal field on a geometrical schema for biological time, as a part of the theory of organisms we are working at, see (Soto, Longo, 2016).

The temporality of the living organisms is very specific compared to the physical treatment of time. Development, aging, biological rhythms, evolution and metabolic cycles attest to this peculiarity (Chaline, 1999). Here, protention and retention will play a constitutive role.

We propose first to take minimal protentional behaviors as mathematically quantifiable observables, without requiring the production of a physical theory of teleonomy. On the contrary, the teleonomic dynamic becomes a simple principle, founded upon the intuition of a movement of retention/protention, which is an observable.

The protentional act indeed represents a temporal specificity, which we observe in the simplest living forms (for example, paramecia, (Misslin, 2003)). It is from the internal rhythms of life phenomena, which we will address, and from this act that we can establish an autonomy of biological time in contrast with physical time. This demonstrates the need for an autonomy of the transcendental principles of biology. It will, therefore, be entirely legitimate to add specific observables as well as a second temporal dimension specific to biological rhythms. We will then construct a geometrical schema for biological time on two dimensions: one to represent the rhythms specific to life phenomena, the other to quantify the usual dimension of the irreversible time of physics. In this dimension, a new observable of time appears to be relevant : the irreversibility specific to biological processes, in particular the setting up and renewal of organization (Longo, Montévil, 2014).

## **2.2 Dichotomous External/Internal Reference Systems**

The transcendental role in the geometric construction of biological time manifests even more radically in the way in which a biological object's two internal/external poles of temporal reference are articulated. Indeed, due to the relativization of the Kantian *a priori*, it is legitimate to consider abstract notions of space and time as able to relate to the mathematical structures of group and semigroup, respectively. In particular, the determination of displacement groups (reversible) is involved in formalizing the abstract notion of space. Analogously, the characteristics of semigroups participate in formalizing the abstract notion of time and, namely, of the properties of compositionality and irreversibility of the flow of time (Bailly & Longo, 2011, p. 169).

In short, we consider, first, physical space, where displacements (reversible, group transformations) are possible, and *within which* we can describe the internal/external spaces for each organism and, second, an irreversible physical time (whose transformations form a semigroup). More generally, by an extension of this correspondence to logic<sup>4</sup>, we

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<sup>4</sup> The notion of group can be put into correspondence with the logical relationship of equivalence, and the notion of semi-group has the same form of ordered relation, (Bailly Longo 2011, p. 163).

can see the outline of a dichotomic structure of constitution taking another step towards abstraction. We have, on the one hand, the *space, group structure*, as the *equivalence relation* pole, and, on the other hand, the *time, semigroup structure*, as the *order relation* pole.

To this ordered time line, we *add* a second dimension specific to the internal time of organisms, the time of biological rhythms. This dimensional extension will be motivated by the relative autonomy of biological rhythms with respect to physical time. They present themselves in fact as “pure numbers”, that is, as invariants common to different biological species. In short, to irreversible physical time (the thermodynamic straight line or, algebraically, a semigroup), we add an orthogonal dimension represented by a compactified straight line (a circle, a “compactification method” that has already been exploited in physics for space by Kaluza and Klein, see (Bailly & Longo, 2011)). It intertwines with it as an iterative and circular observable, that of biological rhythms, which we will address.

Now, these two dimensions articulate with one another through a dichotomy of the internal/external type, which participates, constitutively, in a new conceptual organization of biology. This also comes down to the constitution of a causal field specific to life phenomena, because we will correlate protention with the setting of these internal rhythms, enabling us to conceptualize a form of teleonomy without, nevertheless, referring to a retrograde causality.

To return to our Kantian considerations, the space and time of the *Critique* (2000) were in opposition and, more precisely and respectively, they assumed, within the subject, the *a priori* form of the external sense and the *a priori* form of the internal sense. Let's recall here the progressive rediscovery of the Leibnizian arguments defended by the later Kant of the *Opus Postumum* (1995), according to which space and time can no longer be in such opposition, but themselves possess intrinsic forms on the side of the object. We are then led to rediscover the legitimacy, at least a theoretical one, of the structuring of a proper internal temporal dimension for life phenomena, insofar as both internal/external poles must be found within the same object, the living organism. However, this does not mean that they constitute properties that are intrinsic to the objects, because we are still at an epistemic level of constituting objectivity. What we have, in particular, are forms of constituting the localization of objects coming from their actual determination. In other words, space and

time become the active conditions of constituting the intelligibility of the object: sort of *forms of sensible manifestation* (Bailly & Longo, 2011, p. 153). The external sense, then, determines the form of the manifestation of the relations and the internal sense governs the form of the manifestation of the identification of the objects. By means of this process, and in conjunction with relativisation of the *a priori*, a transformation of the abstract notions of space and time is operated. This transformation, in conclusion, comes down to justifying the epistemic role of the internal spatiotemporal dimensions specific to biology, governing the very conditions of the possibility for individuating the object.

Following (Longo, Montévil, 2014), we reconstruct and elaborate on this process through two movements. First, we identify a proper observable, the time of biological irreversibility, and we place it in the dimension of physical time (thermodynamic time, therefore oriented), anchored upon an extended present (retention/protention). Then, we add a second compactified dimension to this temporal dimension shared with physics. This dimension is supposed to be the proper dimension to describe biological rhythms. This geometrical schema constructs a new intelligibility using the internal constitutive property of the abstract notion of time.

### **3 Retention and Protention**

Husserl undertakes a fundamental analysis of the temporality specific to consciousness, separated from objective time, based upon two opposing temporal directions: memory and anticipation. Memory is characterized as a reconstruction of a distant past and anticipation as the expectation of a possible future. Now, these two poles belong respectively to the past and to the future, but a tendency towards these two directions along the same axis takes place in the *present apprehension* of phenomena.

We will very generally address the movements of retention and protention, even in the absence of intentionality (so also for preconscious activities). Retention and protention are forms of the present: the present instant is therefore constituted as a dialectic situation, which is never simple or defined, a situation *that is not to be described as punctual*.

More specifically, in physics, one can conceive of a punctual (pointwise) present, a singular instant which is a number on Cantor's straight line of real numbers. The temporal singularity of the biological, instead, is *extended*: an extended transition from the past to the future, a union of minimal retention and of the corresponding protention. This change

is fundamental and paradigmatic with respect to physics. With the invention of speed and acceleration as instantaneous values, the limits of a secant that becomes a tangent line (Newton) or of a ratio of which the numerator and denominator tend towards 0 (Leibniz), mathematics sets itself within modern physics. Then, by their punctual values, speed and acceleration also become functions of time<sup>5</sup>.

Now, in biology, in this case and others, the punctuality of a process is devoid of meaning: the snapshot loses what is most important, the functions and action of the living phenomenon, which is *never* devoid of activity. The instantaneous picture of a rock that is falling is identical to the picture of the rock when stationary, the rock being *inert* even during its *inertial* movement. Life can be understood only in its processes, which are constantly renewing and changing, from internal physiological activity to movement. Biological time is therefore not to be grasped based on a possible punctuality; and this will also apply, as far as we are concerned, to *all* biologically relevant parameters and observables. Even more strongly, life is not only a process, a dynamic; it is always (in) a “critical transition”. We have rendered this analysis of the “extension” of biological observables and parameters by the notion of “extended criticality”, which is specific to the living state of matter (Bailly & Longo, 2008, 2011; Longo & Montévil, 2011a), to be briefly hinted below.

In time, retention, directed towards an immediate past, and protention, directed towards the immediate future, constitute an extension of the present that distinguishes itself from the objective time of physics, all the while articulating itself with it. We refer to (Longo & Montévil, 2011b) for the mathematical analysis: retention is described by means of a relaxation function (an exponential that decreases in physical time), whereas protention is described by its symmetrical, corrected by a linear dependence of retention. The composition of these formal symmetrical exponentials formalizes the fact that there is no anticipation without memory, as advanced by Husserl and as confirmed by recent empirical evidence (Botzung, Denkova, & Manning, 2008) (for other works on conscious activity, see (Nicolas, 2006; Perfetti & Goldman, 1976)). Protention is therefore mathematically dependent upon retention, an asymmetry that orients biological time. In short, we consider as if the organism, as elementary as it may be, were capable of protention. Such protention

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<sup>5</sup> Note that H. Weyl, a major mathematician of relativity theory, while working at “Space, time and matter”, a fundamental book for that theory, stresses the limits of the physical description of time. He does so in (Weyl, 1918), in reference to the non-pointwise experience of phenomenal time, where the knowing, living subject plays a role.

is able to govern the behavior of the organism in its present on the basis of prior experience. Even a paramecium manifests clear forms of protention and retention, see (Misslin, 2003).

To conclude, to this construction of objectivity specific to biological time, we added, taking Husserl as a starting point, a temporal observable that is specific to biology based on the interplay between retention and protention. This notion, albeit in the same mathematical dimension as the physical arrow of time, oriented by all irreversible phenomena (at least thermodynamically) does propose a new observable for us: the irreversibility specific to biological time, oriented by the mathematical asymmetry of retention/protention (Longo & Montévil, 2011b).

Notice that within the same physical dimension we can have several observables: energy, for example, can be potential or kinetic. For us, the irreversibility specific to biological time adds itself to that of thermodynamic time. Its irreversibility is not only due to the dispersal of physical energy (entropy), but also to the establishment and maintenance of biological organization (which we have analysed in terms of anti-entropy, see (Bailly & Longo, 2009; Longo & Montévil, 2012)). Evolution and embryogenesis (ontogenesis, in fact) have their own constitutive irreversibility, which adds itself to that of thermodynamic processes. This irreversibility is the observable of time specific to life phenomena; in (Bailly & Longo, 2009), it is considered, mathematically, as an *operator* and not as a parameter as is time in physics, because it operates and constitutes life phenomena, *which is always the result of a history*. In summary, the asymmetry of retention and protention contributes to this new irreversible observable time proper to biological objects and their determination.

#### **4 Biological Inertia**

The minimal protentional capacity of living organisms may be founded upon observing the propensity of any organism to simply extend a situation. This capacity may be more or less pronounced according to the level of evolution and the presence or not of a nervous system. It is, first, the observation of an aptitude to adapt to a situation, by changing and through “self-preservation”, that leads us to introduce a function of retention, a component of identity and “structural stability”. This may be conceived as the possibility of registering a *morphological memory* at various levels, for example at the biochemical, immune,

neural, vestibular or cerebral levels; however, its main biological purpose is precisely to enable protention. In other words, we consider the possibility for an organism to conserve a memory of a comparable previous situation, through learning, even at a very simple level of organization, as the precondition of an *adaptability through anticipation* of a similar situation. In this approach, the genome could be considered as the main retentional component specific to a species. As such, it would play as much of a constraining role with respect to the huge range of hypothetical possibilities as it would the role of an activator with respect to the development of such or such an organism belonging to a given species. This constraint would in a way “canalize” the possibilities of development as a function of the retentional heritage, that is, to the whole biochemical history of the species. The eventual “explosions” associated with the rupture in punctuated equilibria (c.f. the Burgess fauna as analysed by S.J. Gould (1989), for example) would then correspond to the lifting of entire classes of inhibitions with respect to the activating role of genomes. This representation would then correspond to the viewpoint according to which life phenomena, far from selecting singular and *specific* geodesic trajectories as in physics, would evolve within a very *generic* framework of possibilities. Among such possibilities some would be inhibited either by internal constraints (from the genome to the structure of the organism) or external constraints (the environment).

At the level of the organism, we can interpret protentional behavior as an anticipation played upon the activation of memory. Thus, the trace of experience also playing a role of constraint: some consequent reactions become plausible and then generate a related behaviour, even if it then proves to be poorly adapted, thus leading to further learning. Anticipation of this type becomes an instrument for interpreting the behaviour of the organism with respect to the unpredictability that it continuously faces. It can even be seen as a sort of instrument for continuous reorganization in consequence of the impossibility of rendering explicit the whole field of possibilities. Thus, as clearly distinguished by Husserl, retention is not memory itself, but the process of memory activation in the present instant — *in view of action*, we emphasize. Likewise, protentional movement is not anticipation into the future, but the process of projecting the immediate possibilities of a previously lived, yet, in fact, reconstructed state.

By these movements of dynamic extension of the present, we have a sort of inertial principle of life phenomena, which we could call *biological inertia*. In (Longo & Montévil,

2011b), this inertia is mathematically represented as the coefficient of protention: it gives it mathematical “weight”, so to speak, depending on retention, in the same way as (inertial) mass, in physics, is the coefficient of acceleration in the presence of a force.

## **5 Biological Rhythms, a Geometrical Schema for Life Phenomena**

Using the same process of mathematical objectivation, a new dimension of time founded upon the consideration of rhythms that are internal to life phenomena may be added to the dimension of thermodynamics in which retention/protention also resides (Longo, Montévil, 2014). This second dimension of time is compactified (a circle, a loop, instead of the usual straight line of the Cartesian plane), and is thus autonomous in an even more radical way with respect to physical time. In short, as soon as there are life phenomena, there is a rhythm that takes place within: the metabolic rhythm, at least, and then the cardiac, respiratory, and hormonal rhythms, among others. Observation proposes them to us as pure numbers: they give us the time of an organism (life-span, typically), by allometric coefficients, but they do not have the dimension of *time*. For example, the number of heartbeats of mammals is an a-dimensional invariant, a number (approximately  $1.2 \times 10^9$ ) and, by a coefficient given by the mass, it gives the average life-span of the organism in question. Thus, a mouse and an elephant have average life-spans that differ by a factor of 50, but they have the same number of heartbeats, the frequency of heartbeats being 50 times higher in the mouse (refer to (Bailly et al., 2011), (Longo, Montévil, 2014) for the technical details).

This second temporality contributes to establish and justify a specific causal field for life phenomena. Maybe it is this aspect that must in certain respects be interpreted as a *retrograde causality* but without constituting a temporal inversion. It is rather a circular movement which establishes itself and is also at the heart of the minimal retention/protention dynamic: the expectation of the return of a rhythm, as we will argue below.

From a mathematical standpoint, the introduction of a compactified dimension of time gives, for the topology of our biological schemata for time,  $\mathbf{R} \times \mathbf{S}_1$  (a straight line times a circle, a cylinder). Of course, the compactification “radius” remains null in analyses of the inert.

This structure of time breaks certain classical causal aspects, as we were saying: through

protection, there may be a change in the present following an *anticipation* of the future. However, the second compactified dimension is exclusively relative to the biological rhythms and fluxes of the very special component of “information” that is related to protection.

In these analyses, two types of biological rhythms are proposed:

1. “External” rhythms, directed by phenomena that are exterior to the organism, with a physical or physicochemical origin and that impose themselves upon the organism. These rhythms are the same for many species, independent of their size. They express themselves in terms of physical, hence dimensional, periods or frequencies (s, Hz) and the invariants are dimensional; they are described relative to the dimension of physical time (in *exp(it)* ). Examples: seasonal rhythms, the circadian rhythm and all their harmonics and subharmonics, the rhythms of chemical reactions that oscillate at a given temperature, etc.
2. “Internal” rhythms, of an endogenous origin, specific to physiological functions of the organism that therefore depend on purely biological functional specifications. These rhythms are characterized by periods that scale as the  $\frac{1}{4}$ <sup>th</sup> power of the organism’s mass and are related to the life-span of the organism, which scales in the same way; they are expressed as pure numbers. For this reason, these invariants are *numerical*, in contrast with the great constants of physics, which have *dimensions* – acceleration, speed, action... In our description, by a new compactified “temporal” dimension, the numerical values then correspond to a “number of turns”, independent of the effective physical temporal extension (examples: heartbeats, respirations, cerebral frequencies, etc. See the graphical representation in (Longo, Montévil, 2014)).

In short, endogenous biological cycles, which do not depend directly on external physical rhythms that impose themselves, are those which:

1. Are determined by pure numbers (number of respirations or heartbeats over a lifetime) more than by dimensional magnitudes as in physics (seconds, Hertz...).
2. Scale with the size of the organism (frequencies brought to a power  $-1/4$  of the mass, periods brought to a power  $1/4$ ), which is generally not the case with constraining external rhythms, which impose themselves upon all organisms

(circadian rhythms, for example).

3. Can thereby be put into relation with an additional compactified “temporal” dimension (an angle, actually), in contrast with the usual temporal dimension (physical, thermodynamic, more specifically), non-compactified and endowed with dimensionality.

In this framework, the extended critical situation, corresponding to the self-referential and individuated, but changing, character of the organism, presents a topological temporality of the  $\mathbf{R} \times \mathbf{S}_1$  type; whereas the externality of the organism (and the way in which this externality reacts with the organism) preserves its usual temporal topology of  $\mathbf{R}$ .

Without changing the basic question, we can present a somewhat different perspective: for a living organism, the extended critical situation would occupy a volume within an  $n$ -dimension space, with  $n \geq 5$ . Among these  $n$  dimensions we would distinguish the three dimensions of classical physical space ( $\mathbf{R}^3$  topology) and the two dimensions of biological time ( $\mathbf{R} \times \mathbf{S}_1$  topology), of which the compactified dimension would have a null radius beyond this volume. The remaining  $n-5$  dimensions correspond to the compatible values of the vital parameters (temperatures between  $T_1$  and  $T_2$ , metabolisms between  $R_1$  and  $R_2$ , etc.): all intervals of extended criticality in which the limits are those of viability. The metrics of the volume’s space would correspond roughly to the correlation lengths; the metrics of time would maximally correspond to the life-spans (for  $\mathbf{R}$ ) and to pure maximal numbers (maximum endogenous frequencies) for  $\mathbf{S}_1$ . One will notice that the endogenous rhythmicities and cyclicities are not so much rhythms or cycles as such as they are *iterations* of which the total number is set (please refer to the quoted articles and book for the technical details).

Let’s finally return to the play between retention and protention. We propose to situate the primordial or even minimal protentional gesture/experience in the *expectation of the return of a vital rhythm*, as we hinted above. Protention, therefore, presents itself as a consequence of the act intrinsic to life phenomena: as soon as there is life, from its very evolutive or embryonary origin, a rhythm is established, a metabolic rhythm at the least, the other ones afterward. We describe this process as the sudden formation, a sort of “big-bang”, of a new temporal dimension that characterizes life phenomena, the dimension of biological rhythms. They generate the anticipation of their own return, therefore the primary protention/anticipation, which justifies, without teleonomy as such nor retrograde

physical causality, this inverted biological causality we mentioned earlier, that which modifies present action by the preparation of the protentional gesture.

## 6 Conclusion

Knowledge construction is based on a complex friction between the knowing subject and the “real world”. An essential component of its scientific objectivity depends on the invariance of the proposed concepts and abstract thought structures, under transformations of reference system and its metrics (the scale and tools for measuring). This is the main epistemological lesson we draw for the physics of relativity and, even more so, from quantum mechanics.

We proposed above some invariant schemata for the description of the phenomenal time of life. They derive from observations, from references to experiments (from paramecia to neurological measurement) and are based on an explicit philosophical commitment to a phenomenological analysis of life processes and their temporality. Mathematics, even the elementary notions we referred to here, provides useful symbolic tools for describing a relatively stable conceptual invariance. The question remains of the actual generality, effectiveness and independence from the knowing subject of the proposal of those abstract concepts. "But perhaps this question can be answered by pointing toward the essentially historical nature of that life of the mind of which my own existence is an integral but not autonomous part. It is light and darkness, contingency and necessity, bondage and freedom, and it cannot be expected that a symbolic construction of the world in some final form can ever be detached from it." (Weyl, 1927; 1949, p.62)

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