Symmetries and “principles of construction” in Foundations

Giuseppe Longo

LIENS, CNRS – ENS, Paris

http://www.di.ens.fr/users/longo

Francis Bailly, Giuseppe Longo. **Mathematics and Natural Sciences. The physical singularity of Life Phenomena.** Imperial College/World Sci., 2010 (to appear).

Philosophical Preliminaries

A "foundational analysis"
(a fully justified search for "*unshakable* certainties", in 1900)

Frege and **Hilbert**:
the search for (***proof***) ***principles*** and "unshakable certainties"
(*different*: absolute laws and existence *vs.* finite auto-consistency and existence by consistency)
Commun: the solid grounds on Arithmetic
(the **concept** of **finite** number, **induction**)

Today:
A critique of the ***constituting principles*** of a form of knowledge
(*shaking* the principles)

An analysis at the ***interface of disciplines*** (their "foundations").

Mathematics and Physics: Common Construction Principles

1 - Symmetries:

1.1 - *Mathematics:* **Euclid's** geometry, Dualities, Adjunctions …

1.2 - *Physics:* **Geodetic Principle**

(a least action principle, derivable from Energy Conservation properties; Noether's Theorems: conservation of Energy is a translation symmetry in equations)

2- The notion of **generic object**:

The mathematical and the physical "object" are invariant w.r. to the intended law: they are theoretical and/or empirical invariant

(Galileo's falling stone or Einstein's planet or a photon…)

Different "**proof principles**": ***formal*** proof vs. ***empirical*** proof

Symmetries
(from H. Weyl, 1952)

- The mathematical (naive) version:

  *a **transformation** that preserves "some" properties of a figure (those you care of...)*

- … but also an ***invariant*** (e.g. a mirror symmetry preserves symmetries)

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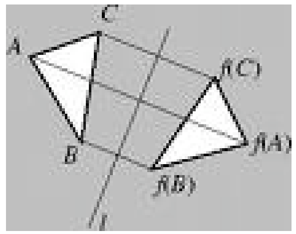
- … but also an ***invariant*** (e.g. a mirror symmetry preserves symmetries)

Symmetries (including *translation symmetries):*





Symmetries
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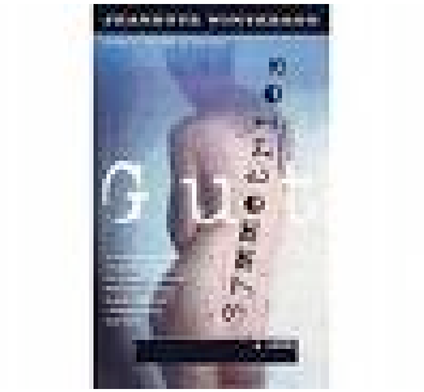
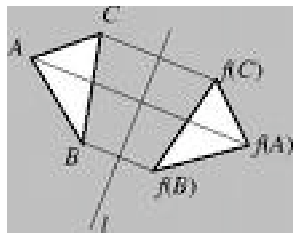
Symmetries and **symmetry breakings**:





In Weyl's terms (1952)

The ***symmetry*** of a figure in space is
"a subgroup of the group of automorphisms"

Back and forwards: Maths, Physics and History:…

The constructive content of Euclid's Axioms

Euclid's ***Aithemata*** (Requests) are the ***least constructions required*** to do Geometry:

1. To draw a straight line from any point to any point.

2. To extend a finite straight line continuously in a straight line.

3. To draw a circle with any center and distance.

4. That all right angles are equal to one another.

5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles [Heath,1908]

Maximal Symmetry Principles

These "Requests" are *constructions performed by ruler and compass*:
an abstract ruler and compass

1. To draw a straight line from any point to any point.

The most symmetric drawing: any other path would break symmetries

(a geodetic)

Cf. Hilbert style's axiom:
"for any two points, there exists one and only one segment…"

In Euclid, **existence** is by construction, **unicity** by symmetry…
(any other path would reduce the plane symmetries)

Maximal Symmetry Principles

2. To extend a finite straight line continuously in a straight line.

Preserving symmetries

3. To draw a circle with any center and distance.

The most symmetric way to enclose a point by a continuous line

4. That all right angles are equal to one another.

*Equality (congruence) is obtained by **rotations** and **translations** (**symmetries** - automorphisms!)*

Note: *right angles are defined as producing the **most symmetric** situations of two crossing lines*

Maximal Symmetry Principles

***5.* That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles**

In Book 3 this is shown to be equivalent to:

***5.Bis* You can draw exactly one line by a point to a line on a plane**

The most symmetric situations:

the two possible negations (non-euclidean geometries) reduce the symmetries, on the euclidean plane (they have less automorphisms)

G. Longo. ***Theorems as Constructive Visions.*** Invited Lecture, Proceedings of ICMI 19 conference on Proof and Proving, Taipei, Taiwan, May 10 - 15, 2009, (Hanna, de Villiers eds.) Springer, 2010.

On evidence

« The original evidence should not be confused with the evidence of the axioms; because the axioms are already the *result of a formation of sense* and have this formation *already behind them* »

[Husserl, The Origin of Geometry, 1933]

Constructions and *Proofs* in Euclid:
a geometry of figures (in the space of senses)

- **Rotations** and **translations**
  (*symmetry* preserving; automorphisms)

- Closure under **homotheties** (*Fifth axiom!)*
  local = global   structure of space
  (space of senses = physical space)

- **Isotropy** as a space symmetry

These "***principles of constructions***", in Euclid, *produce* objects and *give* proofs

Towards Physics:

Archimedes’ Equilibrium Postulate

“Two weights on a balance are in equilibrium
if and only if
for reasons of symmetry no rotation occurs”

[Mach’s rephrasing, 1883]

After the « delirium » of non-euclidian geometry (Frege, 1884)

A remarkable invention (ideas from Aristotle and others…):

Isolate/propose explicit **logical** (*formal* for Hilbert)

Proof Principles

(e.g. rules for implication and quantifiers, arithmetic induction …)

Cf. Euclid's axiom vs. Hilbert's version:

- Trace, construct, rotate, translate …
- *For any* …. *There exists* …. *deduce* ….

(cf. Cauchy …)

Construction Principles and **Proof Principles**

Mathematics	***Physics***
Construction principles: symmetries, (well-)ordering …	*(same)*
Logico/formal **proofs**	Empirical proofs

This distinction (and their interplay): *a leading theme in Bailly-Longo's Book*

From Archimedes to … *XXth* century **Physics**

Back and forth between Mathematics and Physics…

How to relate symmetries and physical "laws" ?

The geodetic principle

From Hamilton to Noether and H. Weyl (in two pages)

(Hamilton's) ***least action*** principle
(**action = energy×times**):

extremize the "action functional"

Informally: minimize a measure (length, surface..) in a suitable metric space (even an Hilbert Space - Schrödinger's equation)

That is, it defines the trajectories as *minimizing the Lagrangian action*, as *the time integral of the Lagrangian*)

The geodetic principle *and* symmetries

Noether's theorems [Noether, 1918 ; Hill, 1951] **pull out of the equations** the *continuous transformation of* ***symmetries*** which **preserve** the *equations of movement*.

Formally: "If a Lagrangian is invariant under an n-parameter continuous transformation (in the sense that the Lagrangian function is invariant), then the theory posses n conserved quantities"

To each of these transformations corresponds **conserved physical quantities**

(e.g. invariance w.r.t. translation in space ≈ conservation of the kinetic moment;
invariance w.r.t. translation in time ≈ conservation of energy)

[van Frassen, 1994; Bailly, Longo, 2010, ch. 4]

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- *In Mathematical Physics:* just equations? Missing the point…

In ***Categories*** and Physics: *equations* ***implicitly*** *contain symmetries*
Theoretical job: pull them out! And **add meaning**… by diagrams.

Summarizing by a "geometric rephrasing"

- *preserve properties* as symmetries

  Example: the optical principle of the *shortest path;*

  Notee: ***Euclid's straight line = a light ray***

Recall: a geodetic in the "plane": a straight line,

it ***preserves*** *the tangent* ***constant*** (momentum)

a pointwise (local) *preservation property* (see next…)

Some consequences of the geodetic principle

The *conceptula priority of the* ***least*** *(or stationary)* **action** in the **intended space** (also very abstract Hilbert…):

A radical change in the notion of "Law":

A **physical law** is the expression of a geodetics

in an *adequate space and with its metric.*

The geometrization of Physics and *the physicalization of Geometry:…*

Back to Geometry: preserving invariance

- **(Riemann-)Klein**:

Back to Geometry: preserving invariance

- **(Riemann-**)**Klein**: a geometry is a set of *invariant properties* and invariance preserving *transformations*

- Various **non-euclidian** geometries (commun property):

  no closure under homotheties (as automorphisms ≈ symmetries)

- Riemann's key distinction: *global vs. local*:
  - **global** (topological, dimension)
  - **local** (metric), *no homotheties*

Local ≠ global, a fundamental ***symmetry breaking***…

(General) **Relativity** will ***unify*** gravitation and inertia along geodetics in Riemann's manifolds …

Summary: symmetry as a fundamental construction principle (and proof principle)

- Why symmetries should count less than « modus ponens » in "foundation"?

Symmetries in Gentzen's and Girard's **Proof Theory**

The role of well ordering (semi-groups):

Longo G (2002) On the proofs of some formally unprovable propositions and Prototype Proofs in Type Theory. …

Conceptual constructions *and* proofs proceed by a blend of proof and construction principles

Some references

http://www.di.ens.fr/users/longo *or* **Google: Giuseppe Longo**

Bailly F., Longo G. **Mathématiques et sciences de la nature. La singularité physique du vivant.** *Hermann, **Visions des Sciences,** Paris, 2006* (**2010**: in English).

Longo G (2002) Reflections on Incompleteness, *or* on the proofs of some formally unprovable propositions and Prototype Proofs in Type Theory. In: **Types for Proofs and Programs**, Durham, (GB), Dec. 2000; Lecture Notes in Computer Science, vol 2277 Callaghan et al. (eds), Springer: 160 - 180

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Giuseppe Longo, Arnaud Viarouge. Mathematical intuition and the cognitive roots of mathematical concepts. Invited paper, **Topoi**, Special issue on Mathematical knowledge: Intuition, visualization, and understanding (Horsten L., Starikova I., eds), Vol. 29, n. 1, pp. 15-27, 2010.

Foundational challenges in Biology

Foundational challenges in Biology

- *Mathematics* as a **science of invariants** and invariance preserving **transformations**

- *Computer Science*: also a science of **iteration** (from primitive *recursion* to *portability* of software)

- *Biology*: a science of **structural stability** *and* **variability** (the main invariant?)

Theories in Biology: changing ***observables*** and ***parameters***?

From Physics to Biology by "Conceptual Dualities"

(or: changing *principles* by dualities)

[Bailly, Longo, 2006-7-8-9-10]

Dualities in Physics vs Biology

1. ***Physics***: *Specific* trajectoires (geodetics) and *generic* objects
   Biology: ***generic trajectories*** (compatible**)** and ***specific objects***

Dualities in Physics vs Biology

1. ***Physics****: Specific* trajectoires (geodetics) and *generic* objects
   Biology: ***generic trajectories*** (compatible) and ***specific objects***

2. ***Physics*****: energy** as operator *Hf***, time** as parameter *f(t,x)* ;

   Biology*:* ***energy*** *as parameter,* ***time*** *as operator* "anti-entropy" (dual to entropy) [Bailly, Longo, 2009]

Methodological (and logical) premises:
How to deal with this new observables?

Physical vs. Biological Theories
in Bailly-Longo three (correlated) approaches:

Theoretical **extensions** (in the sense of Logic) of physical theories

CONSERVATIVE (?) EXTENSIONS

Examples from **Logic**: T ⊂ T' = T+NewAxiom (T' *extends* T)

Formal Arithmetic (PA)

1. **PA + König's Lemma** (any *infinite*, finitely branching tree has an infinite branch) is a ***strict***, ***conservative*** extension: it proves more on infinite trees, but no more *arithmetic* statements.

2. **PA + Axiom of infinity = Set Theory** (Set)

is a ***strict***, ***non-conservative*** extension of PA, since Gödel '31:

an axiom of **infinity** allows to prove Consistency of PA (*Coher*).

Gödel's Theorem:

Set is not conservative over PA (or, PA |-/- *Coher*)

Physical vs. Biological *Theories*

Ontological (bunches of molecules) *vs.* **Theoretical** issue.

What about considering ***extensions*** of Physical Theories by *proper observables?*

- Critical transition not just on (mathematical) "points"
- Levels of biological organization (anti-entropy)
- Various forms of irreversibility of time (+ a two dimensional time)

Reduction to the physical (sub-)theories? Why not …

In Physics:

unification (Newton vs Galileo; Thermodynamics (limit); Relativity/QM …)

Question: **conservative extension?**

Physical vs. Biological Theories
in Bailly-Longo three (correlated) approaches:

1 - ***extended* criticality** *(a physical oxymore),* **JBS**, 16, 2, 2008.

2 - **organization** (a new observable) as **anti-entropy**, **JBS**, 17, 1, 2009.

3 - **extra (irreversible) time** and **two dimensional time** *(not linear time),* ongoing, with M. **Montévil**

Common point to the approaches in 1, 2 and 3:

Strict "Consistent" **extensions**, in the sense of Logic, compatible with current physical theories (Thermodynamics), *but not necessarely reducible:*

1: contract the extension of criticality *(from interval to point);*

2: "=" instead of "≤" in balance equations *(anti-entropy goes to 0)*;

3: collapse the extra dimension *(a time bifurcation).*

Question: are they conservative?

CONSERVATIVE (?) EXTENSIONS

In **Biology**:

1. Preparata, del Giudice (1995-2007): **Water coherence domains** (in phase oscillations of molecules of water) in **cells**: derivable from enclosure of water in *organisms* (10^{14} cells) and *Quantum ED*. ***Strict, conservative extension***.

2. Biot - Pasteur: **asymmetry in chirality** of (levo-)tartaric acid.

So far, no *physical* explanation: ***non-conservative extension*** needed ?

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So far, no *physical* explanation: ***non-conservative extension*** needed ?

Our theoretical attempts: ***strict, conservative*** (?; add new observables):

1 - ***extended*** **criticality** *(a physical oxymore),* **JBS**, 16, 2, 2008.

2 - **organization** or complexity as **anti-entropy**, **JBS**, 17, 1, 2009.

3 - **extra (irreversible) time** and **two dimensional time** (this talk)

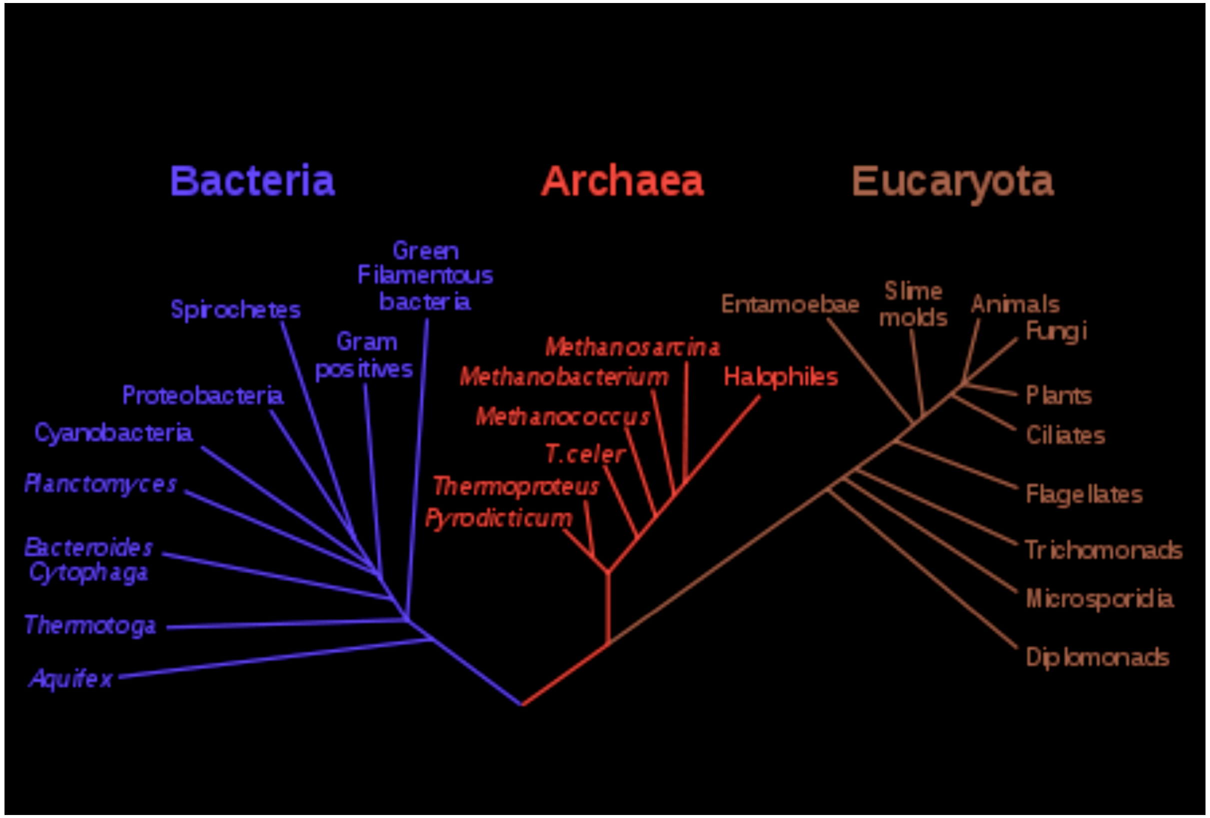
An application to
Biological Evolution and “Complexity”

Evolution and “Complexity”

The ***wrong*** image (progress?):

Growing complexity in Evolution?

Which “complexity”?
Evolutionary complexity?



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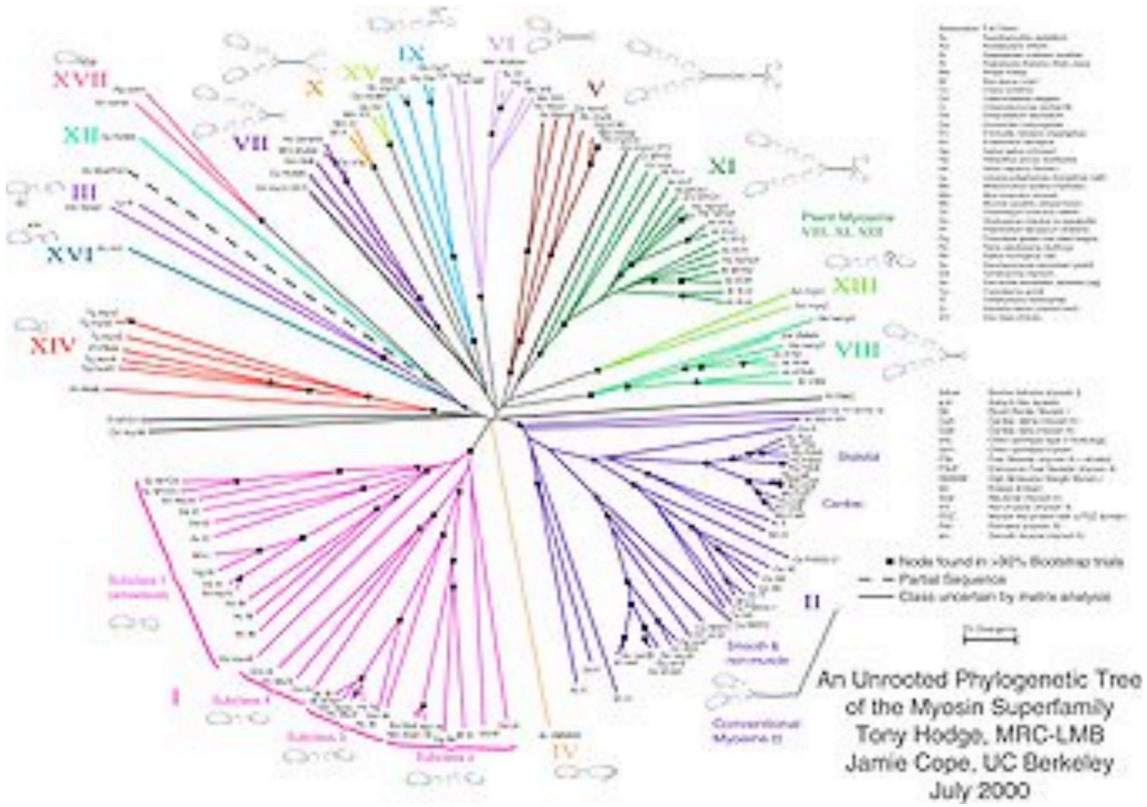
A better representation:



However: Gould’s growth of “morphological” complexity [**Full House**, 1989]

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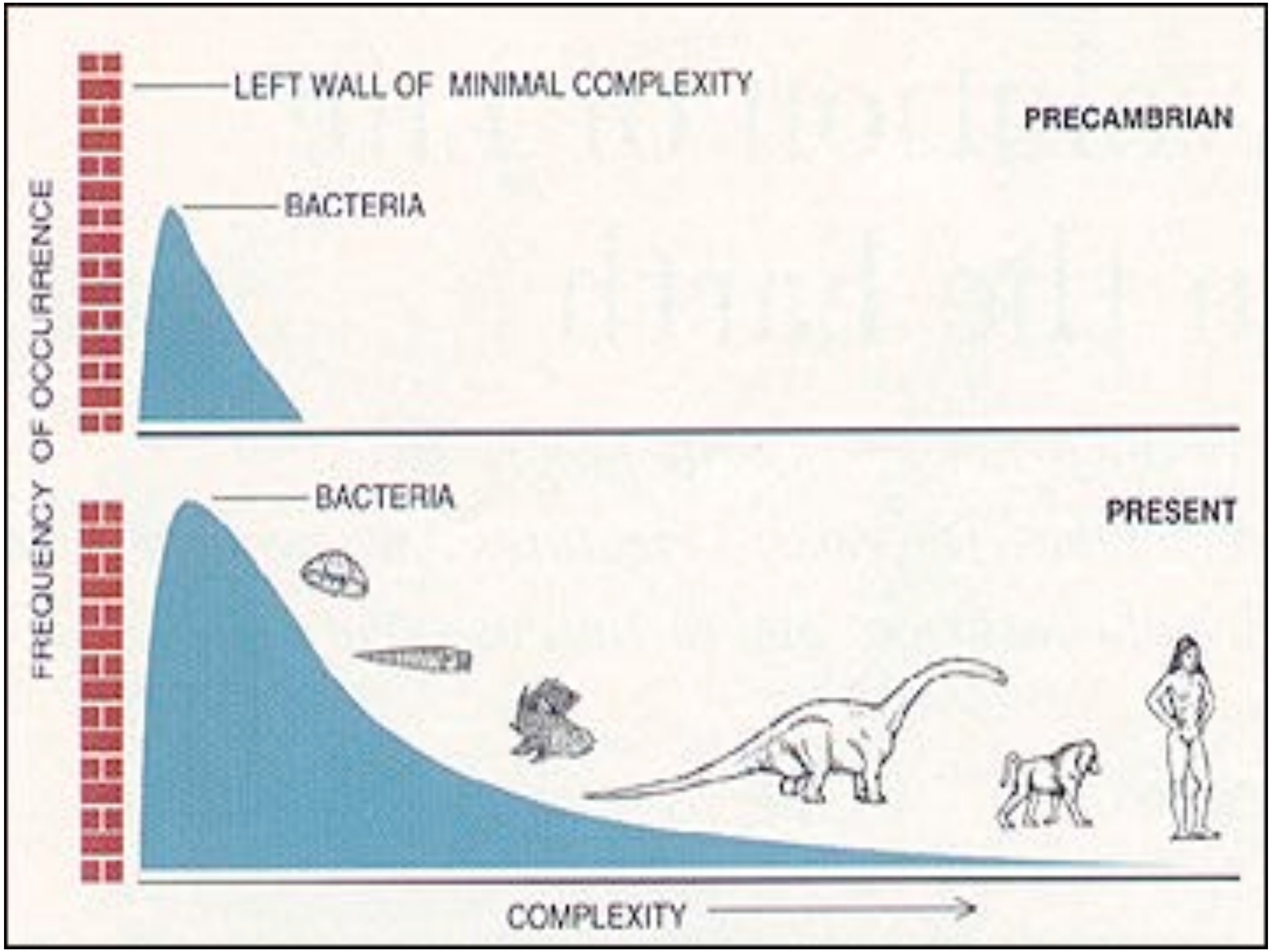

Random increase of complexity [**Gould**, 1989]

Asymmetric Diffusion *Biased Increase*

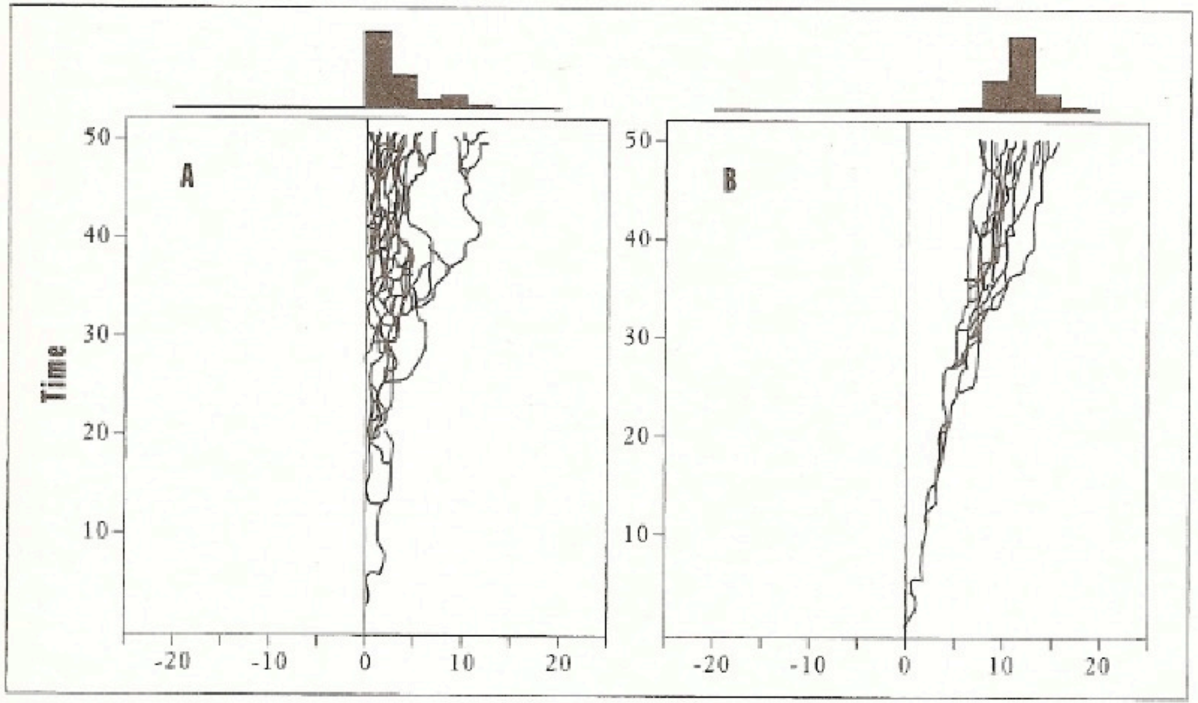

FIGURE 33

Passive and driven trends in McShea's terminology. A passive trend *(A)* begins near a left wall, retains a constant mode at this beginning position, and expands in the only open direction toward the right. In a driven trend *(B)*, both minimum and maximum values increase through time.

How to understand increasing complexity?

No way to explain this in terms of random mutations (only):

1. DNA's (genotype) **random mutations** statistically have probability 0 to cause globally increasing complexity of phenotype (examples: mayfly (ephemeral); equus…[Longo, Tendero, 2007])

2. Darwin's evolution is **selection of the *incompatible*** ("the best" makes no general sense)

3. Greater probabilities of **survival** and reproduction ***do not imply*** greater **complexity** (bacteria, … lizard…) [Maynard-Smith, 1969]

Gould's idea: symmetry breaking in a diffusion…

Mathematical analysis as a distribution of
Biomass (density) over Complexity (bio-organization)

F. Bailly, G. Longo. *Biological organization and anti-entropy*. 2009:

Derive Gould's empirical curb from

- general (mathematical) **principles**,
- specify the phase space
- explicit (and correct) the time dependence

Write a suitable **diffusion equation** inspired by Schrödinger operatorial approach

Note: any diffusion is based on **random paths**!

Morphological Complexity along phylogenesis and embryogenesis

Specify (quantify) Gould's informal "complexity" as *morphological complexity* **K**

$$\mathbf{K} = \alpha K_c + \beta K_m + \gamma K_f$$

$$(\alpha + \beta + \gamma = 1)$$

- K_c (combinatorial complexity) = cellular combinatorics as differentiations between cellular lineages (tissues)
- K_m (phenotipic complexity) = topological forms and structures (e.g., connexity and fractal structures)
- K_f (functional complexity) = metabolic relations, neuronal and cellular (interaction) networks

Main idea: formalize **K** as **anti-entropy** -S …

(*C.aenorhabditis elegans, see* [Bailly, Longo, 2009]*)*

The theoretical frame: analogies

.... by a *conceptual analogy* with **Quantum Physics**:

In ***Quantum Physics*** (a "wave diffusion" in Hilbert Spaces):

- The determination is a ***dynamics*** of a ***law of probability***:

  $$ih\, \partial \psi / \partial t = h^2 \partial^2 \psi / \partial x^2 + v \psi \qquad \text{(Schrödinger Eq.)}$$

In our approach to ***Complexity*** in *Biological Evolution*:

- The determination is a ***dynamics*** of a ***potential of variability*:**

  $$\partial f / \partial t = D_b \partial^2 f / \partial K^2 + \alpha_b f$$

What is f ? a diffusion equation, in *which spaces*?

Random walks …

The theoretical frame: dualities

.... by *conceptual dualities* with **Quantum Physics**:

In ***Quantum Physics*** (Schrödinger equation):

- **Energy** is an ***operator***, *H(f)*, the "main" physical observable.
- **Time** is a ***parameter***, $f(\underline{x}, t)$,

In our approach to ***Complexity*** in *Biological Evolution*:

- **Time** is an ***operator***, identified with entropy production **σ**
- **Energy** is a ***parameter***, $f(\underline{x}, e)$ (e.g. energy as bio-mass in scaling-allometric equations: $Q = kM^{1/n}$)

Our *f* is the density of bio-mass over complexity K (and time t):

m(t, K)

Mathematical analysis as a distribution of
Biomass (density) over Complexity (bio-organization)

Physical analyses at equilibrium (the Hamiltonian):

e. g. **total *energy*** $\mathbf{E = p^2/2m + V(x)}$ **(1)**

with V(x) potential

(e.g. $E = p^2/2m + kx^2/2$*, the harmonic oscillator)*

Biology: *far from equilibrium dissipative* systems:

- Focus on **Entropy production** $\boldsymbol{\sigma}$ (same as "time flow")

And construct an "analog" of the physical Hamiltonian:

$$\boldsymbol{\sigma = (\zeta_b M^2)/T + \sigma_{0b}} \quad (2)$$

or $$\mathbf{T\sigma = \zeta_b M^2 + T\sigma_{0b}} \quad \mathbf{(2)}$$

Some technicalities: how to derive the *diffusion* equation

Schrödinger's operatorial approach:

from **total *energy*** $E = p^2/2m + V(x)$ (1)

with V(x) potential

(*e.g.* $E = p^2/2m + kx^2/2$, *the harmonic oscillator*)

associate $E \Rightarrow ih\,\partial/\partial t$ *and* $\mathbf{p} \Rightarrow ih\,\partial/\partial x$ (operators)

and obtain:

$$ih\,\partial\psi/\partial t = h^2\,\partial^2\psi/\partial x^2 + v\psi \quad \text{(Schrödinger Eq.)}$$

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and obtain:

$$ih\,\partial\psi/\partial t = h^2\,\partial^2\psi/\partial x^2 + \mathbf{v}\,\psi$$ (Schrödinger Eq.)

Our operatorial approach applies to **Tσ**, *a power* :

thus **Tσ** *is a product of forces by fluxes* (~ the square of a mass)

$$\mathbf{T\sigma = \zeta_b M^2 + T\sigma_{0b}}$$ **(2)** (the analog of **(1)**)

Some technicalities: how to derive the *diffusion* equation

Our operatorial approach:

in total speed of **entropy production** $\mathbf{T\sigma = \zeta_b M^2 + T\sigma_{0b}}$,

where $\mathbf{T\sigma}$ has the role of $\mathbf{E}$ and $\mathbf{M}$ the role of $\mathbf{p}$,

associate $\mathbf{T\sigma} \Rightarrow \partial / \partial t$ *and* $\mathbf{M} \Rightarrow \partial / \partial K$ **(*)**

(cf. *Schrödinger's operatorial transformations*:

associate $E \Rightarrow ih\, \partial / \partial t$ *and* $\mathbf{p} \Rightarrow ih\, \partial / \partial x$ **(**))**

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Thus we obtain:

$$\partial m / \partial t = \mathbf{D_b}\, \partial^2 \mathbf{m} / \partial K^2 + \boldsymbol{\alpha}_b \mathbf{m} \qquad (3)$$

where **m** is bio-mass density

(cf. ψ in $ih\, \partial \psi / \partial t = h^2\, \partial^2 \psi / \partial x^2 + \mathbf{v}\psi$ (Schrödinger Eq.)

in the complex field**)**

A **diffusion** equation:

$$\partial m/\partial t = D_b \partial^2 m/\partial K^2 + \alpha_b m(t,K) \qquad (3)$$

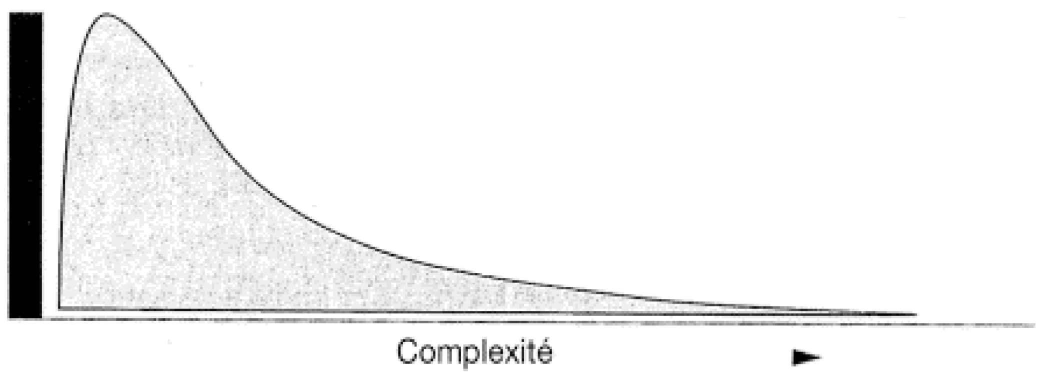
A **solution**

$$m(t,K) = (A/\sqrt{t}) \exp(at)\exp(-K^2/4Dt)$$

models Gould's asymmetric diagram for Complexity in Evolution (diffusion ⇒ *random paths…), **also along t*** :

(biomass and the **left wall** for complexity, archeobacteria original formation)

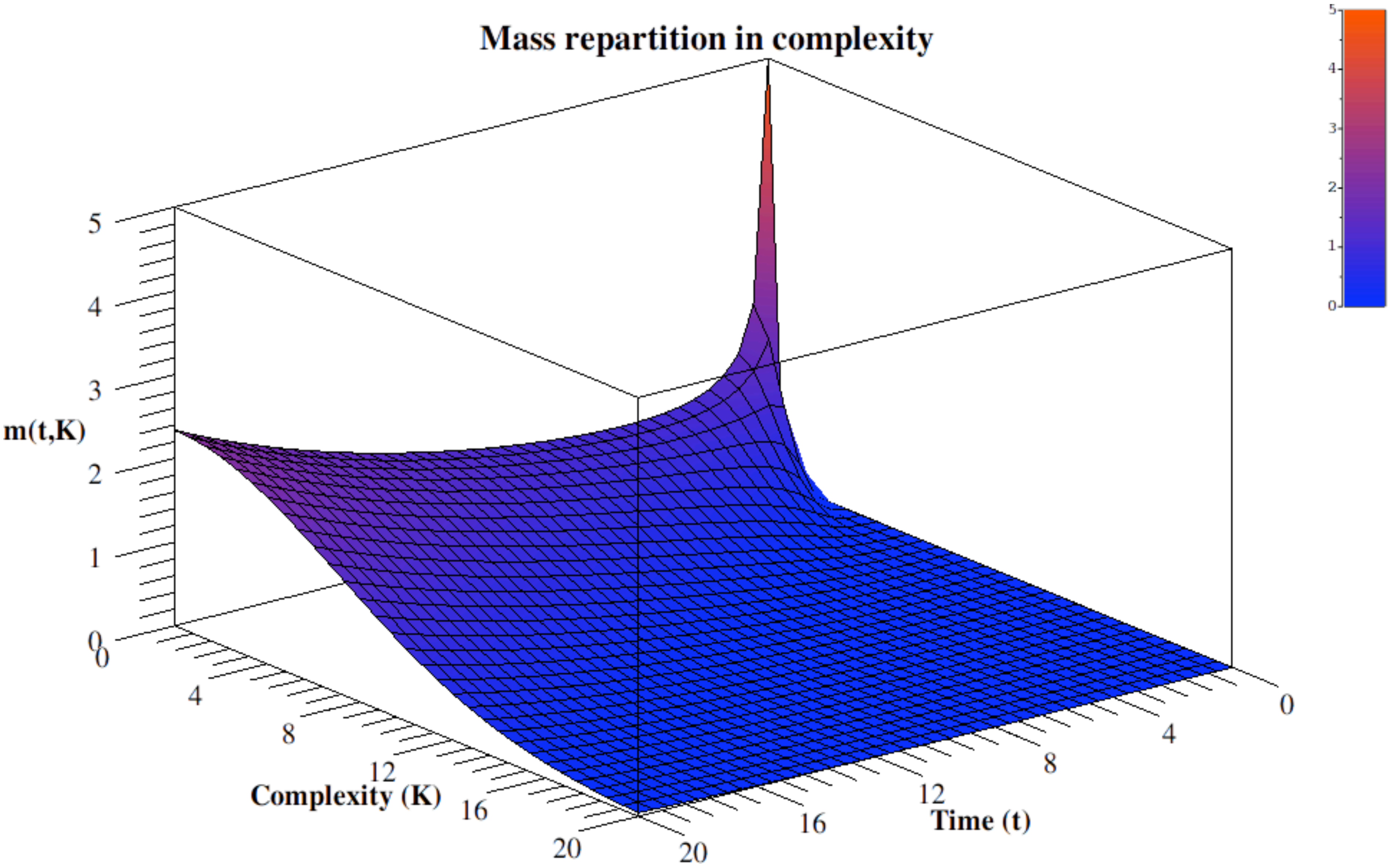
↓



F. Bailly, G. Longo. *Biological Organization and Anti-Entropy…*

Next picture by Maël Montevil:

(Implementation by **Maël Montevil**; “ponctuated equilibria” smoothed out)



Theoretical *changes w.r.t. to Physics*

The physical Hamiltonian $\mathbf{E = p^2/2m + V(x)}$ *vs.* our "analog":

$$\mathbf{T\sigma = \zeta_b M^2 + T\sigma_{0b}}$$

a) **Tσ** plays role of *physical energy* (it is actually a power)

b) **M (bio-mass)** plays the role of *momentum* p (M squared intervenes in Tσ, just as p does in $\mathbf{E = p^2/2m + V(x)}$).

***Theoretical** changes w.r.t. to Physics*

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b) **M (bio-mass)** plays the role of *momentum* p (M squared intervenes in Tσ, just as p does in **$E = p^2/2m + V(x)$**).

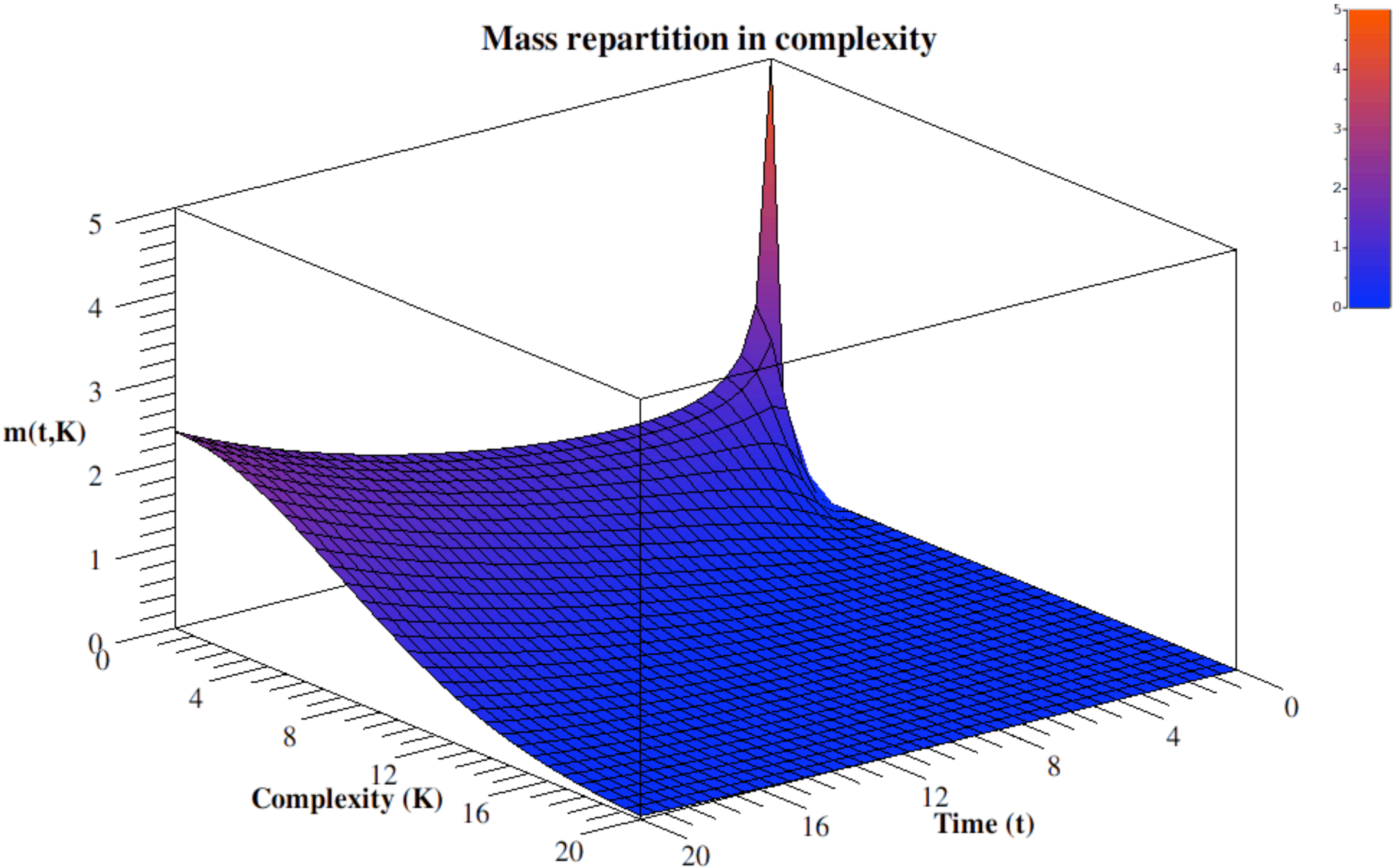
Or, as for the operatorial approach:

1. **Entropy variation**, multiplied by temperature, instead of *physical energy* becomes $\partial / \partial t$
2. **(Bio-)Mass** instead of *momentum* (which is proportional to a mass) becomes $\partial^2 / \partial K^2$, where
3. **Anti-entropy** or complexity instead of *space* (K instead of x *in* $\partial^2 / \partial K^2$) *(real coefficients instead of complex ones)*

A **diffusion** equation:

$$\partial m / \partial t = D_b \partial^2 m / \partial K^2 + \alpha_b m(t,K)$$

(Implementation by **Maël Montevil**; “ponctuated equilibria” smoothed out)



More (possible?) consequences of the dualities from Physics vs. Biology

.... by further conceptual dualities:

In **Physics**, crucial:

absence *of origin of* ***time (****and* ***space)***

constitution of invariants, i.e. conservation of observable quantities (Noether's theorems, '20), e.g. Energy.

In **Biology,** crucial:

existence *of an origin of* ***time (****and* ***space)***

The (proper) **invariants of Biology** are **not** those of Physics (ex. Darwin)

From (Quantum) Physics towards Biology ? …

Pauli's Theorem (1933):

"The lower bound for the *energy operator* implies that *time* is not an operator, but a *parameter*"

In **Physics**: ***no origin of time (***energy conservation***)***

In **Biology**: ***existence of an origin of time (***or "time has a lower bound"***)***

Question: if **time an operator** (= the setting up of organization) does this implies that **energy** cannot be an operator, but a **parameter** ? (a theoretical justification of **scaling laws** ...)

Logical Summary of our view
Physical vs. Biological Theories
in Bailly-Longo three (correlated) approaches:

1 - **organization** or complexity as **anti-entropy**, **JBS**, 17, 1, 2009.

2 - a **two dimensional time** *(not linear time),* ongoing, with M. Montevil.

3 - ***extended* criticality** *(a physical oxymore),* **JBS**, 16, 2, 2008.

Common point to the approaches in 1, 2 and 3:

Strict "Consistent" **extensions**, in the sense of Logic, *not* incompatible with current physical theories*, but not reducible (conservative?)*:

1: "=" instead of "≤" *(anti-entropy goes to 0)*

2: collapse the extra dimension *(a time bifurcation);*

3: contract the extension of criticality *(from interval to point)*.

Some references

http://www.di.ens.fr/users/longo *or* **Google: Giuseppe Longo**

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