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Computer Science at the Interface of Knowledge

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The origins of the discrete state digital machine: between mathematics and philosophy

During the thirties, a very fruitful crossing between the philosophical questions about the foundations of mathematics, the contemplation on human cognition and the new mathematical techniques gave birth to modern computers. At this time, machines that could calculate already existed, beginning from the one invented by Babbage (1850) to analogical machines such as the Differential Analyser of V. Bush (1927); but it is the epistemological problem of deductive completeness of axiomatic formalisms that will lead to the invention of the fundamental concepts of modern digital computation.

The logical analysis of proof in Herbrand (his PhD dissertation, 1930, ENS-Sorbonne) contains a first definition of primitive recursive functions (computable in the strong sense). Gödel (1931) and Turing (1936) took over, giving a definitive answer to the foundational question of the time: does a computation of signs, potentially mechanizable and without reference to meaning, permit one to decide all mathematical statements? Can we demonstrate consistency via "finitary" and formal arguments? And, in fact, can human reasoning be completely reduced to a system of potentially mechanizable dynamics of signs?

In order to answer such philosophical questions, these great mathematicians had to specify what "potentially mechanizable" means. In other words, in order to construct *undecidable* propositions they had to specify what *decidable* or *computable* mean in general, by giving a mathematical formalization (the class of recursive functions) of the informal notion of calculus. Turing, in particular, proposed an original definition, with his Logical Computing Machine (LCM), which is an abstract idea of a "human in the minimal act of calculus", and formally defines by these means non-computable functions. The two ideas at the core of the LCM are the invention and the distinction, purely mathematical in this time and age, of the notion of *software* and the notion of *hardware*.

These two resounding "nos", of Gödel and of Turing, to the hypothesis of completeness of formal deduction constitute thus a technical answer to a philosophical questioning of deductive reasoning and human cognition. And the universe of the computable constructed in this way and well delimitated will allow, ten years later, the materialisation of this discrete state electronic machine which has been changing our forms of knowledge, even our relation to the world. More precisely, what impact does this machine have nowadays upon the construction of scientific knowledge, and of "knowledge" in general?

Imitation and modelling

In two articles, from 1950 and 1952, Turing implicitly proposes a distinction of great importance concerning the intelligibility the machine can give us of the world. In the first, he describes an "Imitation Game" between a woman and a computer, confronted by an interrogator who asks them questions so as to understand, using a Teletype. In the second he proposes a "mathematical model" (a system of equations) of the physicochemical generation of forms ("morphogenesis").

More precisely, in the first article, he specifies the *physical* nature of his logical machine: it is a "discrete-state machine" (DSM) that he uses to fool his interlocutor and let him believe to be a woman. The second one develops an innovating analysis of a chemical system (action-reaction-diffusion) in a "continuous system". The conceptual difference is radical. The first construction doesn't intend to make mental processes intelligible, but rather to imitate them under constraints (one constraint is the discrete linguistic interface, the teletype itself—for example, the examiner is not allowed to touch). The second one, while a simplification and an idealisation, as Turing says in the beginning of the paper, tries to help us to comprehend what is taking place in a process in which we reveal causes and consequences: mathematical equations provide a *structure* for the possible *determination* of the physicochemical system—they make it intelligible and allow the possibility of predicting, at least qualitatively, the evolution (under some conditions, a modular structure, as a strip, will appear, the details of whose form will depend on the initial fluctuations that cannot be observed—a consequence of the "exponential divergence", he says with the foresight of a genius).

This "sensitivity to initial conditions" (the exponential divergence) that Poincaré had understood so well (1890), but that will be described in precise mathematical terms only in the seventies, is at the core of the non-linear dynamic systems. It escapes from the computational *discrete* because the discretization forces a minimal level of accessibility of computation: the approximation proper to its universe of discrete data. Turing stated, explicitly, in 1950: in his discrete-state machines "this phenomenon", the exponential divergence, which interests him particularly in the second paper, "does not occur"; the acces to data is exact, computations iterate identically.

Let's consider in a more general way what is at stake here. The *discrete* and the *continuous* propose different visions of the world; they *organize it mathematically* in different ways. In the following decades, it was clarified by theorems that the one is not an approximation of the other, in general: as soon as we describe a continuous dynamic system of interactions (non-linear, from a mathematical point of view), the trajectories in the *discrete* diverge rapidly from the possible evolutions in the *continuous* and the mathematical predictability changes. "Shadowing Theorems" specify the possibilities and the limits of digital approximations. The world organized in well-separated little boxes is a whole other thing than the smooth (and differentiable) world of Cantor and of differential equations. The DSM allows for the possibility of resolving it like never before, but the modeller must analyze what it loses and appreciate what it gains: the power of computation, of course, but also the possibility of identical iteration through the computational simulation, even in the case of the most wild turbulences and of the most strange attractors. This is inconceivable for physical processes—they never iterate identically: the exponential drift precludes it, as an exponential growth of non-measurable fluctuations, which become observable with time. In the discrete state machine we gain enormously in computation power, but we lose in meaning: we lose the intelligibility proposed by the continuous model.

Quantic mechanics confirms the importance of what is at stake: distance as defined by Planck doesn't divide the space in little cubes phenomenologically separated, because entanglement precludes the separation by measure; the dynamic system is described by the equations of Schrödinger, in the continuum of Hilbert spaces, beyond the world (of infinite dimension, if necessary). And it states the problem of the "next machine", where the nature of the hardware, which will be quantic, will profoundly affect the dynamics of the software and the organization of knowledge that it proposes to us.

The simulation and the interaction

Often, the term "simulation" erases the distinctions we just made, and the machine is supposed to "represent" the world and our knowledge in a more or less faithful way, with no other precisions. However, we should pay attention when using this term, or find other ones that allow the preservation of the subtlety of the analyses. Let's use the term simulation to refer to the integrated and interactive plurality of *the virtual experimentation*, in *the way that this experimentation is pursued noawadays*. In this case, the digital implementation of an image of the world precedes or *guides* the modelling, as a mathematical attempt at causal intelligibility. From a simplistic approach, it has become a scientific method, widely used from meteorology and geophysics to vegetal biology. We insert into the machine a large quantity of data, and especially of parameters and observables, even in some cases (typically in vegetal biology) before being able to conceive a conceptual and mathematical framework capable to seize *in abstracto* the hugeness of these domains of phases. This framework will be in the machine; it will be co-constituted in the interaction between man and machine.

The networks and the interactions between machines

A new level of organization is added over time to the organisation levels of the original machine: that of local and global networks of computers. We cannot mention here the mathematical problems this new organization of material poses; let's just examine some of its consequences.

The sequential machine doesn't model physical randomness. The pseudo-random generators are a (modest) imitation of it; in fact they are one of the most pertinent examples of the *imitation* of physical randomness. Typically, they iterate identically if we reboot them in the *same* data, a meaningless process in dynamics that have some sensitivity to initial conditions. On the contrary, randomness, and in fact a very "powerful" randomness, is at the core of networks. Human interventions, the randomness of people waiting in line, of multiple accesses to the same database… here is "real" randomness. Is the number of people connected at this moment to Skype even or odd? This parity could even depend upon quantic randomness (that differs from classic randomness), because pioneering researchers at the European Organization for Nuclear Research (CERN) can decide to access the network according to the result of a measurement of quantic entanglement… without considering the possible and various relativistic delays when the network surrounds the Earth.

Here is a new role for the randomness which, having been completely absent at the beginning of computer science, has now become a powerful and novel phenomenon: these networks seem to be the only framework where we are obliged to mix the classic, relativist and quantic randomness, as well as that of human activities. Will it be able to help us to grasp, for example, the randomness specific to biological phenomena? During evolution, as well as in embryogenesis, the randomness of mutations can have a quantic origin, but it interacts with frameworks that we understand better in classic terms: the interfaces of various dynamics, or even between phenotypes and between phenotypes and ecosystems. Furthermore, epigenetic effects retroact upon the expression of genes, or even upon the very frequency of mutations. A culture and a science of networks, even if it is about networks of machines, offers us a new point of view for understanding these networks of networks that give form to every organism, indeed every biological species, as well as the possibility, of course, to imitate them, or even simulate them in the best way.

Moving away from science itself, are the *networks of knowledge* foreign to a "thinking of networks"? Can the latter help us to understand what is going on in the realm of our knowledge, in the very structure of our societies, where every individual is (potentially and) directly connected to every other individual, where multiple instantaneous communications, at varying distances, modify the forms of human interaction?

A new symbolic realm

The invention of language and then that of writing have established human history, modifying social interaction. The invention of digital networks is probably as important; the memory and the human presence both accessible to every other human enrich quantitatively and qualitatively human exchanges and, thus, history. It is in fact about a new *symbolic constituted form*. We would like to underline two of its aspects in which the intensive mathematization in the conception and the exploitation of computer networks plays an essential role. The first idea developed in section *Referencing/referenced* will also help to discuss a subsequent and highly general element of the fruitful interaction between knowledge and the role of language, even if it is this (new one) of exchanges through digital networks. The second line of thinking in section *Electronic reticular money* will question the impact of electronic and reticular money.

REFERENCING/REFERENCED

In the same way that we are led to consider reason's double status we are also led to distinguish for every symbolic form two functions: a function that references (*referencing function*) and a function that is being referenced (*referenced function*). Let's try to explain them.

In its referencing function, the symbolic form provides the means to express and set, as in the ordinary language of physics (but this is true for other disciplines as well), the basic principles (theoretical for physics) around which it is organized. Relative to the subject that fixes norms, it thus governs, to a certain extent, the objectivizing activity. By contrast, in each referenced function relative to these models, a symbolic form is characterized mostly by terms rather than by words, in conceptual relations rather than in regards to the issues surrounding signification; it is thus submitted to the very determinations of these mathematic structures that it has contributed to set up and for which it has triggered the proper generativity. This process continues until the movement of scientific theorization uses this referenced state of language in order to confer on it a new referencing function permitting the elaboration of new models, of new principles, more general or more abstract; it is "the final state" of a step becoming in a way "the initial state" of the next step. The mathematical invention of networks and, afterwards, the new formal constituted elements they impose, play this dynamic double role. In this dialectic process which is permanently in action, the mathematical model as such maintains the distance and the distinction—essential for the construction of objectivity—between these two functions of the language, and even every symbolic realm, while ensuring the necessary mediation between them. It is reinforced and modified due to the first one; it is continually transforming the other one by its internal dynamics proper to it due to mathematical generativity. Doing so, the mathematical model contributes to give birth to the new language of knowledge, through the functions that this model makes the new language adopt alternatively; it ensures a regulated circulation between functions (this process, operating here in the domain of rigorous objectivity, resembles what is achieved through poetry in the domain of engaged subjectivity).

A physical example of this process can be found in the status of the universal gravitation of Newton, which follows a straight line from the innovations of Kepler, Copernic and Galileo. The referencing state of language used to call out for an "Aristotelian" representation of the world according to which the "supra lunar" constituted an *absolute* of perfection and permanence (invariability of the course of the planets drawing perfect circles, and corresponding model of Ptolemaic epicycles). However, it is only based on this referencing function (of which the still quasi-mythical state is also found in Newton's alchemical or biblical works) that the mathematical model (as Galileo had wanted it to be) of the universal gravitation that rules every object, infra or supra lunar, is constructed. This radical relativization concerning the forces of interaction, due to mathematization, is certainly accompanied by the maintaining (or even, from a conceptual point of view, by the introduction) of another *absolute*, that of space and time. However, it redefines the language of the planets' course in a state referenced from then on upon this model where the elliptical orbits and the empirical observations are "explained" by the law of universal gravitation. Moreover, this mathematical model enables the deliberation of the adequate physical invariables, which will be used as a structure to support any further consideration and will model the language of this new cosmology. It is this referenced state (referenced to the mathematical model constructed this way) of the language of Newtonian cosmology that will be used afterwards as a new foundation for the research to come, thus playing from then on a referencing role, in order to relativize these absolutes of space and time themselves and lead to Einstein's conception of the theory of general relativity.

Let's return to this distinction from a complementary perspective, one closer more specifically to the logical procedures and formalisms. As a referenced entity, language must be bent to a sort of theory of types in one way or another, in order to make sense and stave off paradoxes; a theory of types capable of discriminating between the different levels of its enunciations. But the construction of such a theory of types calls out for the referencing function of language, which guides in fact the conceptual elaboration and the formulation of formal enunciations. Thus, the referencing function sets the norms and invents. It rules the creative and ordering activity. The referenced function is the object of study and analysis. It demands the mediation of a logico-mathematical language that objectivizes it and enables a rigorous treatment in respect to its proper referencing function.

At this stage, let's note that the demand formulated by constructivist logicians for an "effective" logic concerns then essentially the referenced aspect, as in the case of intuitionism, which sees the activity of thought at work in the process of elaboration and mathematical construction. But as far as it innovates and creates, meaning as far as it enables the emergence of unknown references, thought activity doesn't respond to criteria or norms of constructability, but it produces them.

In a similar way, every symbolic realm, beginning with the modelling of physical theory, is also presented at the same time as second but also as one that determines: its effective construction coming after the thinking (networks and their language have been constructed, they have a scientific history) that enunciates the principles—what there is to model—and to which the anteriority can confer a status of seeming absoluteness, governing every theoretical advance to come, even when the latter goes as far as to contradict principles assumed previously as being established.

It is without doubt this conceptual configuration that allows for an understanding of how situations often addressed against intuition, for example by quantic physics, can nevertheless be "spoken" in a natural language that doesn't spontaneously cease to claim the contrary of the obtained results. This so-called natural language isn't so natural anymore (its words are replaced by terms) and in any case it's no longer supported by its linguistic structures and grammar (and their corresponding mentalities) but much more by the mathematical structures of the model it interprets and comments, without being capable though of giving back the profoundness and, what is more important, the generativity, but still making possible the communication inside culture.

What new objectivity do we constitute through this electronic and reticular language? What is the new role of this symbolic space created by digital networked scripture—a space where informational content is instantaneously shared and always accessible to everyone, due to digital and precise memories—in our culture?

ELECTRONIC RETICULAR MONEY

The establishment of money in the form of coins, in Ancient Greece, and in the form of paper, in Florence, during the Italian Renaissance, were followed, and this is not a coincidence, by the invention of ancient philosophy and science and then in turn by modern philosophy and science. The fact is that these two moments of human creativity required, and contain per se, an extraordinary degree of abstraction. What is the meaning of this coin with the effigy of Croesus, of which the value quantified through geometric signs (squares, triangles, circles…) permits the buying of everything, transforming everything into gold? A value that, a priori, doesn't have a limit—any number can be written, and the amount is transferred very rapidly. The acceptance of these two abstractions is not so evident; it is as powerful as the cultural and scientific inventions that accompanied them.

We are standing now before a third transition. What is the meaning of these stock market values, simple scripts on a screen, far removed from any reference to liquidity (banks' available money)? What happens when the Credit Default Swaps, insurable values, reached in 2008 60 trillion dollars on the screen (i.e. virtual), the equivalent of the GDP of the world? These shares have increased, in fifteen years and out of almost nothing, because… they increased. They aspired to savings, because… they aspired to. But, by increasing out of every "real" value, they made the investor lose every relation to liquidity, even the ones who used to have it before. Of course, when enough agents "ask to see" the King, he is revealed to be naked and everything collapses. These new formal signs of wealth disappear like paper money that burns off—but much more quickly and with global effects.

It is only the massive mathematization of finance and its networking that allow for these effects of purely formal increase we just described, a sort of mathematic divergence of sums of series which express "resonance" effects. What is more, they allow for immediate "arbitrages" (simultaneous sales and purchases in many stock markets, largely automated), which *simulate* wealth productions that can be *observed* but are fictional, based on *infinitesimal fluctuations* (price differences that are insignificant but which, when cumulated by millions of sales/purchases, provide large formal effects).

What is changing in the relation of man to the production of wealth? What impact or correlation can this have with our forms of knowledge? Will this be comparable to the invention of Greek or the Renaissance abstraction, or, escaping from democratic control, will this be used to transfer 80% of the growth towards the 1% of the richest, as was the case in 2000-2007 in the USA, distributing the risks to the world?

Networks and the power of the loss of meaning: a social and ethical problem

The big credit card companies, like Visa or MasterCard, can tell, with a good level of certitude, *who* will divorce during next year. They can tell not only the percentage of marriages that will end, which is a trivial thing for every statistics office, but also the chances that this or that *individual* will divorce. In fact, on the basis of profiles of expenses of some hundreds of millions of holders of credit cards throughout the world, they know the profiles that lead, with well-defined probabilities, towards one or another future behaviour. Another example, easy to understand for every shopper on the Internet: the great pertinence of Amazon's recommendations for other books to buy. The linking by statistical methods of your purchase with those of a large number of other shoppers who bought the same book enables them to propose new ones, almost better than an old friend or a proficient bookseller in the Latin Quarter of Paris would. The latter two might know your preferences and the contents of books; in brief they act with reference to a "meaning". The machines can do so, and even better, on "purely mechanical" bases of formal associations inside huge statistical data bases, with no reference to meaning, of course.

Man has never been confronted with such possibilities of knowledge and of control by those who manage the networks and hold the data. In regards to the permanent war between control and private life, A. Shamir, one of the inventors of cryptography, said recently during a presentation at the École Normale Supérieure (ENS), we win battles for the protection of citizens, but we are losing the war. The economic power of a state, of a large enterprise, always gives it an advantage over an individual. It is only an active thinking, a permanent reference to mankind communicating on values and contents, a strong humanism through the interaction of the different domains of knowledge, that can give meaning and enrich subsequently the potentialities of the DSM of Turing and its networking.

The use of this technical knowledge, information technology on networks, even if it is accessible to everybody, can also lead to dangerous deviations if it is practiced without critical awareness. All domains of knowledge profit, for example, from immediate accessibility to articles, quotations, library catalogues… Thanks to bibliometrics, any bureaucrat can claim to evaluate the production of knowledge by pressing a button. The impact factor (IF) of reviews, the number of citations during two years following the publication of articles, or even the global citations of an author, appear almost instantly on the screen. The scientific community votes in this way, without wishing to do so and via a formal statistical method, to decide in the majority about the quality, and in fact of "the" scientific "truth". The marginal and exploratory works, the difficult new approaches are thus excluded; so too are the researchers who say "no" to dominant theories. However, science and knowledge are constructed around ways of thinking which are always heretical because they are new ways of thinking, since ancient Greece, since Copernic and Galileo. These two innovators, for example, fought against qualified scientists, who worked on a technically feasible but very difficult enterprise, difficult because it was biased from a metaphysical point of view: the analysis of planets' trajectories in the form of epicycles, as it was demanded in the frame of the dominant theory. And what was the impact factor, during the first ten, twenty, thirty years, of the works of Cantor, Poincaré or Boltzman, works that changed science fifty years later? Production of knowledge should be evaluated according to its meaning, so difficult to identify, according to the new audaciousness that escapes the conservative, inert statistics, which are always lagging "behind" novelty, by definition. The history of knowledge, and its future, is a constant changing of ways of seeing, of points of view, in opposition to every majoritarian inertia and dominant theory. It uses from democracy not the majoritarian vote, but its essential complement: the possibility of disagreement.

On the one hand, then, we have built a new world of freedom and exchange, an extraordinary space for the circulation of ideas; on the other, in the absence of strong democratic and ethical controls, unmanageable harmful deviations are possible, in particular because the networks' potentialities are absolutely new. As human sciences constitute the very place of the contemplation on "meaning", the dialogue with the human sciences about what is at stake as well as about the methods is more necessary than ever before; it is only in the interface between domains of knowledge that we can create a situation of awareness which could enrich the multiple forms of knowledge and practices at play, without deforming them or reducing their significance.

Conclusion

Computers are not only instruments with which interaction is unidirectional and one-dimensional (an interaction that has already changed science through imitation and modelling), but thanks to simulation, in the way we defined it above, the computer is an active element of an environment, where machines, engineers, scientists, researchers of all domains interact and dialogue. Moreover, each computer is henceforth in a network, more precisely in a network of networks, which links it to other computers and which links humans to each other. These networks offer us a world of concepts and behaviours that is acquiring the role of a new symbolic realm, a process that modifies not only knowledge, but also *cognition* and human *activities*. Nowadays, a real debate on knowledge and practices must take place about these environments and these networks. We will then better understand and control what happens once the computer and the networks enter the scene of knowledge and behaviours, or even more generally, the scene of history.

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Note: See http://www.di.ens.fr/users/longo/ for further technical reading and numerous references.