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SPACE, TIME AND COGNITION
From The Standpoint of Mathematics and Natural Science[1]

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Premise

The paper offers a twofold epistemological analysis of the concepts of space and time: Part I frames them in the setting of contemporary physics, Part II deals with their role in biology and especially in the project of its mathematisation. Both investigations are closely connected with questions in cognitive science. The issues involved in the analysis of the foundations of mathematics and the natural sciences have profoundly affected approaches to human cognition and treatment of these foundational questions forms an indispensable preliminary to our whole understanding of the cognitive sciences.

Contemporary physical theories have led to a steadily more pronounced geometrisation of physics, the counterpart of which has been a steadily more pronounced physicalisation of geometry. This is clearly illustrated in general relativity, where the geometrisation of gravitation (the trajectories of objects are described as geodesic curves in a Riemannian manifold) can equally well be interpreted as the physical realisation of a mathematical structure (the space-time curvature is determined by the distribution of energy-momentum). This geometrisation is seen even more clearly in quantum field theory, where the introduction of non-abelian gauge fields to give an account of the dynamics of interacting fields has led to the development of an intrinsically non-commutative geometry (see Connes, 1994). As for the epistemological status of space-time concepts, the mathematical specification of geometric notions can be seen as a process of the objectivisation of the forms of intuition of our phenomenal awareness. Indeed these very forms of intuition, just as much as the mathematical specification of the structures of space and time, are to be investigated within the setting of specific contemporary physical theories.

When we turn to the role of mathematics in biology, the *constitutive* role which mathematical concepts play in physics is in contrast to their prevailing *conceptual* status in biology. The various affordances and regularities which experience furnishes are transformed in physics into very rich mathematical structures - structures far richer than suggested by the 'symptoms' through which our senses and/or physical instruments apprehend the physical world. Moreover, these mathematical concepts, rather than being merely descriptive, play a regulative role in constituting our concept of physical reality. One can say nothing of the subject matter of relativity, of quantum theory, or of the general theory of dynamical systems (the heart of theories of critical states and phase transitions) without mathematics.

In biology, by way of contrast, one is struck by the enormous richness of structure with which living systems as given to us in phenomenal awareness are already endowed, and the fact that their theoretical formulation in terms of mathematical concepts suffices to model only certain aspects of that structure, and then in a manner which tends to fragment their organic unity and individuality and fails to do justice to their immersion in wider ecosystems. If we reflect on the role of mathematics in human cognition we are thereby led to re-examine its role in biology, since living systems are the starting point of all reflection on cognition.

Nevertheless, despite these differences and granted the lesser extent of overall mathematisation in biology, one can recognise in many areas of biological research an apparent movement towards what may loosely be termed 'geometrisation'.

Questions involving our understanding of spatial concepts are posed not only in the study of macromolecular structures (e.g. the sequencing of DNA base pairs and the resulting expression of genetic

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[1] *In* **Causality and Mind**, (Peruzzi ed.), Kluwer, 2003. Michael Wright took care of the English translation, by a close understanding of our philosophical view and by proposing several improvements to the original French version (à paraître dans la **Revue de Synthèse,** Paris, n. 1, 2004.)

effects, or the investigation of the spatial structures of proteins or prions) but also within developmental biology (in the study of the effects induced by spatial contiguity in embryogenesis for example) and in the study of organic function (e.g. the fractal geometries affecting the boundaries of the membrane surfaces engaged in the regulation of physiological functions) and also in the study of population dynamics and its associated environmental context.

Alongside these areas involving spatial understanding, the examination of temporal concepts is also strongly implicated in such areas as the study of the response times to external stimuli, the iterative character of internal biorythms, and in the study of synchronic and heterochronic patterns in evolutionary biology, the outcome of which has been a recent formulation of synthetic theory of evolution itself.

What connection can we trace between the roles of spatial concepts in physics and in the life sciences? The conceptual scaffolding of modern geometry is itself rooted in the conditions of possible actions and experiences which are a basic aspect of our presence in the world. It has at its foundation an inseparable intertwining between (i) our presence in the world as sentient creatures and centres of inter-subjective awareness (as suggested by Husserl), through symbolisation and abstraction, and (ii) the evolutionary leap to which this capacity for rational thought and creative imagination has led. Such a constitutive *braid* connects the phylogenesis of humans to their ontogenesis as cultural beings in history, via the stabilisation of inter-subjectivity through language. In this perspective we should also view the semiogenesis of conceptual constructions that arise in mathematics and physics. Without the initial spatiality of actions (especially gestures, with their intentionality) and the dimensionality of our primal imagination and cognition, we could never have arrived at the idea of a ... 10-dimensional manifold, in terms of which the theory of superstrings in quantum physics is elaborated.

In PART 1 we analyse the notions of space and time as characterised by three types of physical theories: relativity, quantum theory and the theory of dynamical systems. In PART 2 and PART 3 each of the authors independently (Part 2: G. Longo; Part 3: F. Bailly) examines the same notions in connection to theoretical biology. We conclude by putting forward a tentative categorisation, in abstract conceptual-mathematical form, of the manner in which space and time operate as invariants in determining our forms of knowledge.

PART 1. An introduction to the space and time of modern physics

1.1 Taking leave of Laplace

The physics of the nineteenth century carries the imprint of Laplace. His achievements in mathematics, physics and philosophy marked the moment at which the development in the direction of modern physics, initiated by Galileo, Descartes and Newton, reached maturity. Laplacean mechanics is organised in the framework of an absolute background space with Cartesian co-ordinates in which the motion of bodies is governed by the laws of Galileo and Newton. The perfection of this *mechanica universalis* is completely expressed through eternal mathematical laws. To cite Laplace: An omniscient being with perfect knowledge of the state of the world at a given instant could predict its entire future evolution with perfect precision. But what counts for even more in Laplace's work, for us earthbound and imperfect beings, is not this divine, and unachievable, knowledge but the approximate analysis of (possibly perturbed) systems. If one knows the state of a physical system to a certain degree of approximation, one can in general determine its evolution to an approximation of the same order of magnitude.

In this sense, according to Laplace, mathematics rules the world and permits the prediction of its future state, by a finite and complete system of differential equations. In fact the analysis of the perturbations of planetary orbits was one of the chief impulses driving the development of nineteenth century mathematical physics. As for causality, in Laplace’s approach, determinism implies predictability.

The development of twentieth century physics has taken a quite different direction. Relativity, quantum theory and general dynamical systems have led to an entirely distinct set of concepts and inspired a quite different philosophy of science from that which prevailed in the nineteenth, in particular as for causality.

We cannot say the same of the mainstream in the cognitive sciences. Turing, in his seminal article founding the strong AI program and setting out the functionalist account of cognition, made the explicit hypothesis underlying his generalised discrete-state machine (the “Turing machine”): by its discrete nature, full predictability is possible, in the sense of Laplacean determinism (Turing 1950). The Laplacean idea of a finite and complete set of rules is thus consciously placed at the heart of the game of simulation (envisaged in the

Turing test) through which he set out to demonstrate that the functioning of the brain was equivalent to that of a Turing machine.[2]

In fact the notion of a deterministic program, as it emerged in the work of the logicians of the 1930s (the theory of computability was developed by Curry, Church, Turing, Kleene and others in the years 1930-36) is inherently Laplacean, as clearly spelled out by Turing. That is to say, it implies complete predictability of the states of a computer running a program (see Longo 2003a). From this ideal model, which stems from the logical calculi of formal deduction rather than from physics, the Laplacean paradigm of the brain as a Turing machine running a program has been transferred to the study of cognition in the biological setting. It is of crucial relevance to the project we are pursuing here that the abstract description of a Turing machine is in no way dependent on our understanding it as a spatial structure. The "Cartesian" dimension of its material being has no influence whatever on its expressive powers. Moreover, its internal clock records a sequence of discrete states in an absolute Newtonian time. It was explicitily invented and behaves as a logical machine, not a physical mechanism (see Turing, 1950; Longo 2003a). By contrast, the analysis of space and time and their dimensionality is at the heart of any analysis of physical phenomena.

In relation to any claim that living systems and their mental activities can be "reduced" to physics we ought to ask: to *which* physical theory? *Which* physical laws have to be employed in the analysis of biological and cognitive phenomena? Functionalism is the still prevailing approach to cognition and biology (the "genome is a program" paradigm, for example) and implicitly refers to a Laplacean causal regime.

1.2 Three types of physical theory: relativity, quantum physics and the theory of critical transitions in the behaviour of dynamical systems

Relativity. Relativistic theories introduce a 4-dimensional spacetime in which conservation laws and relativistic causal principles are described in terms of invariants with respect to the relativity group of the theory. In special relativity (SR), the objects of the theory and the space-time structure are given together with their invariance properties under the group of rotations and translations in this space (the Lorentz-Poincaré group). In general relativity (GR), physical space is described as a Riemannian manifold of all possible locations together with its dimensionality and symmetry properties. The metric coefficients *are* the gravitational potentials just as the local curvature of the Riemannian manifold *is* the energy-momentum. Thus geometry *constitutes* the invariants we name as "objects" and "physical laws". It is not just that physical concepts acquire meaning within the framework of a mathematical space - the latter actually prescribes a thoroughly structuralised notion of *objecthood* and *objectivity* as invariants of geometrical structures.

In metric spaces, which carry the record of and themselves serve to record the cohesion of and between objects (the stability of physical laws and the conservation of energy and momentum), symmetries and geodesics shape the physical content of the theory. Noether's theorem describes these physical invariants in terms of space-time symmetry groups. Energy conservation for example is closely tied to invariance under the symmetry group of temporal translations, just as the geodesic curves furnish the trajectories along which quantities are conserved (inasmuch as they are stable minimal paths). See Bailly *et al.* (1999) and Bailly (2002).

The underlying unity of SR (which unites electricity and magnetism) and GR (which unites gravitation and cosmology) is reflected in the fact that SR may be considered a particular limit of GR. Once again we see geometry providing the framework for actually constituting new structural invariants and unifying them in the same space inasmuch as the stable properties of physical systems with that structure arise in connection with new groups of spatio-temporal transformations.

But there is also another path in the direction of increasing mathematical abstraction: the generative role of mathematical ideas provides the basis for grasping the sense of new physical concepts, indeed constitutes it.

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[2] See the reference to Laplace above and, by contrast, the example of "the displacement of a single electron which could lead to a man being killed in an avalanche a year later or to his escaping" in Turing (1950). Turing is perfectly aware that "the nervous system is certainly not a discrete-state machine. A small error in the information about the size of a nervous impulse impinging on a neuron, may make a large difference to the size of the outgoing impulse". Yet, he believes that, if the interface is limited to a teleprinter, one should not be to distinguish a machine from a man (or a woman?). Unfortunately, Turing was not aware that subsequent results on the geometry of dynamical systems would have confirmed the early work of Poincaré. In particular, no finite grid of inspection can stabilize a (possibly) unstable dynamics (see Longo 2003a, for details).

Take, for example, the physical applications one can find for the compactified (numerical) real line: one takes the infinite real line and transforms it into a circle, by adding one point (which "represents" infinity). On that basis one passes from 4 dimensions (3 of space plus 1 of time) to 5, but this fifth dimension is derived mathematically from the Lagrangean action associated with a field which is both electromagnetic (hence classical, *i.e.*, non-quantum) and gravitational (involving the unification of the Maxwell and Einstein equations). The physical properties carried by this new dimension of space are compactified - the fifth dimension is folded over on itself in the form of a circle: Kaluza-Klein theory (see Lichnerowicz, 1955). The geodesic principles and the symmetries are conserved. The observables of the theory have not changed, because the fifth dimension of this spatial structure is below the threshold of observability - it is a pure consequence of the conceptually generative capacity of the mathematical formalism.

At the same time this new dimension contributes to *explanatory* power, for it allows us to unify the structure of theories which were formerly quite distinct, while exactly preserving the invariants (energy-momentum etc.) which were at the heart of the two approaches. It is mathematical geometry which provides us with this new physically intelligible space; and, through this geometrisation of physics, mathematics plays a role of extraordinary explanatory power. In fact it supplies the models of space and time furnishing *the* framework for physical phenomena and gives them meaning, Kaku (1994).

The required mathematical ideas are not laid up in advance in a Platonic heaven, but are rather constituted within the interface between ourselves and the world which they serve to organise conceptually. Recall the role of Riemannian geometry in organising the framework of relativistic physics.[3] Relativity indeed furnishes one of the most beautiful examples of this *mathematical constitution of phenomena*: the most stable and coherent part of our conceptual apparatus - mathematics - provides the framework for a structuralised conception of objects, space and time which undergoes reciprocal adjustment as it encounters that source of friction (the world) which is continually suggesting/imposing new regularities to be incorporate in the structure, and drawing our active conceptual construction toward some models or deflecting it from others.

Quantum physics. Relativistic theories present space-time as external to physical objects, aiming to understand the latter as singularities of a field, and their evolution as controlled by geodesics. In this case, their phenomenal appearance amounts to nothing more than the mathematical stability of the invariants attached to these geodesics. Quantum mechanics on the other hand adds to this external frame of reference (Minkowski space) an internal frame of reference expressed in terms of quantum amplitudes and their invariants. This internal frame of reference is essential because the atomicity implicit in quantum theory is a matter not, as in classical atomism, of smallest possible bodies in space, but rather an atomicity of the processes determining the evolution of the field (because the dimension of Planck constant is that of an action, *i. e.,* energy multiplied by time). It is thus the variation of energy in time which is discretised in quantum theory and not the structure of matter or of space-time. Space and time remain continuous, as in relativity,[4] and this remains true, in certain respects, of quantum fields, although they behave in a different manner from classical fields. However, the mathematical unification of the theory of quantum fields with that of the gravitational field is far from being accomplished.

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[3] In fact Gauss, Lobachevsky and Riemann all explicitly described the new developments in geometry as leading in the direction of new physics. They did not limit themselves to playing formal games with Euclid's fifth postulate as the formalist caricature of the origins of non-Euclidean geometry would have us believe. Riemann in particular worked explicitly towards the theoretical unification of electromagnetism, heat and gravitation by the geometric rout – see Riemann (1854), Boi (1995), Tazzioli (2000). He thought, like others of the period, in terms of an aether co-extensive with (in fact constitutive of) space, inasmuch as it was conceived as a perfectly elastic and massless medium. For this reason he was able to conceive of physical bodies in space as immersed in an elastic "fluid" subject to deformation by "cohesive forces" related to their presence. The aether notion, later dismissed as erroneous, helped him to the conception of a profoundly original and modern idea of space - a space possessing non-null (and even variable) curvature affected by the energy-momentum of the physical fields within it.

[4] In mathematical terms, the external space-time constitutes the base space of a fibre space, the fibres of which (derived through generalising the notion of the inverse of a Cartesian projection) serve to organise the structure of the internal spaces. But the external space-time of quantum physics, considered as the base space of a family of fibres, displays in general a continuous topology corresponding to the classical representation of special relativity. Discrete processes - such as the quantisation of energy or spin - involve these additional, internal dimensions.

Our understanding of global or external spaces is profoundly bound up with that of local or internal ones: particles, as much as fields, display counter-intuitive non-local effects. Like St. Anthony, it seems quantons[5] can be present simultaneously at widely separated locations. This behaviour is not magic: matter fields are not local - they are not reducible to space-time singularities as in GR. Furthermore, matter includes fermionic fields. On this point, debate centres on the relationship between internal and external spaces - and the debate is very lively, notwithstanding the Einstein-Podolsky-Rosen Paradox which had appeared to demonstrate the opposition between GR and the physics of quanta. Briefly, quantum mechanics, which in first approximation had appeared to bring no essential new element to the determination of our theoretical notions of external space, has nevertheless introduced a novel (and counter-intuitive) perspective on our notion of locality. On the one hand, the physical laws of quantum mechanics remain local in the sense that the evolution of a system between measurements is generated by partial differential equations. On the other hand, the characteristics of the probability amplitudes associated with the state vectors (complex numbers, the superposition principle) engender a non-separability in the properties of quantum systems which is bound up with measurement and corroborated by experiment (Bell inequalities and the Aspect experiment concerning quantons which have interacted in the past).[6]

Despite the absence of theoretical unification, there are mathematical invariants which carry over from the local to the global frame of reference and *vice versa*. For instance a global shift in the frame of reference does not alter the electric charge: certain measurements are locally and globally invariant (in the theory of gauge fields) and the fields themselves are associated with local gauge invariants. Super-symmetric theories best tend to illustrate the connection between internal and external spaces. In these theories one can adjoin further dimensions to the four of relativistic space-time, in the manner of the Kaluza-Klein compactification of space, with the aim of preserving, as far as possible, the space-time symmetries; recent theories of quantum cosmology have sought to unify the theories mentioned here, in a tentative yet very audacious manner, at the level of the Big Bang by representing space as a six-dimensional manifold in which four dimensions would expand (the four-dimensions of the observed universe) while the compactification of the other dimensions provides for the way in which the properties of matter (fermionic fields) and interactions (bosonic fields) are structured.

We should also mention the possible role of the non-commutativity of quantum measurements (the complementarity of position and momentum): a fundamental difference from classical and relativistic approaches. A very promising framework for unification has been proposed via a geometric approach (by Alain Connes, in particular). The idea at bottom consist in reconstructing topology and differential geometry by introducing a non-commutative algebra of measurements (the Heisenberg algebra) in place of the usual commutative algebras, see Connes (1994). Once again, the geometric (re-)construction of space has the effect of making (quantum) phenomena intelligible.

Dynamical systems and their critical behaviour. The physical theories of the type we next consider are concerned with dynamical systems which, for some values of the control parameters (*e. g*., temperature), display discontinuous or divergent evolution (phase transitions such as the freezing of liquids), progressive transition from ordered to disordered states (as in paramagnetism and ferromagnetism) and qualitative change in their dynamical regimes (such as bifurcations of phase-space trajectories or transitions from cyclic to chaotic behaviour). They may be regarded collectively as theories of phase transitions. In approaching the question of causality by the status of space and time in these theories, we must distinguish between two classes.

Firstly the class of theories concerning systems which possess a high number of degrees of freedom – the phase space is therefore very large (as in thermodynamics and statistical mechanics). It was in relation to this class of theories that problems relating to temporal reversibility and irreversibility were first posed. The second class of theories is concerned with non-linear dynamical systems which can be treated only in terms of a small number of degrees of freedom, and the properties of whose dynamics (bifurcations, existence of strange attractors *etc.*) are associated precisely with the nonlinearity (whether treated within the framework of continuous differential equations or via discrete iterative procedures.) These systems also pose questions of reversible or irreversible behaviour, but in slightly different terms from those in the first class. In both cases, and in contrast with the situation prevailing in relativistic and quantum theories (where we find ourselves in a

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[5] The term “quanton” designates a quantum object which is susceptible to manifestation in either its particle or wave aspects depending on the experimental set-up (metaphorically: according to what question is put to the system).

[6] Aspects of this nature have nourished more holistic conceptions such as the ideas of David Bohm, Basil Hiley and their collaborators concerning the so-called “implicate order”.

fairly regular universe), here our attention is more on the *singularities* than the *regularities* of the systems in question.[7]

Both these classes of theories mark an apparent return to more classical conceptions of space and time than those encountered in connection with relativity or quantum theory. In particular, the introduction of spaces with a large number of dimensions (such as the phase space of statistical mechanics) does not involve their fulfilling the sort of constitutive role assigned to space-time structures in relativity or quantum theory. Nevertheless these two classes of theories have also given rise to new approaches to physics, this time relating to aspects which are, on the one hand, in relation to space, markedly morphological and global; and on the other hand, in relation to time, markedly evolving and directional; and this marks the causal relations.

Yet, these systems are characterised by numerous other traits. One is the role they frequently assign to the global aspects. If one takes the "most simple" dynamical system, three bodies together with their associated gravitational fields, the very unity of the system prevents its being analysed in Laplacean terms. One cannot know/predict the position and momentum of each body without at the same time analysing the same parameters for the others. They are correlated through their mutual gravitational fields so as to constitute a sort of *holon*: a global configuration which, evolving in time, determines the behaviour of each of its elements. A step-by-step analysis - that is to say analysis of the behaviour first of one body, then two... or the approximation of that behaviour via Fourier analysis - is simply not possible here. This is what robs the system of the kind of completely predictable behaviour conceived by Laplace. What wrecks this Laplacean predictability is that in sufficiently complex dynamical systems (in the three bodies problem rather than the two) divergences are present (*i. e.*, discontinuities related to control parameters). The nonlinearity of the mathematical representation reflects the intrinsic unity of such systems. The dramatic change, as for knowledge and causal regime, is due to the fact that determination, under a finite set of equations or inference rules, does not imply predictability.

In fact dynamical systems are often assigned their *proper* time in a "peremptory" fashion. Insofar as they exhibit phase transitions, by the bifurcations (particularly that of forms in space) as well as their transitions from cyclic to chaotic regimes, these "impose" directionality on the states of the system, differently from other physical theories. Their time is orchestrated by phase transitions and, irreversibly, by bifurcations and transitions to chaotic behaviour. The essentially irreversible character of time for these systems marks a definite contrast with the picture of time in relativity (where it is intrinsically reversible and its flow is regarded as an epiphenomenon), and it seems to provide a concept of time appropriate to living beings (strongly affected by thermodynamic phenomena amongst others). The irreversibility of time characteristic of such "critical" systems is connected with their unpredictability and their chaotic behaviour.

1.3 Some remarks

We have examined aspects of the geometrisation of modern physics. The mathematics of space and time moulds a framework for the understanding and organisation of phenomena and the unification of different "levels" of their structure. The epistemological and mathematical aspects of space and time turn out to be profoundly bound up with one another in a manner which plays a pivotal role in shaping scientific enquiry, in particular in providing for the unification of physical theories. We have briefly mentioned the (pre-quantum) unification of electromagnetism (governed by the Lorentz-Poincaré group) and gravitation (governed by the group of diffeomorphisms of GR). More recent theories introduce new symmetries (super-symmetries or symmetries of spacetime structure in a generalised sense, associated with the notion of super-space) allowing the articulation within a common framework of the external and internal spaces of quantum systems. From an epistemological standpoint, the unifying aspect of these theories is that they lead to the construction of unfamiliar spaces whose physical relevance is then corroborated by experimental investigation.

More recently still, a non-commutative geometry has been forced on us by quantum measurements and we have hence been led to propose geometric structures even further removed from the ones directly suggested by the sensible world. Geometry provides a mathematical framework organising the practical as well as the theoretical aspects of our spatial experience. Our access to space as expressed in the most developed physical theories is based on measuring instruments very far removed from our naive sensations and hence necessarily follows a route to the (re-) construction of our notion of space very different from what these might suggest. The curvature of the light is detectable only through sophisticated astrophysical measurements, it is not apparent to our raw intuition. The geometry of the universe rests on a geodesic structure quite unfamiliar from

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[7] From a qualitative and conceptual standpoint and clearly in line with processes of increasing complexity which follow from it, there are close connections with the theory of catastrophes (and the related geometry).

the viewpoint of sensory experience. The non-locality of quantum phenomena follows from microphysical measurements quite inaccessible at the level of our physiology.

It is even possible that our geometry itself will take the form of mathematical structures in which the classical notion of a point is no longer basic (*e.g.*, the theory of superstrings or twistor theory). Notice besides this that the generalisation - via homotheties - to all physical scales and dimensions, of all Euclidean properties drawn from our sensory experience is quite arbitrary, see Longo (2003). Straight lines and dimensionless points do not exist (or "exist" in only the same sense as any other mathematical construct or abstraction). They can be replaced by other abstractions which may turn out to hang together better with experimental evidence and with new tools of measurement.

One last word about theories of dynamical systems, near to or undergoing critical change. The treatment of space-time these theories suggest (centred on phase space and the transition of their dynamics from stable to chaotic behaviour) introduces new elements important also for other theories, above all in connection with certain recent cosmological theories (models of the phase transition associated with the Big Bang, singularities and cosmic strings, for example) and also in connection with the relations between local and global structure.

This class of theories forms the key bridge between physics and the life sciences - and also (if we may skate fearlessly over a great many intermediate levels of organisation) with their great associated critical "epiphenomena" – namely, cognitive phenomena.

PART 2. From physics to biology: space and time in the "field" of living systems

2.1 The time of life

As a preliminary, I want to analyse the particular features of time characteristic of living systems. Temporal *irreversibility* is at the heart of the study of dynamical systems exhibiting critical behaviour, but it is also characteristic of living systems. At every stage phylogenesis and ontogenesis are marked by 'bifurcations' and by the emergence of unpredictable phenomena and structures which resemble those observed in critically sensitive dynamical systems, which thus subsume biological phenomena. Moreover, living systems contain a great many subsystems which display this kind of critically sensitive behaviour - dynamical and thermodynamical. These contribute not only to the temporal irreversibility of the system but also to a kind of unity which is apparent in the kind of dynamical systems we touched on above in connection with the three-body problem. Poincaré's three bodies, in exhibiting an example of this kind of *unity*, form a primitive *Gestalt* associated with a purely gravitational interaction. Two bodies exhibit a quite different dynamical behaviour, stable and predictable. It could even be said that what comes into play in the three-body dynamical regime is a kind of emergent behaviour, a unity of non-stratifiable relationships: one cannot analyse first the position, then the velocity, of each body step by step, independent of the unity of the system they form.

In a recent email exchange, F. Bailly remarks:

> The spatial and temporal (and spatiotemporal) terms do not appear to possess the same significance or play the same role within the two principal approaches ("geometric" *vs* "algebraic-formal" which you have distinguished). In the "geometric" approach, space is the correlate of geometry itself, it intervenes at the perceptual level. Time is the time of genesis of structures, the recording medium of their process of constitution. In the "algebraic-formal" approach, by contrast, spatiality is the echo of an abstract linguistic inscription, of formal symbols, while temporality seems to be principally a matter of sequential functioning, of the execution of algorithmic calculation.

This remark refers to the distinction, which in other writings I have drawn in the context of the foundations of mathematics, between *principles of construction* (in particular those with a geometrical aspect) and *principles of proof* (formal principles of logic), see Longo (1999) and (2002). Mathematics is built up on the basis of both types of principles. The philosophical fixation, implicit in the analytic tradition, with logicism and formalism has tended to exclude or sideline the first of these. The "linguistic turn" has given us extraordinarily rich logical/formal machinery (and literally machinery in the form of digital computers) but it has also endorsed the myth of the complete mechanisation of mathematics, indeed of any form of knowledge.

I have argued that incompleteness theorems in formal systems are due to the gulf between these two types of principles. Conceptual constructions based on spacetime regularities possess an autonomy, an essential independence in relation to purely formal descriptions, in a sense made exact by mathematical logic (through the work of Hilbert and his school). Unfortunately physicists are prone to label any "mathematical treatment" of a subject as an instance of "formalisation". For logicians these are quite distinct notions: there is the Gödelian (and other forms of) incompleteness in between, see Longo (2002), Bailly, Longo (2003). The distinction of principles of geometric construction from algebraic-formal principles of proof is in my view one of the crucial factors which underlies the constitutive role of space-time concepts and geometry in the analysis of cognition.

In the conception of time as the medium of algebraic manipulation and formal calculation, as seen in the sequential running of a computer program, one recognises an important fruit of the formalist view of the foundations of mathematics. The 1930s marriage of Hilbertian formalism, together with the problems it addressed (the completeness and decidability of formal systems) and, on the other hand, a mechanistic positivism, was at the origin of the attempt to treat human rationality in terms of a mechanism which indefatigably executes formal algorithms.

But this forgetting of space, which also greatly influenced the characteristic mathematical approach to time (as the medium of the genesis of structure) led to the severe reduction of the analysis of human cognition and, which is a greater distortion, of animal cognition – humans can use logic and formal calculi as supplementary cognitive aids which permit a biased grasp of at least a part of what is involved in understanding, but it is just this part which is least accessible to other living beings.

What marks an interesting historical reversal of this trend is that today we cannot study or seek to construct computers without taking account in a new way of considerations involving space and time. The geometric aspects of the structure of computers enters into the study of distributed, concurrent and asynchronous processing, which areas pose spatio-temporal problems of a kind completely foreign to the theory of Turing machines, the theory which dominated the study of computability from the 1930s to the 1980s and the very interesting mathematical aspects of which for a long time formed my own principal field of study. In these new areas the main problem concerns the *time of structural genesis and the constitution process*. This is a kind of time which involves space and which thus poses a new set of problems for computer science as well as for physics. Is this a further aspect of the new role of geometry in the study of cognition? Should we think of the time of cognitive processing as an inherently distributed time?

Finally, where is the living system which does not exist other than in space and time? Take the dynamical self-organisation of ecosystems for example. Their genesis is above all a genesis of structure, from protein folding to the morphogenesis of an elephant; and their time is the history of a process of constitution. Dynamic irreversibility, Gestalt, systemic unity and cohesiveness – what happens to all these are aspects of living systems which act in space and time?

2.2 Three forms of time

In the foregoing remarks, we have the outline of two ways in which we can regard *phenomenal* time as constituted – *phenomenal*, because it is jointly *construed* by us-and-the-world: it is a constitutive element in our forms of knowledge of a Reality-out-there, but one which must be endowed of structure to become intelligible. This time is at once a real and a rational time, remarkably, but not absolutely objective. It is the co-construction of the knowing subject and the world, as rooted in the regularities which we detect in the world - regularities which are out there but the explanation and the (scientific) objectivity of which are constituted in intersubjectivity - an intersubjectivity with a history. Let us now examine these two forms of phenomenal time more closely, with the aim of suggesting a third.

The first form, algebraic-formal, is that of clock mechanisms - the same clocks which the Enlightenment regarded as a possible model for the operations of the understanding in general - and which later became the time of a (discrete state) Turing machine (see Longo 2003a, for the "discrete *vs* continuous" issue in computational models of mind). A Turing machine tells time by the movement of scanning/reading its tape - to the left or right - tick-tock - like an absolute Newtonian clock. Nothing *happens* between one movement and another (to the left or right as the tape is scanned) nor can anything be said of their duration: these movements are the measure of time itself.

This notion recalls the time of myth inscribed in Homer. To recall an analogy suggested by Bernard Teissier: during the Trojan War, time is marked by the sorties of Achilles from his tent. Achilles leaves his tent, something (the War) happens; he re-enters his tent, everything stops – time stops. Achilles' motions provide the (only) scansion of time. Troy and the Trojan war (in the sense relevant here) lie outside the (space of the) world - they exist in the realm of myth. Their internal time contributes to an extraordinary poetic effect. Turing and Homer are as one in this respect: the time characteristic of 1930s formalism is the time of algebraic-formal construction - the absolute time of a formal mechanism lying outside space, the time of "calculation-in-itself" is the time of one step after another in a void. In fact this time is secreted by the actions of a Turing machine viewed purely as a clock.

But Greek Thought proposed another representation of time as Kronos, son of Ouranus. Kronos (derived from chaos and devouring his children) is "true", physical, time – the "paddle" of the real world. This version of physical time fits well with the analysis of dynamical systems displaying critically sensitive behaviour (*e. g*., characterised by phase transitions). It is a time in a space - the space of the geometry of dynamical systems, a time recorded by their bifurcations, by their irreversible transitions from stable to chaotic regimes. Indeed it is the time of "the genesis of structure", of constitutive process, because a bifurcation, or a catastrophe, can depend on the entire history of a system, and not only its state description at a given instant.

To represent time as a linear continuum, the line of the real numbers, is very convenient; in many contexts one can choose no better model. But I here take up the reasons for the dissatisfaction with it which Hermann Weyl expressed in *Das Kontinuum* (1918). Its "points" cannot be isolated in the manner of points on a spatial line because the present blends into, and indeed has no meaning except in conjunction with, the past and the future. While giving substantial contributions to the mathematical setting of relativity, Weyl recognised the limitations of relativity theory to represent time as an epiphenomenon, given that time is equipped with the same structure as the spatial continuum. Moreover, reversible time, due to the equations of relativistic physics, has nothing to do with phenomenal time as a mixture of experienced and rational time.

The time of dynamical systems theory and theories of critical states seems much better adapted to capture irreversibility than that of relativity theory (and perhaps better than that of quantum physics). Moreover, the time of catastrophe theory can be given no meaning other than in space (in this respect it is like the time of relativity theory): firstly, bifurcations and chaotic behaviour require space for their manifestation; secondly, there is no such thing as the time of a single isolated dynamical system displaying linearity in its bifurcations. No such system exists. The genesis of structures proceeds in parallel, through interaction of a plurality of structures (sub- and super-systems) in a spatial setting.[8]

There are exceptions to the immersion of this second form of time in space. One could say, for instance, that the grammatical structure of natural languages, and other aspects of their structure possess a history and an existence in time without making reference to space. But language is an intrinsically intersubjective phenomenon - it is a plurality of speakers, situated and acting in space, which makes language possible. There is no language of an isolated speaker, language is always spoken within a cultural ecosystem, which is often in friction with other cultures.

As the temporality of physical systems is associated with the genesis of structures in space through the interaction between systems which are both dynamical and distributed, the *synchronisation* of such systems becomes a central problem (though one can have asynchronous physical interactions of course). Already inn relativity it shows up in the exchange of signals between differently accelerated systems. In computer science, this problem is partly bound up with the analysis of concurrence between processing units distributed in space. Both the time of Turing machines and that of Achilles' sorties *requiescant in pace*. Today we have a more "structured" time - that of a plurality of dynamic, distributed and concurrent (or more generally interacting) systems with their own local times, demanding synchronisation where required. But if there is no time apart from this synchronicity, the same holds for asynchronicity, because it is already inherent in any "real" interaction between systems in a not purely local universe. We are today in a position to propose a notion of time better adapted to our scientific understanding of the physical world: one enriched by the consideration of relativistic phenomena and (irreversible) dynamical systems. This time is essentially *relational* in character. Just as the absolute space of Newtonian physics no longer seems to make sense, so the absolute clock of the Turing machine, isolated in an empty universe, no longer seems to define an adequate representation of time. They would be akin to the standard metre of Sevres, isolated in an empty universe: in that universe there is no distance, just the metre.

But, there is yet a third form of time to be discussed and it is one appropriate for biology. The time in question is a phenomenal time, superposing experienced and rational aspects; it is constituted jointly by ourselves and the world, in the very acts of our intentional experiencing the world. That it manifests resistance to our *attempts* to grasp it is essential to its understanding. It is not to be thought of as "already there", yet it is not something arbitrary - because the regularities which supply us with clues and suggest how to speak about this time are certainly there; it is we however who choose how to regard them.

In biology, matters are effectively more complex than in relation to physics, and one is obliged to move away from the idea that our brain (or any living organism) is a logical device or a programmable machine.

First of all the "unity" and the "characteristic" time scales of living systems is related to the autonomy of the biological clocks of which F. Bailly gives a detailed account in the following section. This autonomy is

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[8] Perhaps we can see connections here with the idea of a mean statistical time as proposed by Weyl in the relativistic setting, see Dorato (1995).

even more striking than that of the mechanisms acting as clocks in the case of physics, because of the way in which a living organism strikes us as a unified individual.

In physics, the present and future states of a system, and of the world as a set of dynamically interacting systems, depend "only" on past states. But the situation in the case of organic systems involves even more interactivity than that.

On the one hand, there are autonomous clocks appropriate to the individual system - its metabolic rate, its various biorhythms (heartbeat, respiration etc). These are constants over ranges extending in some cases beyond entire species, even covering an entire phylum (the mammals for example). Evidently these clocks are far from being isolated systems - they regulate the functions of organisms in interaction with their environment; indeed their *raison d'etre* is to constrain and regulate that interaction.

On the other hand there is the phenomenal time of action within space on the part of this same living system - action characterised by aims and purposes, not least that of survival. Before discussing this however, let me review at least two further factors involved in the study of time in biology:

i) the local time of each individual living being, its internal clock(s), which is re-established after any action affecting them (any action within the limits the organism can tolerate). Its clocks indeed exist precisely to permit and to regulate such interactions;

ii) a global time in which possible bifurcations in the system dynamics are determined, according to anticipatory capacities of the organism, by choices on which the possible future states (survival) of the system within its environment depend.

"Intentionality" is thus characteristic of biological time and it extends far below the threshold of consciousness, as is seen in the behaviour of single-celled organisms which move in one direction or another to preserve their metabolic activity. This movement is one of the most elemental forms of choice: constituting bifurcations between possible directions (paths) of the system in phase space. In the case of human beings this choice is made on the basis of explicit awareness and conscious anticipation of the future. It thus depends on the range of possible future states considered – Pauri (1999) makes the same point.

It is thus a "contingent intentionality", related to contingent goals of the kind characteristic of different organisms. There is no organism or species without one implicit goal, that of surviving. But this finality is not metaphysical, rather it is immanent and contingent. If it were otherwise, neither the individual nor the species could long survive. It is essential to the preservation of living systems, from a single cell to multicellular organisms, as they are capable of future-oriented actions. Intentionality in the Husserlian sense of the term, involves an envisaging, a mental act consciously directed towards a target. Here it has a broader meaning: it is thus the *end result* of a network of interactions which plays a *constitutive* role in phylogenesis. Pachoud (1999) also suggests enlarging the Husserlian notion of intentionality in order to revitalise the phenomenological program.

Let us take an example from the study of primates. This example falls midway on the scale between the actions of an amoeba in its metabolic responses and the conscious intentional behaviour of a fully socialised human individual, or even the collective purposeful activity of an entire social group. This is the example: when we switch our attention from one point in our field of vision to another by a saccade (a rapid eye movement), the receptor field of the neurons in our parietal cortex is displaced suddenly, *before* the ocular saccade, in the direction in which we are thereby looking, Berthoz (1997) p. 224. In other words the brain, in order to follow the trajectory of an object, or to escape the claws of a predator whose intentions it has "understood", displaces the receptor field of its neurons and anticipates the consequences of that displacement. This is only one example amongst many which can be given of the role of anticipatory action of the future characteristic of living systems. I consider it of great interest because it is a form of intentional future-oriented behaviour below the threshold of consciousness in animals, but very close to conscious movement. In fact, it seems that the glance actually produces a change in the biochemical (and hence the physical) state of the neurons, in the act of anticipating the future. This new state is imprinted on their structure - the new state in which they are then found does not depend only on their present and past states, but also on their anticipated future state.

In what follows, F. Bailly will develop a suggestive analogy between local curvature in the Riemannian spaces of relativity theory and the locality of the internal time scales of living systems. A constant non-zero curvature provides for a local spatial scale linked to the (local) metric exactly as metabolic or cardiac rhythms appear to provide a time scale, more or less regular but local, observed in the individual system but common to the species or wider phylum. In contrast with the absolute locality of the metric of curved Riemannian space, local biological clocks are embedded in a wider ecosystem, and their *contingent finality* is not what it would be in the case of an isolated organism; rather they contribute to ensuring the stable existence of the organism in a changing milieu. They *synchronise* it with similar systems and maintain it when in interaction with dissimilar ones. Whereas constant local curvature furnishes an invariant, local, metric element, independently of what goes on in the rest of the world, the internal clocks of living systems play a role in interaction. They aid in the establishment of a common time scale and they allow for the regulation and synchronisation of other clocks within an ecosystem.

2. 3 Dynamics of the self-constitution of living systems

Any individual organism, or any species, defines what may be termed a zone of "extended criticality" (F. Bailly), which appears to be a feature impossible in physics, where "critical" states are generally unstable singularities. In this zone of extended criticality numerical invariants characterise the time scales of the autonomous system and reorganise the "unity" of the system in relation to heteronomy.

When one examines a species embedded in an ecosystem much of the conceptual framework taken over from physics appears inadequate. Although the theory of dynamical systems has furnished some effective mathematical tools for biology, the study of a living system with the methods developed in mathematical physics has conceived the evolution of the system as taking place within a "frozen" field of force, or at any rate within a network of fields of force given at the outset. That is to say, the phase space does not change in the course of evolution.

A marble rolling in a cup is a simple classical system. Its field of forces: gravity, the geometric shape of the cup, the frictional resistance - all already in place at the outset. The analysis of the ensuing oscillations follows very straightforwardly. In the case of more complex dynamical systems the mathematical analysis of their behaviour may make reference to so many different forces that the majority of systems turns out to be intrinsically unpredictable. However, *qualitative* analysis allows us some remarkable insights into their possible evolution (the existence of singularities, bifurcations, attractors and so forth), even in the absence of complete predictability. In the case of a living system a further factor is involved: the field of forces acting on the system is itself constituted in the course of the evolution of the system. In analysing that evolution one may have to pass from one phase space to a completely different one.

Take a species within an ecosystem. Doubtless its interactions with the physical aspects of its ecosystem are determined by forces which relate to those aspects (*e.g.,* gravitation, the physics and chemistry of the atmosphere or of seawater) but within an ecosystem one finds also other living beings. They react on the species in question. In fact species co-constitute themselves in conjunction with one another. They may eat one another for example. And these other species were not necessarily present in the ecosystem before the one being studied, nor are they fixed and frozen entities. Their existence and evolution may itself depend on that of the species under consideration. Living systems in their interaction do not form a *given* field of physical forces - no minimum principle, no geodesic principle predetermines their evolution. For modern evolution (and we have for the present no better theory) they rather become more or less compatible with a situation which living systems themselves will have co-constituted and co-modified, rather than with one given in advance.

Neo-Darwinian evolutionary theory refers to the combinatory explosion of life "in all possible directions". That is to say, no overall pattern of development in the system is predetermined, still less predictable, except in the case of small laboratory populations (*e. g.*, of bacteria) under very controlled conditions. But, in general, evolutionary behaviour is compatible only with (and could not exist without) the situation which it itself contributes to determining. Novelty arises on the basis of a given situation (which includes a genetic make-up) but also via the establishment of new patterns of interaction, the significance of which cannot be understood prior to their constitution. S. J. Gould mentions, for example, the tremendous role of "latent potentials" - illustrated by the double articulation of the jaws of certain reptiles 200 million years ago, which became the inner ear of birds and mammals. There was no *a priori* reason why things should have gone this way - no physical field of force and no genetic endowment on the part of reptiles imposed this development - it was made possible in the context of (indeed was co-constituted by) an ecosystem. It would have been impossible to predict. The only reason is *a posteriori*. We find ourselves further than ever from Laplace and there lies the scientific (mathematical) challenge.

Thus novel possibilities modify the field of forces set up by the living ecosystem. It is as if the cup in which the marble was set rolling assumed a shape (even a variety of shapes) from amongst all the physically possible ones, whilst the marble was in motion. But it is even more striking than that, for the marble too becomes extremely malleable whilst at the same time seeking to safeguard its unity and autonomy, just as all living individuals and species endeavour to do. Briefly, the biological "field" is co-constituted in time. In this respect it is something over and above physical fields; it *depends* on the latter of course, but is *not reducible* to them; at any rate we are a very long way from being able to produce such a reduction. The unification of biology with, rather than its reduction to, physics remains a principal aim. But it may be that this looked-for unification will come about from a quite different theoretical direction. It may be that an account of quantum phenomena will emerge within the framework of a general account of systems, including anticipatory capabilities.[9] In this connection one will need to enrich the very concepts of "causal determination", "system" *etc.* Our aim at this juncture is a conceptual analysis which pinpoints the parallels and divergences between new mathematical models of space.

What can be meant by a shift/enrichment of our concepts of "causal determination" and "system"? Let me illustrate it by means of a dialogue at a distance between Galileo and Kepler.

Kepler, a mathematician and astronomer of extraordinary gifts, did not disdain the task of compiling almanacs and casting horoscopes and mingled his talents in order to make his living. It was thus he came to think that the moon had an influence on the character of women, and also on the pattern of the tides. Galileo, a man of science through and through, did not agree with these ideas. The first of the problems, however important, had nothing in his eyes to do with physics, and as for the tides, to claim that a distant body like the moon could be implicated in their cause seemed to him to smack of magic and astrology. Sooner than admit this, he set out, in his *Dialogo sopra i massimi sistemi* to explain tidal motion in physical terms, for the tides were clearly physical phenomena: the tides are the result of inertial forces acting within the framework of Galilean relativity. The combined forces acting on the earth - its rotation around its axis and its orbital motion around the sun - are the cause of the inertial motion of its waters.

Galileo's theory of inertia and the relativity of motion marked the debut of modern physics. But his theory of tides took no account of countervailing empirical data: Galileo's reasoning would lead one to expect a 24 hour cycle in the tides. His error was one of methodology; an error which with mild abuse of language one could label "physicalism": a (misconceived) reduction located at the heart of physics itself. By "physicalism" what I intend here is not so much the position that, "in the final analysis", all the phenomena are of a physical nature and supervene in principle on the physical description of the world by means of (a final) physical theory, but rather the reduction of phenomena to one *given physical theory*, constructed on the basis of *a priori* considerations around a restricted and well-defined range of phenomena other than those which are the target of reductive explanation.

To speak in a modern idiom, Galileo's difficulty was that he lacked the *field concept*. (What was more serious, he could give no account of what it was his theory lacked or suggest any measurements which could be carried out to test it as it stood.) Granted, he would have had to cover a great deal of ground to arrive at the concept of a field and its accompanying mathematical representation - ground including Newtonian gravity. The modern notion of field did not *reduce* supra-lunar phenomena to sub-lunar galilean motion: it proposed instead new mathematical concepts and a novel synthesis.

The difficulties involved in the analysis of living systems (and the methodological youth of biology) suggest that in the life sciences (perhaps) and in the cognitive sciences (surely) to we are at a stage analogous to that seen in the Kepler-Galileo debate.

Anyone who observes that the range of biological phenomena displays aspects which elude description in terms of current physical theory risks being branded an obscurantist and accused of believing in magic. The situation is not helped by the fact that one does indeed encounter terminology of a magical-poetical flavour in some writings on this subject. Confronted with this position, some tough-minded commentators cling to the notion of a deterministic program (in the sense of Laplace and Turing) and see it encoded into the brain, as the hardware on which the program, or rather a whole set of interlocking programs, is run. Others take up the issue of quantum non-locality, locating its manifestation at the level of the microtubules of neurons and claiming that this will turn out to form the reductive basis of consciousness. Others again turn to the study of dynamical systems and take this as the framework for modelling the evolution of neural networks and the plasticity of their behaviours.

Clearly there are very important differences between these approaches. The first of them nowadays comes within a hairsbreadth of being a swindle. It has long been clear that we see less and less evidence in current physics of the kind of determinism embraced by Laplace and Turing, and even less in biology. This is not to deny the importance of both Laplace and Turing for rational mechanics and information sciences respectively.

The second approach sets out a challenge to be taken up, but is currently lacking in experimental evidence, or in linkages between the scales of the structures and systems involved in the hypothesis: between the activity of neurons, which are very large scale structures, and that of the quanta, intermediary levels of description are altogether lacking. The third approach is founded on a strong body of evidence concerning the workings of the brain - the observed reinforcement of synaptic connections and, more generally, the effectiveness of the dynamical systems framework for the treatment of any interactive system. Here progress has

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[9] The step from tunnel effects in microtubules to moral freedom, as argued by appeal to the incompleteness of arithmetics, is problematic indeed (for logical and experimental reasons).

been remarkable, yet the reduction is performed towards a specific physical theory: no novel conceptual unity is proposed.

In these three approaches we also see the change of the notion of "determinacy". For Laplace (and for the sequential programming of computers) any deterministic system is completely predictable.[10] Within dynamic systems theory, determinism does not necessarily imply predictability. Quantum physics introduces a further and deep-going modification of the concept, via its dual, the notion of intrinsic (non-epistemic) indeterminacy.

In less than two centuries our notions of what it is that determines what, and our notion of what is a system evolving in time, have undergone a profound shift. But we still have no equivalent general notions in biology (see Rosen 1991). We cannot say in what manner DNA determines the ontogenesis of living systems, nor in what way the state of a nervous system determines its later states. In an attempt to tackle these issues with the concepts of present mathematical physics, researchers have entered the conceptual kitchen, so to speak, and are busily drawing up a menu based on the recipes and cooking utensils they have already mastered, a menu drafted, where possible, in collaboration with the biologists. But to make better progress we stand in need of a robust notion of *biological field* - which is still lacking.

2.4 Morphogenesis

Let us now turn again to the the notion of space appropriate to the study of living systems. One of the areas in which we see the richest use of geometrical concepts in the study of living beings and their associated ecosystems is in the study of morphogenesis, in which I include the study of the evolution of the forms of living beings and the influence of form on the structure of life in general.

"The stability of living forms is geometric in character" (Thom 1972, p. 171). The topological complexity of a form is for Thom the locus of its "meaning" and of its organisation. Thom assigns an almost exclusive explanatory role to topology: the topological evolution of the form of a living individual provides the explanation for its biochemistry, rather than the other way around, Thom (1972) p. 175. The form in question contains information in two ways: it determines an equivalence class of topological forms under the action of a group of transformations; and it also supplies a measure of the computational complexity of a system via the number and evolution of its singularities. Here we can glimpse the idea of a "morphogenetic field" which fashions living systems, in the course of their phylogenesis as well as their ontogenesis. *Global* structure and operations of a *global* character occupy centre stage in Thom's view. In the embryo, he emphasises, we already have the global pattern of the organism, from which the specialisation of organs and their function follows. As it has been said (Jean, 1994, p. 270): plants form cells, not cells plants.

But just what is this "morphogenetic field"? This expression could lead us astray if we think in terms of the physical fields. The morphogenetic field must be thought of as in some sense containing all the known physical fields at once, together with new fields characteristic of co-constituted organisms. In particular, each field – physical, biological or cognitive, acts at a certain level of organisation, *conceptually* independent of others: the phenomenal level and its conceptual structuring by our forms of scientific knowledge are completely distinct. However the individual organism achieves *de facto* integration of this plurality of levels: its unity results from this integration of physical, biological and cognitive levels. These different levels of structure and

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[10] Turing in 1935 himself demonstrated that a Turing Machine was subject to a kind of unpredictability: one cannot decide the halting problem (whether the machine will halt or not in executing a given program). In fact we can decide no "interesting" property of programs (Rice's Theorem: see Rogers 1967). However, this unpredictability only becomes apparent in the limiting evolution of the system. The non-halting shows up only when a system performs infinitely many steps, and the undecidability of programs is a property of functions inasmuch as they admit infinite values and arguments. By contrast, the unpredictability of deterministic physical systems, investigated by Poincaré, is manifested already in finite levels. Given the initial state (defined with the due approximation), there exists a finite time (the Poincaré relaxation time) after which one *cannot* predict the state in which the system will be found. Despite the well-known undecidability results, classical computability theory thus conceives of a computation as a deterministic process of calculation and as completely constrained to follow a predictable evolution at any finite instant. Such theory is a *logical* theory and Turing machines are *logical* machines. In neither case are they to be thought of as constrained by any physical limitations. The issues of determinism and indeterminism involved in the halting problem are issues of logic, not of physics and not subject to the hazards of physical approximation (see Longo 2003a).

organisation, analysed by quite different scientific methods and concepts, interact with each other via spatial and temporal linkages. Each level displays plasticity with respect to the others.

However no current physical theory supplies the concepts needed to describe these forms of mutual action, control and constraint, operating between the different levels (the ascending and descending linkages between them at all levels of the system and its biological, chemical and physical components and subsystems). Cybernetics, the first theory of control automation, has certainly furnished remarkable models of the linkages involved in self-regulation. But these models have been located specifically at the physical level, and are constrained by the range of the theoretical tools they employ, whereas living systems establish linkages between conceptually wholly distinct levels of description.

Thom's analysis, subsequently enriched by the work of many other researchers, is also directed at the physical aspects of the topological plasticity of living forms, including those aspects induced by their “virtual” interactions. His work deals with an extremely informative *physis* of living systems, but still a *physis*. While, on the one side, it views the plasticity seen in the evolution of living forms as constrained by the dynamic fields operating in their morphogenesis, on the other hand, topological evolution is regarded as developing within a physical schema which takes no account of such phenomena as latent potentials or the combinatory explosion of life in all *compatible* directions. But, compatible with what? We do not mean compatible with the forces acting on a system at a given instant, but also with those it *will* experience. This poses a (mathematical) problem which is at the heart of the developmental plasticity of living forms.

As has been understood by those who have contributed to the most fully worked-out aspects of the theory of morphogenesis, namely phyllotaxis, it is possible to induce forms very similar to those seen in phyllotaxis by means of superconducting currents imposed on a magnetic field, see Jean (1994), p. 264. For instance, the Fibonacci sequence, which is observed very frequently throughout the vegetable kingdom, can be reproduced by this method on any mesh of “soft objects” under repulsive forces and strong deformations (*ib*., p. 265). In this sense, such an analysis does indeed consider living forms with respect to their being as purely physical systems - that is to say as bodies subject to the influence of physical fields. But although an important and necessary investigation, this is not exhaustive as an analysis of the forms of living systems.

Morphogenesis also has an important role to play in helping biology break out of the stranglehold of “genetic chauvinism”. The latter in the writing of some authors takes the form of a near maniacal expression of the Laplace-Turing vision of an absolutely deterministic causality, legislated in advance by the initial configuration of the system’s components. This vision of a closed future is strongly rooted in currents relating genetics and socio-biology (and unhappily congenial to religious believes in predestination).

In contrast to such a picture, H. Atlan replied: “the program of a living organism is everywhere except in its genes”: certainly the patterns and the forms seen in phyllotaxis are not entirely in the genes. They are also in the structure of space and time and of physical matter and energy. The genes do *not* contain all the information on the symmetries which are set up in a system in interaction with its environment, such as are observed in crystals and minerals, Jean (1994) p. 266. The so-called “program” for the development of an organism is to be found in the interface between its phylogenetic record (its genetic legacy) and its physical and biological environment (its ecosystem).

An example of the greatest importance is provided by the brain, which in the course of ontogenesis manifests a developmental pattern which is both Darwinian and Lamarckian. The immense number of possible connections between its neurons (each one of around 100 billion neurons has up to 10.000 synaptic connections, maybe more) could not be (or at any rate very little of it could be) encoded in the genes. Of the numerous connections established very rapidly during the growth of the foetus or the new-born child, most disappear through selection effects, Edelman (1987). On the other hand, throughout the course of our entire life, stimuli lead the brain to establish new connections and reinforce existing ones, jettisoning and replacing existing connections as it does so, selecting certain neurons and leaving others to die off. Cerebral plasticity, at all levels, is at the heart of the continuity between phylogenesis and ontogenesis, and is what permits continuing individual identity: "the structure of the nervous system carries the material traces of its individual history", Prochiantz (1997).[11]

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[11] The constitution of geometrical patterns of neural networks is a typical result of this complex compositional activity and permanent dynamism, but it is not the only result. See for instance the remarks in Edelman 1987 on the fine structure of synaptic connections and other aspects of neural plasticity which go beyond those modelled by the dynamics of neural networks. Consider also the interactive genesis of the forms of such networks in the context of an ecosystem. Here the stimuli are of a physical or biological nature, grounding the mental activity in the material of living systems. Animal intelligence involves a dynamic of such forms distributed over many different levels of structure (from that of proteins to that of synapses to that of entire neural networks) all

2.5 Information and geometric structure

In an epoch of free-floating bits, the picture of information (even of intelligence) as purely a sequence of bits enjoys great currency. The digital encoding of information is of great effectiveness for certain purposes: once encoded, such information can be safeguarded and transmitted with unrivalled accuracy and speed. No method is superior to that of bit-storage in the construction of digital computers and the networks they form, which are now in the course of transforming our world.

Moreover a number of notable mathematical results of the 1930s demonstrated that *all discrete encodings and their effective treatment are equivalent*. Kleene, Turing, Church *et al.* demonstrated the equivalence of (very) different formalisations of "computability": the numerical functions calculated by using the systems of Herbrand, Curry, Gödel, Church, Kleene and Turing were the same. By means of an astonishing philosophical sleight of hand, trading on the surprising and technically difficult nature of these results, and influenced by the surrounding intellectual climate of formalist and positivist ideas, the claim was *later* made that *any physical form in which information is processed,* and thus any biological form of information processing or any form of intelligence, *can be encoded in any such formal system*, thus it can be encoded in the form of the strings of 1s and 0s used in the memory stores of digital computers – see Longo (2003a) for more on some parodies of Church-Turing thesis.

A quite different way in which information can be thought of as structured, one involving geometric principles, is through equivalence classes of continuous deformations. These provide for the transfer and processing of much of the information essential to the make-up of living beings and, more generally, of physical systems. Continuous, differentiable or isometric transformations and the regularities they preserve or fail to preserve may help to structure and make intelligible living phenomena, as can be seen not least in the geometric structure of DNA or of proteins and their evolution. To these transformations the discrete and quantitative structure of bits of information serves as an addition; bits behave as singularities and thus as a possible measure of the topological complexity of the geometric structure. Information has both a qualitative and quantitative nature. The concentration on only its quantitative, digital, nature has become a severe limitation when information is assigned an explanatory role.

A frequent reaction is: yes, granted the role of these kinds of transformation and this kind of continuity, nonetheless, in the last instance, the geometric structures involved are reducible to very minimal discrete units.

Such a reaction hides many problems. Firstly, there is a problem about complexity: suppose one tries to describe, by a string of 1s and 0s, for example the three-dimensional structure of proteins exchanged in post-synaptic cascades, plus the biochemical flux in the brain fluid of an animal and the convection currents which accompany it. One faces extraordinary difficulties of principle as well as of practice. Physical and mathematical principles prevent our modelling this continuous and tri-dimensional information in discrete and linear form. The discrete-bits-representation becomes demonstrably intractable. Briefly, information in digital form, even when encoded in our tiniest microprocessors, would cover an area larger than the whole surface of the earth and there would thus be problems arising from relativistic effects obstructing its synchronisation. Moreover, if one entertains the idea of the possible discretisation of all spatio-temporal magnitudes, accuracy of approximation would it yield? The smallest living phenomena comprise dynamical systems (thermodynamic systems, systems with critical points). But this does not imply that a discrete mesh laid down *a priori* will be sufficient for their analysis. This is because sensitivity to initial conditions typically generates far-reaching consequences at or above the threshold of discernibility, triggered by a variation below that of the fineness of the measure.

But what kind of discreteness are we really talking about here? On the assumption we can push the encoding right down into the microphysical realm (so as not to have to cover the whole earth with processors) it seems the discreteness in question will come from quantum physics and will arise at the scale of the Planck length. One then encounters a fallacy – well explained in Bitbol (2000) – of the same stamp as that involved in the case of the formalist and mechanist reduction of mathematics to formal manipulation (processing) of discrete symbolic inputs. The reference to well-defined and ultimate discrete level of "material points" is the *conditio sine qua non* of Laplacean mechanics. No such appeal is possible in quantum theory, because it is a theory of continuous (quantum) fields, where Planck's constant, the only possible referent for these notions of ultimate discretisation, has the dimension of an action – a fibration orthogonal to the continuum of space-time. Moreover quantum indeterminacy and the epistemological debate which has raged around it has involved

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of which are in mutual interaction. Its unity is that of a subtle and complex kind of interacting and self-reacting field which it is still very difficult to grasp.

assigning a role to the knowing subject of a profoundly *anti-mechanist* kind. In approaching this topic armed with digitised version of microphysical structure one is lending support to the myth of Laplacean mechanism, as a matter of the bit-strings programmed by formal laws of thought.

We cannot renounce the mathematics of continua. The deformation of geometric structure can reproduce information in analogue form. And *the analogy involved is intentional* - one *chooses* what to represent or reproduce in analogous form, one selects those aspects of the original form which are to undergo processing or simulation. The choice of analogy is the outcome of a controlling vision or an aim, conceptually appropriate to living systems, of a kind which is missing in physics, where the phenomenal arrow of time is oriented without backwards linkages; see, nonetheless, Novello (2001). The fact that the reproduction and transformation of information via geometrical forms is analogue in character and may better accommodate intentionality, appears to be just what is crucial for biological representation. The eventual greater instability over time of geometrical forms by comparison with binary bit-strings is actually an *enriching* factor, because it corresponds to the possibilities of evolutionary change. The analysis of the geometric structuring of living systems (particularly of the brain) permits us to grasp a factor essential in information: selective analogical simulation carries with it evolutionary possibilities. By contrast with this, the perfect stability of bit-by-bit information processing renders such an elaboration impossible (whether this is a practical impossibility or one of principle is unclear).

To conclude: in studying living phenomena, from the most elementary systems all the way up to cognitive agents, it is not so much a matter of denying the important role played by formal and mechanical aspects (“bits” are key singularities in relation to information) but rather of enriching this analysis through the phenomenal richness of geometric structures and their effectiveness in information processing. Once again, formalism and mechanistic physicalism are seen to be not a variety of scientific reduction of the kind we should expect to meet in scientific practice, but rather a philosophical monomania, which has lost touch with the plurality of forms taken by our knowledge of and interaction with the world.

Research on both morphogenesis and architecture of dynamical neural networks (see e.g. Hertz 1991, Amari and Nagaoka 2000), despite its incompleteness (arising from the limitations of a physicist’s, though neither formalist nor mechanist, standpoint), at the very least suggests the richness of the geometrical structures implicated in any account of the organisation of living systems.

2.6 Globality and circularity in space and time

One of the great difficulties for a mathematical analysis of living systems and their evolution lies in the aspects of circular co-constitution. To the dialectic tension of the individual and the ecosystem we must add that between the present and the future.

As already pointed out, and contrary to certain theories of mind, I assign a very elementary sense to the notion of intentionality. The intentionality of our knowledge and our will is the ultimate and non-compositional epiphenomenon, the journey’s end of an intentionality characteristic of all living beings. It is the sense of intentionality illustrated by an amoeba moving in one spatial direction or another to preserve or ameliorate its metabolism. This kind of motion implies the unity of a living system, an individual with its membrane, so crucial to isolate it as a biological unity and the essential condition of autopoiesis, Varela (1989).

Intentionality suggests an analysis of time which incorporates the description of structural loops: the self-defining structuring of the ecosystem, where anticipations of possible future situations contribute to determining the present evolution and its bifurcations. Interactions within the ecosystem take place in 3-space and time: that is to say, the local times or internal rhythms of the individual organism as well as the time of its spatial interactions – the unity of an ecosystem relies on spatio-temporal cicles within it.

Mathematical methods appropriate for some kinds of structural loops have already been proposed. Mathematical Logic, for example, has suggested impredicative definitions and non-well-founded sets, amongst others, see Barwise (1996), Longo (2000). An impredicatively defined set contains elements, parts, the definition of which depends on the set itself (the local depends on the global). In fact topology very often employs impredicative notions (an intersection of sets containing the sets to be defined *etc.*). In a certain sense, impredicative definitions are “formally unstable”, in a manner reminiscent of the way a dynamical system is unstable: its global structure dynamically determines components which in their turn serve to constitute it.

It is not clear whether these approaches can tell us anything about the unity of living systems, because that unity clearly goes a long way beyond the forms of circularity they capture. However they do provide conceptual hints, for the properties which can be expressed and the functions which can be computed in such formal languages greatly outnumber those seen in their predicative or stratified versions, not to mention the gain in simplicity (see Girard et al. 1989). Their representation in the setting of category theory brings a richer structural framework in which closure properties play an important role. (See Asperti and Longo 1991, for an application of the notion of “internal category” to the semantics of impredicativity.)

Non-stratified or impredicative systems have received relatively little attention in the study of the foundations of mathematics, because of the hegemony of foundational/mechanistic trends taking the stratification of world structure as the only reliable source of explanation. It is in this way that the predicativistic approach long neglected or discarded tools which, by contrast, bring us closer to a “mathematics of the real world” (from “complex” dynamical systems to systems forming organic unities).

The obstacles posed to the development of such a mathematics are profound. In the first place the need of “expressive and constitutive circularity” of the kind already seen has to be enriched and put to better use. The reasons why biologists have resurrect “teleonomic” arguments may provide a clue. The notion of *telos* is always close to hand in the description of living systems (see the “contingent finality” above). The prevailing mechanistic outlook gets rid of these teleonomic traits. By contrast, the analysis of the mutual dependence between states and aims needs to be integrated into the mathematical framework within which we formulate the description of living systems.

PART 3: Spatio-temporal determinacy and biology

3.1 Biological aspects

The question of space has played a very important, even a foundational role in biology: one which has not always received due appraisal. Take for example the concept of *milieu interieur* (internal environment) introduced by Claude Bernard, which allowed an essential topological separation between the interior and exterior of an organism. Consider also the question of chirality in biology, highlighted by Pasteur. In the wake of his experiments on the tartrates and the manner in which their biological activity differed depending on whether they coiled to the left or the right, he stated unhesitatingly: “Life, as it is manifested to us, is a function of the asymmetry of the universe and a consequence of it”.

Indeed Pasteur anticipated both developments in his own field of scientific inquiry and, *mutatis mutandis*, what later came about in physics with the discovery of the asymmetry of matter and anti-matter, which cosmology now views as the precondition or the existence of the universe and of the actual material structures we see all around us.

Biological structures are subject to organising processes leading to the emergence of complex forms, such as those studied in developmental biology; furthermore, they display physiological functions which sustain the part/whole mutual dependence which mediate their integration as organisms and regulate the linkages between the different levels of organisation typical of organic existence, These facts clearly have a connection with theories of the critical behaviour of dynamical systems, such as that seen in phase transitions.

It was not by chance that the first mathematical models of biological systems appealed to and borrowed from those of thermodynamics, in particular models of cascade effects in bifurcations of thermodynamical systems (see Nicolis 1986, Nicolis *et al.* 1989), followed by models of emergence of self-organised critical behaviour (see Haken 1978, Kauffman 1993 and Varela 1989), and application of fractal geometry (see Mandelbrot 1982, Bailly *et al.* 1989, Bouligard 1989) and chaotic regimes (Babloyanz *et al.* 1993, Auger *et al.* 1989, Demongeat *et al.* 1989) to an organic context. Alongside these developments, it had been clear that the character and genesis of processes of formation could in many cases be modelled using the elementary theory of catastrophes (Thom 1977) and, more generally, singularity theory. What clearly shows up in the analysis of selfregulation and homeostasis (but also in the analysis of pathology and death) is what may be termed the “extended criticality” i.e. the enduring sensitivity to critical parameters of systems in that situation - a situation which is limited in spatial and temporal extent, but which nonetheless is extended (see Bailly 1991).

As a comparison, recall that in the framework of quantum theory, energy and time are conjugate variables. But an asymmetry nevertheless holds between them. While energy is a well-defined observable of the quantum system, associated with a Hamiltonian operator, time appears only as a parameter: one seemingly less essential and less well incorporated into the theory. In biology we seem to have the inverse: it is the time characteristic of biological systems (an iterative time which regulates biological clocks and internal rhythms) which seems to be the essential observable; whereas energy (the size or weight of an organism for example) appears simply as a parameter (an accidental parameter at that). In this sense one might even say that biology, relative to the energy/time conjugacy is quasi-dual to quantum mechanics.

3.2 Space: laws of scaling and of critical behaviour. The geometry of biological functions

Since the pioneering work of D'Arcy Thompson (D'Arcy Thompson 1961), recent studies (Peters 1983, Schmidt-Nielsen 1984, West *et al.* 1997) have shown that numerous macroscopic biological characteristics (as distinct from genetic traits at the biomolecular level) are expressed at the same scale across the range of entire species, indeed across genera, taxa and in some cases the entire animal kingdom. This scale-invariant parameter picks out the organism by its mass W or in some cases by its volume V. Furthermore, characteristic time scales for organisms (lifetimes, gestation periods, heart rates and respiration) all seem to obey a scaling law. They are typically in a ratio of one fourth of the mass ($T \sim W^{1/4}$). Just as these frequencies scale as 1:4 of W, metabolic rates typically scale in a ratio of 3:4 of W and many other properties display similar scaling. Such scaling laws call to mind the behaviour of dynamical systems where critical transition in regime is associated with fractional exponents of some key parameters. What differentiates one group of organisms from another is simply the value of the ratios seen in the expression of these scaling relations. These remain the same across numerous species and even across much wider biological groupings. Perhaps the most spectacular example is that of lifespan, which is in the same ratio to body mass.

Other kinds of scaling laws - allometries - link geometric properties of organs (as distinct from organisms) across numerous species, or within a single organism at different stages of its development. Here, however, our principal point is bound up with the display of fractal geometry in certain organs, engaged directly in the maintenance of physiological functions, such as respiration, circulation and digestion. This fractal geometry appears to be the objective trace of a change in the level of “organisation” and of top-down regulation of the parts by the whole, in conjunction with the bottom-up integration of the parts within the whole.

The fractal geometry in question falls into two distinct kinds. On the one hand the examples seen in the interfacing membranes of the organism. The metric dimensions of these are between 2 and 3. Examples are the membranes of the lungs, the brain and the intestines. The other class is formed by branching networks, such as the bronchial tubes, or the vascular and nervous systems. Here the metric dimensions of the extremities may be greater than 2. These fractal geometries permit the reconciliation of opposed constraints associated with spatial properties. On the one hand, because the organs involved are engaged in the regulation of exchanges such as respiratory or cardiac function, their effectiveness and their corresponding size must be maximised in order to support and to fine-tune these exchanges; on the other hand the fact they are incorporated into an organism containing many parts means that their bulk must be minimised to ensure their overall viability. To the extent that organs are clearly individuated and alloted wholly to certain specialised functions, they must present a certain homogeneity throughout their spatial extent. These various constraints are clearly antagonistic and only the fractal character of the geometry of the organs in question allows them to be reconciled.

Another aspect which raises interesting questions is the intrinsic three-dimensionality of living systems. If one enquires into the abstract possibility of developing biology in dimensions other than three, one recognises that the choice of three dimensions again allows the reconciliation of antagonistic constraints. On the one hand it is required that the organism present sufficient local differentiation to permit different concurrent functions across its whole structure; on the other hand it needs to be the site of sufficient internal connectivity to co-ordinate the activity of all its parts. In a space of only two dimensions, if the differentiation were sufficient, the connectivity and co-ordination between the different parts could not be established because the required connections of the components would intersect so greatly as to disrupt their separated functioning. On the other hand, in a space of four dimensions, the degree of possible connectivity is clearly greatly enhanced, but it is known that in four dimensions mean field theories[12] become applicable and the constraints of local differentiation become insufficient to allow for the establishment of systems stratified into different levels of organisation. Development of the system in three dimensions serves to reconcile these constraints at the cost of producing fractal geometries (their emergence reflects the existence of certain dynamical attractors). All these considerations concern the internal space of biological systems - they have no bearing on the dimensionality or topology of external space.

Following on from the earlier presentation of the notion of space in physics in terms of fibrations (carrying the internal symmetries of the system) over a base-space (the external space-time) one might consider the existence, in similar terms, of internal “spaces” associated with different levels of biological organisation. These should be distinguished from the different levels of scale structure in physical systems. What is distinctive in the biological context is the way these levels are connected with the regulation of the lower by the higher level of organisation and with the manner in which the different levels of structure act in constraining the formation of the integrated wholes which together they constitute.

3.3 Three types of time

To a first approximation, in describing the actual state of an organism (or a population) one can consider two types of temporality, jointly implicated in its survival. The first type, which carries echoes of time as seen

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[12] Intuitively, mean field theories (for example Landau's theory of ferromagnetism) are theories in which an approximation consists of replacing the effect of an element of the system, in the ferromagnetic case a spin, by the sum of all the individual interactions due to all the other elements by a “mean field” integrating their effects. These theories become better adapted for the description of the system in question the higher the number of near neighbours of any given spin is raised, because then the spin in question is better seen as the mean effect of these others. In a 4-dimensional space their number is sufficient to provide a model of a mean field theory.

in classical physics, is associated with the stimulus-answer coupling between an organism and its environment. It is manifested chiefly by relaxation processes (in the quasi-canonical form $e^{-t/\tau}$ and exponential combinations thereof).[13] The second (of a very different nature) is associated with internal clocks which administer the biorhythms of a living system and ensure its continued functioning (Glass *et al*. 1988; Reneberg 1989). It takes the form e^{iwt} and its combinations.

But the most important aspect of biological time is perhaps irreducible to these distinct forms: the internal "temporality" of organisms is iterative rather than historical. The measure of duration in this internal time is no longer a dimensional magnitude, as in physics, but rather a pure number registering the iterations already effected and those still remaining for an organism which experiences a finite number of these within a range fixed in advance, depending on its class. Thus, all the mammals, from mice to elephants or whales, form one such class. This is characterised by the number of heartbeats per average lifetime (around 10^9 for mammals) or the number of corresponding breaths (around 2.5×10^8). The variation in these frequencies between species is traceable to a single parameter - the body mass of the average adult. This striking trait is directly connected to the scaling law mentioned earlier.[14]

The importance of this aspect of biological time is emphasised by recent attempts to re-think the principal features of evolutionary theory in terms of the "living clocks" approach (see Chaline 1999). By interpreting evolutionary transformations in terms of their synchronic and diachronic effects, this approach concerns both the developmental level of individual organisms and the evolution of species.

But as G. Longo has proposed in section 2, it seems that in biology it is necessary to take into account a *third* type of temporality, connected with what he terms "contingent finality". This expression means a degree of (non-reflexive) intentionality which may help to explain the evolutionary and adaptive aspects of living systems. It is an *anticipatory* form of temporality, linking the current state of the organism and the future state of its environment, to which the organism contributes by its behaviour; and it is a form *specific to biology*, termed as "teleonomic" by Monod, which arises in connection with a coupling between the rhythms registered by inner biological clocks of the system and the stimuli-responses the system undergoes while interacting with its environment. (This coupling may introduce delay effects.)

Unlike the two previous dimensions, this "third dimension" corresponds to aspects of biological systems not seen in physics, in that it concerns the variety of integration-oriented factors involved in determining the present state of the system with those involved in determining its future state.

Perhaps this is another reason why, as G. Longo has underlined, one cannot define the notion of a trajectory traced out in the course of the evolution of a biological system in the manner one does for phase spaces in physics. There is no scope in the biological case for the application of a geodesic principle which extracts and determines one trajectory (that obeying an extremal principle) from amongst all the virtual possibilities. It seems the logic of biological systems operates in a quite different fashion, in a manner designed to display a *Bauplan* selected by external criteria.

As Gould claimed in his account of the organisms found in the Burgess Shale where it seems all the virtual possibilities – every possible pattern of development – saw the light of day. Here it seems we are dealing with criteria operating not so as to secure the emergence of a single possible form of the system (as with minimum principles in physics) but rather so as to secure the elimination of impossible forms so as to produce a maximal variety of system structure within a limiting "envelope of possibility".[15]

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[13] The simplest example of a relaxation process is the return to equilibrium of a system that has been subjected to a small perturbation. The speed of return is proportional to the departure from equilibrium the system has undergone. If P is a quantity of equilibrium-value p, with $P > p$, then $dP/dt = -r\ (P-p)$, where r is the inverse of a time; this leads to an exponential decrease in the departure of the system from equilibrium with time. The inverse of r is the characteristic 'relaxation time' of the system.

[14] This scaling law tends to confer objectivity on the more or less intuitive fashion in which we undertake the taxonomic classification of living beings.

[15] G. Longo cites the fact that biological systems, rather than following an evolutionary trajectory (tracing out a geodesic) explore all the possibilities compatible with their continued existence in a manner at once passive (*i.e.*, subject to the effects of natural selection) and active (in modifying the environmental conditions in which selection operates). One can find analogies in physics for the first aspect, but not at present for the second, which appears specific to biology. In quantum field theory, path integrals (Feynman integrals) are constitutive of entities, still not yet everywhere well-defined, seeking to take account of *all* the paths (with their appropriate weighting) from the initial to the final state (and not only privileged trajectories such as geodesics, albeit the probability of non-geodetic paths is very low). It is rather as if we could take account, in the case of biological systems, of all the transformations which a given form of the

Without seeking to formulate premature conclusions, we can nevertheless draw some lessons for our understanding of our notions of space and time, in the light of recent developments in theoretical biology. The distinction between internal and external space is connected with the distinction between the autonomy of an organism (the homeostatic stabilisation of its functioning and its identity) and its heteronomy (its dependence on and adaptation to its environment), see Bailly (1998).

Equally, the internal/external articulation of a space physically determined in structure and another space, determined by the functionalities of the organism and its complex morphology, leads directly to issues of the relationship between the genetic programming of an organism and the epigenetic factors involving its interaction with its environment in the course of its typical development. The essential new element distinguishing the situation in biology from that in physics lies in the fact that in biology the articulation of this internal/external distinction applies also to time and in a crucial way, involving the relationship between the different types of temporality: one of them akin to that in physics and with the character of a dimension, the other specifically biological, iterative and expressed in pure numbers, which seems to play a quasi-constitutive role with regard to our concept of a biological system, in that it supplies the basis for the characterisation of the invariants operating in the definition of equivalence classes in the biological setting (the invariants of mammalian biorhythms illustrate this well).

We could go a little further in tracing this comparison with physics, and seek to locate the distinguishing feature of biological systems at an even more fundamental level, that of the dimensionality of its elements (in a topological sense). With the rise of string theory, ultimate entities have gone from being punctate to being linear. In biology an inverse, but curiously convergent, development occurs. It seems that what is regarded as lying at the most fundamental level of the organisation has undergone a change from being something which occupies a volume to being something linear. What appears fundamental to the genetic programming and the resultant life forms, is held to be a linear sequence of macromolecules, aligned in an order determined by the base pairs of DNA which constitute the genetic endowment and to a large degree govern the development and functioning of the organism.

The biological activity of these macromolecules depends very strongly on their 3D spatial structure (as is shown by the activity of prions) but it is nonetheless remarkable that the linear chains of macromolecules have come to assume such importance, and that the spatial structure of their enfolding appears to be so largely dependent on their linear sequences, which effectively control the interactions regulating that enfolding. The manner in which the *concepts* of space and time are treated is, once more, fundamental to the constitution of biology as a science. Beyond the analysis of perception, any epistemological account of the status of such concepts - to the extent it is based on the results of natural science and aims to be objective - cannot ignore their role in the framework of theoretical biology.

3.4 Epistemological and mathematical aspects

Now, it may be instructive to run over the epistemological ramifications of space and time by means of an analysis “transversal” to the theoretical frames of reference considered so far. Three pairs of concepts appear important for such an analysis. Firstly, *local vs global* concepts in relation to space; second, *iterative vs processual* aspects in the understanding of time (with an aside examining how both these pairs of concepts are intimately bound up with the topic of *causality*). Third, *regular vs singular* in connection with our system of representation and reference.

In fact, by considering physical and biological aspects of space and time, one cannot evade the epistemological issues involved in their purely formal definition. Spatial and temporal concepts are “abstract” concepts in two different senses of that term. The first relates to the process of abstraction which takes its cue from common features of the theoretical treatment of space and time and the second, concomitant, sense relates to their being formal, quasi-*a priori* notions which come to be imposed as the result of that process of abstraction, as an intrinsic component of our notion of objectivity itself.

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system could undergo (together with their probabilities), as suggested by Gould’s description of the PreCambrian explosion to which the fauna of the Burgess Shale bear witness.

All the quantum paths which determine the state of a system in the Feynman formalism are located in spaces (and involve modes of interaction) completely defined in advance and which do not really depend on a specific path (even if they can be regarded as depending on the final state, once reached). In biological systems, by contrast, any stage in the evolution of a system (even of an individual) modifies the conditions in which all subsequent stages are produced and these conditions are not defined in advance.

(I) LOCAL *vs* GLOBAL. In passing from relativistic to quantum theories and then to general dynamical systems theory, and finally to biology, we recognise a shift in the relative importance and pertinence of the local/global opposition (broadly speaking, a shift from the former to the latter). Despite the stress on a global interactive point of view seen in Mach's Principle, general relativity completely preserves the principle of locality inasmuch as it is essentially and exhaustively expressed through partial differential equations. In this respect, it lends itself to an interpretation in terms of *local causes* propagating within the light cone.

Quantum physics can equally well be presented as a theory of local interactions and their propagation by state vectors. The Schrödinger Equation and that of Dirac are just as much partial differential equations as those of Einstein. Hence it could be taken to involve a notion of causality of the same apparent kind. But quantum measurements on the one hand, and non-separability on the other stand in the way of a completely local interpretation of the theory. Classical causality is affected by the fact that measurement leads to intrinsically probabilistic results while non-separability disrupts any purely local representation of the propagation of effects.[16]

The case of theories of the critical states and dynamical systems takes us a step further: here non-locality plays a twofold role. Firstly the fact that interactions can now take place at long distance leads to correlations becoming infinite. Local variations and effects lose their relevance both for analysis and measurement in favour of the global behaviour of the system. This even reaches the point that our notion of what counts as an object needs to be re-defined. Furthermore this global behaviour is itself governed by critical exponents and scaling laws which are in no sense local (since they are dependent on the dimensionality of the embedding space and on an order parameter).

A concomitant of this situation is that the usual notion of causality (even when there exists a linear correlation between cause and effect - small causes giving rise to small effects) is undermined (in critically sensitive systems, infinite effects can arise from finite causes which lead to discontinuities in the evolution). Curie's Principle (that symmetry of causes is mirrored in symmetry of effects) is thus called in question (at least for systems displaying singularities and discontinuities in their behaviour) by the symmetry breaking which accompanies phase transitions.

In biology, locality seems pertinent chiefly to the description of underlying physico-chemical processes, while the definition of biological systems and their manner of functioning involves global concepts associated with the fundamental non-separability of living systems and their complexity. This global level of structural organisation becomes decisive for the representation of processes of regulation and integration which stabilise the functioning of a biological system. To this there corresponds a more complex notion of causality involving an entangled and interactive hierarchy and its associated 'agonistic or antagonistic' effects. In brief, the notion of local causality cannot be called in question without the global notion being affected as well (and the global notion is associated with “contingent finality”). This opens the way to a distinction - one meaningless in physics - between the normal and the pathological. A locally pathological mode of functioning can co-exist with the preservation and global functioning of an organism.

(II) ITERATIVE *vs* PROCESSUAL. Relativistic theories, with their characteristic metric structure, display an almost completely spatialised type of temporality. They introduce the concept of an event as a “marker flag” in a generalised space. Only physical causality - the fact that interactions between point-events cannot propagate outside the light cone, or to reformulate that requirement from a mathematical standpoint, the fact that the signature of the metric is fixed - serves to introduce a distinction between spatial and temporal dimensions. Conceptually, this distinction is intimately bound up with the fact that from the viewpoint of the symmetries of the system, Noether's theorem classifies time as a conjugate variable with respect to energy (or conversely, views energy as the conjugate of time), just as the spatial variables are conjugate to the components of the momentum. But essentially, relativity, via the group of general covariant transformations, treats time as a notion of the same kind as space.

Contrasting with this situation, in quantum theory time is treated as a simple parameter. Its status as the conjugate of energy is preserved (as is seen in the Heisenberg indeterminacy relations) but it does not appear as an observable of the theory. Moreover it seems that certain phenomena - those connected with quantum state transitions or even measurement (setting to one side the decoherence approach) - do not easily lend themselves to an interpretation in terms of temporal concepts of the kind connected with our experience of passage and duration. One sees something similar in the apparently instantaneous connections associated with the behaviour

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[16] Theories of hidden variables, proposed to overcome certain of these “a-causal” aspects, are themselves non-local.

of non-separable quantum systems. This further stresses how greatly the conceptualisation of causality is bound up with that of time.

In theories of dynamical systems, time regains more classical characteristics, but ones made apparent in connection with different aspects of phenomena than in the classical case. Besides being the time of events (as state transitions), it plays further roles, distinct from the role it plays as a parameter: in the definition of stability, in the definition of irreversibility, in the characterisation of attractors (asymptotic behaviour, as seen in fractal geometries, the Lyapunov exponents *etc.*); and when a bifurcation takes place, it can establish a cycle and thus acquire an iterative character.

In biology, temporality displays two quite distinct aspects: the external time, *i.e.,* the relaxation time of stimulus and response, of functional adaptation to an exterior environment, and the iterative time of pure numbers associated with internal biorhythms involved in the regulation of physiological functions. The corresponding notion of causality, adaptive and intentional in character, seems closely connected with the mutual articulation of these two aspects.

(III) REGULAR *vs* SINGULAR. Relativistic space-time is "regular", continuous and differentiable, and singularities (whether of the Schwarzschild or the initial manifold) play a quasi-incidental role which assumes central importance only in certain astrophysical and cosmological contexts.

The situation is different in quantum physics, where the regularity of certain spaces is associated with the discretisation of others and where space-time structures can be envisaged as fractal at very small scales. The regular/singular couple carries the traces of the old debate about the interpretations of the theory, namely in terms of fields or in terms of particles.

In theories of "critical" systems, the interest in singularities is accentuated. They are associated with increase in complexity, and also with the particular consequences of nonlinear dynamics (*e. g.*, for what concerns solitons and their propagation). In fact, these theories are essentially singular since critical situations all involve singularities (divergence, discontinuity, bifurcations). The mathematics of singularities (singular measures, catastrophes) plays a predominant role in modelling the behaviour which gives rise to complexity. Nevertheless, it appears that the outcome of these features is in fact a new form of regularity, one located at a more general level of analysis, revealed in laws of scaling and leading to a universal classification embracing very different systems, which nonetheless manifest identical behaviour with respect to the singularities in their dynamical evolution. The critical transitions in these systems are typically restricted to a very narrow range, even to a single point in phase space, a single value of the control parameter, and on either side of this very narrow critical zone regular behaviour of the system once again becomes dominant.

Precisely this last aspect seems to contrast with the position prevailing in biology. Organisms and ecosystems can survive and maintain themselves within a range of values of critical parameters - an extended zone of criticality. Exit from this zone implies the death of the organism: its underlying physico-chemical structure can no longer sustain biological functions. Any biological system behaves in a manner characterised by a *dense* distribution of critical points in the space of control parameters, and not a discrete or isolated one. Homeostasis then, corresponds to a sort of structural stability of the trajectories, relative to the attractor-basins of the dynamics.

Space and time, especially as they feature in the framework of modern physics, are neither *objects* nor *categories*. To recall Kant's formulation (Kant, 1986) they are "*a priori* forms of sensible intuition", and as such the preconditions of any possible experience. In the light of the most deep-going analysis our current physical theories allow us to make of them, they seem to reflect the mathematical structure of a group and a semi-group, respectively (See Bailly 1999). Indeed, the mathematical properties postulated for space, inasmuch as it is the medium and support of displacements in general, necessarily connect with and exemplify the group structure. Given the tight connection between the group structure and its associated equivalence relations, an abstract, epistemically basic frame of reference emerges which acts as a kind of pole of attraction for any representation of objectivity, namely the frame: <space, group structure, equivalence relation>.

Similarly in respect to time, the property of possessing an orientation – "time's arrow" as the index of change - is reflected in the abstract structure of a semi-group. That structure can be put in correspondence with an order relation. This leads us to the recognition of a second epistemically basic frame of reference: <time, semi-group, order relation>.

To repeat, for the avoidance of all confusion: the space and time which feature in these frames are not so much an aspect of entities with an intrinsic nature, but rather of the conceptual grid presupposed by any natural science: they are conditions of possibility rather than of concrete actuality. If this approach is correct, these two "poles" leave their mark in our notions of permanence and change, stability and evolution, identity and differentiation. They delimit the “field” of preconditions for any natural science, to the extent that the phenomena studied therein are manifested in a spatio-temporal setting.

3.5 Closing remarks

This *Tableau General* is still incomplete (notably in the life sciences). But what clearly stands out is that contemporary theories across a whole range of scientific domains involve conceptions of space and time that have not yet been fully stabilised or clarified (although in physics super-symmetric string theories are seen as leading to unification). Will we continue to distinguish space radically from time despite the merging of their status in the setting of relativity theory? Shall we continue to refer to a single notion of space in view of the variety of topological and other structural properties (compactification, non-commutativity, internalisation, fractal dimension) envisaged for it in current physical theory? Or in the face of the complexification and fractalisation of the forms of space arising in biology?

Likewise shall we retain the representation of time as a unique parameter in physics or as intrinsically irreversible? And how will the time of physics turn out to be connected with that of biology? By jettisoning its formalisation in terms of isolated bifurcations perhaps, in order to take into account the synchronic and diachronic effects in biological systems?

One reason for this rather confused state of affairs (albeit one closer to the often counter-intuitive nature of reality than our spontaneous perceptions can bring us) is that the very epistemological status of space and time remains relatively problematic despite the formal categorisation introduced in the foregoing. One additional difficulty has been introduced by the fact that since recent developments in physics (mirroring what has long been the situation in biology) we now have to take account of spaces external and internal to the systems under investigation.

We here encounter a distinction long made by philosophers – and notably by Kant - but this time in a form lying on the side of the objects themselves, whereas for Kant it was conceived as lying on the side of the epistemic subject. It is the distinction between space as the form of external and time as the form of internal sense (Kant 1986). Such a distinction with respect to objects was unacceptable to the Kant of the Critique, for having broken with an ontological characterisation of the objects of natural sciences, any such internality was denied and location was considered something totally external. However, towards the end of his life, in re-examining Newton’s *Principia*, Kant could not refrain, on the evidence of the *Opus Postumum* (Kant 1986a) from thinking through this point afresh, in particular in relation to the question of energy. He renewed his investigation of certain aspects of Leibniz’ thought on this subject which he had largely avoided beforehand and he would perhaps have found in the latter-day evidence for a spatialisation (and temporalisation?) *internal* to the objects of physics and biology, if not the answer to his puzzle, at least a spur to new investigations.

Any attempt to make the internal/external dichotomy correspond in a straightforward way to the distinction between space and time is infected with artificiality – the more so if the distinction is seen as lying within reality itself rather than the knowing subject. Nevertheless, taking into account the Kantian view of space and time as the very conditions for the constitution of objectivity, it does not seem too extreme to speak of them, rather than of subjective “forms of sensible intuition”, of objective “forms of sensible manifestation”. Such forms could themselves be connected with the concepts of externality and internality. In the case of “the external”, one essentially considers the phenomenal manifestation of *relations* between objects (interactions and corresponding measurements). In the case of “the internal” one considers, rather, constraints concurrent with the phenomenal manifestation of the continuing *identity* of an object – or better, of its *identification.*

Notice that the distinction between internal and external aspects is one of the conceptually distinguishing features of biology. But it can be expected to be an element in our conception of the objects of theoretical physics as well, since it now has an objective, mathematically expressed counterpart in the distinction between the external space-time (the base space) and internal spaces (the fibres over the base) which is now a fully developed aspect of the mathematical formalism of key areas of physics. Briefly (and ironically, in the light of certain epistemological tendencies which have sought the reduction of biology to physics), it is on the side of biology that one now looks for the conceptual clarification allowing the development of a more comprehensive abstract framework for the understanding of physical phenomena (see Rosen 1991).

We have thus arrived at a kind of conceptual “re-normalisation”. The external space-time of physics can be seen as a manifestation of the couple <space/relation> and as providing a ‘base’ relative to which the corresponding couple <time/identifiability> can be viewed as the fibration (the internal spaces). The further articulation of these two elements (via the introduction of super-symmetry and super-space) gives rise to what might be termed a space-time of a kind which allows us to take account of all the aspects in which an object becomes determinate: both its relational properties and its internal structuring activity which permits its continuing identity. Whereas for biology it is the external space-time which corresponds to the manifestation of the <space/relation> couple, it is morphogenesis and biorhythms which correspond to the manifestation of the <time/identifiability> couple, associated with the identity and functioning of the organism over its lifetime.

All these considerations are extremely speculative and demand detailed elaboration to test their pertinence. On the other hand, the status conferred on our concepts of space and time in the constitution of objectivity has to do with epistemological issues, and here it impacts at a really profound level on the status of categories as basic as causality and on conceptual pairs as important as that of local and global. These categories and concepts are to a greater or lesser degree derived from the intuitive or the theoretical representation of space and time and it is in connection with both intuitive and theoretical considerations that we must give an account of the role of space and time in explanation.

The mathematical formalism of scientific theories confers our frame of reference a status and a structure which is objective yet increasingly counterintuitive: that is to say, basic categories and derived concepts are both increasingly accurate and increasingly unfamiliar in character. We have arrived at a stage where, alongside the development of mathematical formalism (whether in symbolic or diagrammatic form), *new structural intuitions* are achieved – intuitions inherently generated by the formal system itself and further and further lacking in directly empirical content.

The task confronting us today is the rational articulation of connections between these new types of (theory-generated) intuition and the experimental results with which we are presented in the realm of physical and biological phenomena. To recall an old distinction from hermeneutics, explanation may make progress, but comprehension may be poorly equipped to follow. I recall René Thom remarking, in one of his barbed quips, that quantum mechanics was unintelligible and that everything about it that was rigorous was insignificant.

What is the connection between increasingly abstract spaces and times, theoretically constructed and formally specified, and the intuitively given ones which preside over the development and regulation of our own cognitive capacities? The permanence of our vocabulary, while it constitutes an index of familiarity, certainly does not suffice to justify or explain these relationships. It is to the existence of profound cognitive schemata and invariants of our mental representation and to the way they are transformed that we must appeal. The two epistemic frames proposed (<space, group, equivalence> and <time, semigroup, order >) suggest a way of approaching that difficult question. But this approach as yet remains largely inadequate.

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